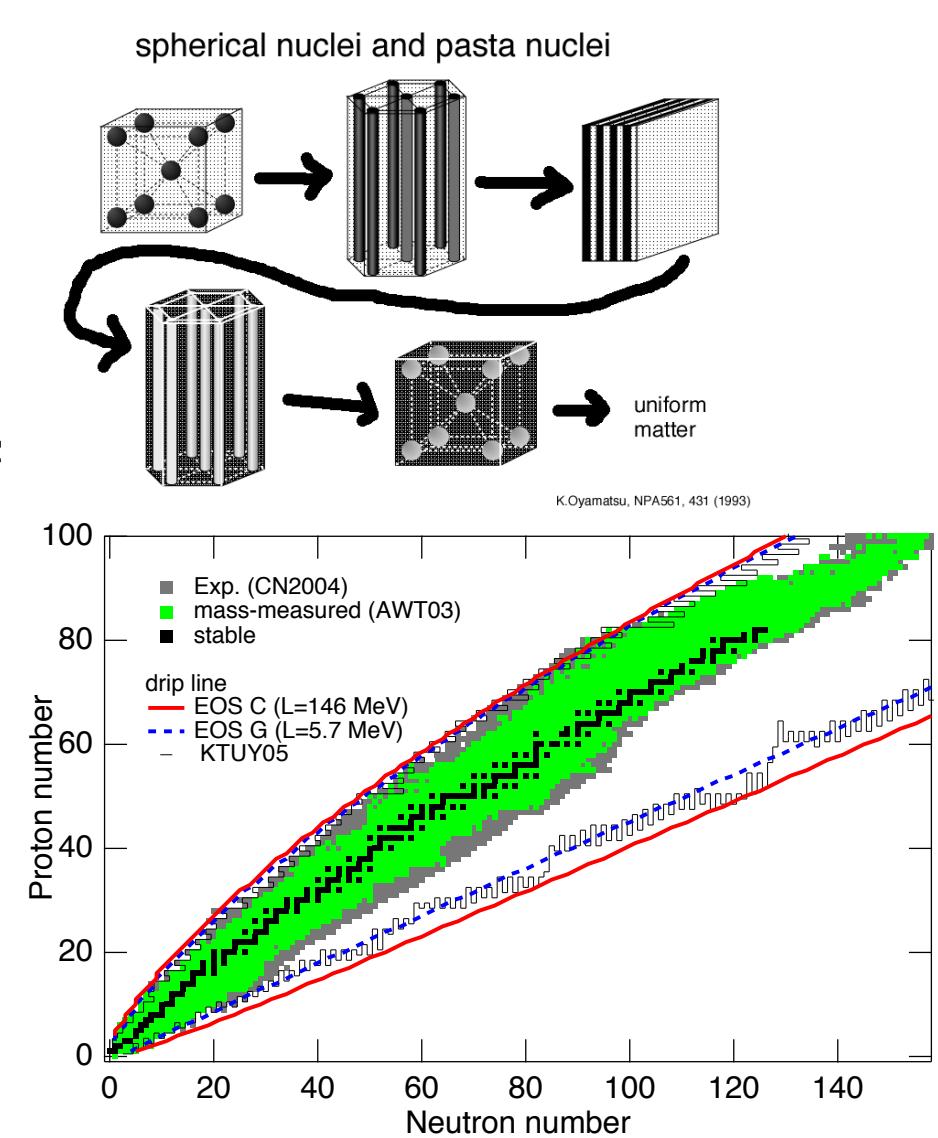
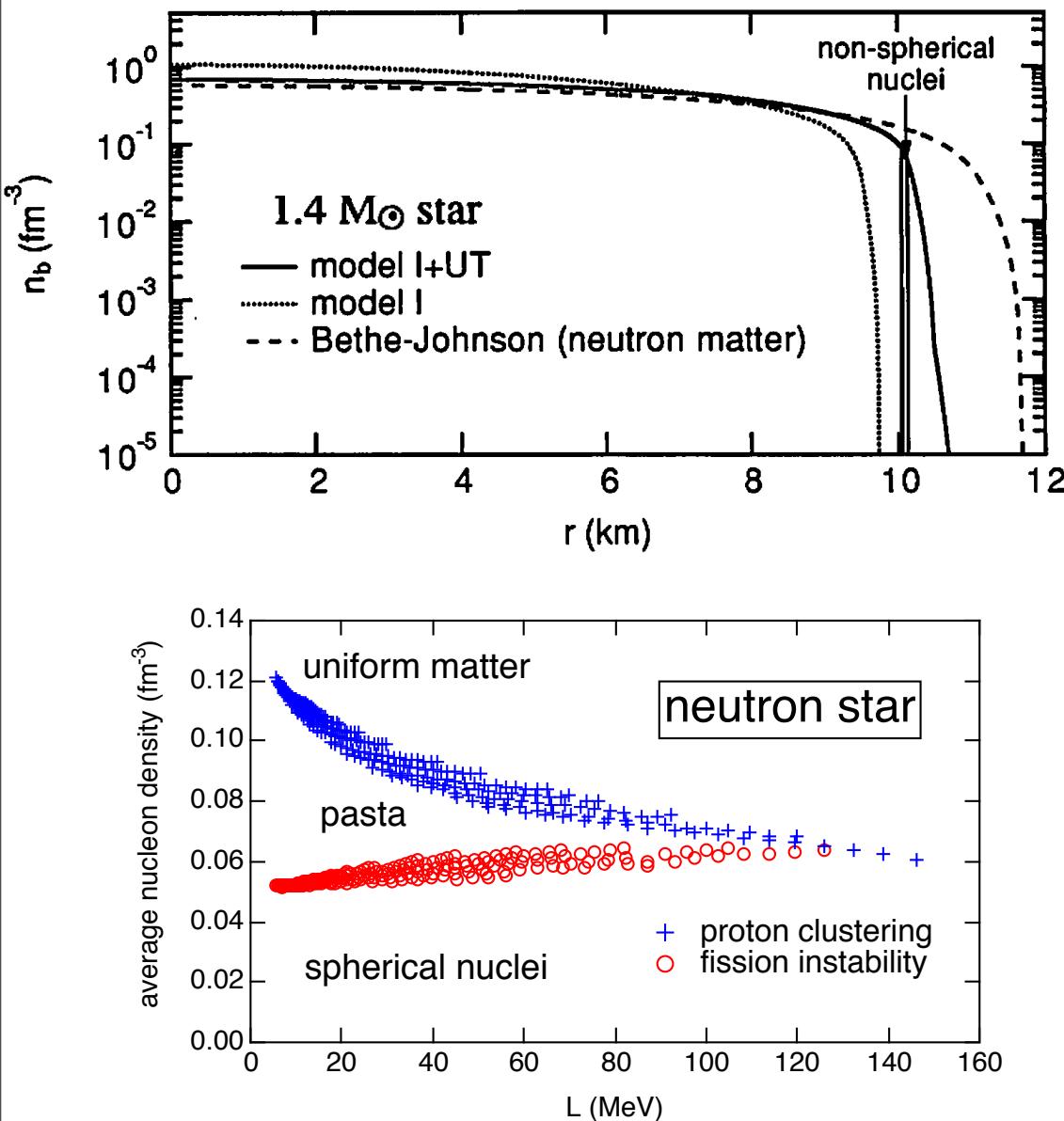


# 対称エネルギーが不安定核と中性子星クラストを結ぶ

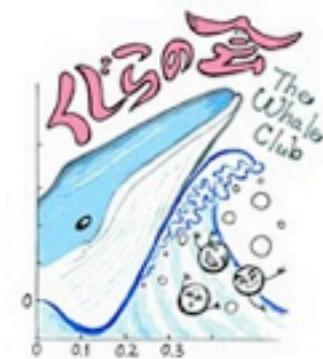
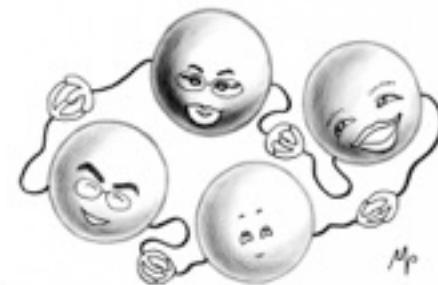
親松和浩（愛知淑徳大）



ハドロン物質の諸相と状態方程式—中性子星の観測に照らして—、2012年8月30日、京都大学基礎物理学研究所

# Collaborator Meeting

## くじらの会 at 入野



Kurotama  
radius  
 $\sigma_R$

Kohama  
( $\sigma_R$  : Kurotama)  
(mass formula)

2008年10月29日(水)～31日(金)

Koura lida Oyamatsu

Empirical  
EOS

mass  
radius  
pasta



12年9月22日土曜日

# Which EOS parameter dominates macroscopic properties of neutron-rich nuclei in laboratory and in neutron-star crusts?

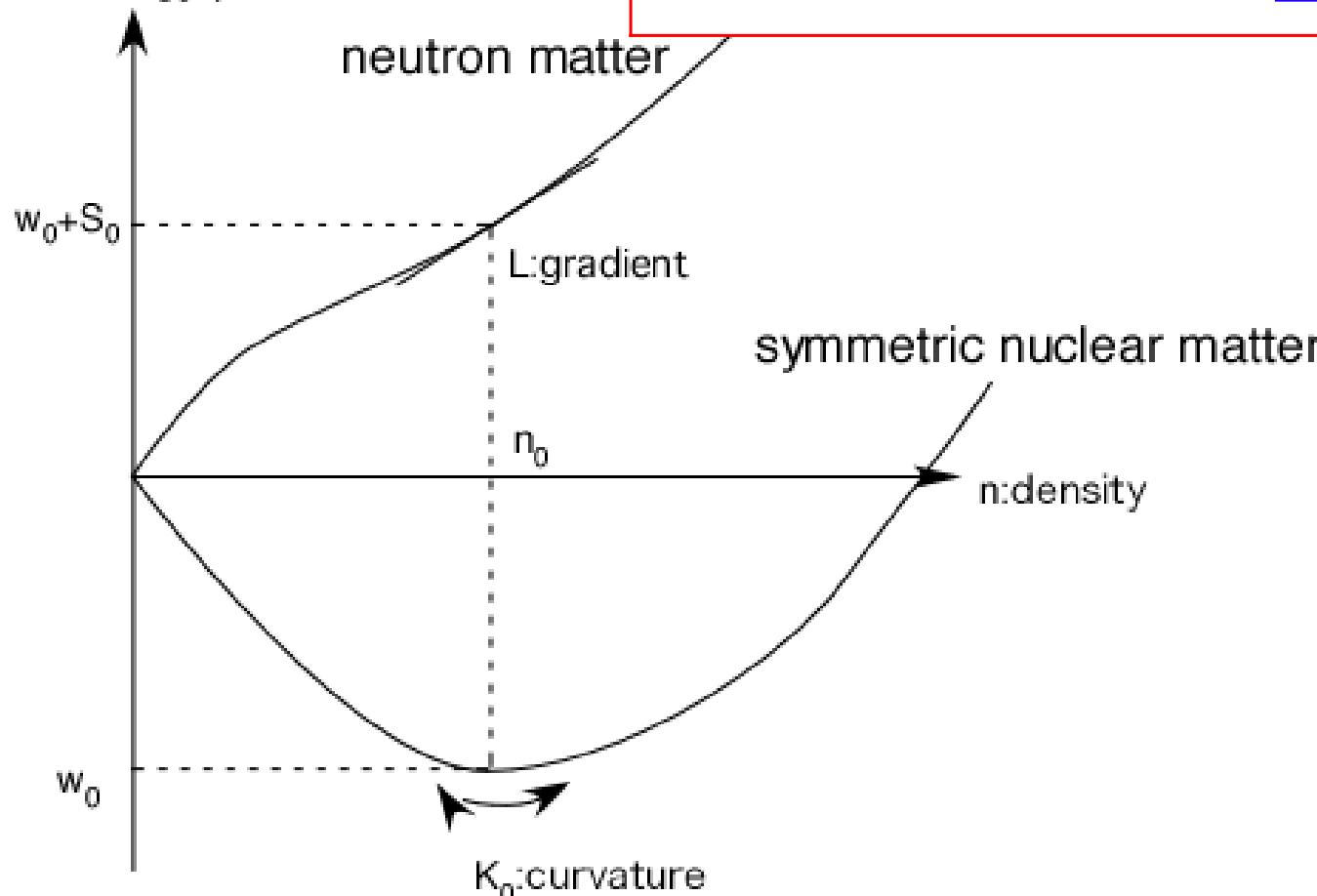
Energy per nucleon of nearly symmetric nuclear matter

$$w(n, x) \approx w_0 + \frac{K_0}{18n_0^2} (n - n_0)^2 + (1 - 2x)^2 \left[ S_0 + \frac{L}{3n_0} (n - n_0) \right]$$

$n_0$ : nuclear density,  $w_0$ :saturation energy,  $K_0$ : incompressibility

$S_0$ : symmetry energy at  $n=n_0$ ,  $L$ : its density derivative coefficient

$w$ :energy per nucleon



$$L = 3n_0 \frac{dS(n)}{dn} \Big|_{n=n_0}$$

$$S_0 = S(n_0)$$

原子核内部では  $x \approx 0.5$  で

$$\alpha = 1 - 2x$$

$$w_s = w_0 + S_0 \alpha^2$$

$$n_s = n_0 - \frac{3n_0 L}{K_0} \alpha^2.$$

# Approaches to obtain the EOS of (uniform) nuclear matter

approach	starts from	ingredients	Theory/Model
empirical	parametrized EOS	nuclear mass, size, ...	Liquid-Drop Model Droplet Model Thomas-Fermi Theory .....
Phenomenological	effective NN int. (Hamiltonian, Lagrangean)	nuclear mass, size, ...	Skyrme HF RMF AMD .....
microscopic	bare NN int. (AV18, Bonn, Paris,...)	NN scattering, ...	Variational Calc. DBHF .....

# 不安定核と中性子星クラスト

- 不安定核
  - 半径、反応断面積
  - 質量
- 中性子星
  - クラストの原子核
  - クラスト・コア境界
- EOS
    - 対称核物質
      - 飽和密度、飽和エネルギー、非圧縮率
    - 対称エネルギー
      - 密度依存性、非対称度依存性

## Step 1

Generate all empirically allowed EOS's systematically

K. Oyamatsu and K. Iida, Prog. Theor. Phys. 109, 631 (2003).

## Adopted macroscopic model

Energy per cell (or Energy of a nucleus)

$$W = \int_{cell} d\mathbf{r} \left[ \varepsilon_0(n_n, n_p) + m_n n_n + m_p n_p \right] + \int_{cell} d\mathbf{r} F_0 |\nabla n| ^2 + \left( \text{electron kinetic energy} \right) + \left( \text{Coulomb} \right)$$

$n_n$  ( $n_p$ ) : local neutron (proton) density,  $n = n_n + n_p$  : total density

$\varepsilon_0(n_n, n_p)$  : EOS of uniform nuclear matter (energy density)

$F_0$  : surface energy parameter

Parametrization of the EOS (energy density)

$$\varepsilon_0(n_n, n_p) = \frac{3}{5} \left( 3\pi^2 \right)^{2/3} \left( \frac{\hbar^2}{2m_n} n_n^{5/3} + \frac{\hbar^2}{2m_p} n_p^{5/3} \right) + \left[ 1 - (1 - 2Y_p)^2 \right] v_s(n) + (1 - 2Y_p)^2 v_n(n)$$

Fermi kinetic energy density                                    potential energy density

potential energy densities of symmetric and neutron matter

$$v_s(n) = a_1 n^2 + \frac{a_2 n^3}{1 + a_3 n} \quad v_n(n) = b_1 n^2 + \frac{b_2 n^3}{1 + b_3 n}$$

★  $a_1 \sim b_2$  and  $F_0$  : masses and radii of stable nuclei ( $b_3 = 1.59 \text{ fm}^3$ , a fit to FP EOS)

★ very flexible function form:  $a_3$  can vary  $K_0$  widely. (better than Skyrme)

The function can be fitted to SIII and TM1 EOS very well.

# Simplified Thomas-Fermi calculation

(The same method as Shen EOS)

energy minimization with respect to parameters of  $n_n(r)$  and  $n_p(r)$  (and lattice constant)

neutron (proton) density distribution  $n_n$  ( $n_p$ )

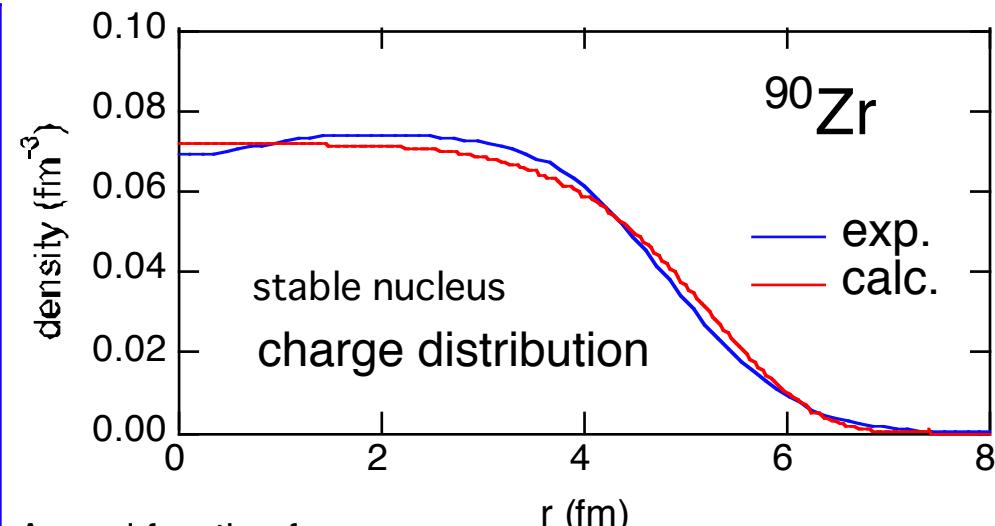
$$n_i(r) = \begin{cases} \left(n_i^{in} - n_i^{out}\right) \left[1 - \left(\frac{r}{R_i}\right)^{t_i}\right]^3 + n_i^{out} & r < R_i \\ n_i^{out} & r > R_i \end{cases}$$

$R_n$  ( $R_p$ ) : neutron (proton) radius parameter

$t_n$  ( $t_p$ ) : neutron (proton) surface thickness parameter

$n_i^{in}$  : central density

$n_n^{out}$  : neutron gas density ( $n_p^{out}=0$ )



A good function form

The n and p distributions are independent.

=> neutron skin

The empirical information is limited: radius and thickness.

The gradient term in Euler Eq. is continuous.

The density is zero beyond the classical turning point.

The values of parameters  $a_1 \sim b_3$  (EOS) and  $F_0$  are determined

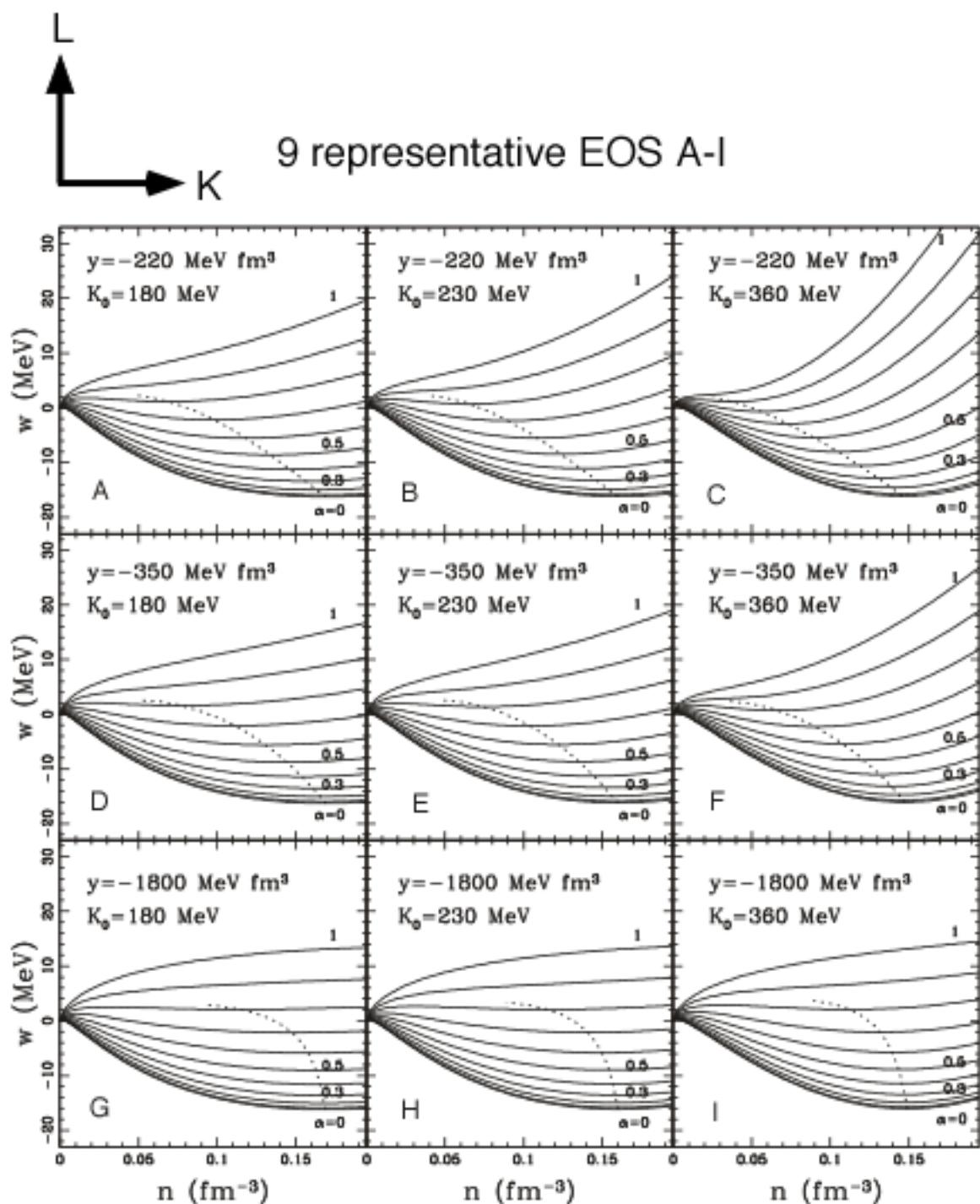
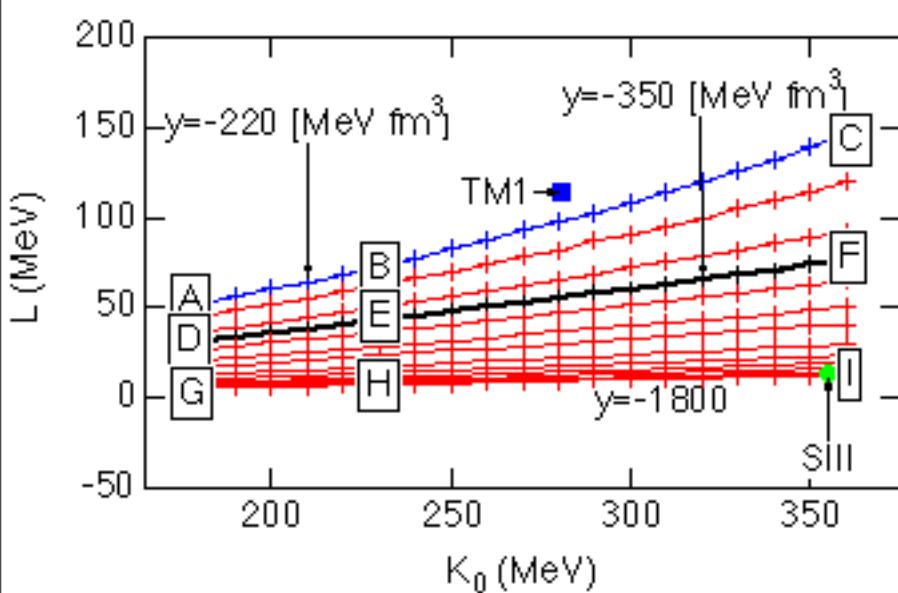
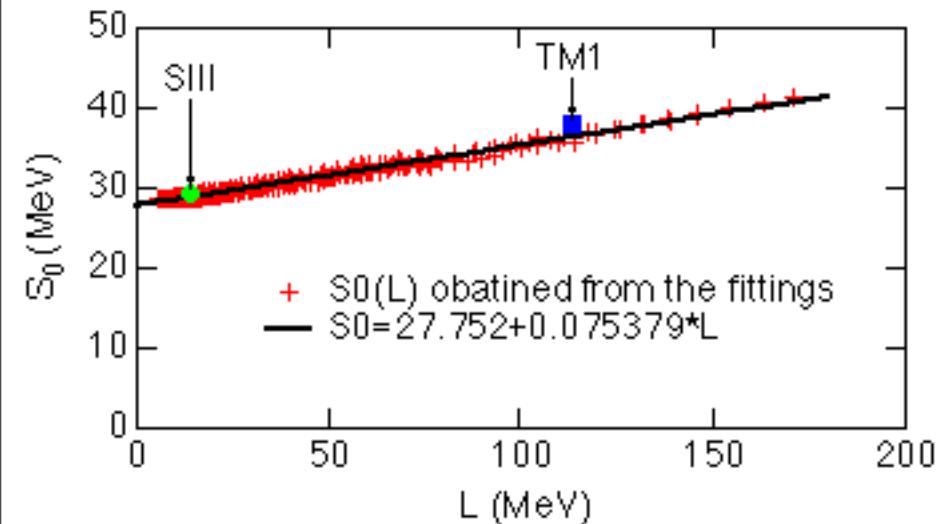
to fit masses and radii of stable nuclei.

=> about 200 sets of empirical EOS+ $F_0$

# EOS parameter values obtained from stable nuclei

$S_0$ :symmetry energy

L : density symmetry coefficient



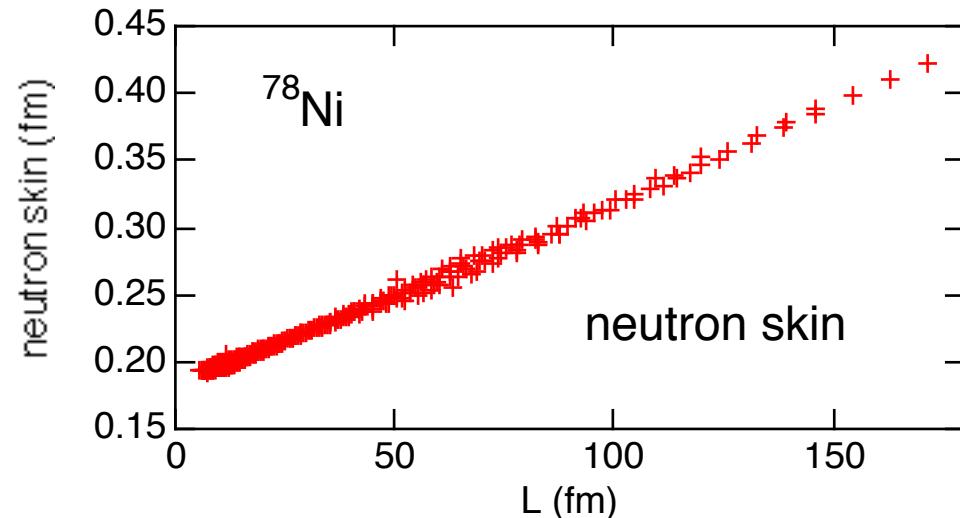
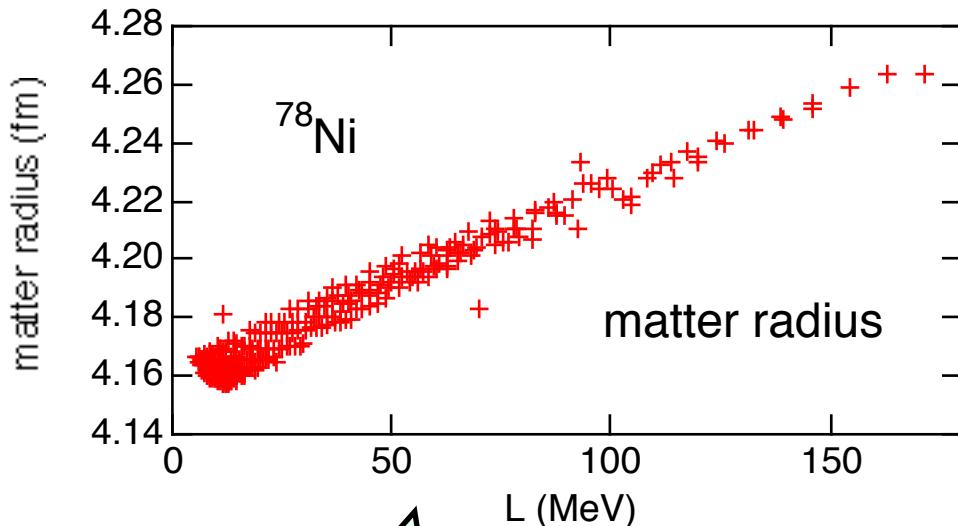
## Step 2

Calculate neutron-rich nuclei in labs  
with the 200 EOS's

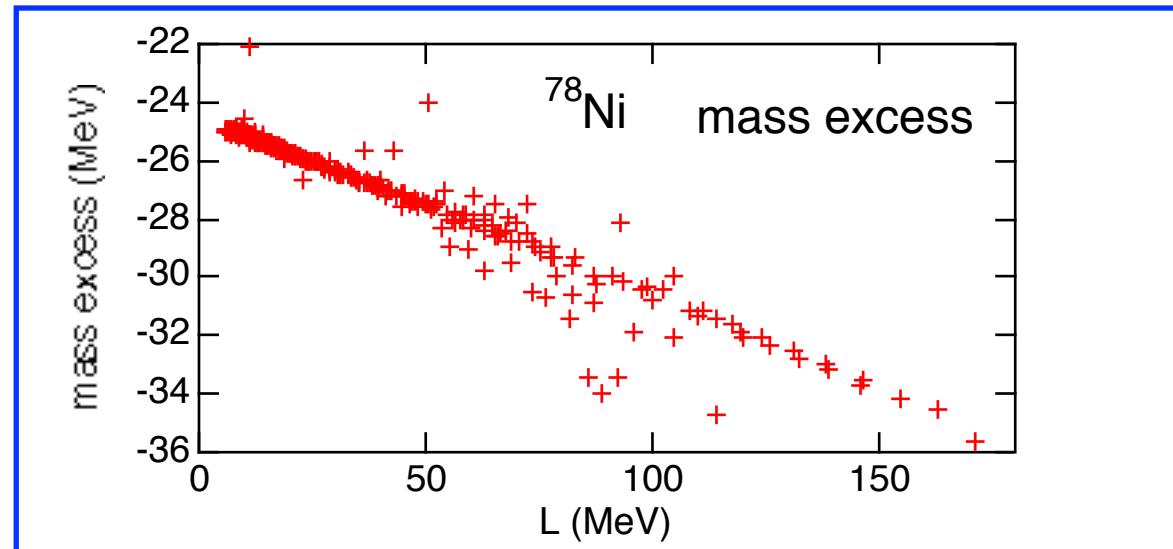
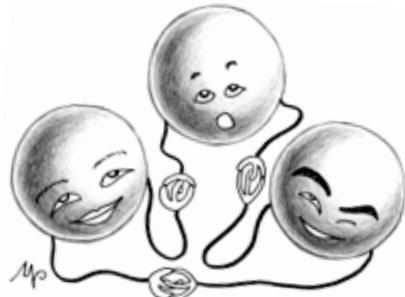
K. Oyamatsu and K. Iida, Prog. Theor. Phys. 109, 631 (2003).

Oyamatsu and Iida, PRC81, 054302, 2010.

The mass, radius and neutron skin are dependent on L but not on  $K_0$ .

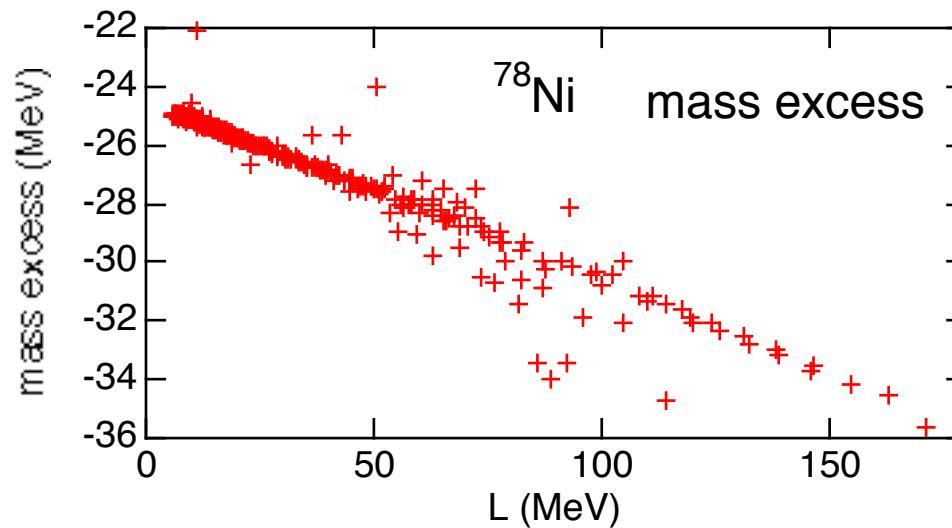


To be studied  
by Kurotama



Let's examine the L dependence of mass.

# L dependence comes from surface symmetry energy



Larger L => smaller mass

(>\_<) volume symmetry energy

Larger L => larger volume symmetry energy  $S_0 \Rightarrow$  larger mass

(^\_^) surface symmetry energy

Oyamatsu and Iida, PRC81, 054302, 2010.

# Surface energy comes from ...

in the cases of beta-stable nuclei  
in neutron-star crusts and in laboratories

1/2 from

$$F_0 \int d\mathbf{r} |\nabla n(\mathbf{r})|^2$$

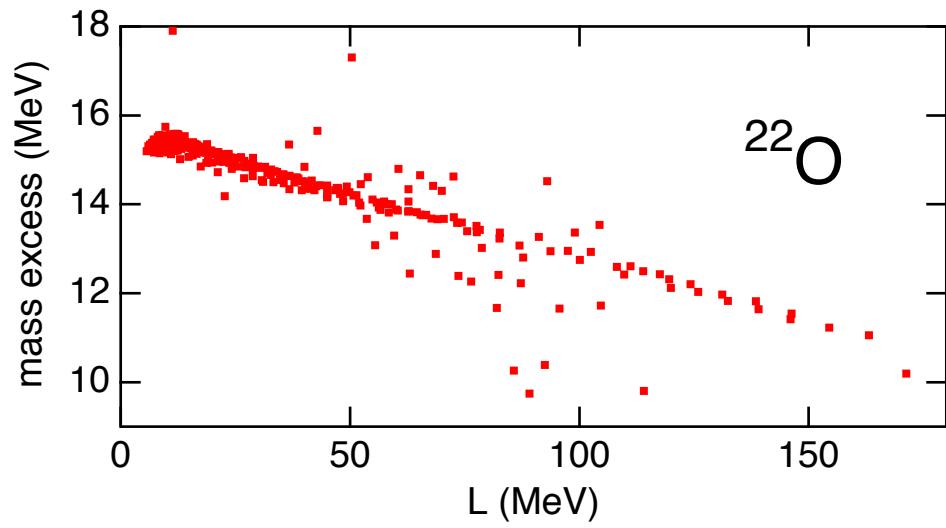
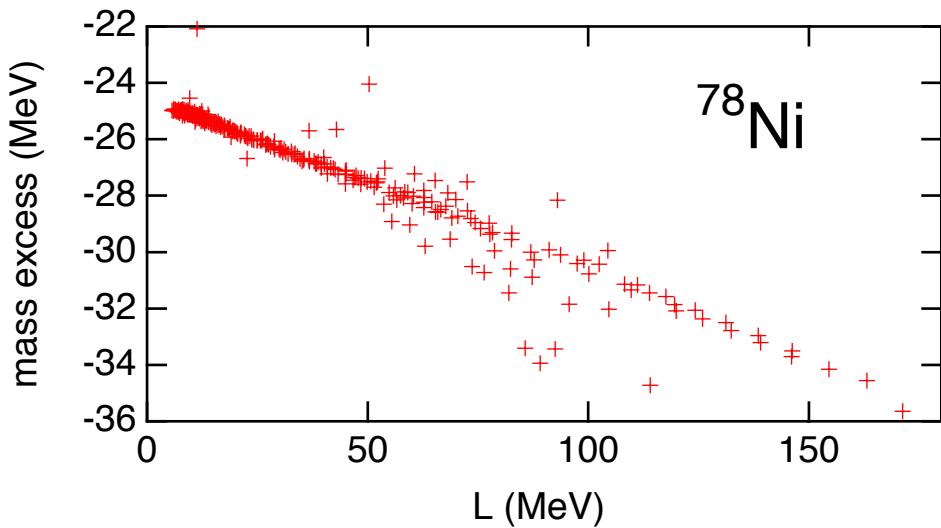
the remaining  
1/2 mainly from

$$\int d\mathbf{r} \varepsilon(n_n(\mathbf{r}), n_p(\mathbf{r})) \quad (\text{EOS})$$

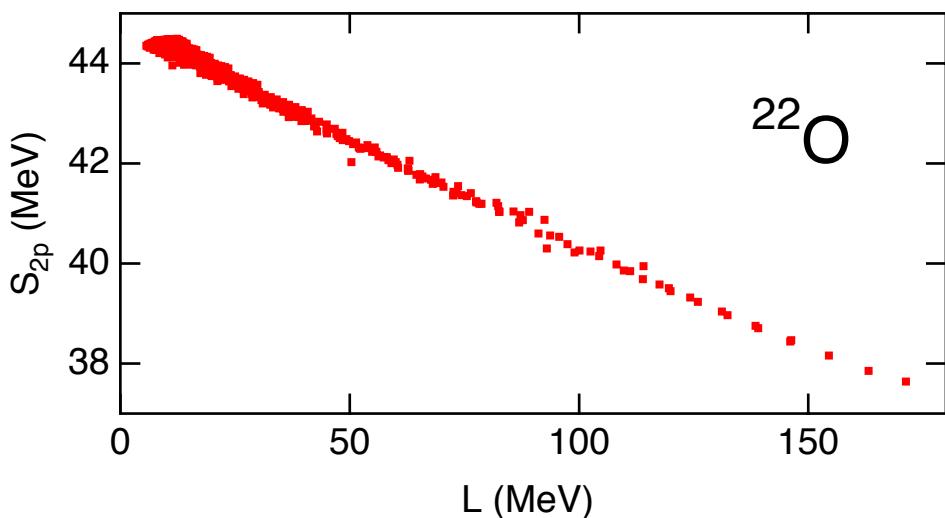
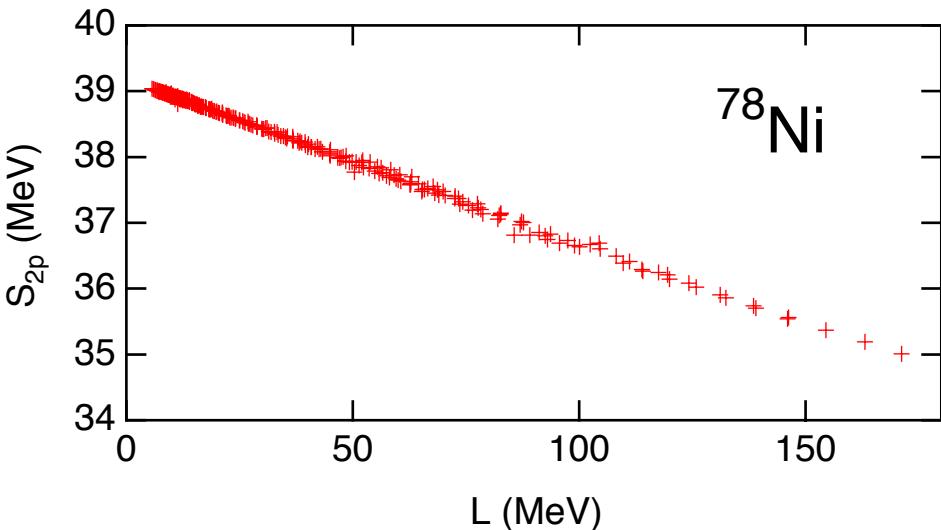
Anyway, L dependence emerge through density distribution.

Oyamatsu and Iida, PTP109, 631-650, 2003.

# $S_{2p}$ , $S_{2n}$ : clear L dependence better than mass

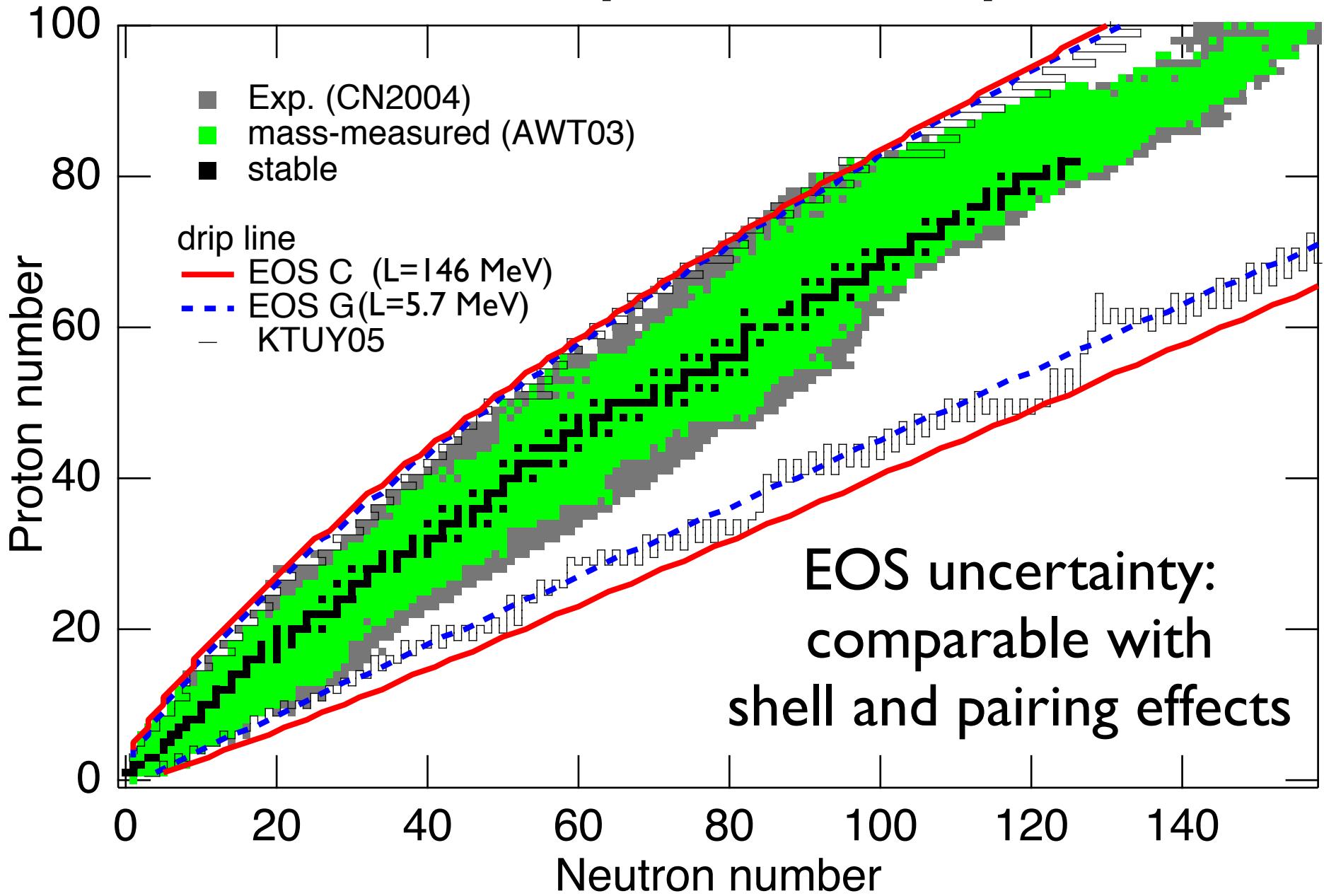


scatterings due to numerical errors in optimizing  $n_0$ ,  $w_0$ , and  $K_0$



Oyamatsu and Iida, PRC81, 054302, 2010.

# neutron and proton drip lines



Oyamatsu, Iida and H. Koura, PRC 82, 027301, 2010.

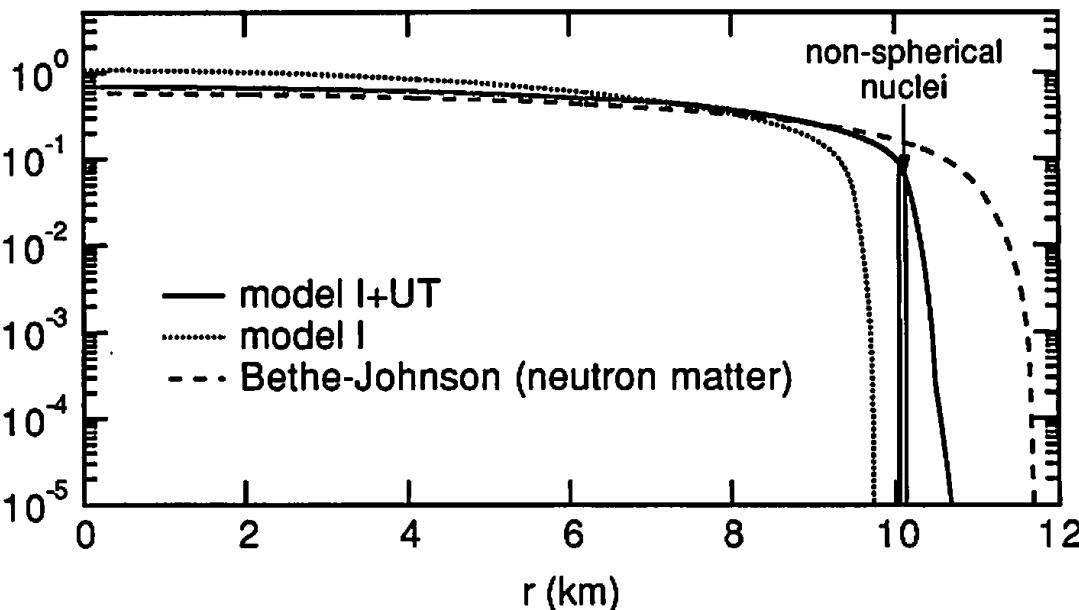
## Step 3

Calculate nuclei in neutron star crusts  
with 200 EOS's

Proton number and fraction  
Density region of pasta nuclei

K. Oyamatsu and K. Iida, Phys. Rev. **C75** (2007) 015801.

# 中性子星クラスト計算例



**1.4 M<sub>⊙</sub> star**

( $n, n_b$ : 核子数密度、 $\rho$ : 物質密度)

原子核密度

$$n_0 = 0.16 \text{ (fm}^{-3}\text{)}$$

$$\rho_0 = 3 \times 10^{14} \text{ (g/cm}^3\text{)}$$

コア・内殻境界密度 : Lに感度

$$\rho = (1/3 - 2/3)\rho_0$$

$$n_b = (0.06 - 0.1) \text{ (fm}^{-3}\text{)}$$

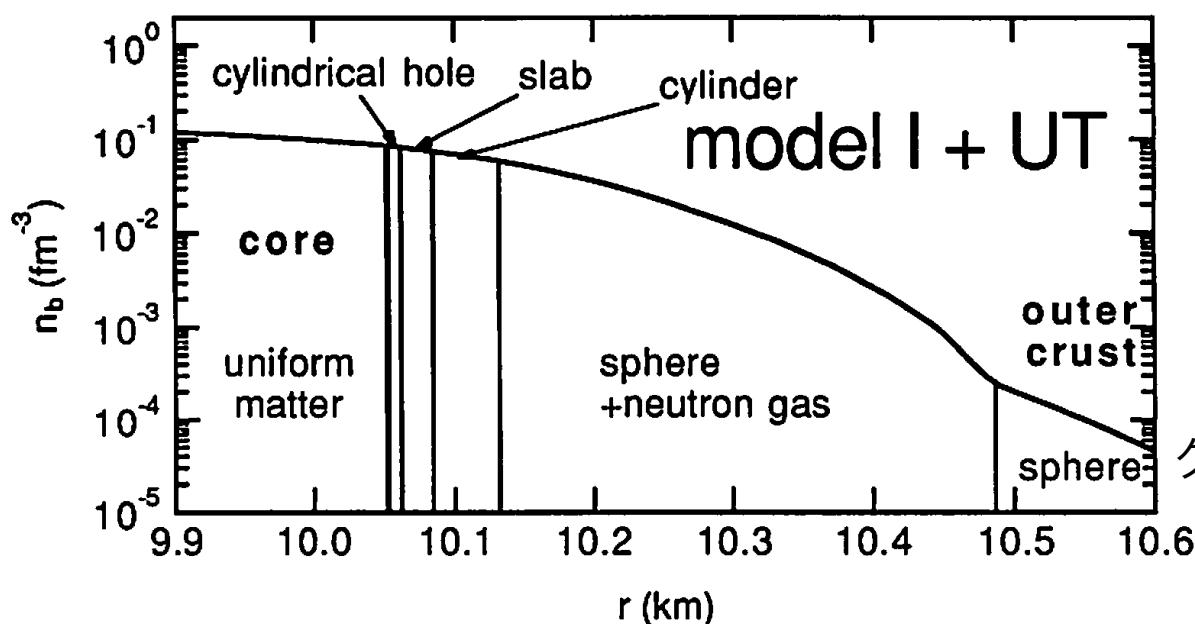
内殻・外殻境界密度 :  $S_0$ で決まる

$$\rho_{NDP} = 4 \times 10^{11} \text{ (g/cm}^3\text{)}$$

(NDP: Neutron Drip Point)

パスタ原子核層 : 70 m

クラストの質量、慣性モーメントの大部分



K. Oyamatsu, 1994 (unpublished).

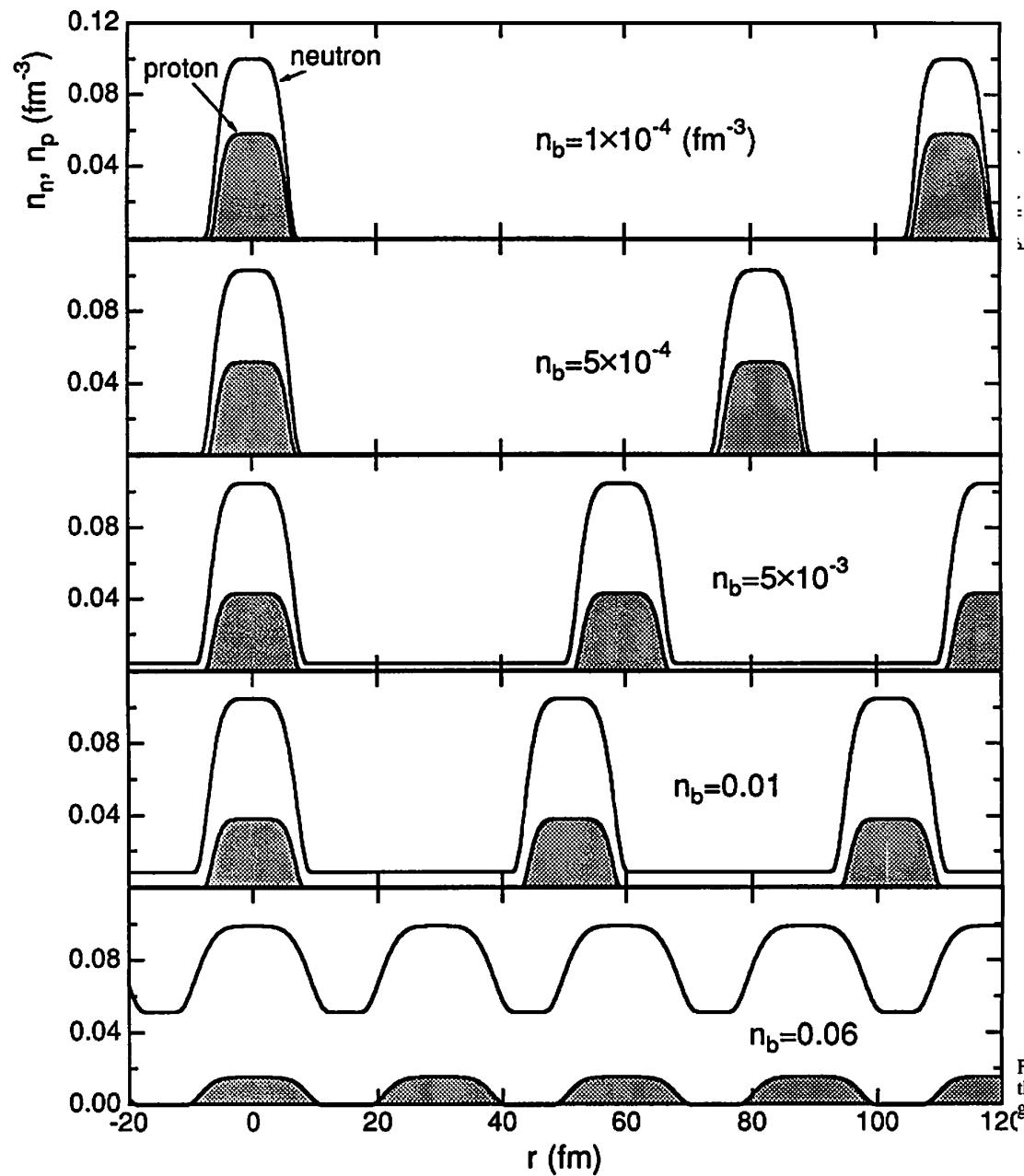


Fig. 2-4. The neutron (upper) and proton (lower, shaded) distributions along the straight lines joining the centers of the nearest nuclei. Calculations are performed in the case of spherical nuclei with model I interaction in chapter 4.

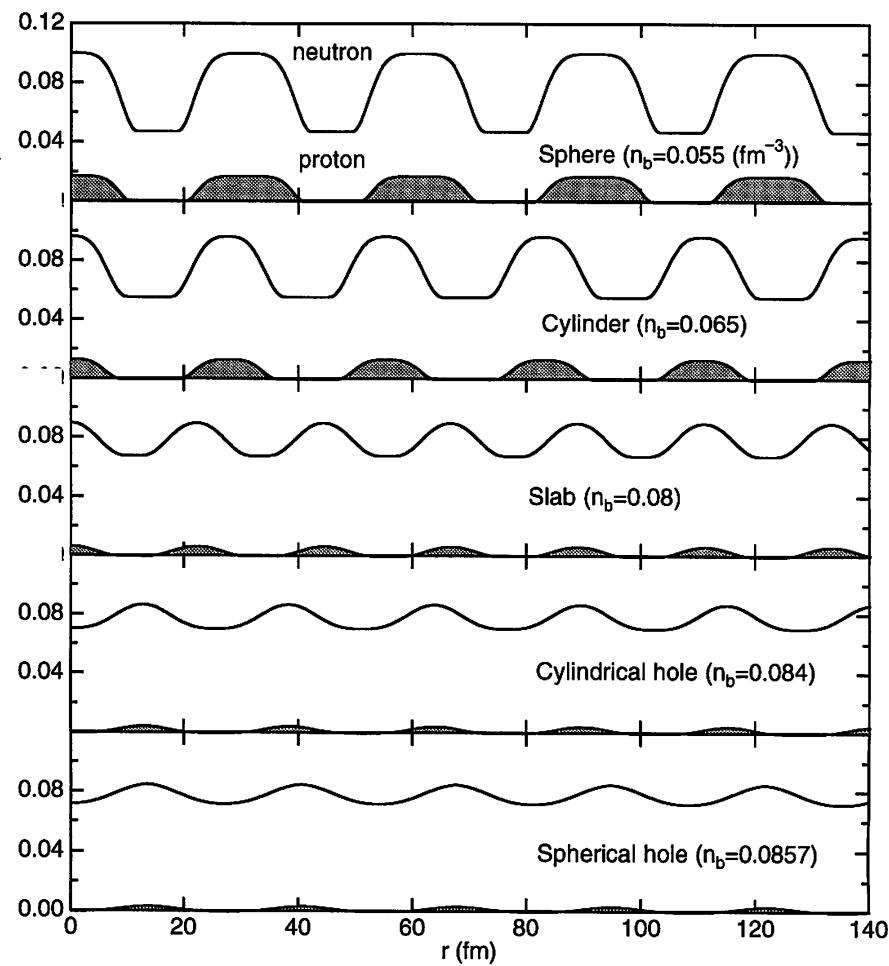
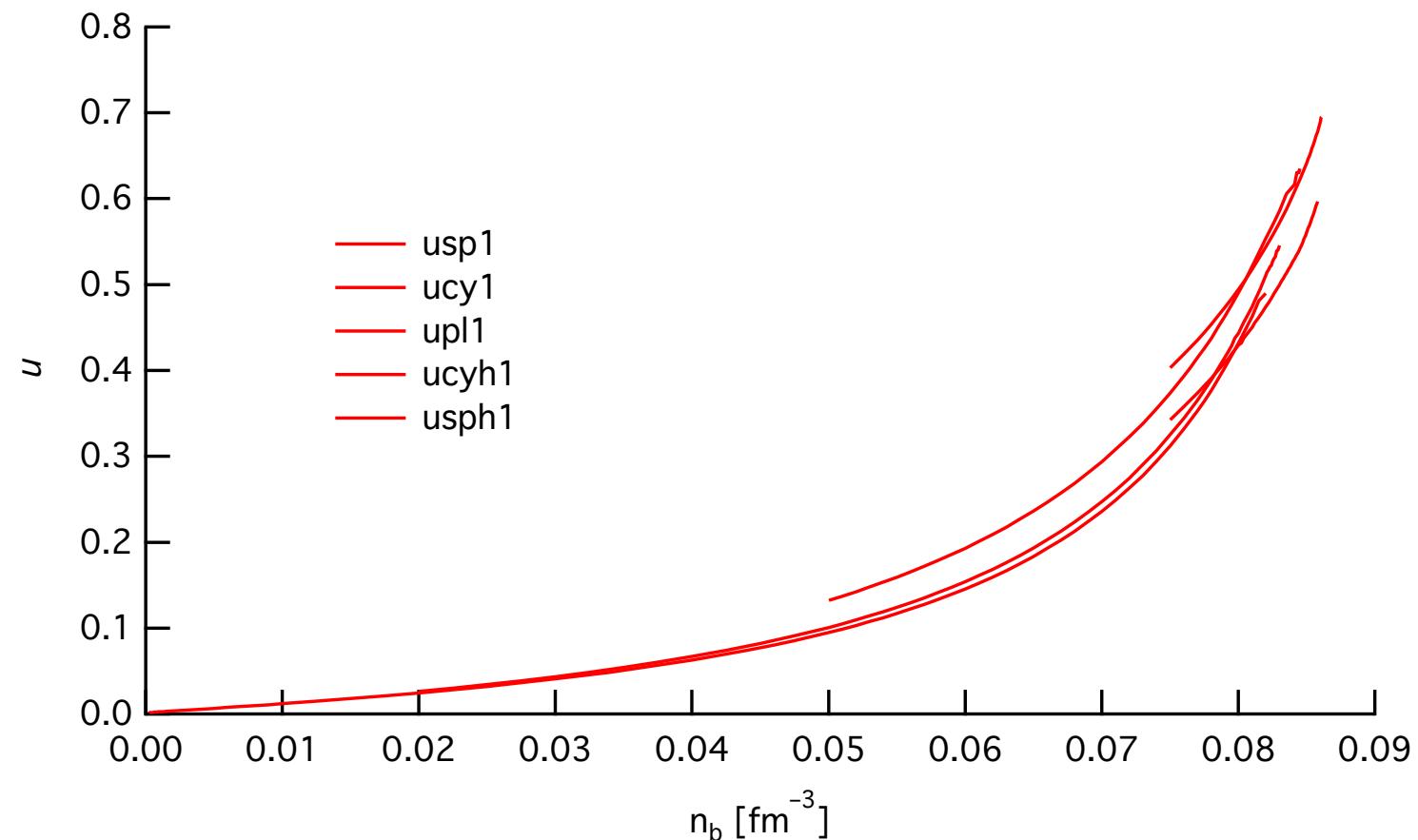


Fig. 4-14. The neutron (upper) and proton (lower, shaded) distributions along the straight lines joining the centers of the nearest nuclei or hole. Calculations are made with model I, but the other models give similar results.

# 原子核の体積比



# Estimate of density region of pasta nuclei

C.J. Pethick and D.G. Ravenhall, Annu. Rev. Nucl. Part. Sci. 45, 429 (1995).

## lower boundary

stability against fission of spherical nuclei

In the liquid drop model, (Coulomb self energy)=2\*(surface energy)

==> (volume fraction of nucleus) = 1/8

## upper boundary (core-crust boundary)

instability against forming proton clusters

$$v(Q) = v_0 + 2(4\pi e^2 \beta)^{1/2} - \beta k_{TF}^2 > 0$$

$$Q^2 = \left(\frac{4\pi e^2}{\beta}\right)^{1/2} - k_{TF}^2$$

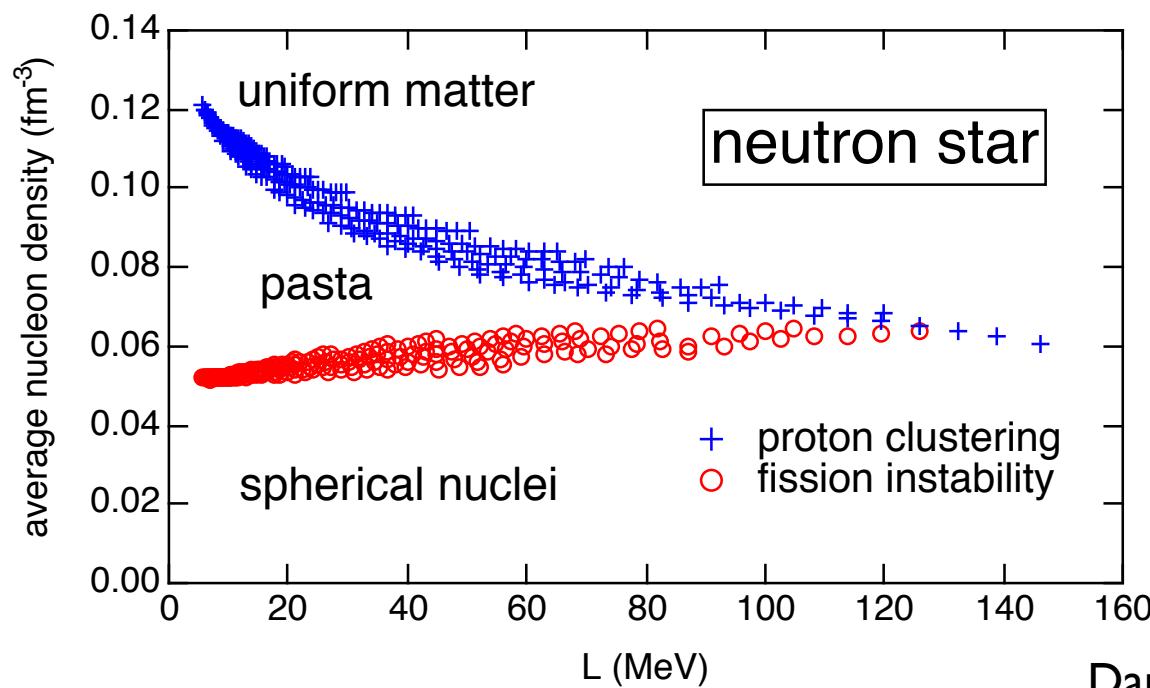
$$v(Q) \approx v_0 = \frac{\partial \mu_p}{\partial n_p} - \frac{(\partial \mu_p / \partial n_n)^2}{\partial \mu_n / \partial n_n} = \left(\frac{\partial \mu_p}{\partial n_p}\right)_{\mu_n, \mu_e}$$

$\mu$ の微分=>L依存性を示唆

$$\beta = D_{pp} + 2D_{np}\zeta + D_{nn}\zeta^2, \quad \zeta = -\frac{\partial \mu_p / \partial n_n}{\partial \mu_n / \partial n_n},$$

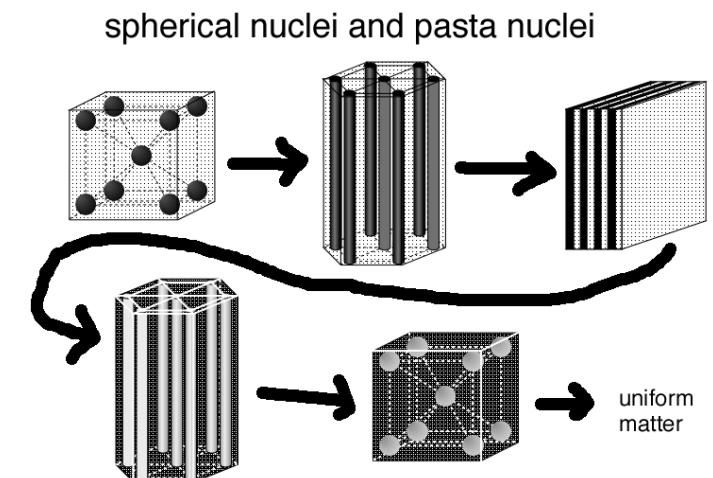
$$k_{TF}^2 = \frac{4\pi e^2}{\partial \mu_e / \partial n_e} = \frac{4\alpha}{\pi} (3\pi^2 n_e)^{1/3}.$$

# 中性子星のコア・クラスト境界



クラストコアの境界密度 (青)  
パスタ原子核の存在領域  
球形核存在領域の最大密度 (赤)

Oyamatsu and Iida, PRC75, 015801, 2007.

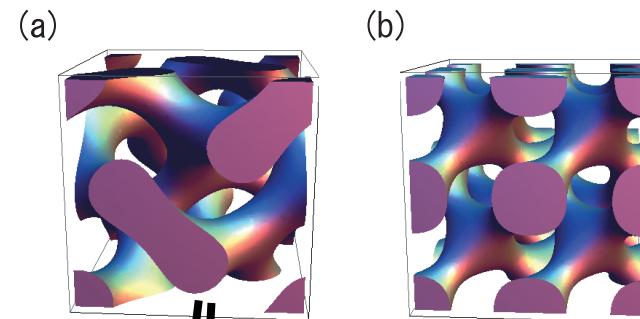


Dark domains means nuclei (proton clusters).  
At low densities in neutron-star crusts, we have nuclei which are more or less spherical.  
In the core we have uniform matter. Pasta nuclei could exists in between.

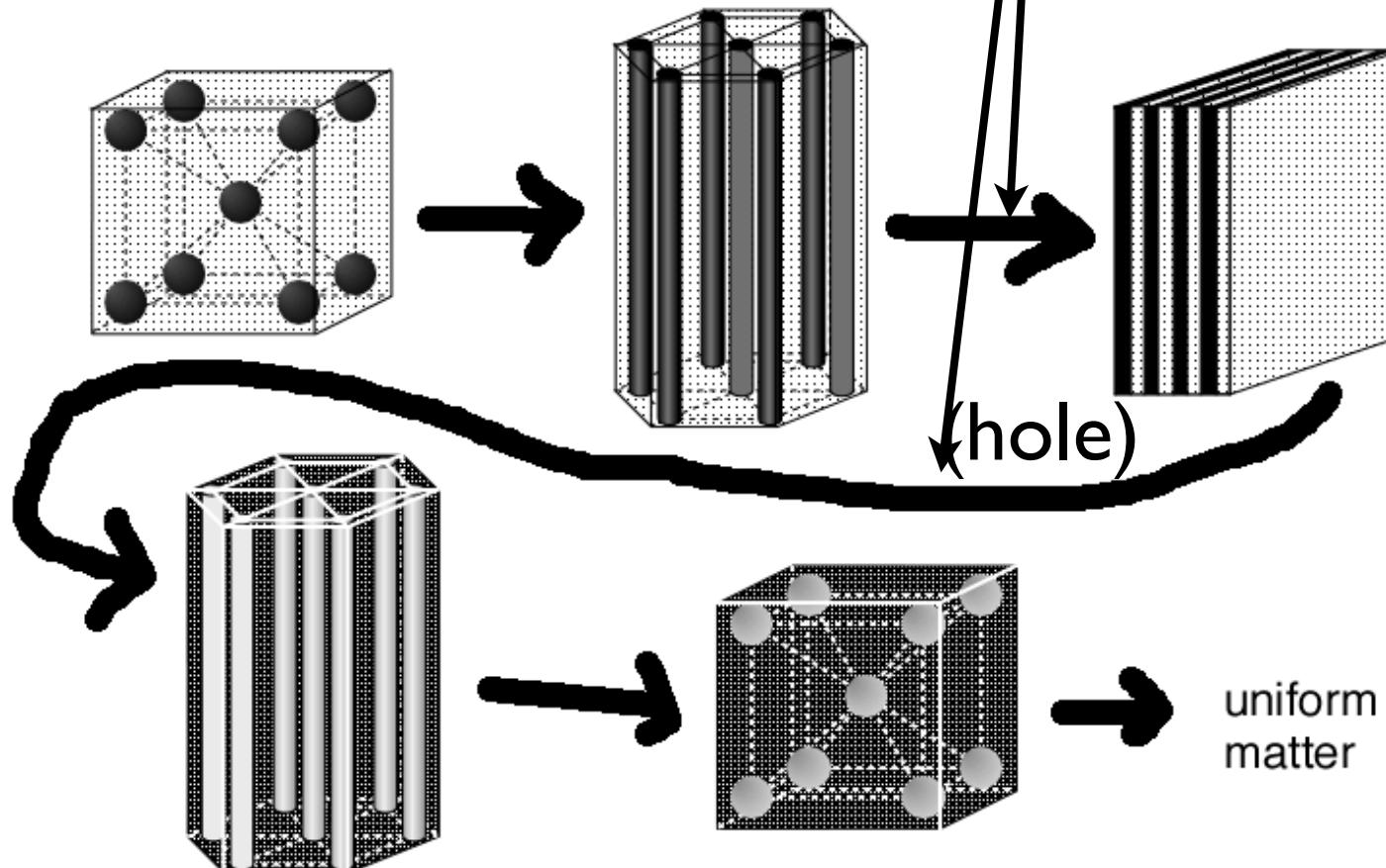
Existence of pasta nuclei depends on the EOS.

# If $L < 100$ MeV, gyroid could appear at finite temperature.

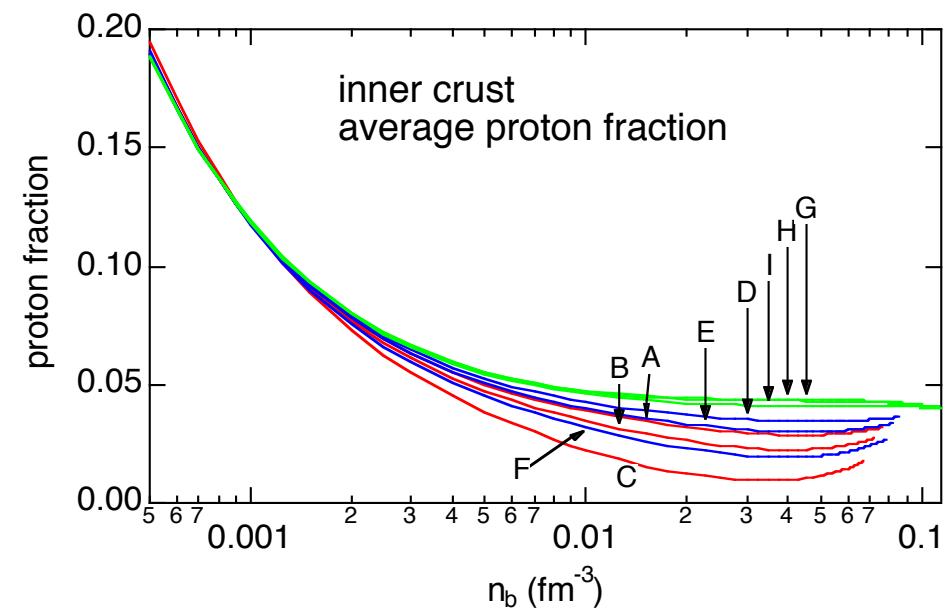
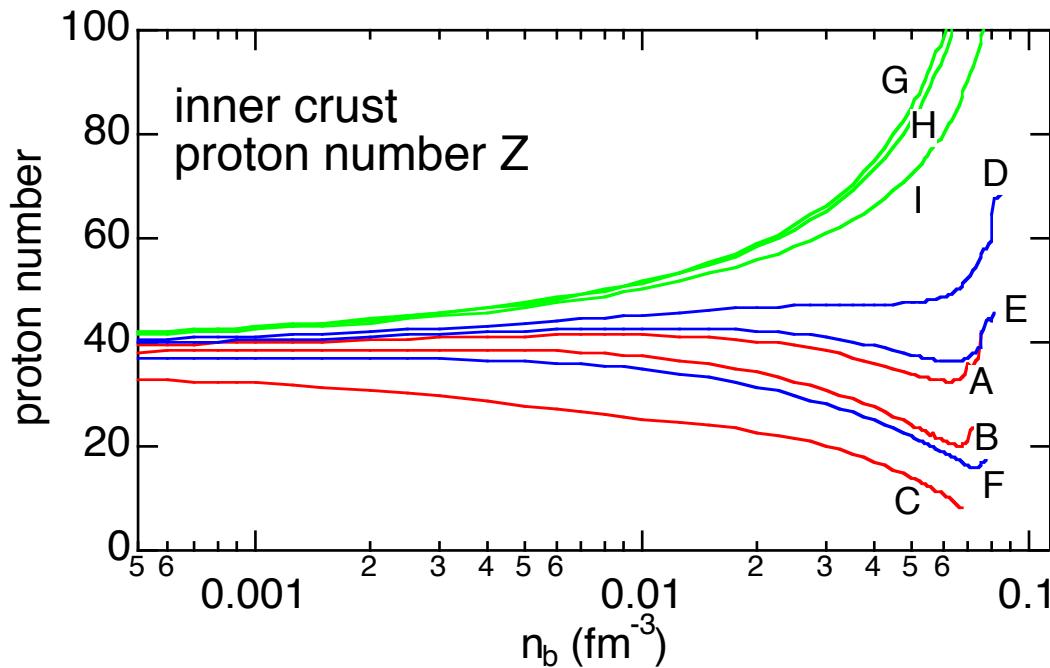
Nakazato, Oyamatsu and Yamada, PRL103, 132501, 2009.



spherical nuclei and pasta nuclei



# Inner-crust nuclei: Proton number and fraction decrease with L



For large L,  $S(n)$  at  $n < n_0$  is small  
so that nuclei become more neutron-rich.

# まとめ：対称エネルギー $S_0$ と $L$

- 通常は $S_0$ が支配的
  - 安定原子核の質量・大きさ
  - $L$ の感度はそれほど強くない：核図表のdrip line、中性子星のdrip point
- $L$ を見るには工夫（微分）が必要
  - 中性子過剰核
    - $S_{2n}, S_{2p}$ のN,Z依存性
    - 原子核の大きさ、中性子スキンのN,Z依存性
  - 中性子星物質：コア（核密度）に近づくほど感度が増す
    - 陽子混在度
    - 中性子星のコア・クラストの境界、パスタ原子核の存在
  - 中性子星の観測から $L$ の値を評価する試み => 祖谷さん