基研研究会「ハドロン物質の諸相と
状態方程式　ー中性子星の観測に照らして」，2012.8.30-9.1

クォーク物質の磁気的性質とコンパクト星

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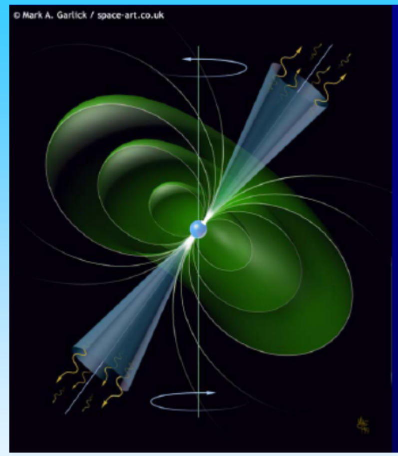
Introduction

Microscopic origin of the magnetic field in
compact stars

Inhomogeneous chiral phase and its magnetic property

SSB in the presence of the magnetic field
+
Dimensional reduction

Strong magnetic field
in compact stars

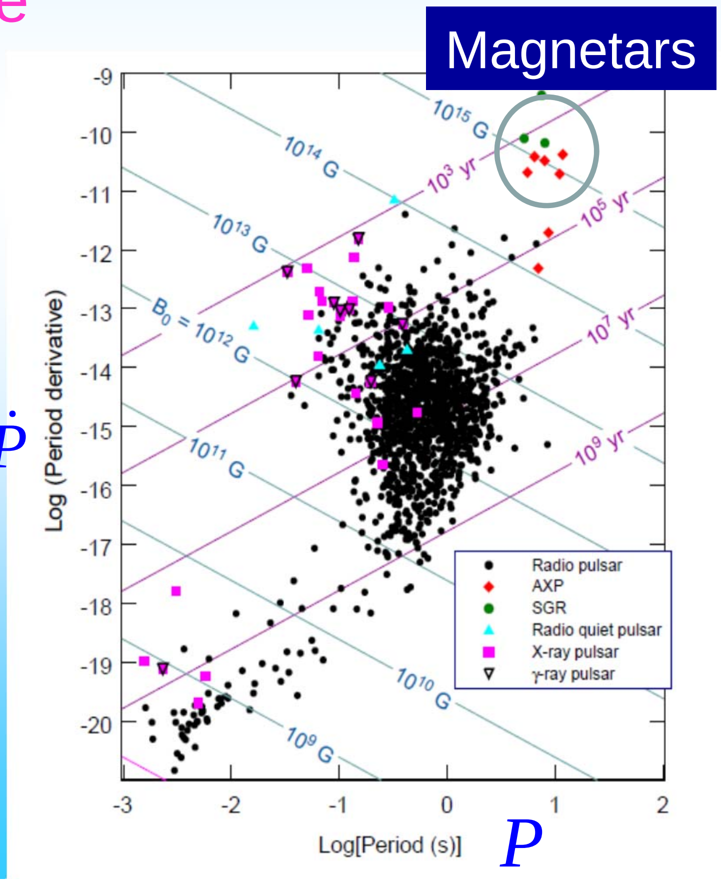


最初のパルサー発見いらい
磁場の起源は長い間の謎になっている。

→ Recent discovery of magnetars seems to revive this issue.

Origin:
(i) Fossil field
(ii) Dynamo scenario (crust)
(iii) Microscopic origin (core)

$P-\dot{P}$ curve



その起源を強い相互作用に帰する
ことが興味深いであろう。

Ferromagnetism or spontaneous spin polarization

微視的な核物質計算はすべて否定的な結果をあたえる
ことがわかっている。

→ For recent references,
I.Bombaci et al, PLB 632(2006)638
G.H. Bordbar and M. Bigdeli, PRC76 (2007)035803

(cf. ^{3}He は常磁性だが、高圧力下では強磁性に近い。これは
^{3}O状態が強い斥力芯を避けることができるため。)

→ クォーク物質の自発的磁化または強磁性相転移
の可能性

Some ideas in QCD

Perturbative calculation
(one-gluon-exchange interaction)

⟶ Fermi liquid theory

T.T. PLB489(2000)280
T.T. and K. Sato., Phys. Lett., B663 (2008) 322; B672(2009) 132.
K. Sato and T.T., Nucl. Phys. A826 (2009) 74

A.Niegawa, PTP113(2005)581,
K. Pal et al., PRC79(2009) 015205; PRC80(2009)024903; PRC80(2009)054911

Non-perturbative calculations

Gordon identity $(\mathbf{A}=\mathbf{B}\times\mathbf{r}/2)$:

$$\int \mathrm{d}^4 x L_{\mathrm{int}}^{\mathrm{QED}} = e_q \int d^4 x \bar{\psi} \boldsymbol{\gamma} \cdot \mathbf{A} \psi$$

$$= \mu_q \int \mathrm{d}^4 x \bar{\psi} \left[-i\mathbf{r}\times\nabla + \boldsymbol{\Sigma} \right] \cdot \mathbf{B} \psi .$$

Magnetization in relativistic theories

T. Maruyama, T.T., NPA693 (2001) 701.
S. Maedan, PTP 118(2007) 729.
Y. Tsue et al, arXiv:1205.2409

$$\mathbf{M} = \langle \boldsymbol{\sigma} \rangle ,$$

Axial-vector mean-field:

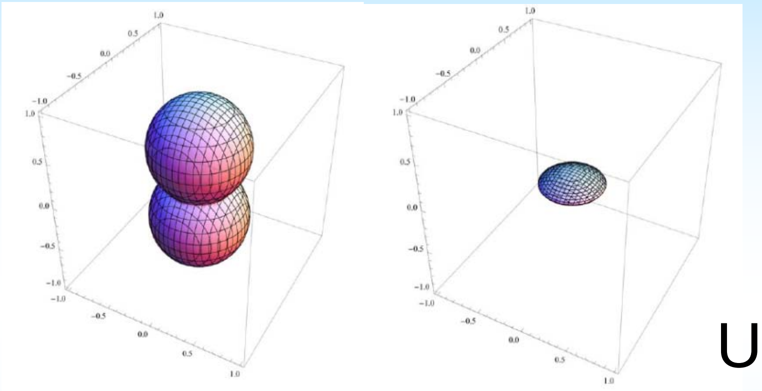
$$\mathbf{U}_A = \langle \bar{q} \gamma_5 \boldsymbol{\gamma} q \rangle ,$$

Tensor mean-field:

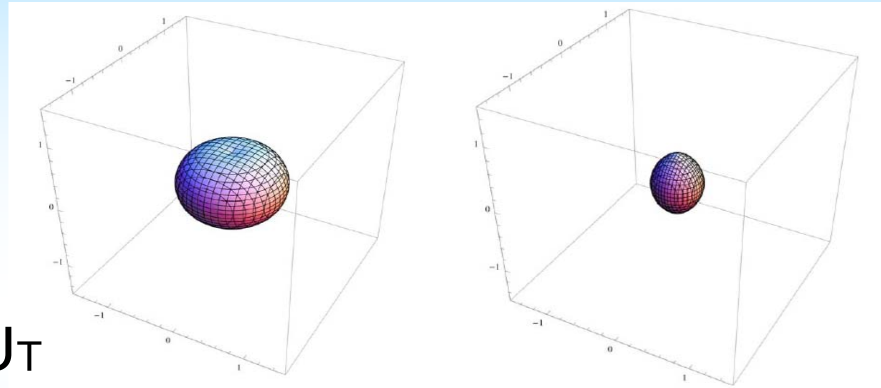
$$\mathbf{U}_T = \langle \bar{q} \gamma_0 \gamma_5 \boldsymbol{\gamma} q \rangle$$

Topology of Fermi surface:

$$e(\mathbf{p},\zeta) = \sqrt{E_p^2 + U_A^2 + 2\zeta U_A \sqrt{m^2 + p_z^2}}, \quad e(\mathbf{p},\zeta) = \sqrt{E_p^2 + U_T^2 + 2\zeta U_T \sqrt{m^2 + p_\perp^2}}.$$



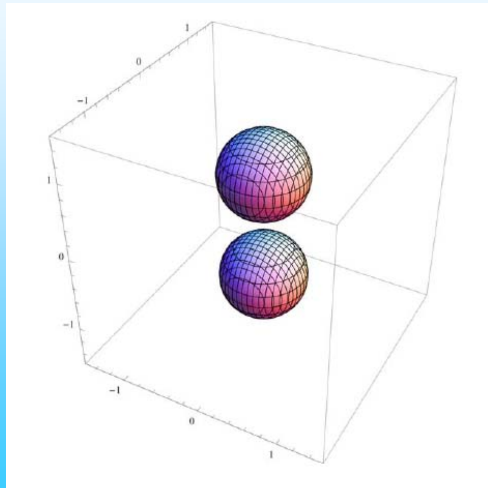
prolate oblate

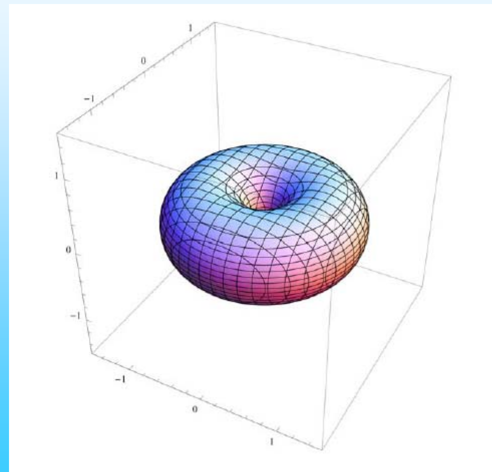


oblate prolate

U_A, U_T

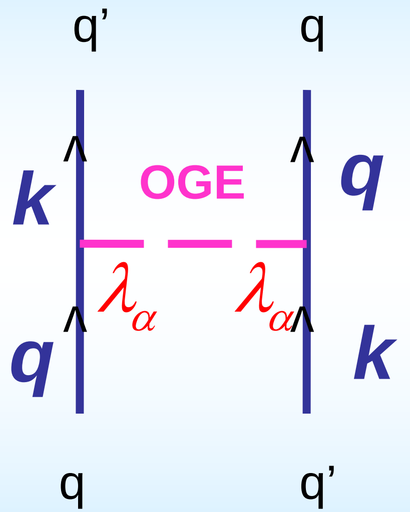
large





Bloch mechanism
(repulsive int. + Pauli pr.)

Fock exchange interaction is responsible to ferromagnetism in quark matter



c.f. Ferromagnetism of itinerant electrons (Bloch,1929)

Relativistic spin (Landau-Lifshitz)

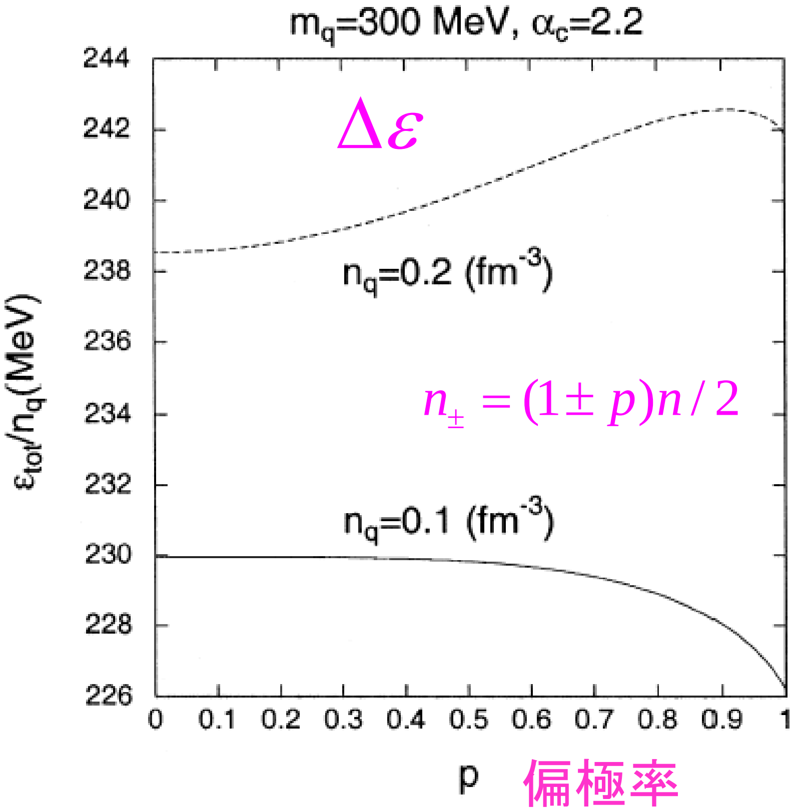
$$\mathbf{a} = \zeta + \frac{\mathbf{k}(\mathbf{k}\cdot\zeta)}{m(E_k+m)}, \quad a_0 = \frac{\mathbf{k}\cdot\zeta}{m}$$

$$(\lambda_\alpha)_{ab}(\lambda_\alpha)_{ba} = 1/2 - 1/(2N_c)\delta_{ab} > 0$$

$$M_{\mathbf{k}\zeta,\mathbf{q}\zeta'}\Big|_{\text{Fermi surface}} \simeq g^2\frac{N_C^2-1}{4N_C^2E_F^2}\left[2m^2 - E_F^2 + k_F^2\cos\theta - m^2 a\cdot b\right]\frac{1}{-k_F^2\left(1-\cos\theta\right)}$$

$$\to g^2\frac{N_C^2-1}{4N_C^2E_F^2}\left[m^2 + \zeta\cdot\zeta'\frac{(m+E_F)^2}{4}\right]\frac{1}{-k_F^2\left(1-\cos\theta\right)}$$

Weakly first order



偏極率

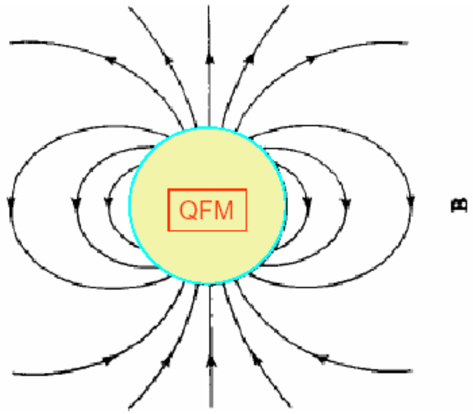
Fig. 1. Plot of the energy density as a function of the polarization parameter at $n_q = 0.1\,\text{fm}^{-3}$ and $n_q = 0.2\,\text{fm}^{-3}$. The critical density is around 0.16fm^{-3} in this case.

$$\Delta\varepsilon \equiv \varepsilon_{\text{ferro}} - \varepsilon_{\text{para}}$$

T=0での臨界密度

$$n_B^c \sim O(n_{nuclear})$$

c.f. A.Niegawa, PTP113(2005)581,
K. Pal et al., PRC79(2009) 015205; PRC80(2009) 024903;PRC80(2009)054911



$$B_{max} = \frac{8\pi}{3}\left(\frac{r_Q}{R}\right)^3 \mu_Q n_Q$$

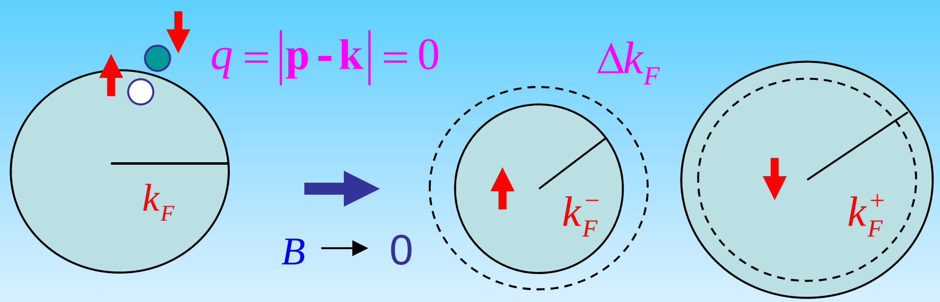
$$\approx O(10^{15-17}G)$$

for

$$n_Q = O(0.1\text{fm}^{-3})$$

Magnetars as quark stars

Magnetic susceptibility by way of Fermi liquid theory



T.T. and K. Sato., Phys. Lett., B663 (2008) 322.
K. Sato, T.T.,Prog. Theor. Phys. Suppl. 174 (2008) 177

T.T. and K. Sato, Phys. Lett. B672(2009) 132.
K. Sato and T.T., Nucl. Phys. A826 (2009) 74.
T.T., Proc. of CSQCDII (2010) in press.

$$\langle M \rangle \propto \Delta N = N_C V k_F^2 \Delta k_F / 2\pi^2$$

spin susceptibility

$$\chi_M = \left. \frac{\partial \langle M \rangle}{\partial B} \right|_{N,T,B=0}$$

$\chi_M \rightarrow \infty$ or $\chi_M^{-1} \rightarrow 0$

for spontaneous magnetization (ferromagnetism)

$$\chi_M = \left(\frac{g_D \mu_q}{2} \right)^2 N(T) / \left(1 + N(T) \bar{f}^o \right) = \left(\frac{g_D \mu_q}{2} \right)^2 / \left(\frac{\pi^2}{N_C k_F E_F} - \frac{1}{3} f_1^s + \bar{f}^a \right)$$

μ_q : Dirac magneton

Quasiparticle interaction:

$$f_{\mathbf{k}\zeta,\mathbf{q}\zeta'} = f^s_{\mathbf{kq}} + \zeta\zeta' f^a_{\mathbf{kq}} \longrightarrow$$

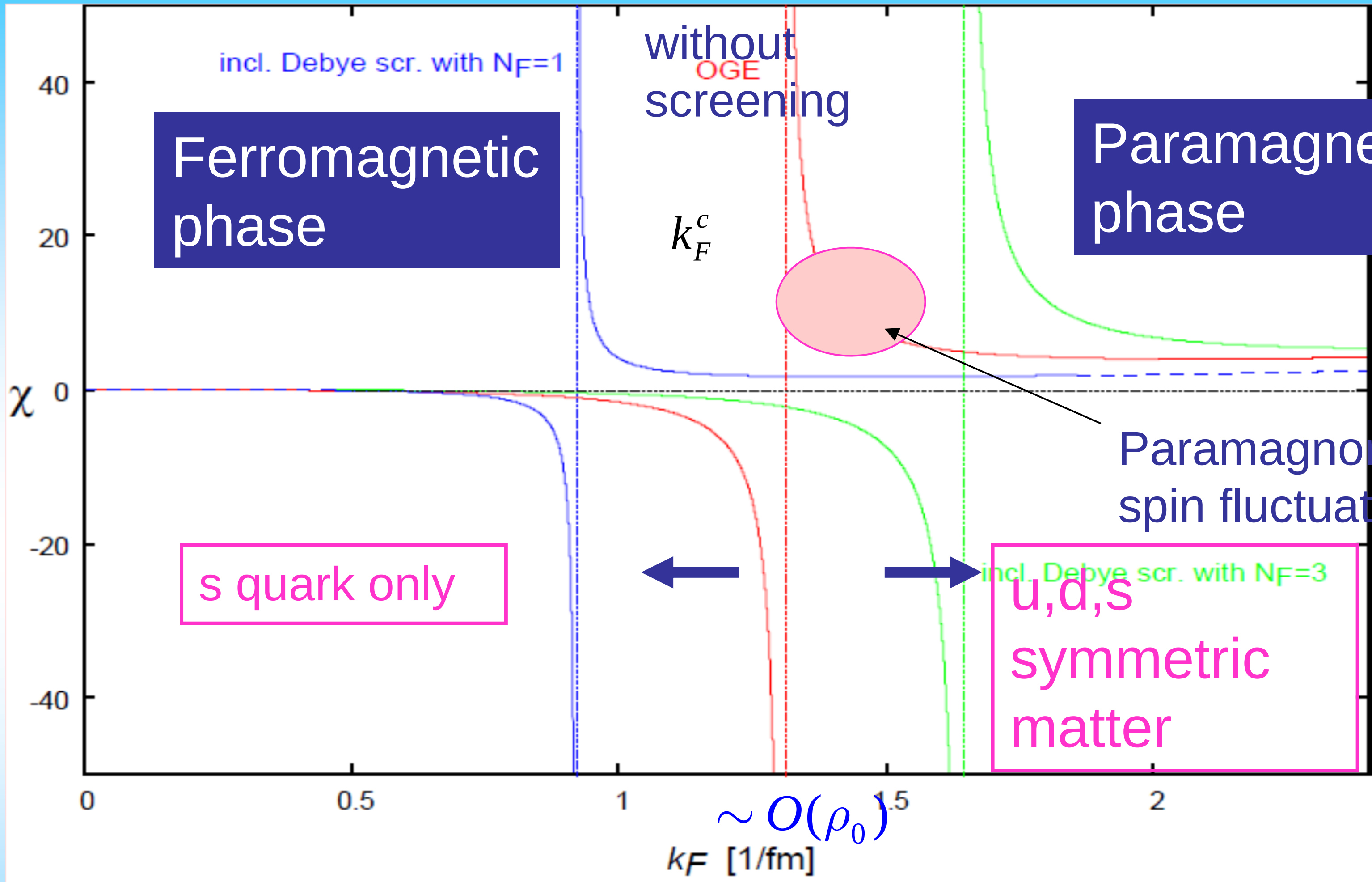
Spin dep.

Infrared (IR) singularities in QCD/QED

$f_1^s, \bar{f}^a \propto m \rightarrow 0$

Spin susceptibility at T=0

T.T. and K. Sato., Phys. Lett., B663 (2008) 322.
K. Sato and T.T. , Prog. Theor. Phys. Suppl. 174 (2008) 177



NF dependence

$$\Delta\chi^{-1} \propto \kappa \ln(2/\kappa)$$

$$\kappa = m_D^2 \Big/ 2k_F^2 \qquad m_D^2 = \sum_{\text{flavors}} \frac{g^2}{2\pi^2} k_{F,i} E_{F,i}$$

Screening favors spontaneous magnetization in large NF

Magnetic susceptibility at T>0

$$\chi^{-1}(T) \sim \chi^{-1}(T=0) + \frac{\pi^4}{6N_C k_F^5 E_F}\left(2E_F^2 - m^2 + \frac{m^4}{E_F^2}\right)\left(T^2 - \frac{C_g N_C v_F}{3\pi^2} T^2 \ln\left(\frac{T}{M_D}\right)\right)$$

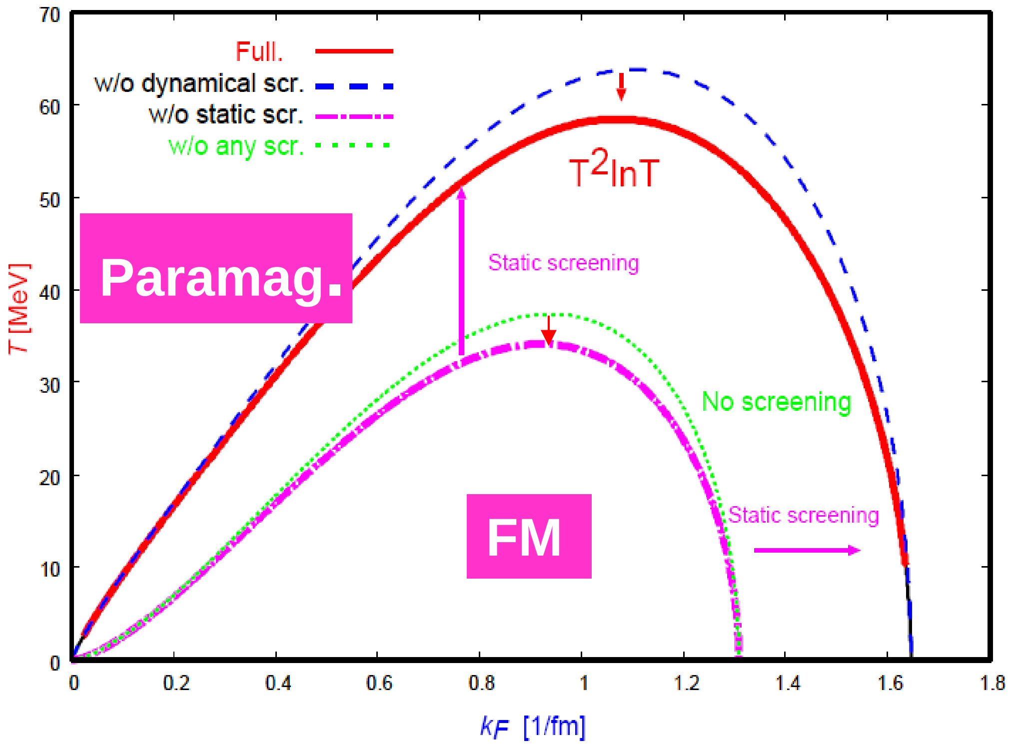
usual T^2 term

Non Fermi-liquid effect

cf. ・Specific heat $C_V \approx T \ln T$ (A. Ipp et al., PRD 69(2004)011901)

・Gap equation $\Delta \approx \exp\left[-(\pi^2+4)(N_C-1)/16\right]\exp(-3\pi^2/\sqrt{2}g)$

(D.T. Son, PRD 59(1999)094019)



Curie (critical) temperature should be order of several tens (40-60) MeV.

T.T. and K. Sato, Phys. Lett. B672(2009) 132.
K. Sato and T.T., Nucl. Phys. A826 (2009) 74.
T.T., Proc. of CSQCDII (2010) in press.

Remarks

Bloch 機構に基づく強磁性相の可能性 （低密度領域）

臨界温度は比較的大きい（数十MeV）
ゲージ相互作用による非フェルミ液体効果の発現

非摂動的効果による平均場の形成（Stoner,Weiss）

軸性ベクトル、テンソル平均場 → 高密度でのスピン偏極の可能性

他の相、e.g. CSCとの競合、共存

(E.Nakano, T. Maruyama, TT, PRD 68(2003) 105001.)

現象的意義

マグネター、初期宇宙起源磁場、...

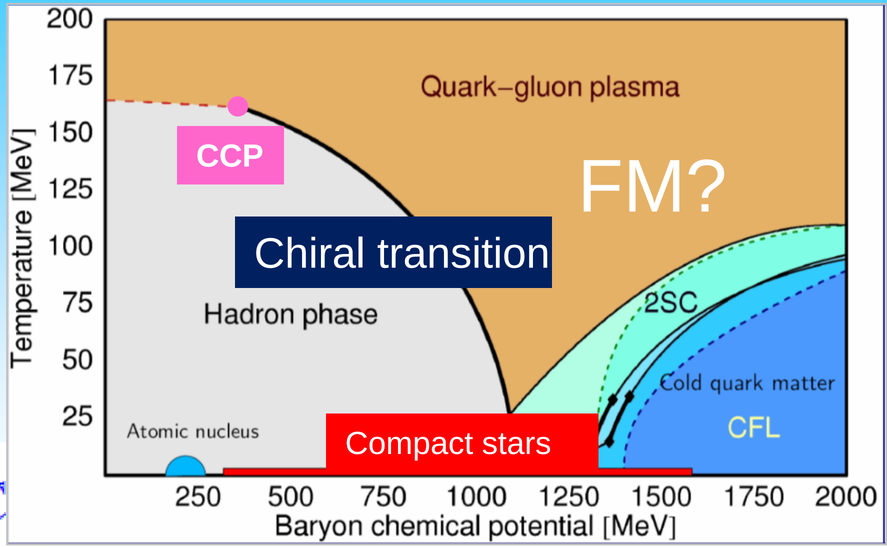
・Lattice simulations or model studies have suggested a chiral transition

$$\Delta(\text{or } M) \to 0$$

Chiral order-parameter:

$$M \equiv \langle \bar{q}q \rangle + i\langle \bar{q}i\gamma_5\tau_3 q \rangle = \Delta \exp(i\theta) \in \mathbb{C}$$

$$(M \in \mathbb{R} \text{ for } \langle \bar{q}i\gamma_5\tau_3 q \rangle = 0)$$



(B. Ruester)

QCD phase diagram

inhomogeneous chiral phases in the vicinity of the chiral transition:

CCP as Lifshitz point

Dual chiral density wave (DCDW):

$$\Delta = \text{const.}, \theta = \mathbf{q}\cdot\mathbf{r}$$

(T. Tatsumi and E. Nakano, hep-ph/0408294.
E. Nakano and T. Tatsumi, Phys. Rev. **D71** (2005) 114006.

Real kink crystal (RKC): $\Delta(z), \theta = 0$

(D.Nickel, PRL 103(2009) 072301; PRD 80(2009) 074025.)

Inhomogenous chiral phase

A new paradigm ? in the QCD diagram

DCDW

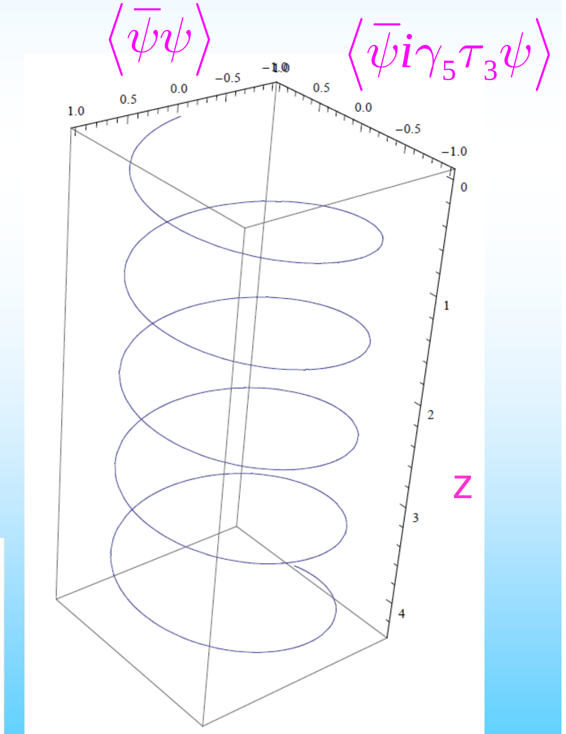
ref. T.T. and E. Nakano, hep-ph/0408294
PRD71(2005)114006.

M. Sadzikowski and W.Broniowski,
PLB488 (2000)63.)

Dual Chiral Density Wave (DCDW)

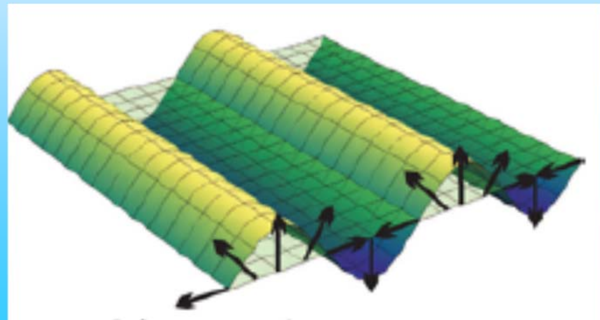
$$\langle\bar{\psi}\psi\rangle = \Delta\cos\mathbf{q}\cdot\mathbf{r}$$
$$\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle = \Delta\sin\mathbf{q}\cdot\mathbf{r},$$

cf　　スパイラル状態 →



Spin density wave （SDW）

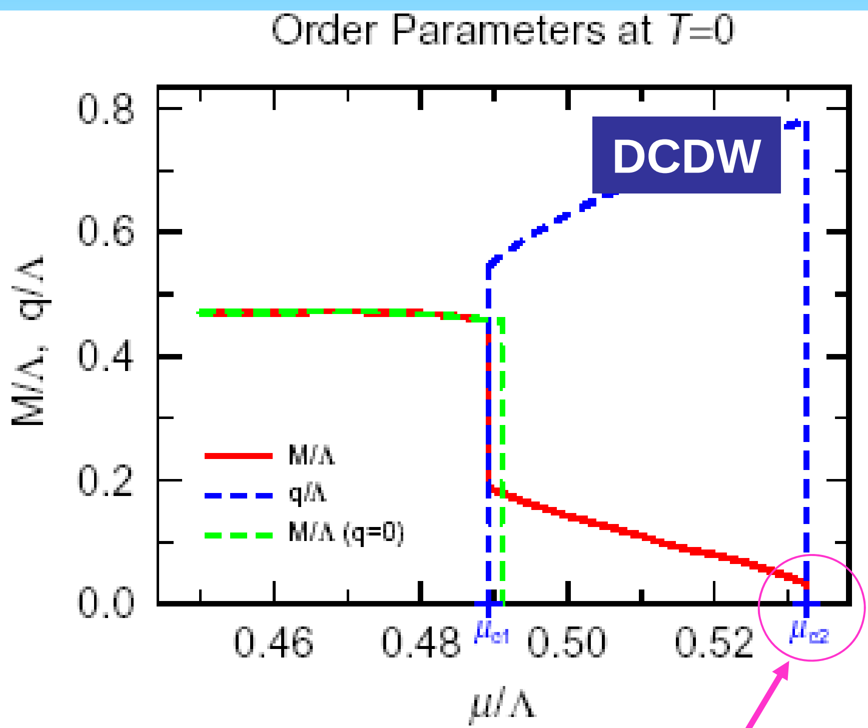
(c.f. Overhauser)

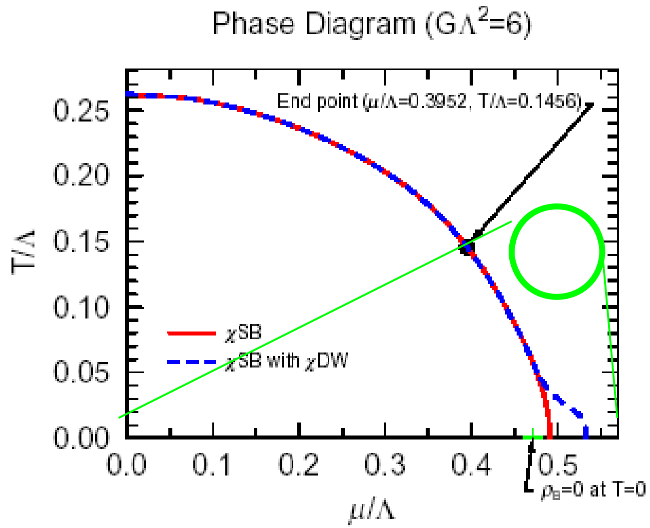


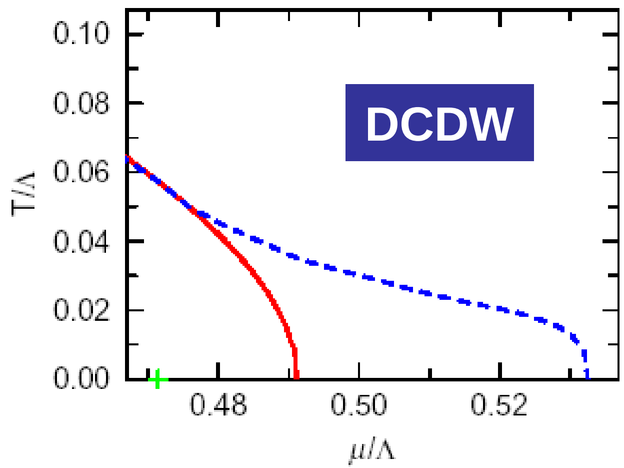
Magnetic moment:

$$\langle\sigma_{12}\rangle = M\cos(\mathbf{q}\cdot\mathbf{r})$$

NJL modelを用いた結果II:







weakly first-order
≈ second-order

Strong magnetic field in compact stars

cf. magnetars ($B_{\rm surf} \sim 10^{15} G$)

Relativistic heavy-ion collisions (high T)

cf. chiral magnetic effect ($B \sim m_\pi^2 \sim 10^{17} G(?), \quad 10^{13}{\rm G} \simeq 1{\rm MeV}^2$)

(G. Basar, G.V. Dunne, D.E. Khazeev, PRL 104 (2010) 232301
"*Chiral magnetic spirals*")

Electroweak phase transition in the early universe;.

$$B \sim 10^{24} G(?)$$

Effects of the magnetic field for inhomogeneous phases

DCDW as an inhomogeneous chiral phase

Dirac Hamiltonian in the presence of the magnetic field as well as DCDW

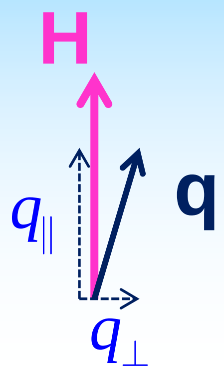
What direction is most favorite ?

$$H_D = \boldsymbol{\alpha}\cdot\mathbf{P} + \gamma^0 m - \Sigma_3 q/2, \leftarrow$$ CPT violating

$$\mathbf{H} = (0,0,H)$$

$$\mathbf{P} = -i\nabla + e\mathbf{A} \quad \text{with the Landau gauge, } \mathbf{A} = (0, Hx, 0)$$

after the Weinberg transformation.



cf quantum Hall effect in graphene (E.V. Gorbar et al., Loe Temp. Phys. **34**(2008) 790)

standard model extension (D. Colladay and V.A. Kostelecky, PRD **58** (1998) 116002.)

Energy spectrum (dimensional reduction):

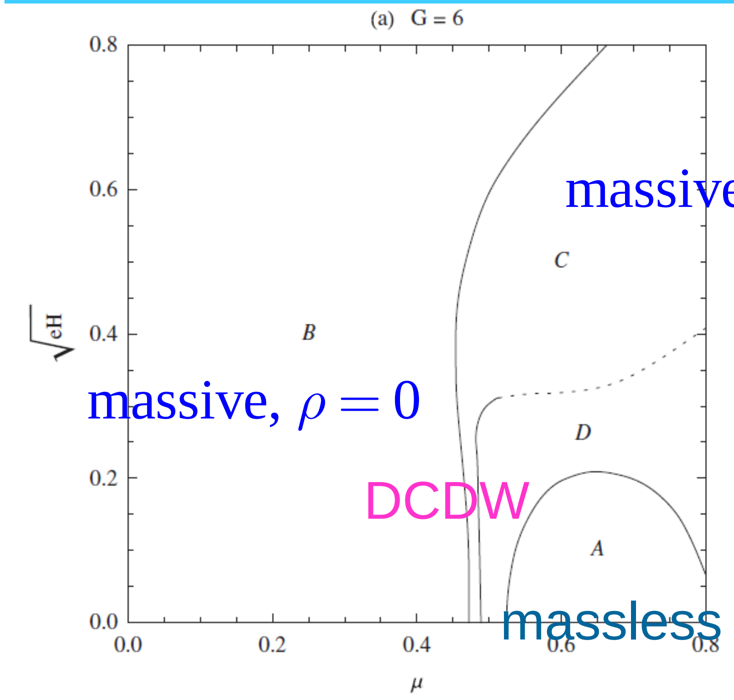
I.E. Frolov et al. (PRD 82(2010) 076002)

$$E_{np\zeta\varepsilon} = \begin{cases} \varepsilon\sqrt{\left(\zeta\sqrt{m^2+p^2}+q/2\right)^2+2eHn}, & n=1,2,\ldots \quad \text{(Landau levels)} \\ \varepsilon\sqrt{m^2+p^2}+q/2, & n=0, \quad \text{(LLL)} \end{cases}$$

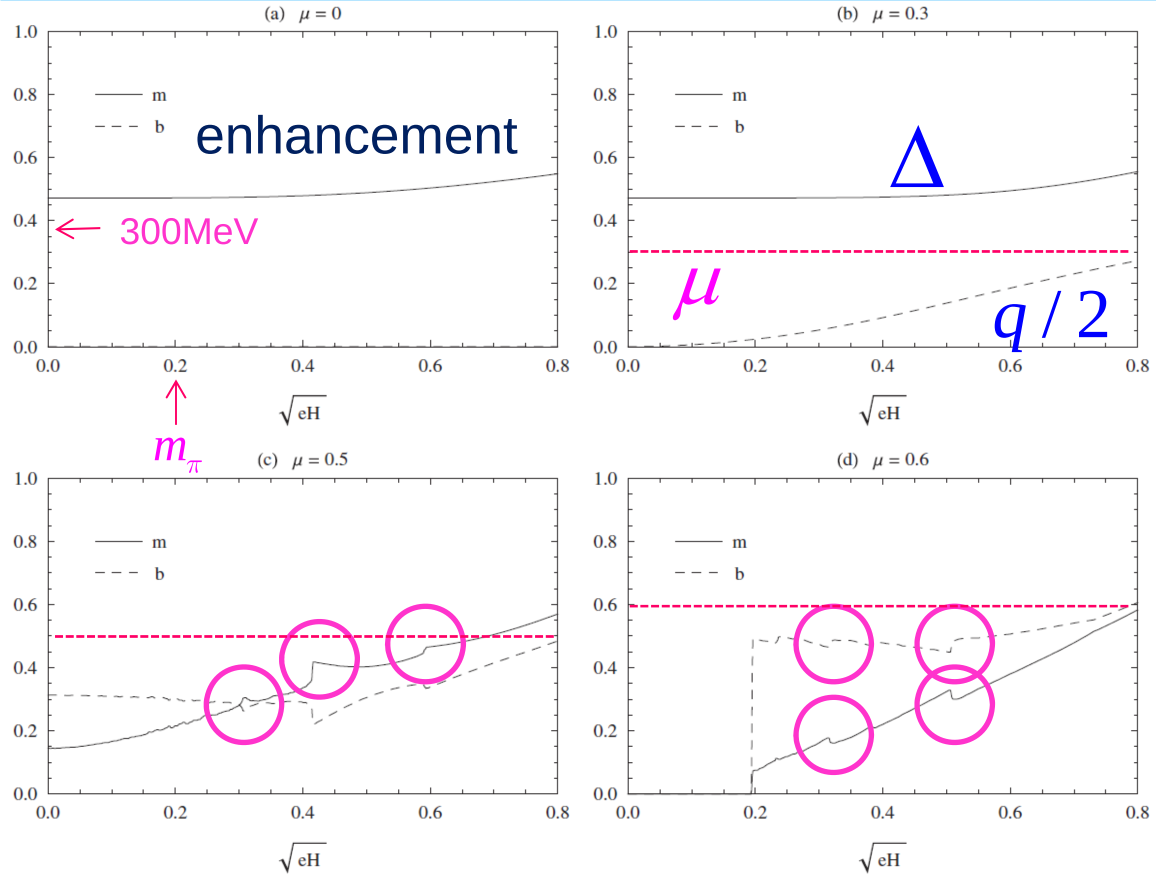
cf. $$E_{\pm}^{\mathrm{DCDW}} = \varepsilon\sqrt{\left(\pm\sqrt{p_z^2+m^2}+q/2\right)^2+p_\perp^2}$$

prolate or oblate deformation
of Fermi sea due to spin-orbit coupling

(scaled by the cut-off)



phase diagram



De Haas-van Alphen effect

(D. Ebert et al., PRD 61 (1999) 025005.)

$$q \to 2\mu \quad \text{as} \quad H \to \infty$$

Dimensional reduction

$B \to$ large
$1+3 \to 1+1$

In the large B limit, only LLL contribute, so that

$1+3 \longrightarrow$ 1+1(complete)

Energy spectrum

$$E_{p\varepsilon} = \varepsilon\sqrt{m^2 + p^2} + q/2, \quad \varepsilon = \pm 1$$

cf

$$E_{p\varepsilon} = \varepsilon\sqrt{m^2 + |eB|(2n+\alpha) + p^2}, \quad n = 0,1,2,..., \quad \alpha = \pm 1$$

(the Landau levels)

This is exactly the same with NJL_2 model in 1+1 dimension.

↓

Chiral spiral is the most favorite solution in 1+1 dim.

(G. Basar, G.V. Dunne, M. Thies, PRD 79(2009)105012)

cf. D. Nickel, PRL 103(2009) 072301; PRD 80(2009) 074025.

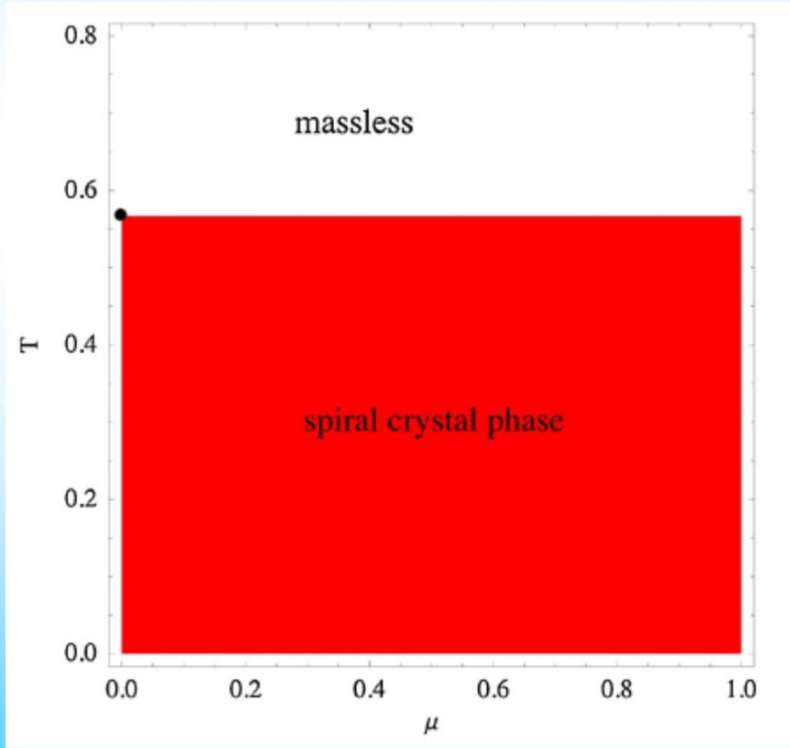
RKC is the lowest in 1+3 dim.?

Some results:

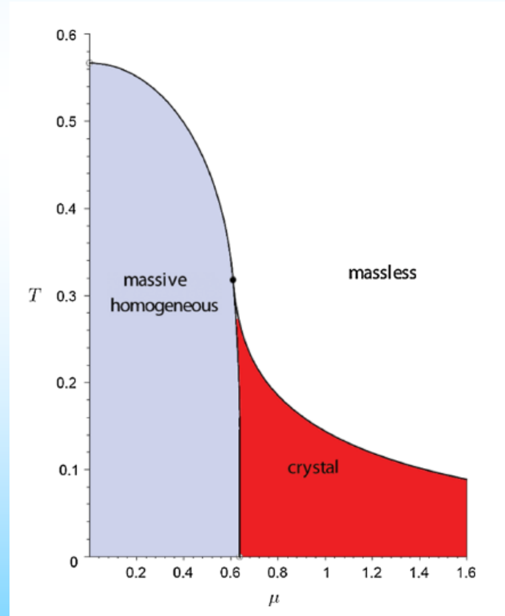
(i) critical point is the Lifshitz point for kink crystal

(ii) q=μ for spiral crystal

(iii) Spiral crystal is the most favorite configuration



NJL2

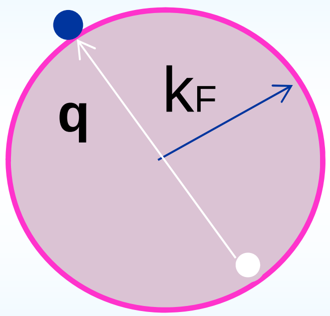


GN model

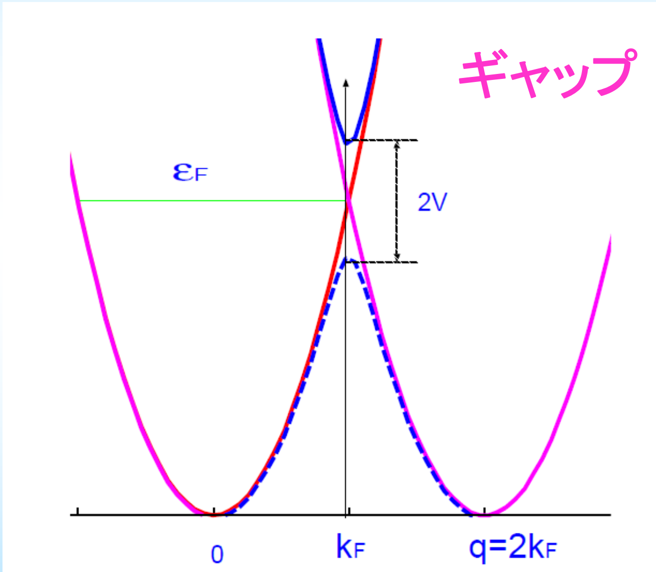
平均場の範囲での非一様相の起源はネスティング　(cf. 量子相転移)

Nesting　(Overhauser, Peiels) is the key mechanism for generating SDW

Level crossing of the energy spectrum near the Fermi surface



粒子ー空孔不安定性



平均場

$V \cos \mathbf{qx}$

$e^{i\mathbf{kx}} \to e^{i\mathbf{kx}} + e^{i(\mathbf{k}\pm\mathbf{q})\mathbf{x}}$

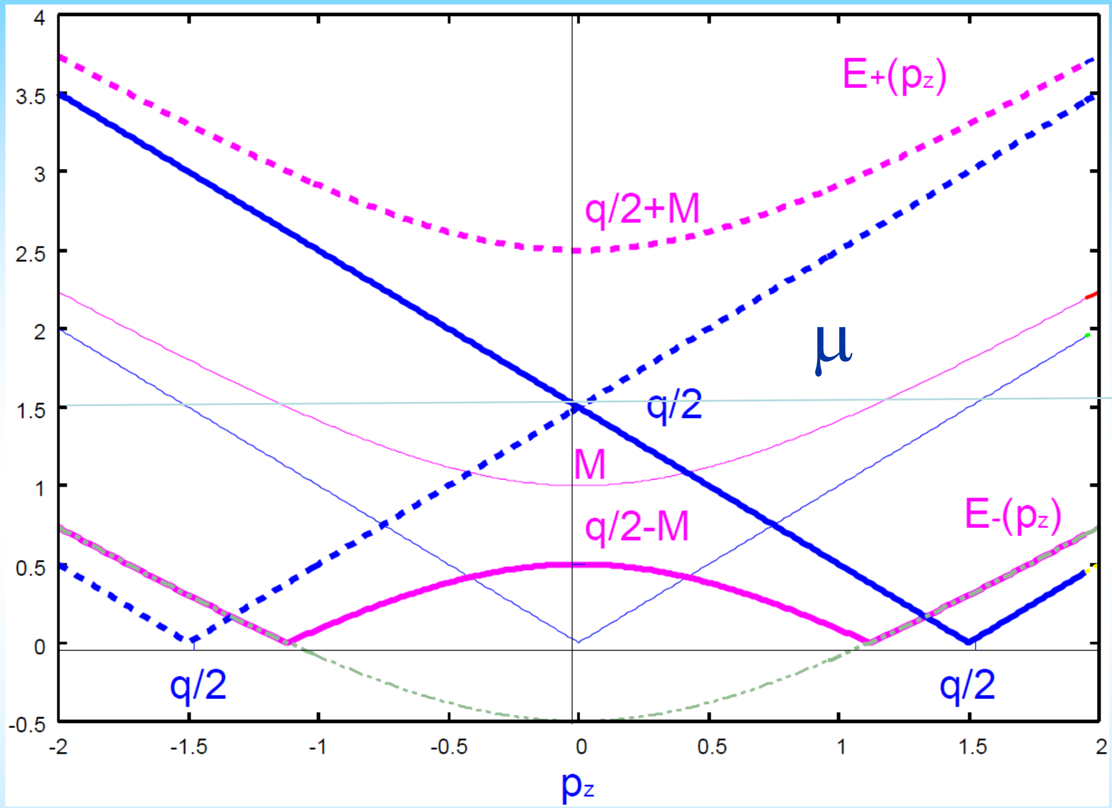
$|\mathbf{q}| = 2k_F$

Model indep.

→　SDW,CDW　→　Floerich superconductivity

A.W. Overhauser, PRL 4(1960) 462.
R.E. Peierls, *Quntum Theory of Solids* (1955)

Nesting mechanism

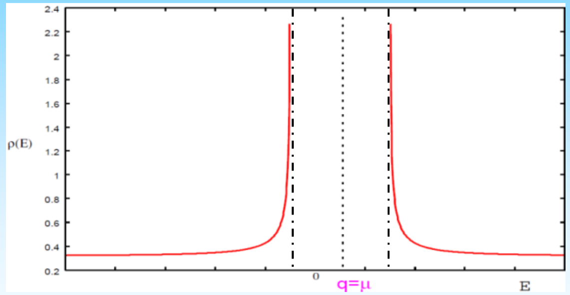


$$M < q/2:$$

$$E_{1,2} = \left| \sqrt{p_z^2 + M^2} \pm |\mathbf{q}|/2 \right|$$

NJL2

Density of state

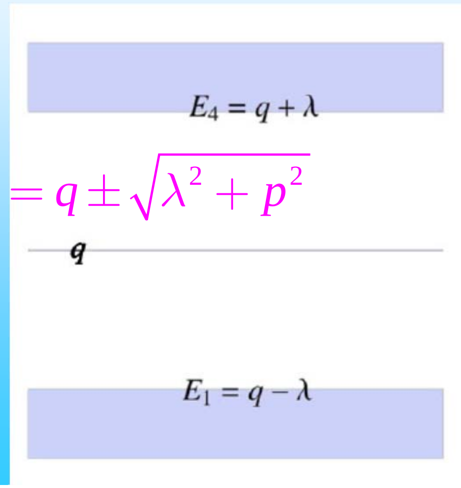


$q-\lambda$ $q+\lambda$

$$\rho(E) = \frac{1}{\pi} \frac{|E-q|}{\sqrt{(E-q)^2 - \lambda^2}}$$

(cf BCS)

Energy spectrum



$$E_{1,2} = q \pm \sqrt{\lambda^2 + p^2}$$

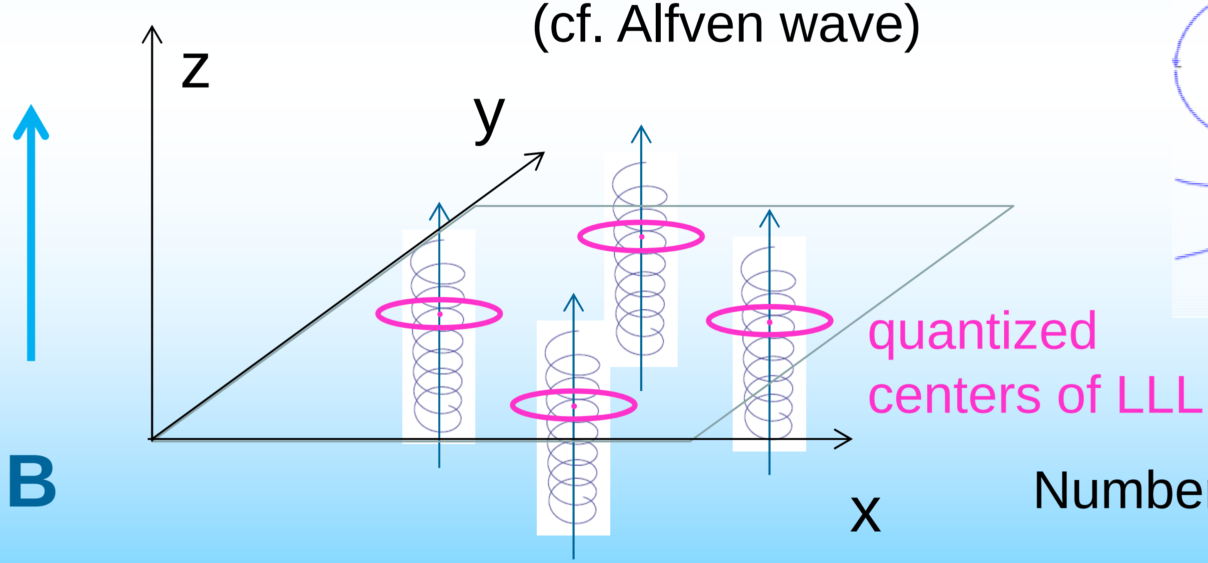
cf. 1+1 QCD in the large-N limit (B. Bringoltz, PRD 79 (2009) 125006)

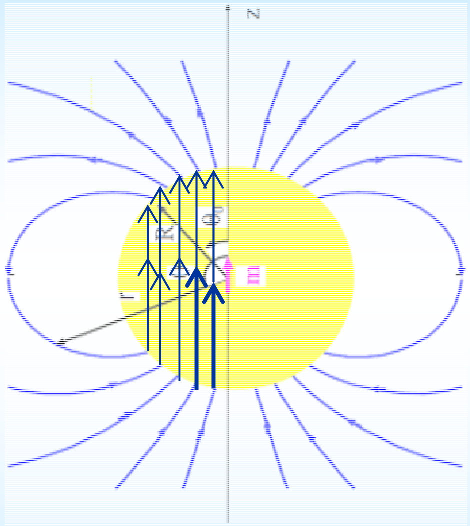
We have a model-independent result in the large B limit.

DCDW is the most favorite state

Coupling of B and DCDW

(cf. Alfven wave)





$$7.1\times10^{17}\,G\left(\frac{\rho}{\rho_0}\right)^{2/3}\leq|eB|$$

Number of states

$$dN_0=N_C S_{12}\frac{|eB|}{2\pi}L_3\frac{dk_3}{2\pi}$$

Magnetic catalysis

V.P. Gusynin, V.A. Miransky, I.A. Shovkovy, PRL 73 (1994) 3499;
NPB 462 (1996) 249.

$$E_{p\varepsilon} = \varepsilon\sqrt{m^2 + |eB|(2n+\alpha) + p^2}, \quad n = 0,1,2,..., \quad \alpha = \pm 1$$

(the Landau levels)

- “一次元系の集合(n<∞)体”
- IR singularity

↓

SSB in the presence of magnetic field,
irrespective of the strength of the interaction
like Cooper instability

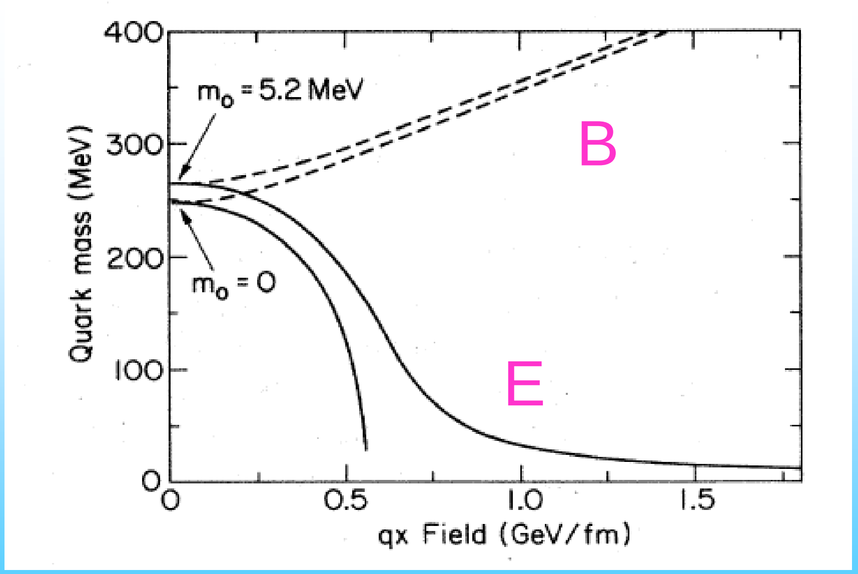
LLL contribution is essential

Critical coupling in 1+3 NJL

$$G_c\Lambda^2 = \frac{2\pi^2}{N_C N_F}$$

G<Gc: "weak" coupling (理論的面白さ)
G>Gc: "strong" coupling （現実的状況）

Enhancement of SSB



S.P. Klevansky and R.H. Lemmer, PRD 39(1989) 3478.

See also
H. Suganuma, T.T., Ann. Phys.208 (1991) 470.

Weak coupling

Gap equation (LLL):

$$m_{\rm dyn} = GN_C \frac{|eB|}{2\pi^2}\int_0^{\Lambda} \frac{m_{\rm dyn}}{\sqrt{k^2+m_{\rm dyn}^2}}dk \qquad \text{(IR sigularity)}$$

$$= N_C G \frac{m_{\rm dyn}|eB|}{2\pi^2} \ln\frac{\Lambda^2}{m_{\rm dyn}^2}$$

⟶ Non-trivial sol.

$$m_{\rm dyn}^2 \simeq \Lambda^2 \exp\left(-1/2\nu_0 G\right), \quad \nu_0 \equiv V^{-1}\frac{dN_0}{dE}\bigg|_{E=0} = \frac{|eB|N_C}{4\pi^2}$$

(cf BCS)

$$\Delta^2 \simeq 4\omega_c^2 \exp\left(-1/N(0)g\right)$$

Remarks:

In the strong B limit, chiral spiral (DCDW) is the most favorable phase for $\mu \neq 0$
(almost model independent?)

Clarify the situation where the LLL is dominant, where DCDW is realized:

thermal effect is also elucidated.

Compare DCDW with RKC (or more general) solution.

Consider some implication of DCDW on astrophysical phenomena, especially *magnetic* phenomena;

thermal effect and elasticity are also important

Elucidate the relation with axial-anomaly

D.T. Son, M.A. Stephanov, PRD 77(2008)014021.

Summary and concluding remarks:

We have discussed two subjects about magnetic properties of quark matter, spontaneous magnetization and inhomogeneous chiral phase.

More studies are needed about tensor mean-field.

Magnetic catalysis by Milansky et al should be reexamined in the context of DCDW.

DCDW should be examined in the light of pion condensation.

Phenomenological implications on compact stars: strand structure, new cooling mechanism.

(iii) From ferromagnetic phase to SDW phase (Magnetism)

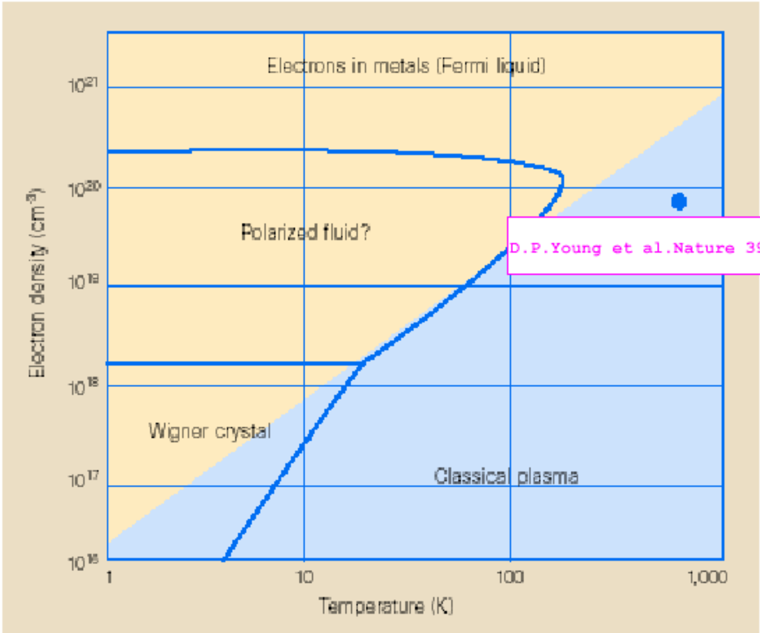


Figure 1 Phase diagram of the electron gas. The two colours divide the classical (blue) from the quantum (yellow) regimes. The phase transition boundaries are estimates from ref. 6. The dot is the transition temperature measured by Young *et al.*[1].

 NATURE | VOL 397 | 4 FEBRUARY 1999 | www.nature.com

FM at low density (Bloch, 1929) $\langle \mathbf{M} \rangle = \text{const.}$

↓

SDW at higher densities (Overhauser, 1962) $\langle \mathbf{M} \rangle = \mathbf{M}_0 \sin qz$

Globally AFM but locally FM

(iv) Similarity to LOFF state in superconductor or vortex in superfluid

Δ (uniform) ↔ 0

↑

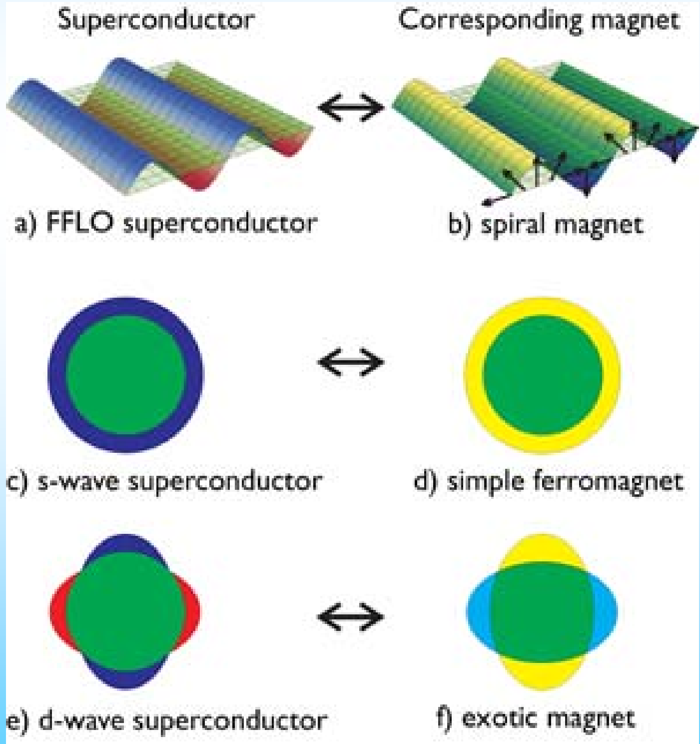
Δ(r) (non-uniform)

(superconductivity)

pp   ph

unventional

unconventional



(A.J. Schofield, *Physics* 2(2009) 93.)

← 相対論的効果?

(P.Monthoux, D.Pines, G.G. Lonzarich, *Nature* 450(2007) 1177.)

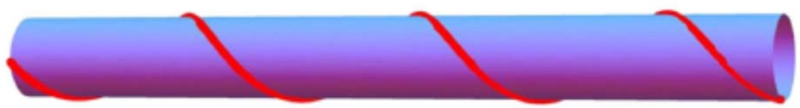
NJL2 model

G. Basar et al., PRD 79 (2009) 105012)

FIG. 1 (color online). The twisted kink crystal condensate of (2.20), shown as the solid (red) curve. The (blue) skeleton surface is shown just to illustrate the periodic amplitude modulation and phase winding.

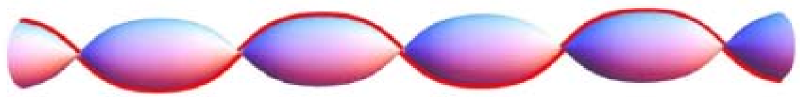
Twisted kink crystal condensate

G.Basar and G.V.Dunne, PRL 100(2008) 2004004; PRD 78(2008) 065022.



$$\Delta = \lambda e^{2iqx}$$

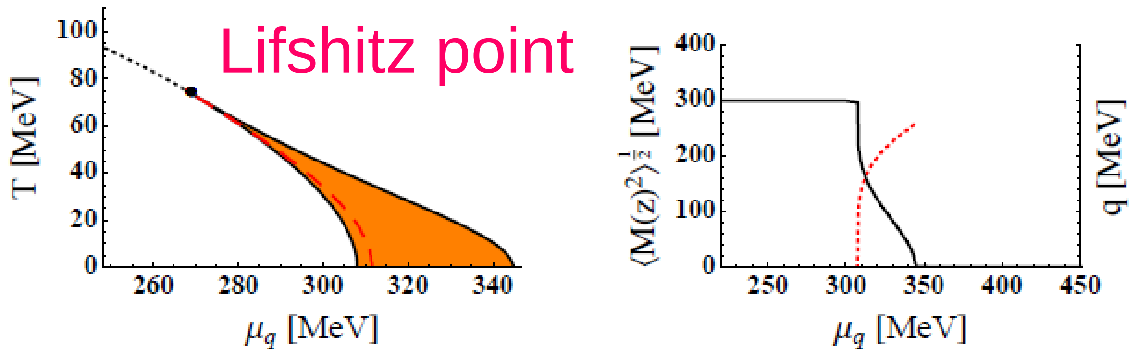
Spiral condensate



$$\Delta(x) = \lambda\left(\frac{2\sqrt{\nu}}{1+\sqrt{\nu}}\right)\mathrm{sn}\left(\frac{2\lambda x}{1+\sqrt{\nu}};\nu\right)$$

$$\to 2\lambda\sqrt{\nu}\sin(2\lambda x) \quad \text{as } \nu \to 0$$

Reak kink crystal



$q \equiv \pi\lambda / 4K(\nu)$

Fig. 4. Left: Phase diagram in the chiral limit. The orange shaded region marks the inhomogeneous regime. Right: Wave vector $|\vec{q}|$ (dashed line) and average amplitude $\sqrt{\langle M^2\rangle}$ (solid line) at $T = 0$ as functions of the quark chemical potential μ_q. Adapted from Ref. [19].

reminiscent of nesting?

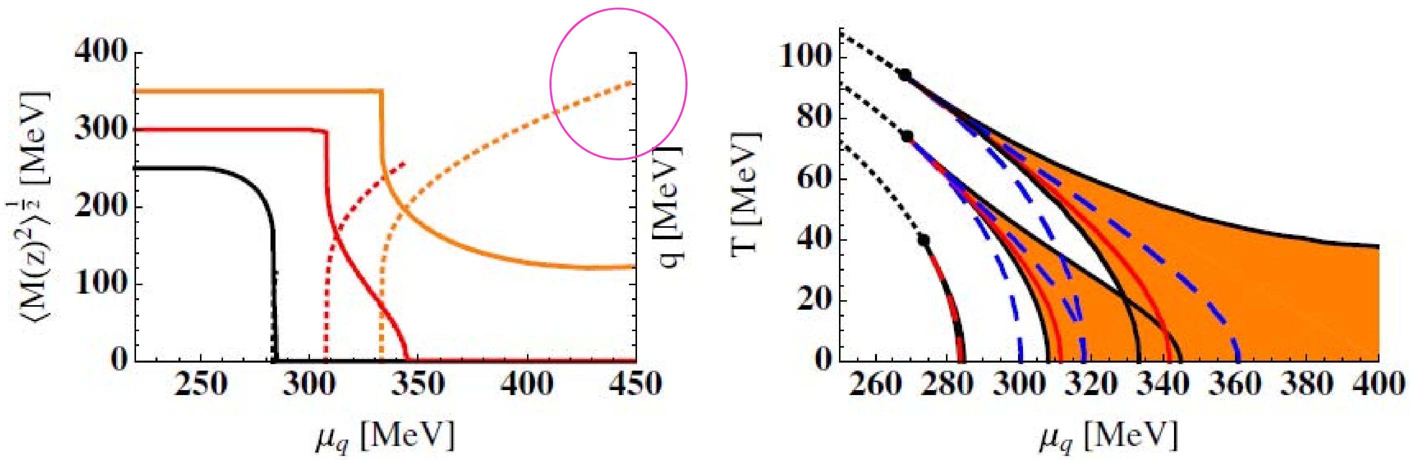
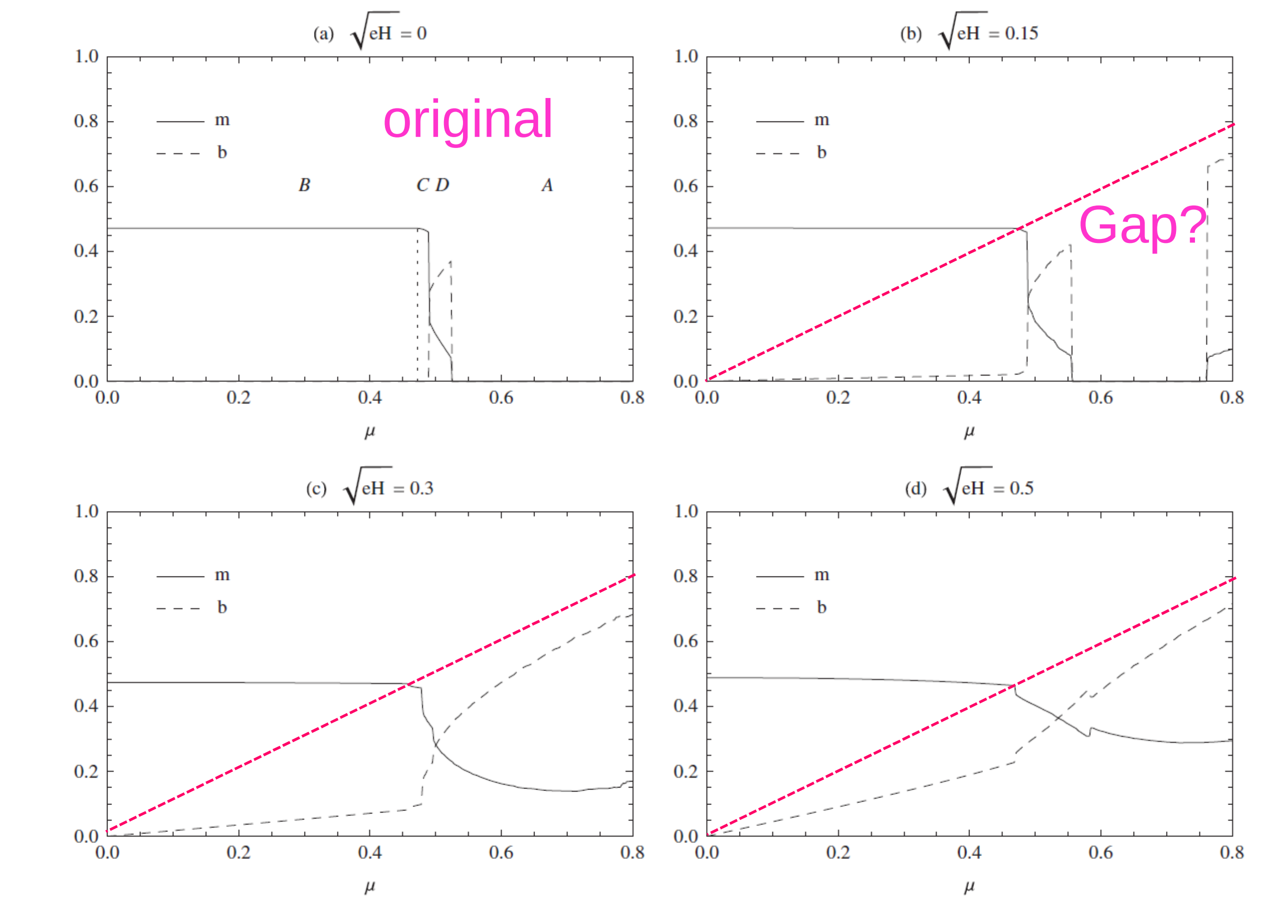


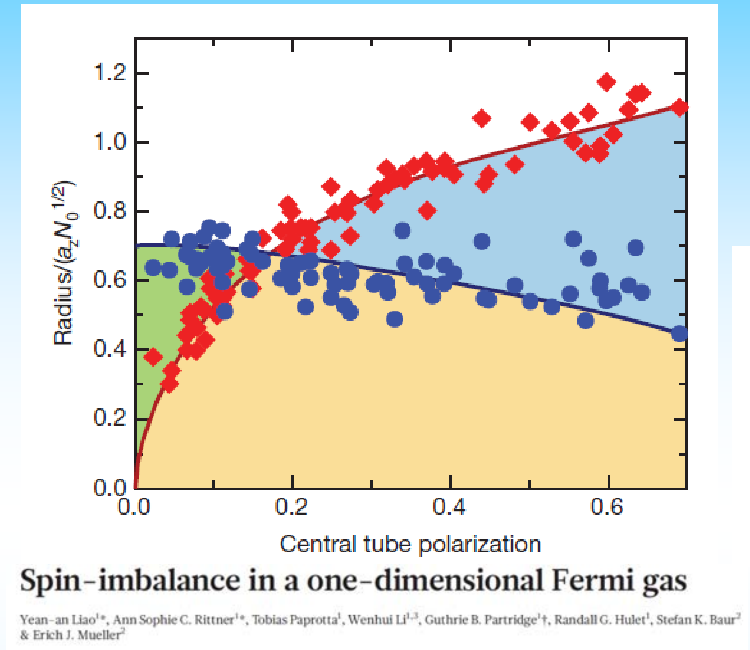
FIG. 2 (color online). Left: Wave vector q (dashed lines) and average of constituent mass $\sqrt{\langle M(z)^2\rangle}$ (solid lines) at vanishing temperatures as function of quark chemical potential μ_q for $M_q = 250$ MeV (black lines), $M_q = 300$ MeV (red, dark grey lines lines) and $M_q = 350$ MeV (orange, light grey lines lines). Right: Same plot as on the right of Fig. 1, now including results for $M_q = 350$ MeV (upper branch) and $M_q = 250$ MeV (lower branch).



Enhancement of DCDW

finite temperature?

FFLO state in superconductor or vortex in superfluid



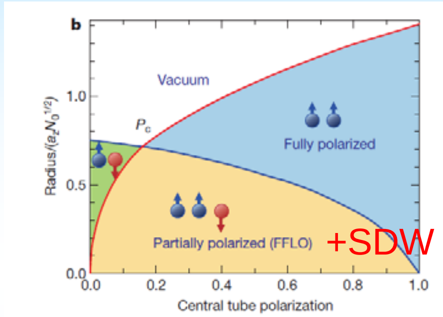
(Nature **467**(2010) 567.)

(superconductivity)

Δ (uniform) $\leftrightarrow$ 0

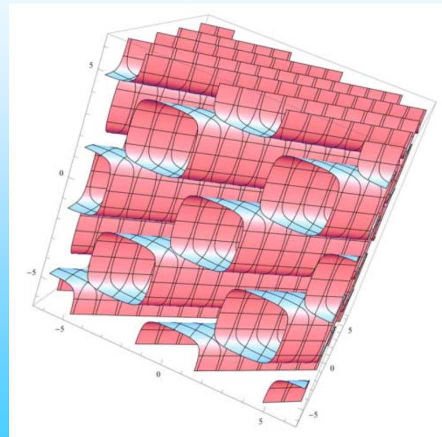
$\uparrow$

Δ(r) (non-uniform)



QLRO Nesting?

Crystalline structures of CSC



M.G. Alford, J.A. Bowers, K. Rajagopal,
PRD 63 (2001) 074016.
J.A. Bowers, K. Rajagopal,
PRD 66 (2002) 065002.

Imbalanced population

Consider the energy scale:

$$H_{int} = \mu_i B, \qquad \mu_i = e_i \hbar / 2m_i c$$

For $B = 10^{15}$**G**,

	e^-	p	q
m_i[MeV]	0.5	10^3	$1 - 100$
E_{int}[MeV]	$5 - 6$	2.5×10^{-3}	$2.5 \times 10^{-2} - 2.5$
E_{typ}	KeV	MeV	MeV

$\longrightarrow$

$E_{typ} \ll E_{int}$ (electrons), $E_{typ} > E_{int}$ (nucleons and quarks)

Ferromagnetism or spin polarization

→ **Nuclear matter calculations have shown negative results.**

For recent references,
I.Bombaci et al, PLB 632(2006)638
G.H. Bordbar and M. Bigdeli, PRC76 (2007)035803

→ Spontaneous magnetization of quark matter

Two major fields in the condensed matter physics

magnetism and superconductivity

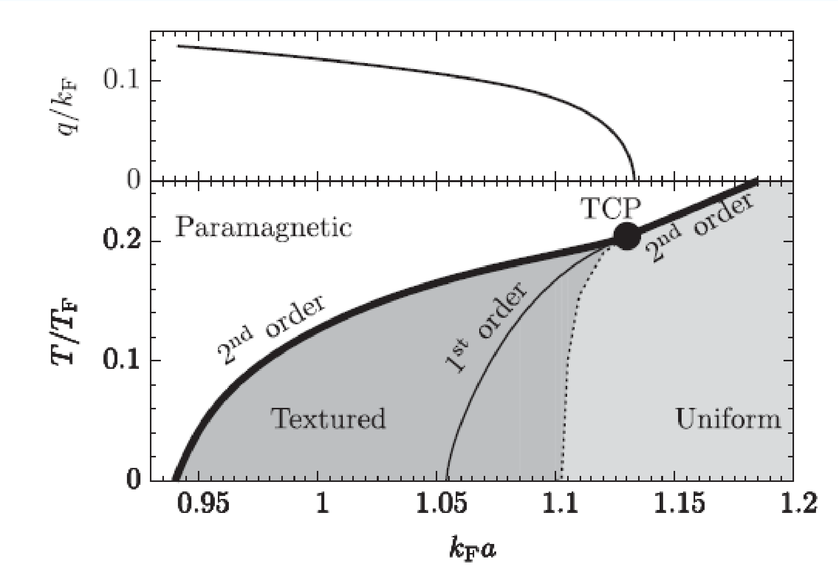
From ferromagnetic phase to SDW phase (Magnetism)

FM at low density (Bloch, 1929) $\langle \mathbf{M} \rangle = \text{const.}$

↓

SDW at higher densities (Overhauser, 1962) $\langle \mathbf{M} \rangle = \mathbf{M}_0 \sin qz$

Globally AFM but locally FM



Nesting mechanism

(G.J. Conduit et al.,
PRL 103(2009) 207201)