Asymmetric Neutrino Emission from Strongly Magnetized Neutron Stars

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§ 1. Introduction

High Density Matter Study ⇒ Exotic Phases inside Neutron Stars

Strange Matter, Ferromagnetism, Meson Condensation, Quark matter

Observable Information ・・・・Neutrino Emissions

S.Reddy, et al., PRD58 #013009 (1998) Influence from Hyperons Λ, ∑

Magnetar 10^{15}G in surface $10^{17\text{-}19}$G inside (?) → Large Asymmetry of ν ?

P. Arras and D. Lai, PRD60, #043001 (1999) , S. Ando, P.R.D68 #063002 (2003)

(Non-Relativistic)

Our Works ⇒ Neutrino Reactions on High Density Matter wih Strong Mag. Fields in the Relativistic Mean-Field (RMF) Approach

TM et al., PRD83, 081303(R) (2011)

Application to Phenomena related with ν－Asymmetric Emission

Pulsar Kick in Poloidal Magnetic Field

Spin Deceleration in Toroidal Magnetic Field

§ 2. Formulation

Magnetic Field : $\vec{B} = B\hat{z}.$

$$\text{Lagrangian}: \mathcal{L} = \mathcal{L}_{RMF} + \mathcal{L}_{lep.} + \mathcal{L}_{mag} + \mathcal{L}_{int}$$

Baryon **Lepton** B & L – Mag.

1. **Proto-Nuetron-Star (PNS) Matter without Mag. Field**
2. **Baryon Wave Function under Mag. Field in Perturbative Way**
3. **Cross-Sections for ν reactions**

Weak Interaction

$$\mathcal{L}_{int} = G_F \{\bar{\psi}_{l'}\gamma_\mu(1-\gamma_5)\psi_l\}\{\bar{\psi}_{B'}\gamma^\mu(c_V - c_A\gamma_5)\psi_B\}$$

$\nu_e + B \longrightarrow \nu_e + B$ **: scattering**

$\nu_e + B \longrightarrow e^- + B'$ **: absorption**

S.Reddy, M.Prakash and J.M. Lattimer, P.R.D58 #013009 (1998)

§ 2-1 EOS of Proto Neutron-Star-Matter in RMF $N, \Lambda, \sigma, \omega, \rho$

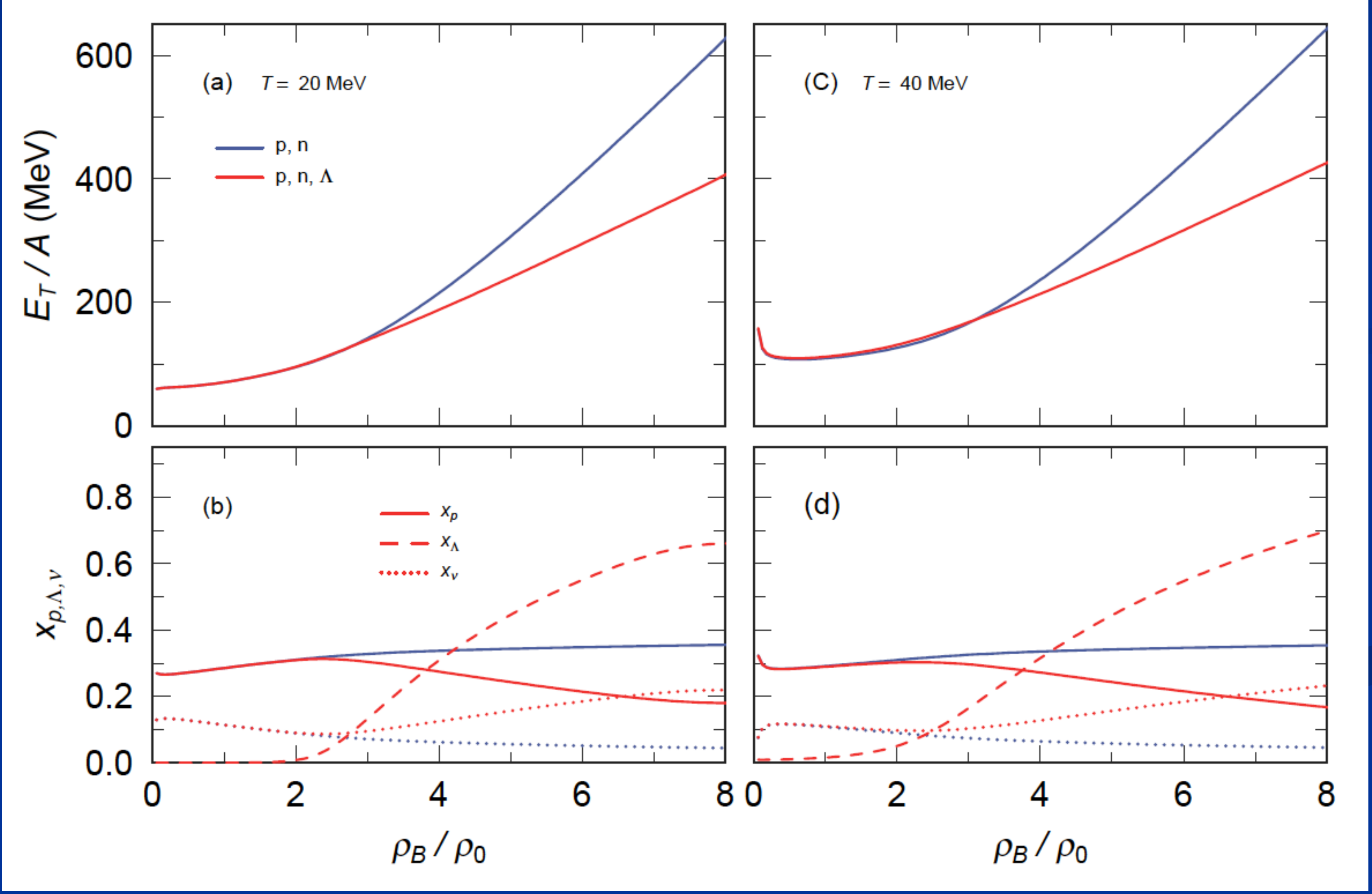
PM1-L1

$BE = 16\,\mathrm{MeV},\ M_N^* / M_N = 0.7,\ K = 200\,\mathrm{MeV}$ at $\rho_0 = 0.17\,\mathrm{fm}^{-3}$

$g_{\sigma,\omega}^{\Lambda} = \frac{2}{3} g_{\sigma,\omega}$ **SU(3)**

T.M, et al. PTP. 102, p809 (1999)

Charge Neutral ($\rho_p = \rho_e$) & Lepton Fraction : Y_L = 0.4



§ 2-2 Dirac Equation under Magnetic Fields

$\mu_N B \ll \varepsilon_N$ **(Chem. Pot)** $\rightarrow$ ***B* can be treated perturbatively**

$B \sim 10^{17}$ **G** **Landau Level can be ignored**

Lagrangian
$$\mathcal{L}_{mag} \approx -\sum_b \mu_n B \bar{\psi}_b \sigma_z \psi_b$$

Dirac Eq.
$$\hat{K}(p)u(p) = [\gamma_\mu p^\mu - M^* - \mu B \sigma_z]\, u(p) = 0$$

Single Part. Eng.
$$e(\boldsymbol{p}, s) = \left[\left(\sqrt{p_x^2 + M^{*2}} + s\mu B\right)^2 + p_z^2\right]^{\frac{1}{2}} \approx E_p^* + s\mu B \frac{\sqrt{p_T^2 + M^{*2}}}{E_p^*}$$

Dirac Spinor
$$u(\boldsymbol{p}, s)\bar{u}(\boldsymbol{p}, s) \approx \frac{(\not{p} + M)(1 + \gamma_5 \not{a}(p) s)}{4E_p^*}$$

Spin Vector
$$a_z = \frac{E_p}{\sqrt{p_T^2 + M^2}} \qquad \boldsymbol{a}_T = 0 \qquad a_0 = \frac{p_z}{\sqrt{p_T^2 + M^2}}$$

The Cross-Section of Lepton-Baryon Scattering

$$\frac{d^2\sigma}{dk' d\Omega'_k} = \frac{G_F^2}{8\pi^2}\boldsymbol{k}'^2 \sum_{s_i, s_f} \int \frac{d^3p}{(2\pi)^3} \tilde{W}_{BL} (2\pi)\delta(|\boldsymbol{k}| - |\boldsymbol{k}'| + e_i(\boldsymbol{p}) - e_f(\boldsymbol{k} + \boldsymbol{p} - \boldsymbol{k}'))$$
$$\times [1 - f'_l(\boldsymbol{k}')] n_B(e_i) [1 - n_{B'}(e_f)]$$

$$\tilde{W}_{BL} = \mathrm{Tr}\left\{ \frac{(\not{k}' + m_f)(1 + \gamma_5 \not{a}_{l'})}{4|\boldsymbol{k}'|} \gamma^\mu (1 - \gamma_5) \frac{\not{k}'}{2|\boldsymbol{k}|} \gamma^\nu (1 - \gamma_5) \right\}$$
$$\times \mathrm{Tr}\left\{ \frac{(\not{p}' + M_f^*)(1 + \gamma_5 \not{a}_f(p'))}{4E_f^*(\boldsymbol{p}')} \gamma_\mu (c_V - c_A\gamma_5) \frac{(\not{p} + M_i^*)(1 + \gamma_5 \not{a}_i(p))}{4E_i^*(\boldsymbol{p})} \gamma_\nu (c_V - c_A \gamma_5) \right\}$$

$$m_f = 0 \quad \text{when } l_f = \nu \qquad m_f = m_e \quad \text{when } l_f = e$$

Fermi Distribution

$$n(e(\boldsymbol{p}), s) \approx n(\varepsilon(\boldsymbol{p}, s)) + n'(\varepsilon(\boldsymbol{p}, s)) \frac{\sqrt{p_T^2 + M^{*2}}}{E_p^*} \mu B s.$$

Deformed Distribution

Perturbative Treatment ⇒

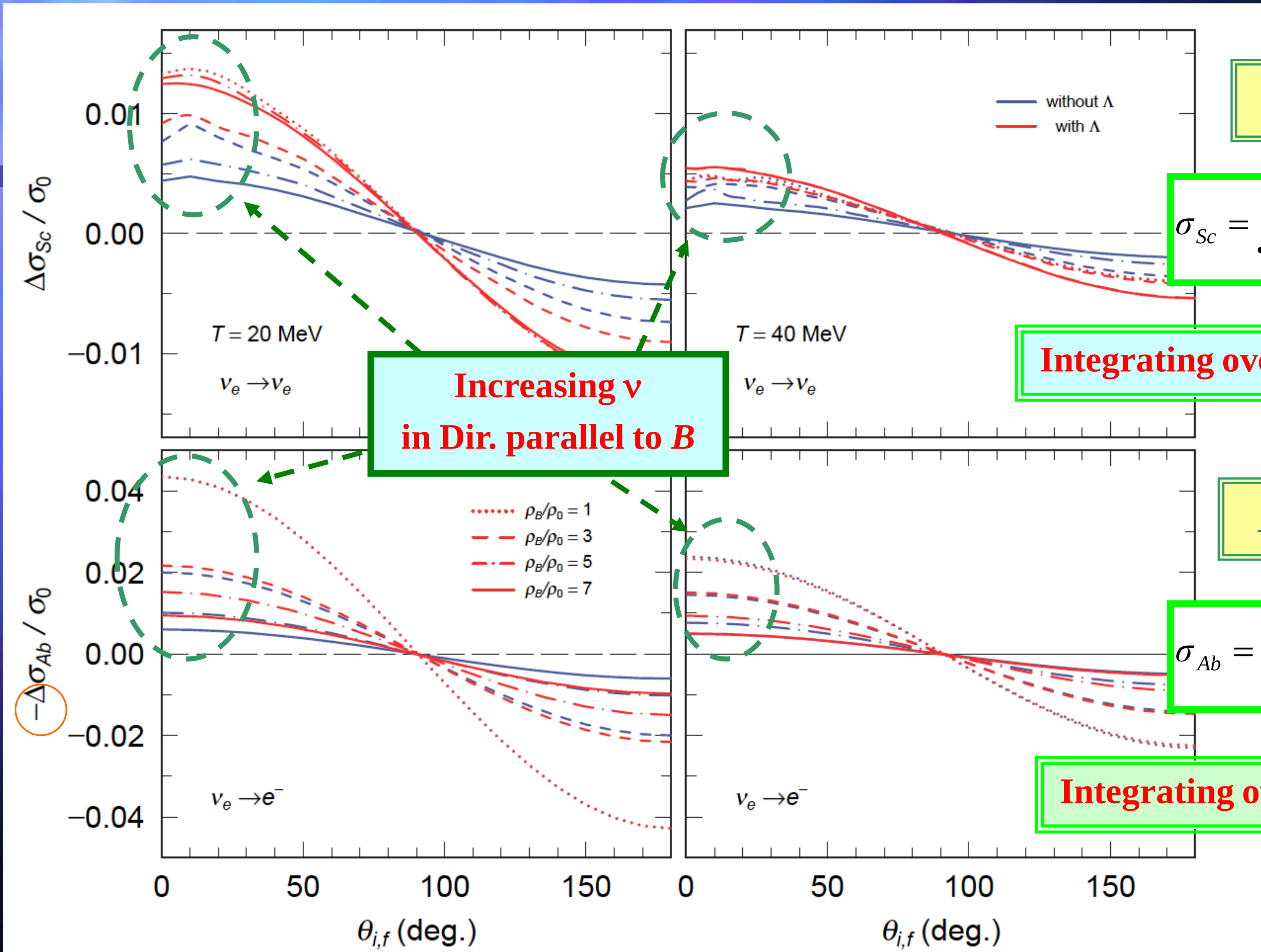
$$\sigma = \sigma_0 + \Delta\sigma \qquad \Delta\sigma \propto B$$

Non-Magnetic Part

Magnetic Part

§ 2-3 Magnetic parts of Cross-Sections

$$\sigma = \sigma_0 + \Delta\sigma \qquad \Delta\sigma \propto B$$



Scat.

$$\sigma_{Sc} = \int d\Omega_i \frac{d\sigma(\nu_e \to \nu_e)}{d\Omega_f}$$

Integrating over the initial angle

Absorp.

$$\sigma_{Ab} = \int d\Omega_f \frac{d\sigma(\nu_e \to e)}{d\Omega_f}$$

Integrating over the final angle

$$k_i = \varepsilon_\nu \text{ (neutrino chem. pot.)}, \quad B = 2\times10^{17}\,\mathrm{G} \text{ and } \theta_i = 0^\circ$$

§ 2-4 Neutrino Transportation

Neutrino Phase Space Distribution Function

$$f(\boldsymbol{p},\boldsymbol{r}) \approx f_0(\boldsymbol{p},\boldsymbol{r}) + \Delta f(\boldsymbol{p},\boldsymbol{r}), \quad f_0(\boldsymbol{p},\boldsymbol{r}) = 1 \big/ \left\{1+\exp\left[\left(\left|\boldsymbol{p}\right| - \varepsilon_\nu\right)/T\right]\right\}$$

Equib. Part

Non-Equib. Part

Neutrino Propagation ⇒ Boltzmann Eq.

$$c\frac{\partial}{\partial \boldsymbol{r}} f_0(\boldsymbol{p},\boldsymbol{r}) \approx c\frac{\partial}{\partial \boldsymbol{r}} f_0(\boldsymbol{p},\boldsymbol{r}) + c\frac{\partial}{\partial \boldsymbol{r}} \Delta f(\boldsymbol{p},\boldsymbol{r}) = I_{coll} \approx -c b_\nu \Delta f(\boldsymbol{p},\boldsymbol{r}), \quad b_\nu = \frac{\sigma_{ab}}{V}$$

Neutrinos Propagate on **Strait Line**

only absorption

Solution ⇒

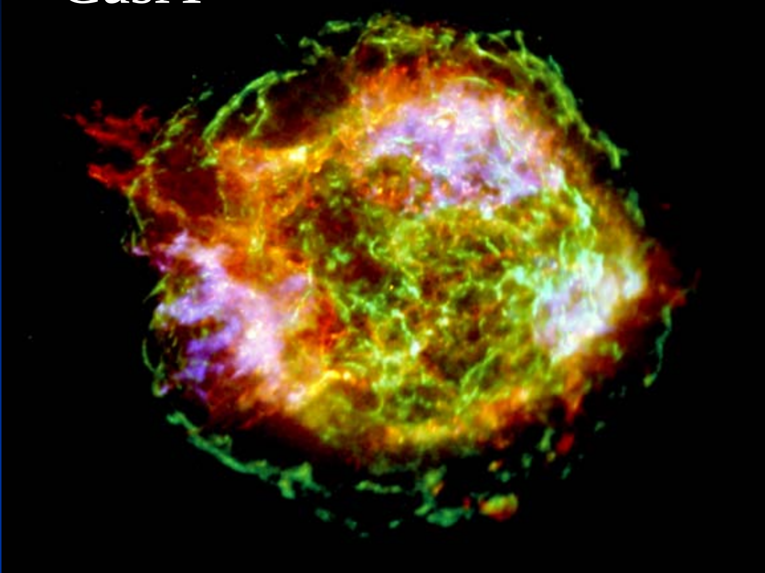
$$\Delta f(\boldsymbol{p},\boldsymbol{r}_T,z) = \int_0^z dx \left[-\frac{\partial}{\partial x} f_0(p,r_T,x)\right] \exp\left[-\int_x^z dy\, b_\nu(y)\right],$$

$$z = \boldsymbol{r}\cdot\hat{p}, \qquad \frac{\partial}{\partial z} f_0(p,r_T,z) = \frac{d\varepsilon_\nu}{dz}\frac{\partial}{\partial \varepsilon_\nu} f_0(p,r_T,z)$$

§ 3 Estimating Pulsar Kick Velocities of Proto-Neutron Star

A.G.Lyne, D.R.Lomier, Nature 369, 127 (94)

CasA



http://chandra.harvard.edu/photo/2004/casa/casa_xray.jpg

Asymmetry of Supernova Explosion

kick and translate Pulsar with

Kick Velocity: Average … 400km/s,

Highest … 1500km/s

Explosion Energy ~ 10^{53} erg

(almost Neutrino Emissions)

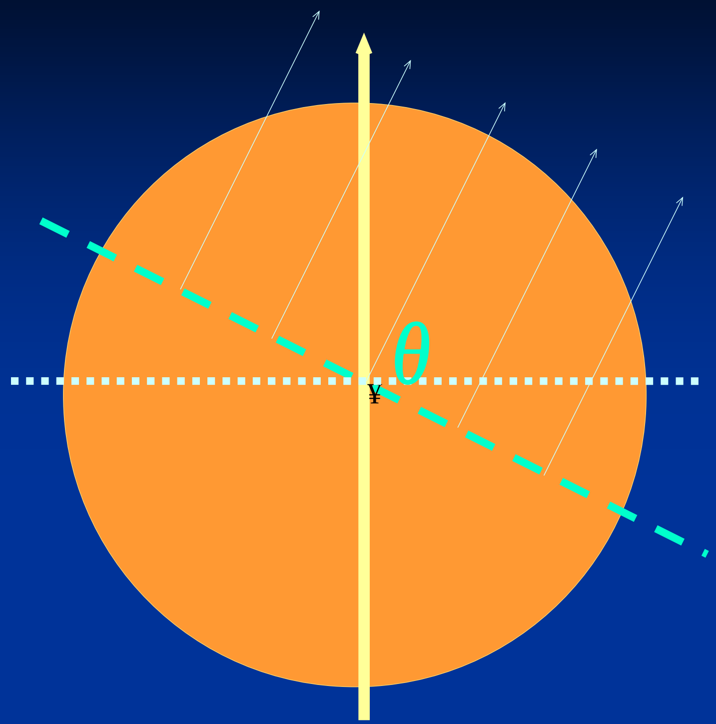
***1% Asymmetry* is sufficient to explain the Pulsar Kick**

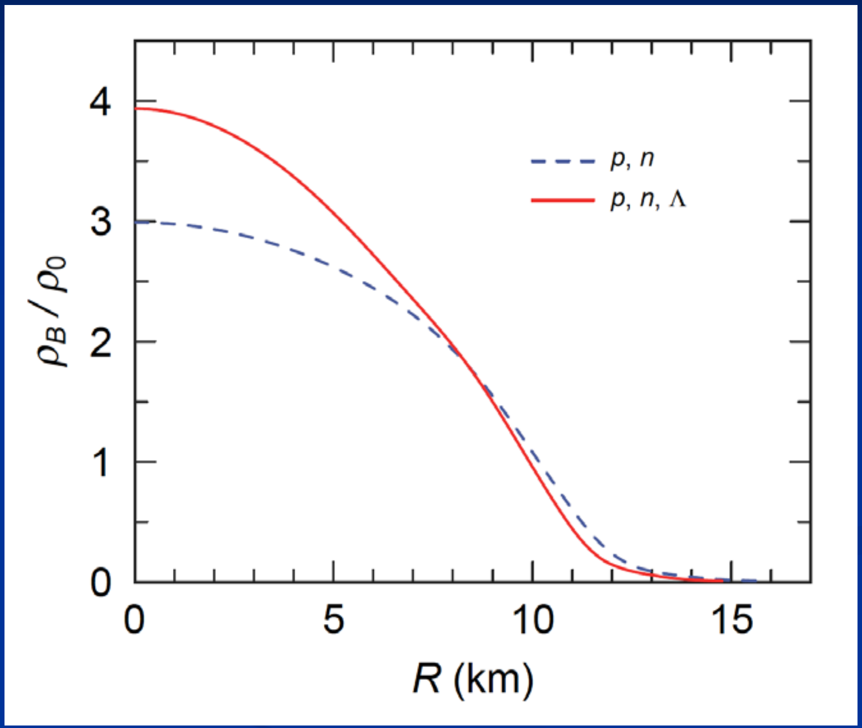
D.Lai & Y.Z.Qian, Astrophys.J. 495 (1998) L103

Estimating Kick Velocity of PNS with T = 20 MeV and $B = 2 \times 10^{17}$G

Poloidal Magnetic Field 2 - 3% Asymmetry in Absorption

Baryon density in PNS





1) Neutrinos propagate on the straight lines

2) Neutrino are produced and absorbed at all positions on the lines

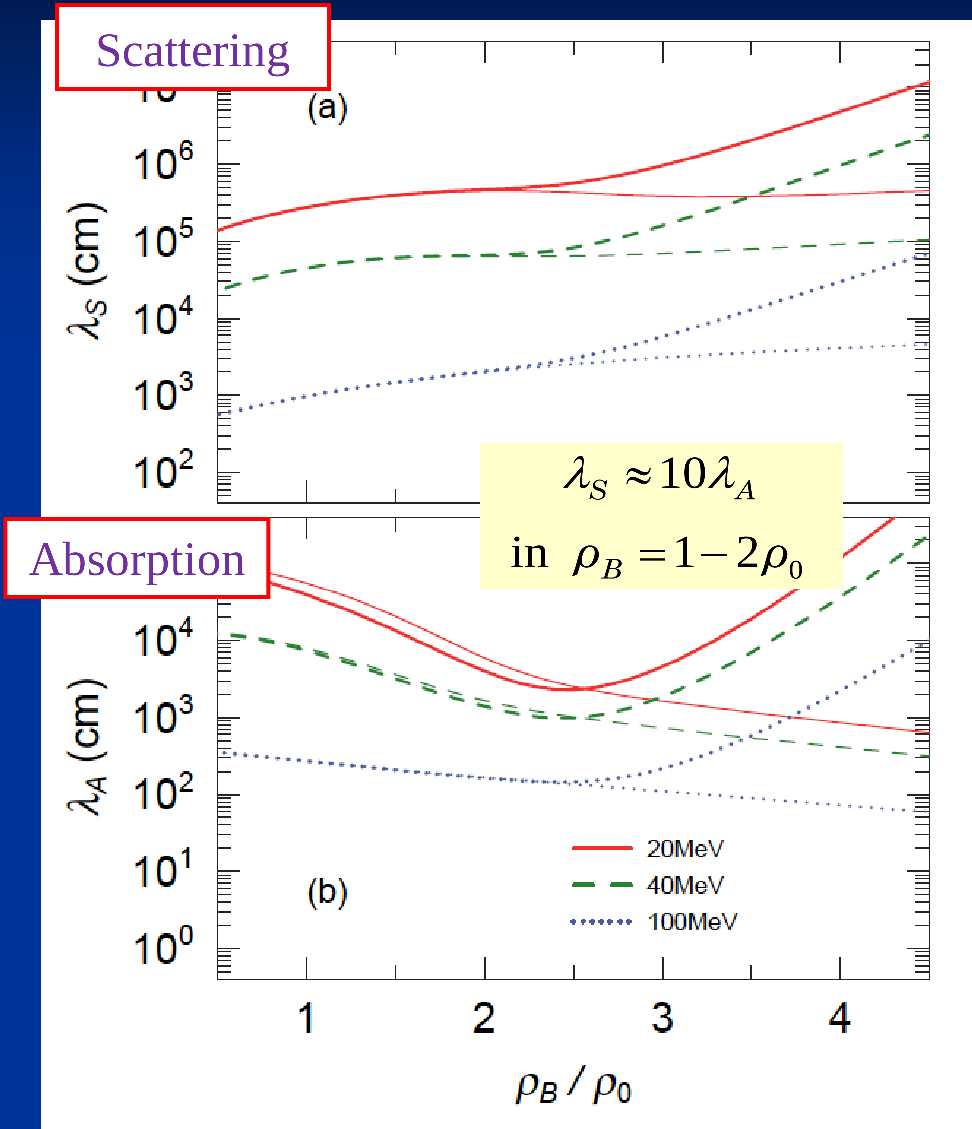
$M = 1.68M_{solar}$, $Y_L = 0.4$, $T = 20$ MeV

Mean-Free path : V/σ_{ab}

Calculating Neutrino Propagation above $\rho_B = \rho_0$ ρ_0

Mean-Free Paths

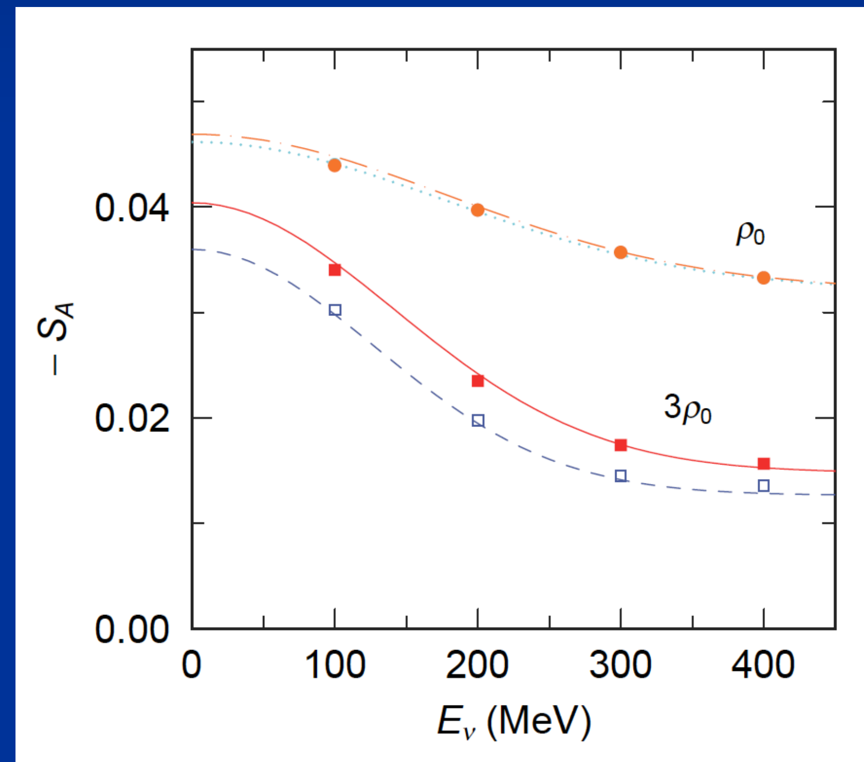
$$\lambda_{S,A} = V / \sigma^0_{S,A}$$



Magnetic Parts

$$\sigma_A = \sigma^0_A \left(1 + S_A \cos\theta_{i,f}\right)$$

S_A: fitting function



Angular Dep. of Emitted Neutrinos in Uniform Poloidal Mag. Field

$$M_{NS} = 1.68 M_{solar} \ [\mathrm{g}],$$
$$E_T = 3\times 10^{53} \ [\mathrm{erg}]$$

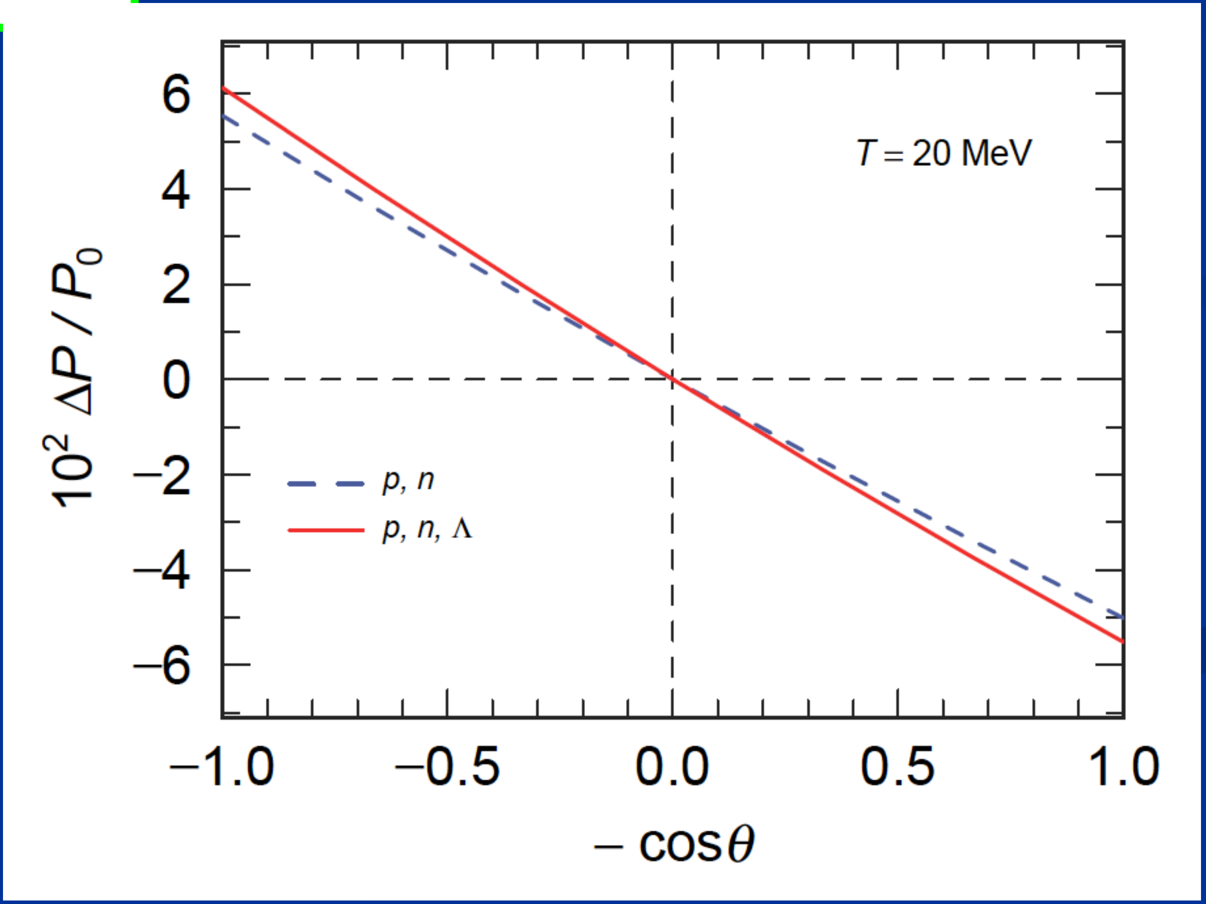
$$\frac{\langle p_z \rangle}{E_T} = \frac{P_1}{3P_0} = 2.0\times 10^{-2} \qquad T = 20 \text{ MeV}$$

$$< p(\boldsymbol{n}) > \approx P_0 + P_1 \cos\theta$$

$$B = 2\times 10^{17} \quad [\mathrm{G}]$$

$$v_{kick} = \frac{\langle p_z \rangle}{M} \approx 600 \, [\mathrm{km/s}]$$

$T = 20$ MeV



Observable
Average … 400 km/s,
Highest … 1500 km/s

§ 4 Angular Deceleration in Toroidal Magnetic Field

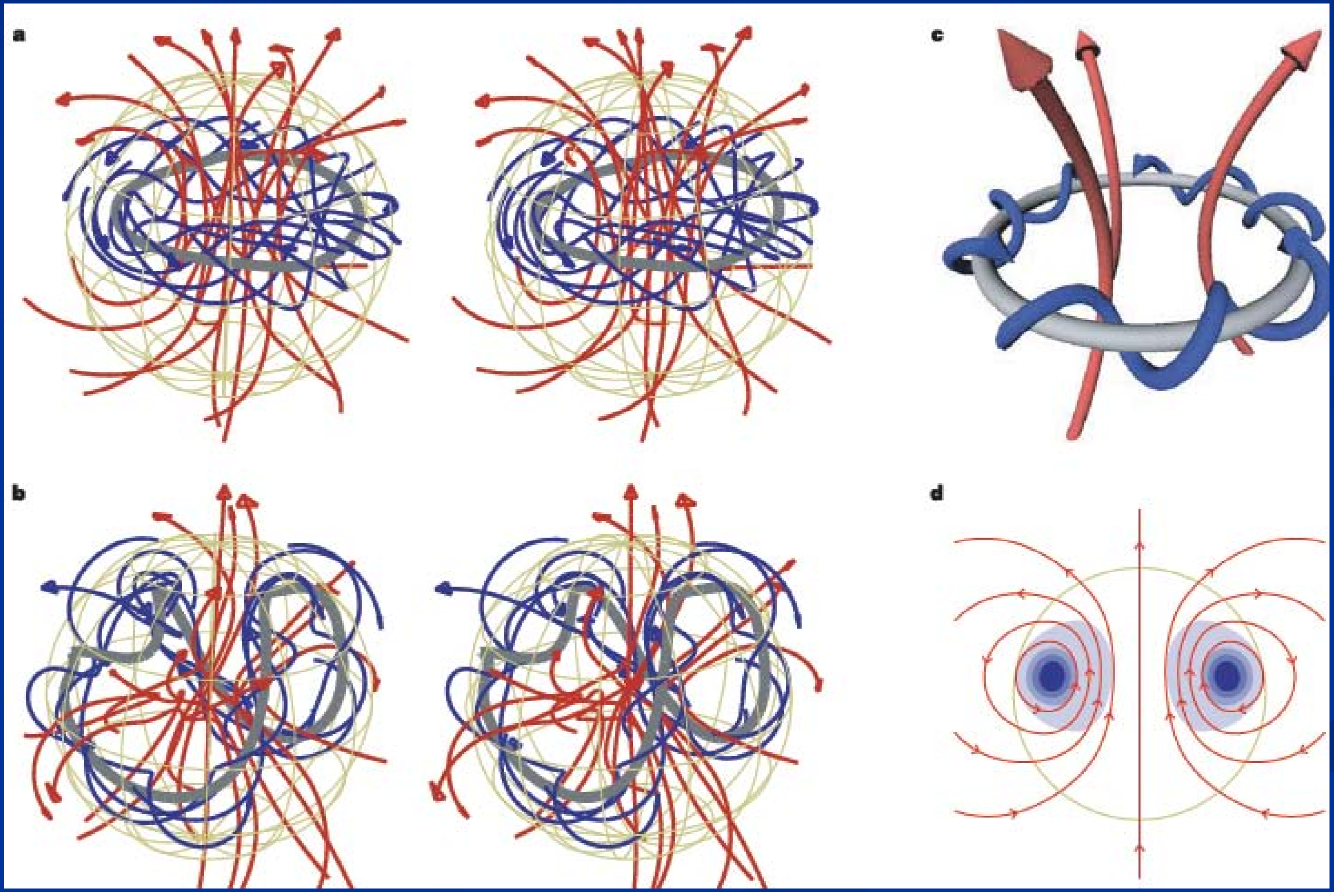
Stability of Magnetic Field in Compact Objects

(*Braithwaite & Spruit 2004*)

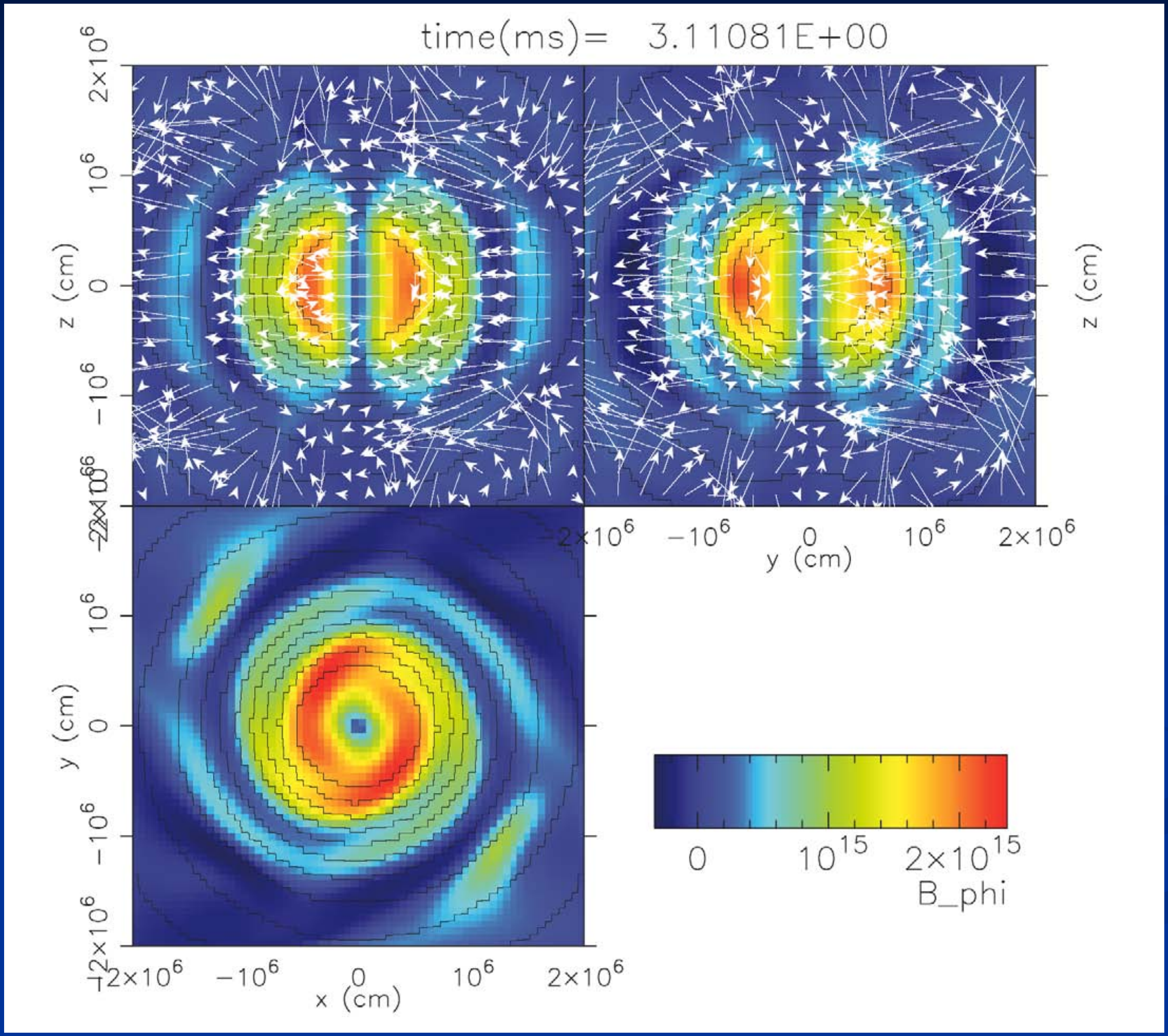
Toroidal Magnetic Field is stable !!

Mag. Field Parallel to Baryonic Flow

Assym. of ν-Emit. must decelerate PNS Spin



No poloidal Magnetic Field at the beginning



Single Toroidal

by T. Kuroda

Toroidal Magnetic Field

$$\boldsymbol{B}(r_T, z) = B_0 G_T(r_T) G_L(z) \hat{e}_\phi$$

$$G_T(r_T) = \frac{16 \exp[-(r_T - R_0)/\Delta r]}{\{1 + \exp[-(r_T - R_0)/\Delta r]\}^2}$$

$$G_L(z) = \frac{\exp[-z/\Delta r]}{\{1 + \exp[-z/\Delta r]\}^2}$$

$$\hat{e}_\phi = (-\sin\varphi, \cos\varphi, 0)$$

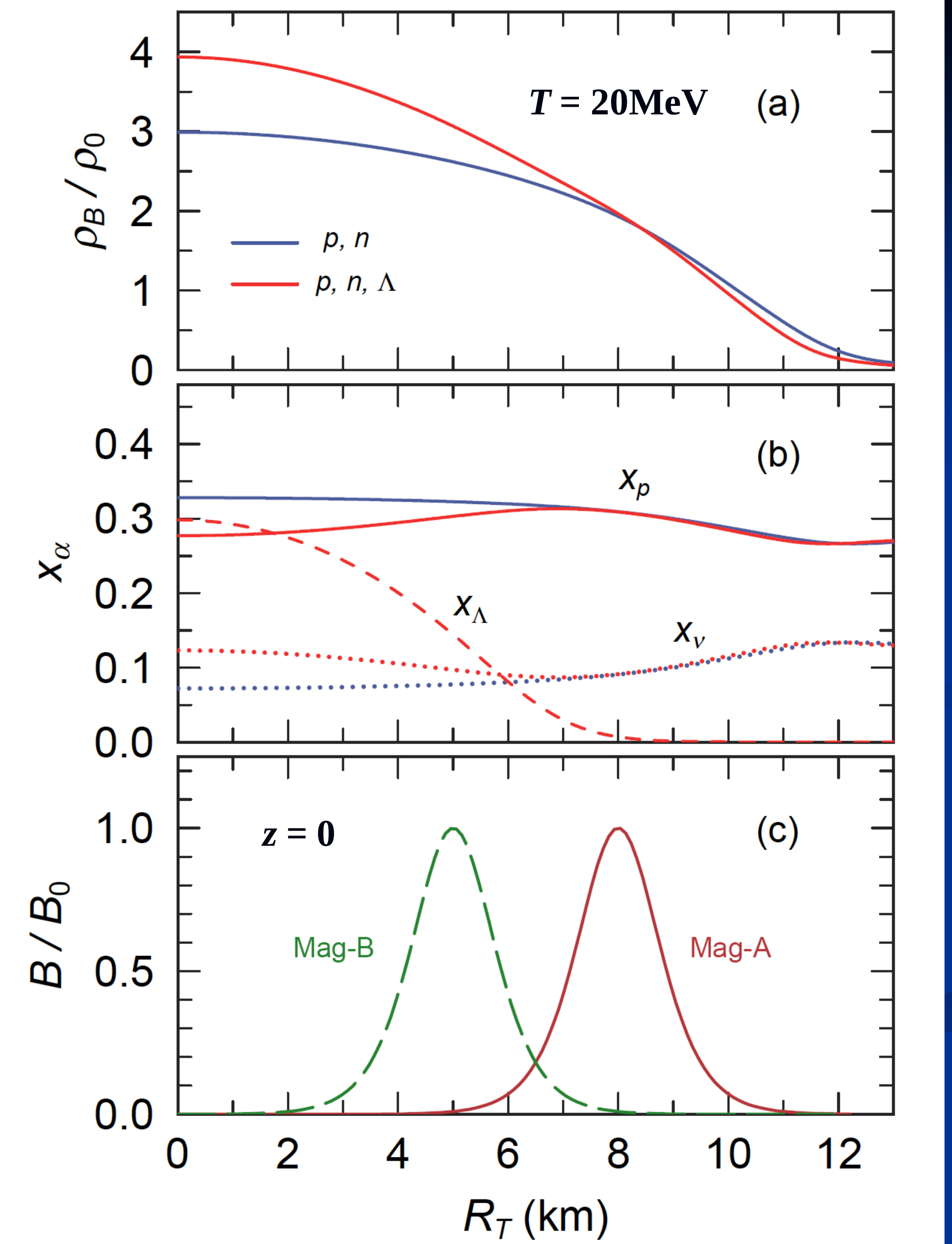
$\Delta r = 0.5$ (km)

$R_0 = 8$ (km)

Mag. - A

$R_0 = 5$ (km)

Mag. - B



Angular Deceleration

Neutrino Luminosity

$(dE_T/dt)_\nu \sim 3\times 10^{52}$ erg/s

$$\frac{dL_z}{dt} = c\int_{S_N} d\boldsymbol{r}\int d\boldsymbol{n}\int dp_L \Delta f(\boldsymbol{r}, p_L, \boldsymbol{n})(\boldsymbol{r}\times\boldsymbol{p})_z$$

$M_{NS} = 1.68 M_{solar}$

Period $P = 10$s

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{1}{I_{NS}}\frac{dL_z}{dt} = \frac{1}{I_{NS}}\left(\frac{dE_T}{dt}\right)_\nu \frac{cdL_z/dt}{dE_T/dt}$$

Magnetic Dipole Rad. $\frac{\dot{P}}{P} = \frac{P}{2\pi c I_{NS}}\left(\frac{cdL_z/dt}{dE_T/dt}\right)\mathcal{L}_\nu.$

Mag Distr.	Bary.	$-\frac{cdL_z/dt}{dE_T/dt}$ (cm)	$-\dot{\omega}$ (rad/s)	$\dot{P}/P$		
				ρ_0	$0.1\rho_0$	(Mg.Dpl-rad.)
Mag-A	p,n	66.9	4.94×10^{-2}	7.82×10^{-2}	1.03×10^{-2}	5.32×10^{-12}
	p,n,Λ	109	7.07×10^{-2}	11.26×10^{-2}	1.11×10^{-2}	6.77×10^{-11}
Mag-B	p,n	9.64	7.09×10^{-3}	1.13×10^{-3}	4.50×10^{-5}	5.32×10^{-12}
	p,n,Λ	7.81	5.07×10^{-3}	8.07×10^{-4}	2.29×10^{-5}	6.77×10^{-11}

In Early Stage (~ **10 s**) ν Asymmetric Emission must affect PNS Spin

More Significantly than Magnetic Dipole γ-Radiation

§ 5 *Summary*

- EOS of Neutron-Star-Matter with p, n, Λ in RMF Approach
- Exactly Solving Dirac Eq. with Magnetic Field in Perturbative Way
- Cross-Sections of Neutrino Scattering and Absorption
  under Strong Magnetic Field, calculated in Perturbative Way
- Neutrinos are More Scattered and Less Absorbed
  in Direction Parallel to Magnetic Field

⇒ More Neutrinos are Emitted in Arctic Area

Scattering 1.7 %

Absorption 2.2 % at $\rho_B = 3\rho_0$ and T = 20 MeV

⇒ Convection, Pulsar-Kick

Pulsar Kick in Poroidal Mag. Field $B = 2\times 10^{17}$G $\Rightarrow$ **Perturbative Cal.** Antarctic Dir.

v_{kick} = 580 km/s (p,n) , 610 km/s (p,n,Λ) at T = 20 MeV
= 230 km/s (p,n) , 270 km/s (p,n,Λ) at T = 30 MeV

400 km/s (Average of Observed Values)

Spin Deceleration in Toroidal Magnetic Field

$\dot{P}/P\,[\mathrm{s}^{-1}]$

Asymmetric Neutrino Emit. $10^{-2} \sim 10^{-5}$ when B ~ 10^{17}G

Magnetic Dipole Radiation $10^{-11} \sim 10^{-12}$

Asymmetric ν-Emission plays an important role in PNS Spin in Early Time

Future Plans

ν-Scattering

Iso-Temp. $\Rightarrow$ Iso-Entropy

Exact Solution of Dirac Eq. in Non-Perturbative Cal.

$\rightarrow$ Landau Level at least for Electron

Neutrino Propagation in Low Density

$$e^- + p \rightarrow \nu_e + n$$

Making Data Table and

Applying it to Supernovae Simulations