磁場を持つ相対論的回転星の数値解

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ハドロン物質の諸相と状態方程式－中性子星の観測に照らして－研究会, 基研, **31/Aug/2012**

- Introduction
- Formulation
- Numerical method – COCAL code
- Very preliminary result
- Discussion

STRONG MAGNETIC FIELD IN COMPACT OBJECTS

- Comparing the energy of strong magnetic field (B-field) with the gravitational energy of NS:

  T : Kinetic energy
  W : Gravitational energy
  $\mathcal{M}$: Magnetic energy

  $$T/|W| \sim 0.1 \qquad \mathcal{M} \sim \frac{1}{8\pi}\int B^2 d^3x$$

  B-field with the energy around $\mathcal{M}/|W| \sim 0.01$ may change the structure of NS, which is

  $$B \sim 4.4 \times 10^{16} \left(\frac{M\,[M_\odot]}{1.4\,[M_\odot]}\right)\left(\frac{10\,[\mathrm{km}]}{R\,[\mathrm{km}]}\right)^2 [\mathrm{G}]$$

- Magnetar
  - Observations of anomalous X-ray pulsar or soft γ repeater suggest NS with $B \sim 10^{14} - 10^{15}$ G at the NS surface.
  - B-field of NS interior may be much stronger.
    - Possibility of highly anisotropic B-field, or strong toroidal B-field.

SOME RESENT STUDIES ON THE STRUCTURES OF MAGNETIZED NS

- Simulations -- stable evolution of strongly magnetized NS.
  - Braithwaite and collaborators (2006 – 2010)
    - Twisted-torus B-field, Newtonian.
  - Kiuchi, Kyutoku, Shibata (2012)
    - Differential rotation. A few tens of periods.
  - cf) HMNS with B-fields: Kyoto group, Illinois group
- Equilibrium solutions of strongly magnetized NS.
  - Many earlier works.
    - Formulations, perturbative or numerical solutions, stability analysis.
  - Eriguchi and collaborators (including Fujisawa, 2005 –).
    - Newtonian, numerical solutions with mixed toroidal and poloidal B-field.
  - Ioka, Sasaki (2004); Yoshida, Kiuchi, Shibata (2012)
    - GR formulation. Mixed poloidal and toroidal B-fields. Slow rotation.

COMPACT OBJECTS IN (QUASI)EQUILIBRIUM

- We are developing a code for computing equilibriums and quasi-equilibrium initial data of compact objects in GR.
  - Rapidly rotating NS in equilibrium.
  - BNS, BHNS, BBH initial data in quasiequilibrium circular orbits.
  - BH-toroid systems.

  All of these with magnetic fields (and with GW, stellar winds etc.).
- Applications of the code include:
  - models of millisecond pulsar, magnetar, proto NS, post-merger objects of binary coalescence.
  - initial data for numerical relativity simulations.
  - economical computation of the GW waveforms.
  - studies on secular instabilities such as Chandrasekhar Friedman Schutz instability.

BRIEF LOOK AT FORMULATIONS FOR COMPUTING QUASIEQUILIBRIUMS OF MAGNETIZED COMPACT OBJECTS

- Equations for Einstein-Maxwell perfect MHD spacetime.
  - Einstein's equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$
  - Maxwell's equations $\nabla_\beta F^{\alpha\beta} = 4\pi j^\alpha \quad (dF)_{\alpha\beta\gamma} = 0$
  - (Current conservation $\nabla_\alpha j^\alpha = 0$)
  - MHD-Euler equation $u^\beta d(h\underline{u})_{\beta\alpha} - T\nabla_\alpha S = \frac{1}{\rho} F_{\alpha\beta} j^\beta,$
  - Rest mass density conservation $\nabla_\alpha(\rho u^\alpha) = 0$
  - Normalization of the 4 velocity $u^\alpha u_\alpha = -1$
  - Isentropic flow $\pounds_u S = 0$
  - Perfect MHD condition $F_{\alpha\beta} u^\beta = 0$
  - EOS $p = K\rho^\Gamma$

where

$$F_{\alpha\beta} = (dA)_{\alpha\beta} \qquad h := \frac{\epsilon + p}{\rho}$$

$$d(h\underline{u})_{\alpha\beta} := \nabla_\alpha(hu_\beta) - \nabla_\beta(hu_\alpha)$$

BRIEF LOOK AT FORMULATIONS FOR COMPUTING QUASIEQUILIBRIUMS OF MAGNETIZED COMPACT OBJECTS

- For a magnetar model, time symmetry and axisymmetry are assumed. t^α and ϕ^α are Killing vectors.
- 3+1 decomposed Einstein's and Maxwell's equations are written a system of elliptic equations (Poisson/Helmholtz).
  - Stationary and axisymmetric spacetime is not assumed to be circular. (Symmetry under a simultaneous $t \to -t$, $\phi \to -\phi$ is not assumed.)

✕ $$ds^2 = -e^{2\nu}dt^2 + e^{2\psi}(d\phi - \omega dt)^2 + e^{2\mu}(d\bar{r}^2 + \bar{r}^2 d\theta^2)$$

○ $$ds^2 = -\alpha^2 dt^2 + \psi^4 \tilde{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

  - Waveless formulation/Helically symmetric formulation. (Shibata, Uryu, Friedman 2004; Uryu, Limousin, Gourgoulhon, Friedman, Shibata 2009).
  - e.g.

$$\mathring{\Delta} h_{ab} = \mathcal{S}_{ab}$$

$$\mathring{\Delta} A_a = S_a$$

Flat Laplacian.

Solve Cartesian components.

Non-linear sources.

BRIEF LOOK AT FORMULATIONS : CASE WITH t^α AND ϕ^α SYMMETRIES

Gourgoulhon, Markakis, Uryu, Eriguchi (2011)

Integrations of MHD-Euler eq., rest mass conservation eq., and perfect MHD condition (and u.u=-1) are tricky when the time symmetry is imposed (as they are not one of specific types of PDE).

- Assume these equations are analytically integrable.
  - To derive a set of first integrals (sufficient conditions), we introduce a master potential Υ (Soloviev, 1967) , and assume

$$\boxed{A_t = A_t(\Upsilon), \quad A_\phi = A_\phi(\Upsilon), \quad \Psi = \Psi(\Upsilon), \quad \text{and} \quad S = S(\Upsilon)}$$

Then, MHD-Euler eq. and perfect MHD condition become

t or ϕ components: $d[\text{terms}] \wedge d\Upsilon = 0 \quad \Leftrightarrow \quad [\text{terms}] = \Lambda(\Upsilon)$

meridional components: $[\text{terms}] d\Upsilon = 0 \quad \Leftrightarrow \quad [\text{terms}] = 0$

where [terms] are algebraic terms. (We call them first integrals.)

  - The master potential Υ is solved from an elliptic equation called transfield equation.

BRIEF LOOK AT FORMULATIONS : TECHNICAL DETAILS

- A_ϕ is chosen as a variable ($\Upsilon = A_\phi$).
- Components of current is written in terms of arbitrary functions appears in the integrability conditions and first integrals.

$$j^A\sqrt{-g} = \left([\sqrt{-g}\Psi]''hu_\phi + [\sqrt{-g}\Lambda_\phi]'\right)\delta^{AB}B_B - [\sqrt{-g}\Psi]'\delta^{AB}\omega_B$$

$$\begin{aligned} j^\phi\sqrt{-g} + A_t'\,j^t\sqrt{-g} \;&=\; \left([\sqrt{-g}\Psi]''hu_\phi + [\sqrt{-g}\Lambda_\phi]'\right)B_\phi - [\sqrt{-g}\Psi]'\omega_\phi \\ &\quad - \left(A_t''\,hu_\phi + \Lambda'\right)\rho u^t\sqrt{-g} \end{aligned}$$

- Functions are chosen similarly to Yoshida, Yoshida, Eriguchi (2006)

$$A_t = -\Omega_c A_\phi + C_e\ , \qquad \Lambda = -\Lambda_c A_\phi - \mathcal{E}$$

$$[\sqrt{-g}\Lambda_\phi]' = a(A_\phi - A_\phi^{\max})^k\Theta(A_\phi - A_\phi^{\max})$$

$$[\sqrt{-g}\Psi]' = a_\Psi(A_\phi - A_\phi^{\max})^p\Theta(A_\phi - A_\phi^{\max})$$

  - So far, forms of some arbitrary functions and parameter values are chosen heuristically.

METHOD OF SOLUTION
COCAL CODE

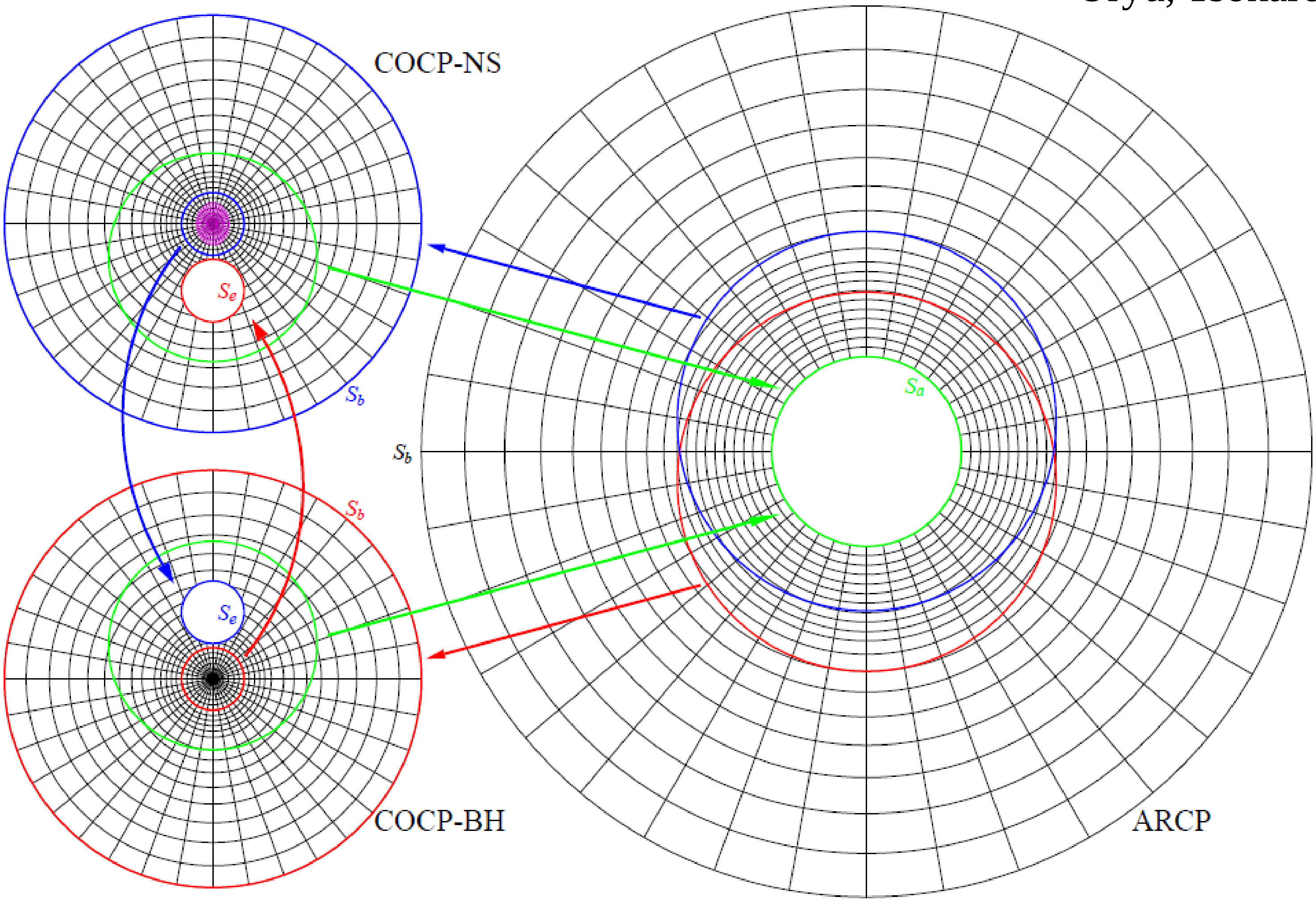
- COCAL (Compact Object CALculator)
  - “Cocal” means a “seagull” in Triestino (Trieste dialect of Italian).
  - Magnetized RNS code is under construction on the COCAL code.

- COCAL code project is aim to develop a “minimalistic” code for computing compact objects, including
  - Rapidly rotating neutron star
  - Binary neutron stars
  - Black hole – neutron star binary
  - Binary black holes
  - Black hole – toroid system
  - And all the above with magnetic fields

- COCAL – as simple as possible
  - COCAL code is simple, minimalistic, and robust.
  - No symmetry is assumed for the 3D spherical computational domain.
  - 2nd order finite difference scheme on the spherical grids is used.
  - Multipole expansion of Green’s function is used in the Poisson/Helmholtz solver.
  - A set of subroutines written in Fortran 90 are reasonably structured.

MULTIPATCHES FOR BINARY SYSTEMS

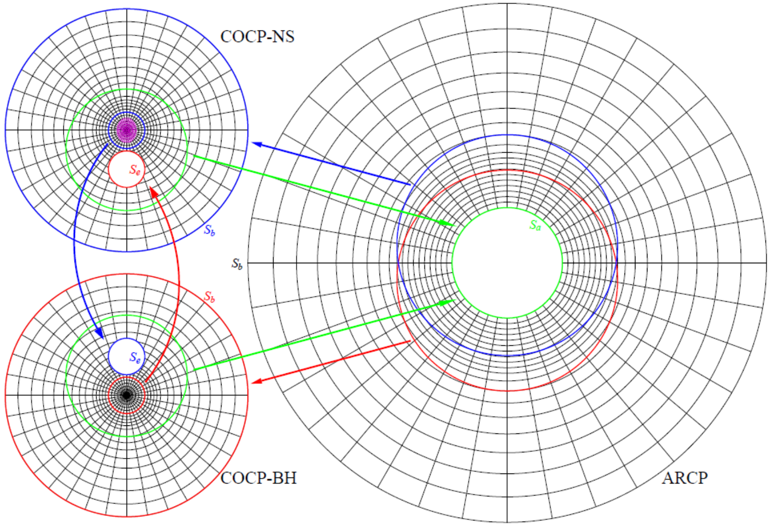
Uryu, Tsokaros (2012)



- COCP – compact object coordinate patch
- ARCP – asymptotic region coordinate patch

- S_a – inner (BH excised) boundary sphere.
- S_b – outer boundary sphere.
- S_e – binary excision boundary sphere.

ELLIPTIC SOLVER



$$\Delta\Phi = S \qquad \Delta G(x,x') = -4\pi\delta(x-x')$$

- Superimpose Green's formula and a homogenous solution.

$$\Phi(x) = \Phi_{\rm INT}(x) + \chi(x)$$

$$\Phi_{\rm INT}(x) = -\frac{1}{4\pi}\int_V G(x,x')S(x')d^3x' + \frac{1}{4\pi}\int_{\partial V}\left[G(x,x')\nabla'^a\Phi(x') - \Phi(x')\nabla'^a G(x,x')\right]dS'_a$$

$$\partial V = S_a \cup S_b \cup S_e$$

- Determine the homogeneous solution to have Φ satisfies the boundary condition at surfaces S_a (BH) and S_b.

$$\chi(x) = \Phi(x) - \Phi_{\rm INT}(x)$$

$$\chi(x) = \frac{1}{4\pi}\int_{S_a\cup S_b}\left[G^{\rm BC}(x,x')\nabla'^a(\Phi_{\rm BC}-\Phi_{\rm INT})(x') - (\Phi_{\rm BC}-\Phi_{\rm INT})(x')\nabla'^a G^{\rm BC}(x,x')\right]dS'_a$$

- Data on S_e is transferred from the corresponding sphere of the other patch.

COCAL CODE STATUS

- Einstein's equation (initial data) solver is ready for
  - IWM formulation

$$ds^2 = -\alpha^2 dt^2 + \psi^4 f_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

  - Waveless/Near-zone helically symmetric formulations

$$ds^2 = -\alpha^2 dt^2 + \psi^4 \tilde{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

- Maxwell's equation solver is ready.
- Multi-coordinate patches for binary system are introduced.
- Parametrized EOS (piecewise polytropic EOS) is implemented.
- Rotating NS, BBH initial data codes are ready.

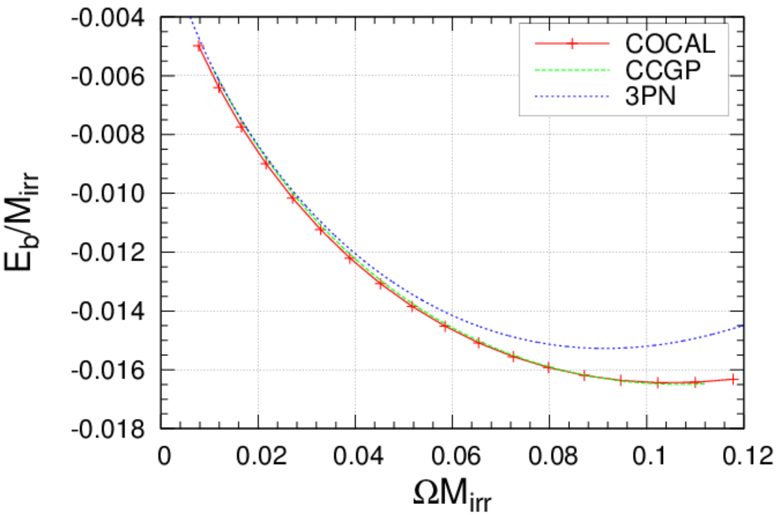
Also several tools are available including,

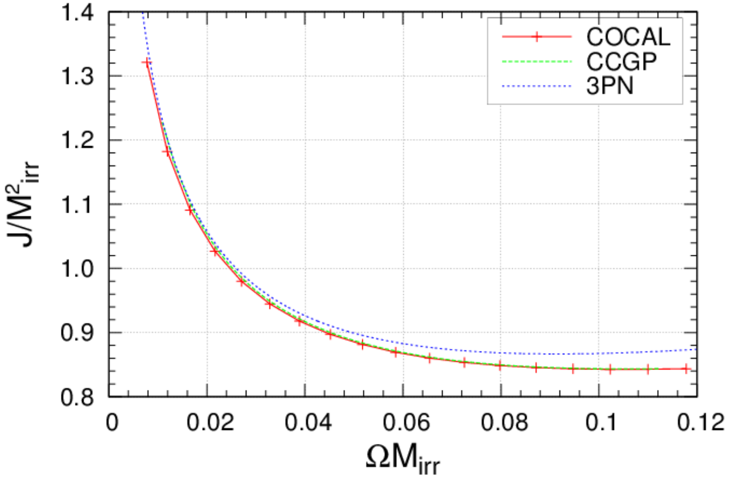
- 1D spherical NS solver (TOV solver, and NS solver on isotropic coordinates).
- Subroutines for the mass, the angular momentum and the other physical quantities.
- Apparent horizon finder.
- Helmholtz solver for outgoing waves, or half-outgoing+half-incoming waves.
- Quadrupole formula to estimate GW amplitude.

CODE TEST: CONFORMALLY FLAT BBH INITIAL DATA

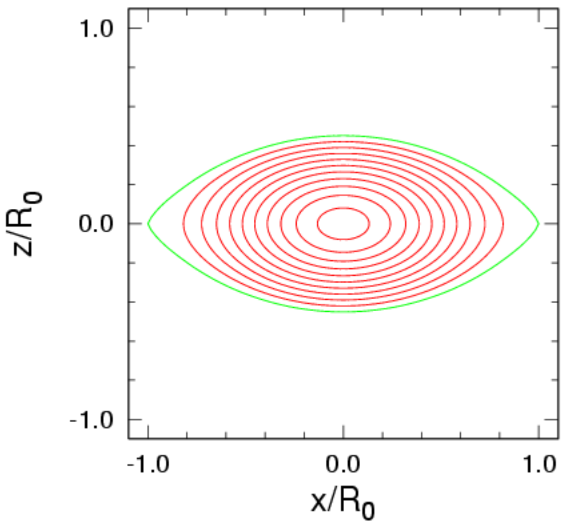
Uryu, Tsokaros, Grandclement, preprint (2012)

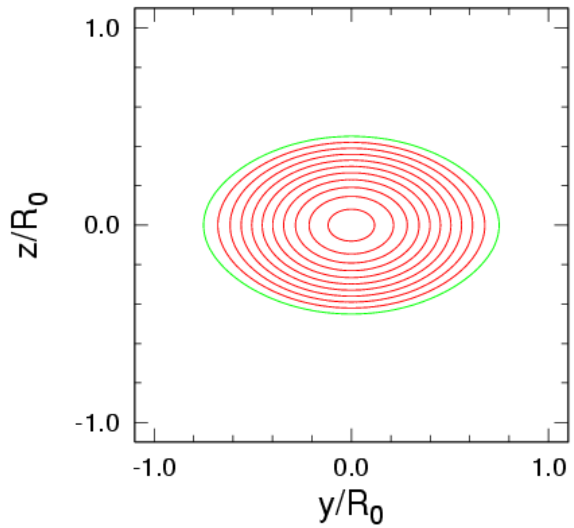
- Convergence tests are performed for conformally flat BBH inital data.
- Solution sequence agrees well with the results by Caudill, Cook, Grigsby, Pfeiffer (2006).
  - Corotating spin. Apparent horizon boundary conditions are imposed at the excised sphere.

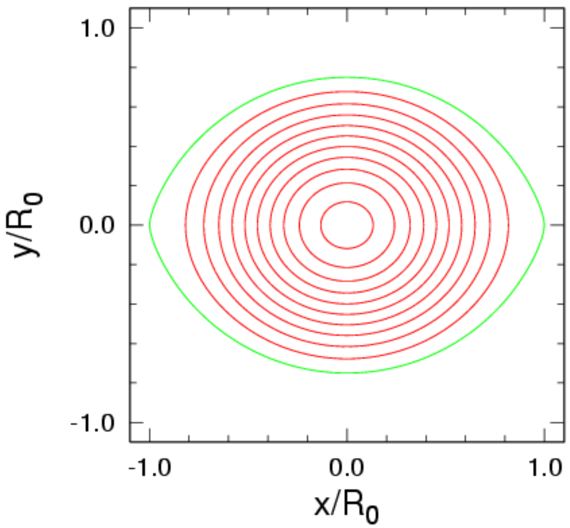


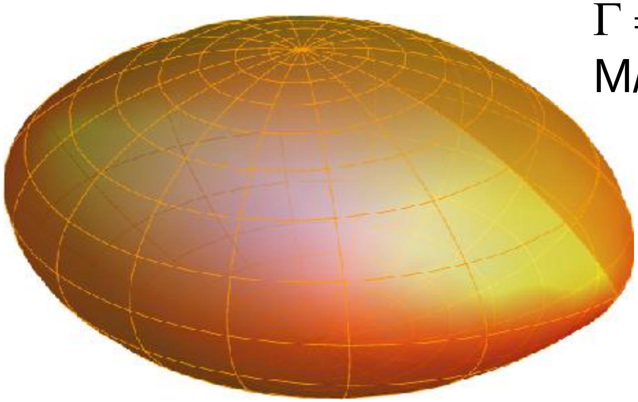


Example of a solution from Cocal code: Relativistic Jacobi ellipsoid









Huang et.al. PRD (2008)

Triaxially deformed rapidly rotating NS

$$p = K\rho^{\Gamma}$$

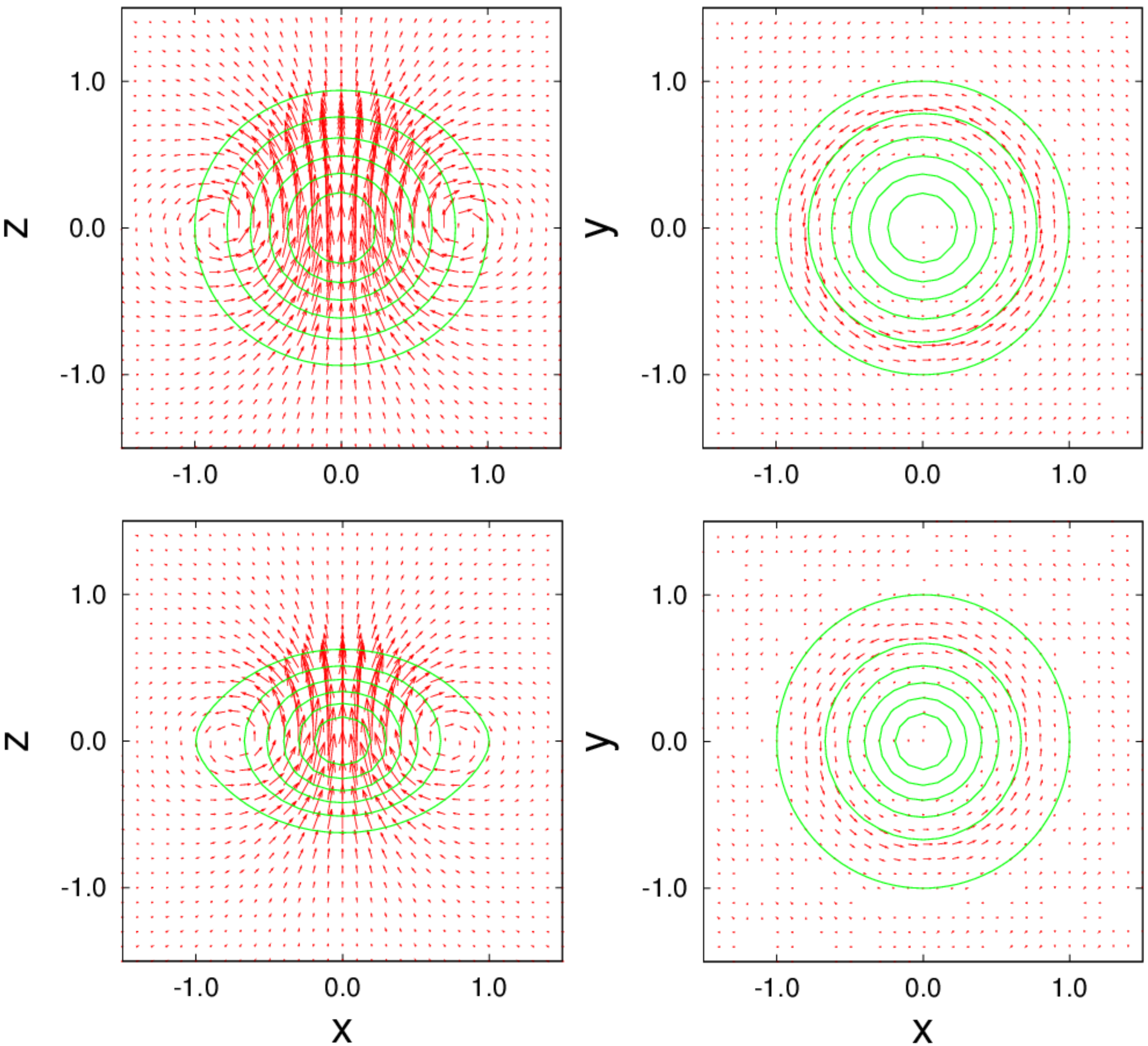
$\Gamma = 4.33$

$M/R = 0.2$

SOLUTIONS OF MAGNETIZED NS (PRELIMINARY)

$$[\sqrt{-g}\Lambda_\phi]' = a(A_\phi - A_\phi^{\max})^k \Theta(A_\phi - A_\phi^{\max})$$

- Some solutions are calculated for magnetized NS.
- Γ=2.0, M/R = 0.14, k = 1
- Top: R_p/R_e=0.9375
  $B_P \sim 1.4\times 10^{17}$G
  $B_T/B_P \sim 0.16$
- Bottom:
  R_p/R_e=0.625
  $B_P \sim 1.3\times 10^{17}$G
  $B_T/B_P \sim 0.12$



DISCUSSION

- A new code for stationary and axisymmetric NS with mixed poloidal and toroidal B-field in full GR is under development as a part of COCAL code.
- Preliminary solutions of strongly magnetized and relativistic NS are calculated.
  - GR solutions are qualitatively the same as those of Newtonian.
- Subroutines to compute GR and EM fields are not restricted to the case with axisymmetry.
- Solutions of magnetized compact object in (quasi) equilibriums can be used for various problems.
  - Unperturbed state for stability analysis. Initial data.