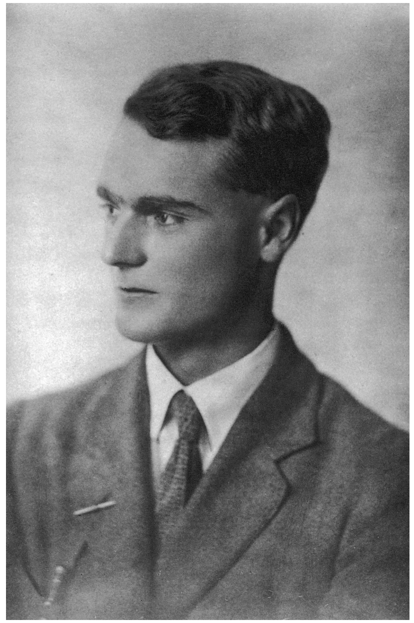
Non-Abelian statistics for non-Abelian vortices in color superconductivity

安井繁宏 (KEK)

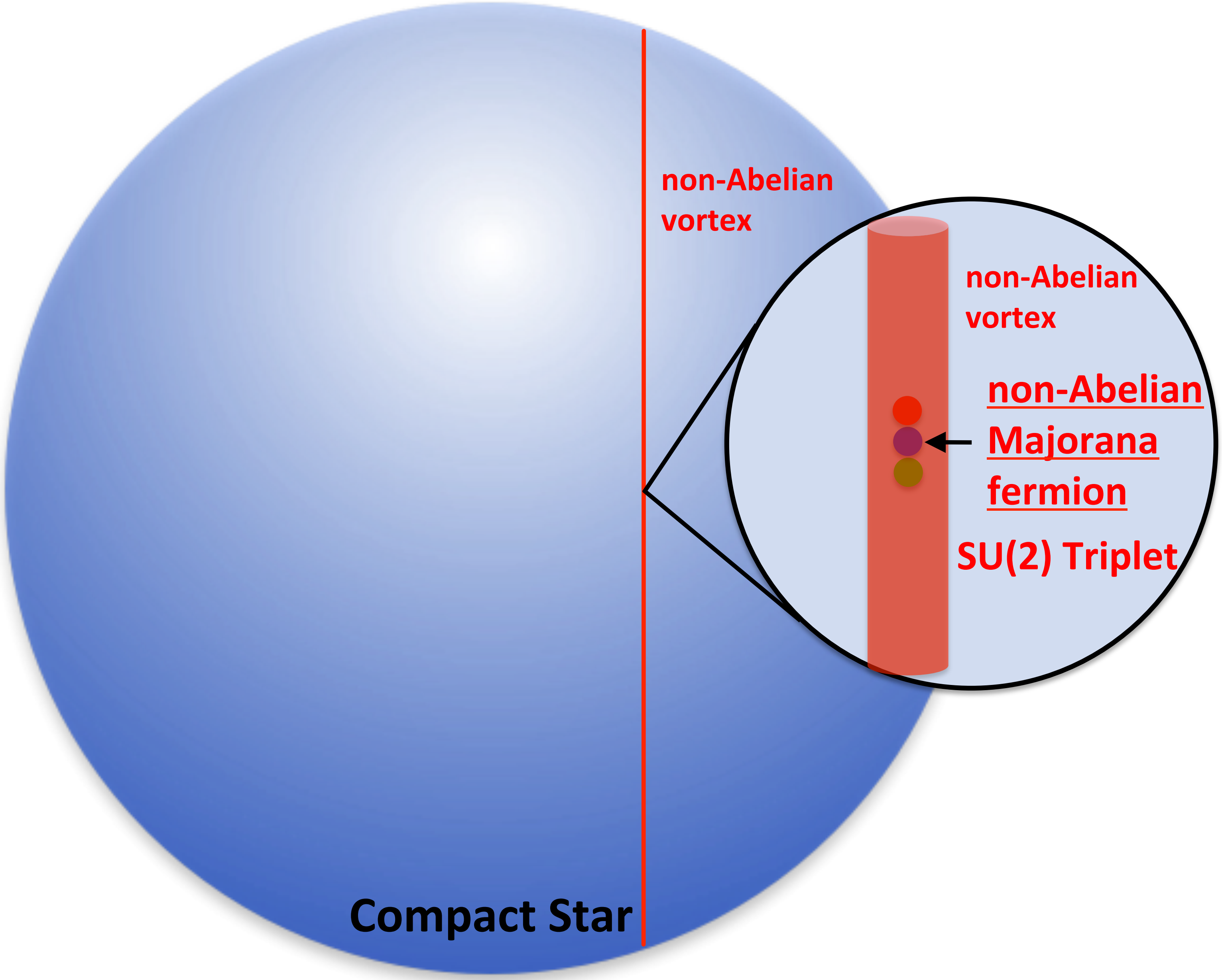
Collaborators
板倉数記 (KEK)
新田宗土 (慶応義塾大学)
広野雄士 (理化学研究所)

E. Majorana
(1906-1938?)



H. S. M. Coxeter
(1907-2003)

基研研究会「ハドロン物質の諸相と状態方程式-中性子星の観測に照らして-」
@基礎物理学研究所 30 Aug. - 1 Sep. 2012



Color-Flavor Locked (CFL) Color Superconductor

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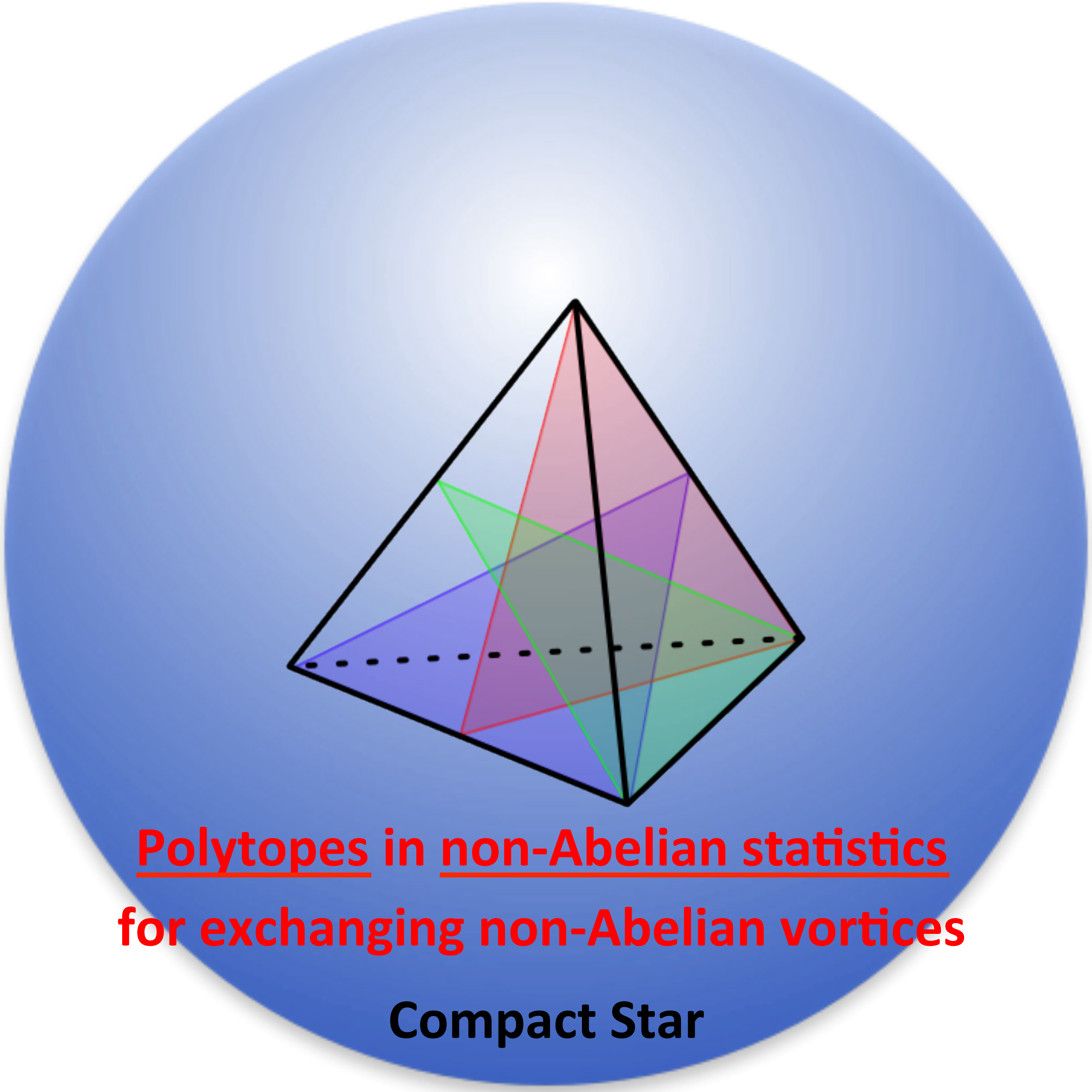
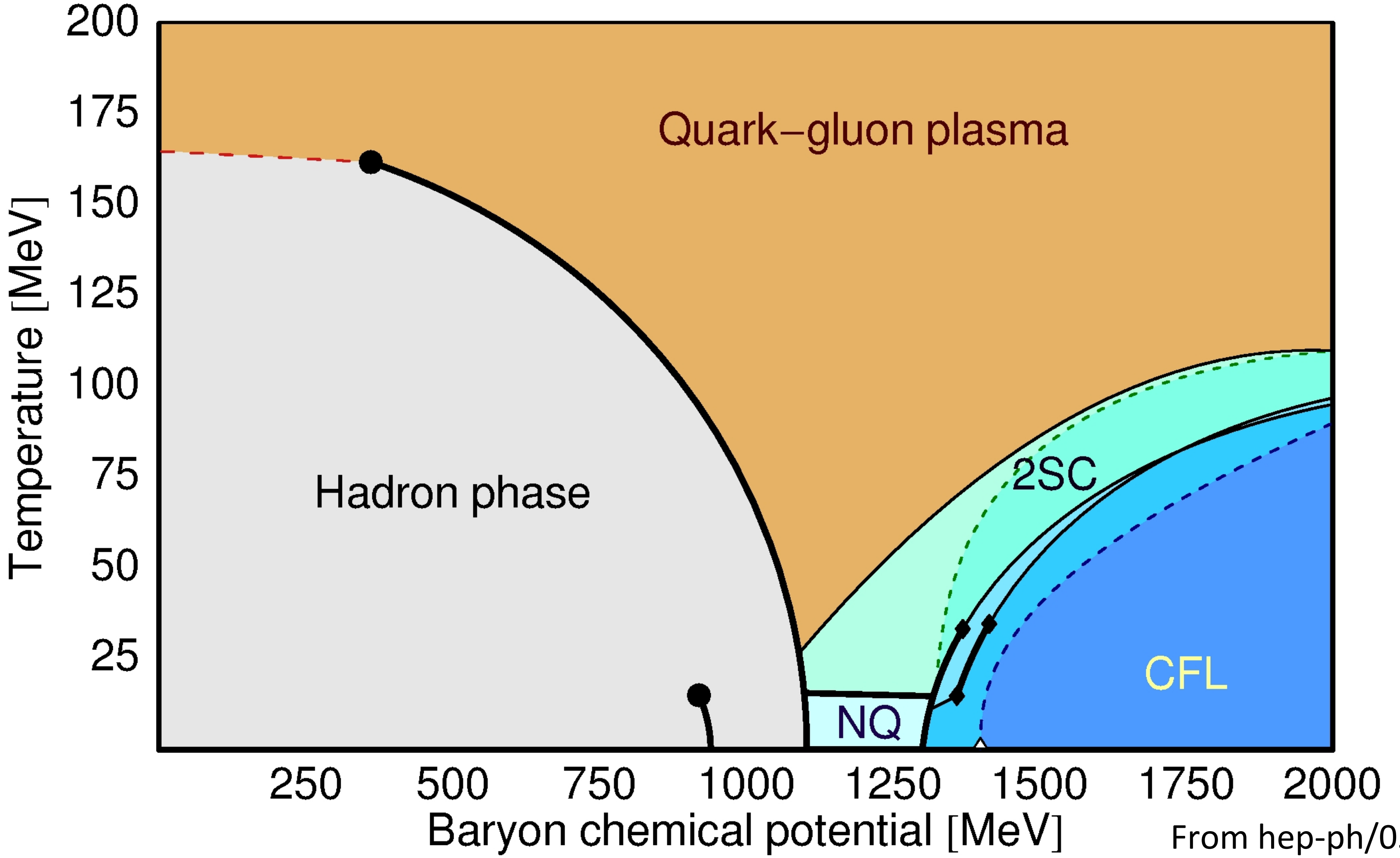


Color-Flavor Locked (CFL) Color Superconductor

Contents

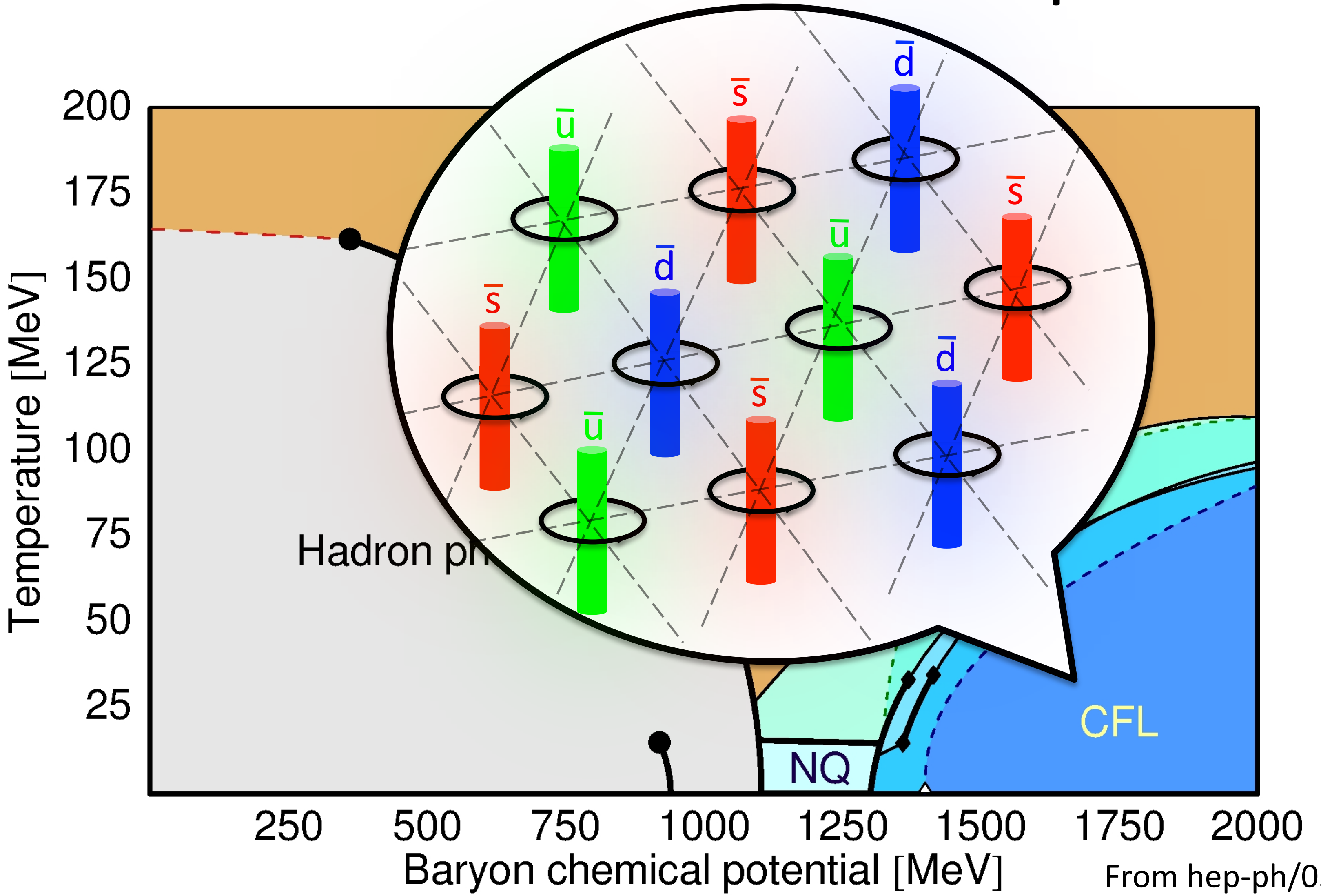
[1] SY, K. Itakura and M. Nitta, Phys. Rev. D81, 105003 (2010)
[2] SY, K. Itakura and M. Nitta, Phys. Rev. B83, 134518 (2011)
[3] T. Fujiwara, T. Fukui, M. Nitta, SY, Phys. Rev. D84, 076002 (2011)
[4] SY, Itakura, Nitta, Nucl. Phys. B859, 261 (2012)
[5] Y. Hirono, SY, K. Itakura, M. Nitta, PRB86, 014508 (2012)
[6] SY, Y. Hirono, K. Itakura, M. Nitta, arXiv:1204.1164

Non-Abelian vortices in CFL phase



From hep-ph/0503184

Non-Abelian vortices in CFL phase



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Non-Abelian vortices in CFL phase

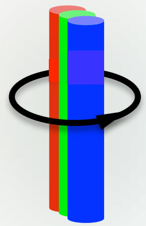
Color-Flavor Locked (CFL) phase

$SU(3)_C \times SU(3)_F \rightarrow SU(3)_{C+F}$

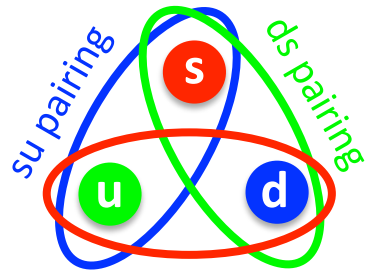
$$\langle \psi_{j\beta}\psi_{k\gamma} \rangle = \Delta_{i\alpha}\varepsilon_{ijk}\varepsilon_{\alpha\beta\gamma}$$

$$i=\alpha \downarrow \quad \begin{array}{l} i=u,d,s \\ \alpha=r,g,b \end{array}$$

$$\Delta_{i\alpha} = \begin{pmatrix} \Delta & & \\ & \Delta & \\ & & \Delta \end{pmatrix} \xrightarrow[\text{vortex}]{} \begin{pmatrix} \Delta e^{i\theta} & & \\ & \Delta e^{i\theta} & \\ & & \Delta e^{i\theta} \end{pmatrix}$$



Abelian vortex ?



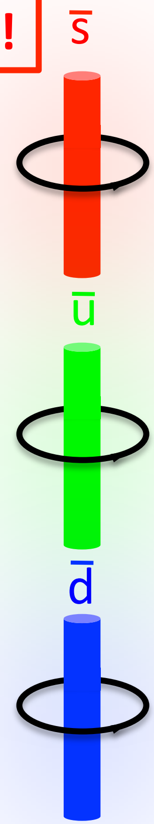
- M. Alford, K. Rajagopal, F. Wilczek, Nucl. Phys. B537, 443 (1999)
- T. Schäfer, F. Wilczek, Phys. Rev. Lett. 82, 3956 (1999)

- M. M. Forbes and A. R. Zhitntsky, Phy. Rev. D65, 085009 (2002)
- K. Ida and G. Baym, Phys. Rev. D66, 014015 (2002)
- K. Iida, Phys. Rev. D71, 054011 (2005)

$SU(3)_{C+F} \rightarrow SU(2)_{C+F} \times U(1)_{C+F}$

non-Abelian vortex !!

$$\begin{pmatrix} \Delta_1 e^{i\theta} & & \\ & \Delta_0 & \\ & & \Delta_0 \end{pmatrix} \qquad \begin{pmatrix} \Delta_0 & & \\ & \Delta_1 e^{i\theta} & \\ & & \Delta_0 \end{pmatrix} \qquad \begin{pmatrix} \Delta_0 & & \\ & \Delta_0 & \\ & & \Delta_1 e^{i\theta} \end{pmatrix}$$



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Non-Abelian vortices in CFL phase

Color-Flavor Locked (CFL) phase

$SU(3)_C \times SU(3)_F \to SU(3)_{C+F}$

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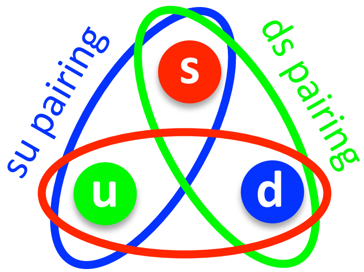
$i = u, d, s$; $\alpha = r, g, b$; $i = \alpha$ ↓

$$\Delta_{i\alpha} = \begin{pmatrix} \Delta & & \\ & \Delta & \\ & & \Delta \end{pmatrix} \xrightarrow{\text{vortex}} \begin{pmatrix} \Delta e^{i\theta} & & \\ & \Delta e^{i\theta} & \\ & & \Delta e^{i\theta} \end{pmatrix}$$



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Abelian vortex ?

Energy ≈ n^2
n : winding number

E ≈ 1^2

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$SU(3)_{C+F} \to SU(2)_{C+F} \times U(1)_{C+F}$

non-Abelian vortex !!

$$\begin{pmatrix} \Delta_1 e^{i\theta} & & \\ & \Delta_0 & \\ & & \Delta_0 \end{pmatrix} \quad \textbf{E} \approx \mathbf{(1/3)^2}$$

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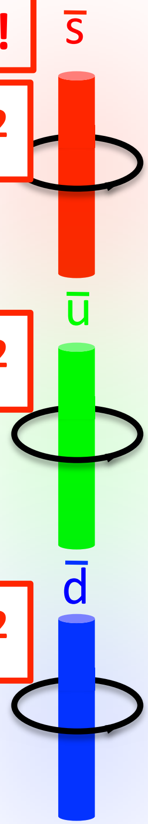
$$\begin{pmatrix} \Delta_0 & & \\ & \Delta_0 & \\ & & \Delta_1 e^{i\theta} \end{pmatrix} \quad \textbf{E} \approx \mathbf{(1/3)^2}$$



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1. Introduction

2. Majorana fermions in non-Abelian vortices

3. Non-Abelian statistics in non-Abelian voritces

4. Summary

Majorana fermions in non-Abelian vortices

Majorana (particle=hole; zero mode) fermion → triplet of $SU(2)_{C+F}$ in CFL

Bogoliubov-de-Gennes equation : S. Y., K. Itakura and M. Nitta, Phys. Rev. D81, 105003 (2010)
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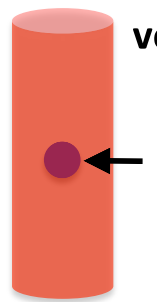
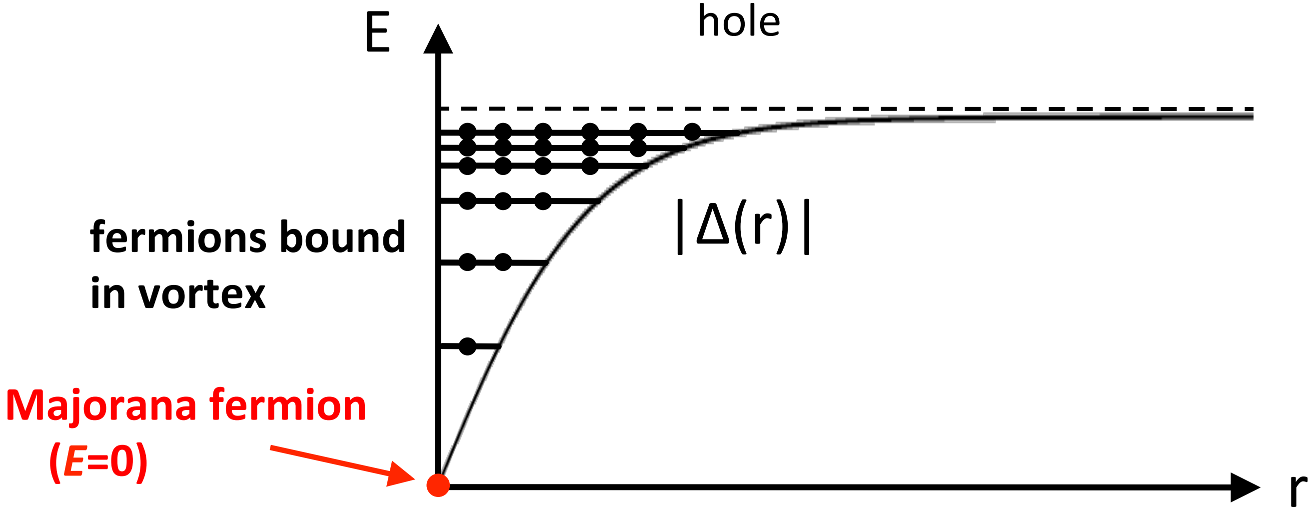


Bogoliubov-de Gennes (BdG) equation for single flavor

particle

$$\begin{pmatrix} -i\gamma_0\vec{\gamma}\cdot\vec{\nabla}-\mu & |\Delta|e^{i\theta}\gamma_0\gamma_5 \\ -|\Delta|e^{-i\theta}\gamma_0\gamma_5 & -i\gamma_0\vec{\gamma}\cdot\vec{\nabla}+\mu \end{pmatrix}\begin{pmatrix} \varphi_{\pm,n} \\ \eta_{\mp,n-1} \end{pmatrix} = E\begin{pmatrix} \varphi_{\pm,n} \\ \eta_{\mp,n-1} \end{pmatrix}$$

hole



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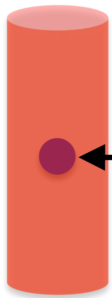
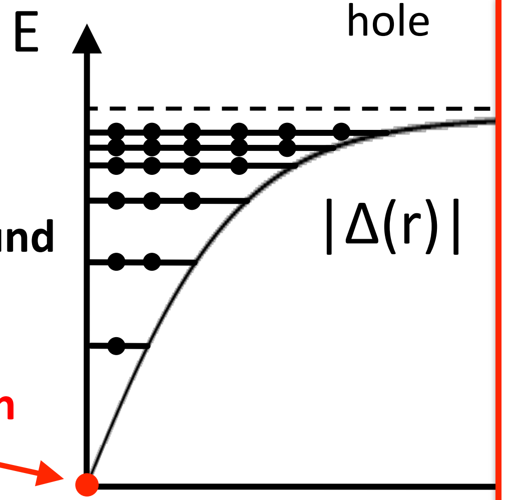
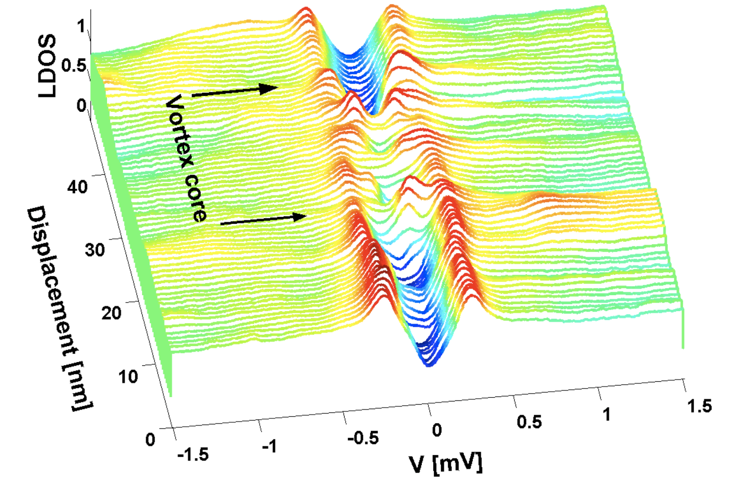


Bogoliubov-de Gennes (BdG) equation for single flavor

$$\begin{pmatrix} -i\gamma_0\vec{\gamma}\cdot\vec{\nabla}-\mu & |\Delta|e^{i\theta}\gamma_0\gamma_5 \\ -|\Delta|e^{-i\theta}\gamma_0\gamma_5 & -i\gamma_0\vec{\gamma}\cdot\vec{\nabla}+\mu \end{pmatrix} \begin{array}{c} \text{particle} \\ \begin{pmatrix} \varphi_{\pm,n} \\ \eta_{\mp,n-1} \end{pmatrix} \\ \text{hole} \end{array} = E \begin{pmatrix} \varphi_{\pm,n} \\ \eta_{\mp,n-1} \end{pmatrix}$$





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Majorana fermions in non-Abelian vortices

Majorana (particle=hole; zero mode) fermion → triplet of SU(2)$_{C+F}$ in CFL

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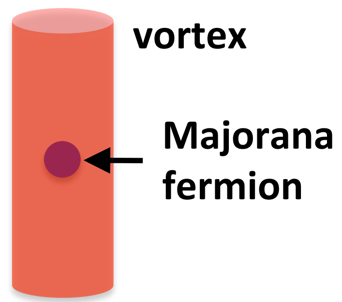
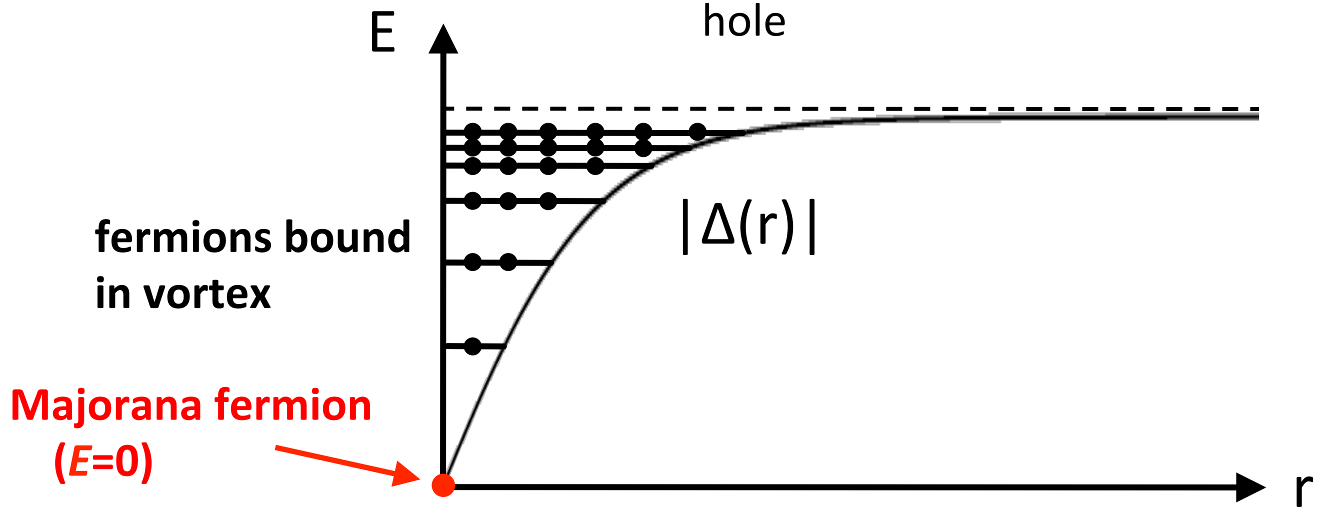


Bogoliubov-de Gennes (BdG) equation for single flavor

$$\begin{pmatrix} -i\gamma_0\vec{\gamma}\cdot\vec{\nabla}-\mu & |\Delta|e^{i\theta}\gamma_0\gamma_5 \\ -|\Delta|e^{-i\theta}\gamma_0\gamma_5 & -i\gamma_0\vec{\gamma}\cdot\vec{\nabla}+\mu \end{pmatrix} \begin{array}{c} \text{particle} \\ \begin{pmatrix} \varphi_{\pm,n} \\ \eta_{\mp,n-1} \end{pmatrix} \\ \text{hole} \end{array} = E \begin{pmatrix} \varphi_{\pm,n} \\ \eta_{\mp,n-1} \end{pmatrix}$$



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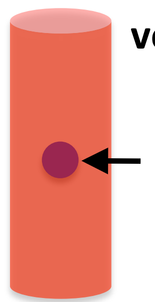

Bogoliubov-de Gennes (BdG) equation for single flavor

particle

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hole

Majorana fermion (*E*=0)

particle

|| c.c.

hole

$$\varphi_{+,0}(r,\theta) = Ce^{-\int_0^r |\Delta(r')|\mathrm{d}r'}\begin{pmatrix} J_0(\mu r) \\ iJ_1(\mu r)e^{-i\theta} \end{pmatrix}$$

$$\eta_{-,-1}(r,\theta) = Ce^{-\int_0^r |\Delta(r')|\mathrm{d}r'}\begin{pmatrix} -J_1(\mu r) \\ iJ_0(\mu r)e^{-i\theta} \end{pmatrix}$$

- Localization with $e^{-|\Delta|r}$
- Oscillation with $J_0(\mu r)$, $J_1(\mu r)$

For vacuum (μ=0), see R. Jackiw and P. Rossi, Nucl. Phys. B190, 681 (1981).

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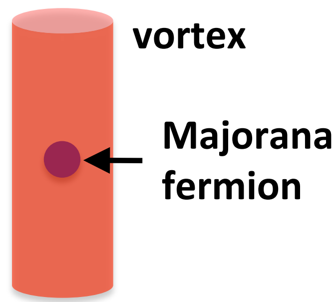

Bogoliubov-de Gennes (BdG) equation for <u>single flavor</u>

$$\begin{pmatrix} -i\gamma_0\vec{\gamma}\cdot\vec{\nabla}-\mu & |\Delta|e^{i\theta}\gamma_0\gamma_5 \\ -|\Delta|e^{-i\theta}\gamma_0\gamma_5 & -i\gamma_0\vec{\gamma}\cdot\vec{\nabla}+\mu \end{pmatrix} \begin{array}{c} \text{particle} \\ \begin{pmatrix} \varphi_{\pm,n} \\ \eta_{\mp,n-1} \end{pmatrix} \\ \text{hole} \end{array} = E \begin{pmatrix} \varphi_{\pm,n} \\ \eta_{\mp,n-1} \end{pmatrix}$$

Majorana fermion (*E*=0)

$$\boxed{\text{particle}}\ \varphi_{+,0}(r,\theta) = Ce^{-\int_0^r |\Delta(r')|\mathrm{d}r'} \begin{pmatrix} J_0(\mu r) \\ iJ_1(\mu r)e^{-i\theta} \end{pmatrix}$$

|| c.c.

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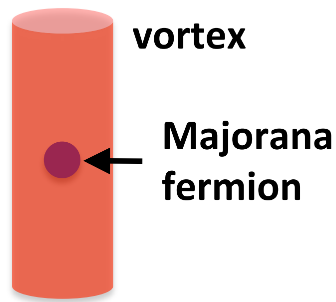

Bogoliubov-de Gennes (BdG) equation for single flavor

$$\begin{pmatrix} -i\gamma_0\vec{\gamma}\cdot\vec{\nabla}-\mu & |\Delta|e^{i\theta}\gamma_0\gamma_5 \\ -|\Delta|e^{-i\theta}\gamma_0\gamma_5 & -i\gamma_0\vec{\gamma}\cdot\vec{\nabla}+\mu \end{pmatrix} \begin{array}{c} \text{particle} \\ \begin{pmatrix} \varphi_{\pm,n} \\ \eta_{\mp,n-1} \end{pmatrix} \\ \text{hole} \end{array} = E \begin{pmatrix} \varphi_{\pm,n} \\ \eta_{\mp,n-1} \end{pmatrix}$$

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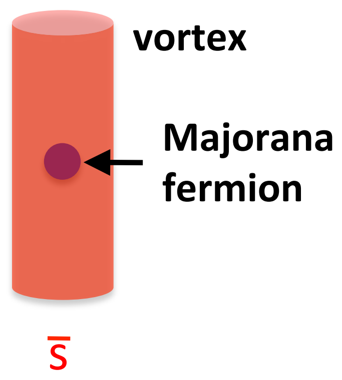
Bogoliubov-de Gennes (BdG) equation for $\underline{SU(2)_{C+F}\text{ flavor}}$



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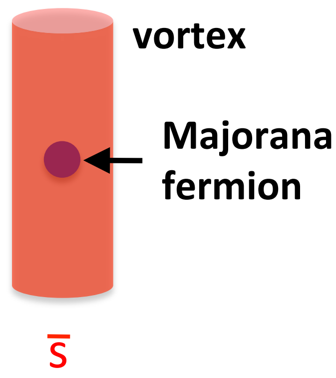

Bogoliubov-de Gennes (BdG) equation for $SU(2)_{C+F}$ flavor

$$
\begin{pmatrix}
\hat{H}_0 & \hat{\Delta}_1 & \hat{\Delta}_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hat{\Delta}_1 & \hat{H}_0 & \hat{\Delta}_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hat{\Delta}_0 & \hat{\Delta}_0 & \hat{H}_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \hat{H}_0 & -\hat{\Delta}_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\hat{\Delta}_1 & \hat{H}_0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \hat{H}_0 & -\hat{\Delta}_0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\hat{\Delta}_0 & \hat{H}_0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{H}_0 & -\hat{\Delta}_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\hat{\Delta}_0 & \hat{H}_0
\end{pmatrix}
\begin{pmatrix} u_r \\ d_g \\ s_b \\ d_r \\ u_g \\ s_r \\ u_b \\ s_g \\ d_b \end{pmatrix}
= E
\begin{pmatrix} u_r \\ d_g \\ s_b \\ d_r \\ u_g \\ s_r \\ u_b \\ s_g \\ d_b \end{pmatrix}
$$

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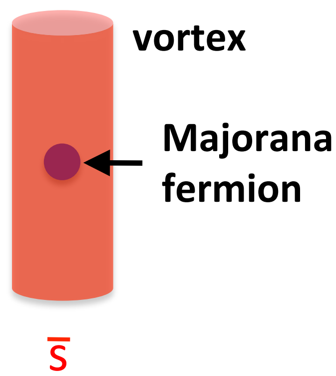

Bogoliubov-de Gennes (BdG) equation for <u>$SU(2)_{C+F}$ flavor</u>

$$\begin{pmatrix} u_r & u_g & u_b \\ d_r & d_g & d_b \\ s_r & s_g & s_b \end{pmatrix} = \sum_{A=1}^{9} \frac{\lambda^A}{\sqrt{2}} \Psi^{(A)}$$

$SU(3)_{C+F} \rightarrow SU(2)_{C+F} \times U(1)_{C+F}$

Triplet (normalizable) $\Psi_t = \Psi^{(1)}\lambda_1 + \Psi^{(2)}\lambda_2 + \Psi^{(3)}\lambda_3$

Singlet (non-normalizable) $\Psi_s = \Psi^{(8)}\lambda_8 + \Psi^{(9)}\lambda_9$

Doublet (no zero mode)

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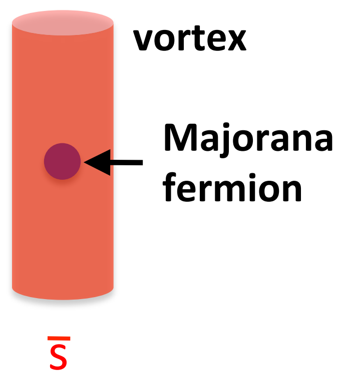

Bogoliubov-de Gennes (BdG) equation for $\underline{SU(2)_{C+F}\text{ flavor}}$

$$\begin{pmatrix} u_r & u_g & u_b \\ d_r & d_g & d_b \\ s_r & s_g & s_b \end{pmatrix} = \sum_{A=1}^{9} \frac{\lambda^A}{\sqrt{2}} \Psi^{(A)}$$

$SU(3)_{C+F} \to SU(2)_{C+F} \times U(1)_{C+F}$

Triplet (normalizable) $\Psi_t = \Psi^{(1)}\lambda_1 + \Psi^{(2)}\lambda_2 + \Psi^{(3)}\lambda_3$

Singlet (non-normalizable) $\Psi_s = \Psi^{(8)}\lambda_8 + \Psi^{(9)}\lambda_9$

Doublet (no zero mode)

Majorana fermions in non-Abelian vortices

Majorana (particle=hole; zero mode) fermion → triplet of $SU(2)_{C+F}$ in CFL

Bogoliubov-de-Gennes equation : S. Y., K. Itakura and M. Nitta, Phys. Rev. D81, 105003 (2010)
Index Theorem : T. Fujiwara, T. Fukui, M. Nitta, S. Y., Phys. Rev. D84, 076002 (2011)

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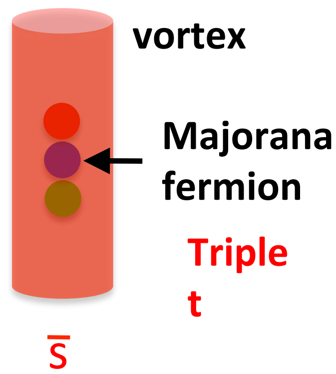

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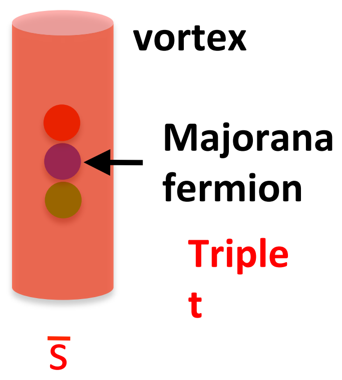
Index Theorem→ Triplet Majorana fermions only



Bogoliubov-de Gennes (BdG) equation for $SU(2)_{C+F}$ flavor

$$\begin{pmatrix} u_r & u_g & u_b \\ d_r & d_g & d_b \\ s_r & s_g & s_b \end{pmatrix} = \sum_{A=1}^{9} \frac{\lambda^A}{\sqrt{2}} \Psi^{(A)}$$

$SU(3)_{C+F}$ → $SU(2)_{C+F}$ x $U(1)_{C+F}$

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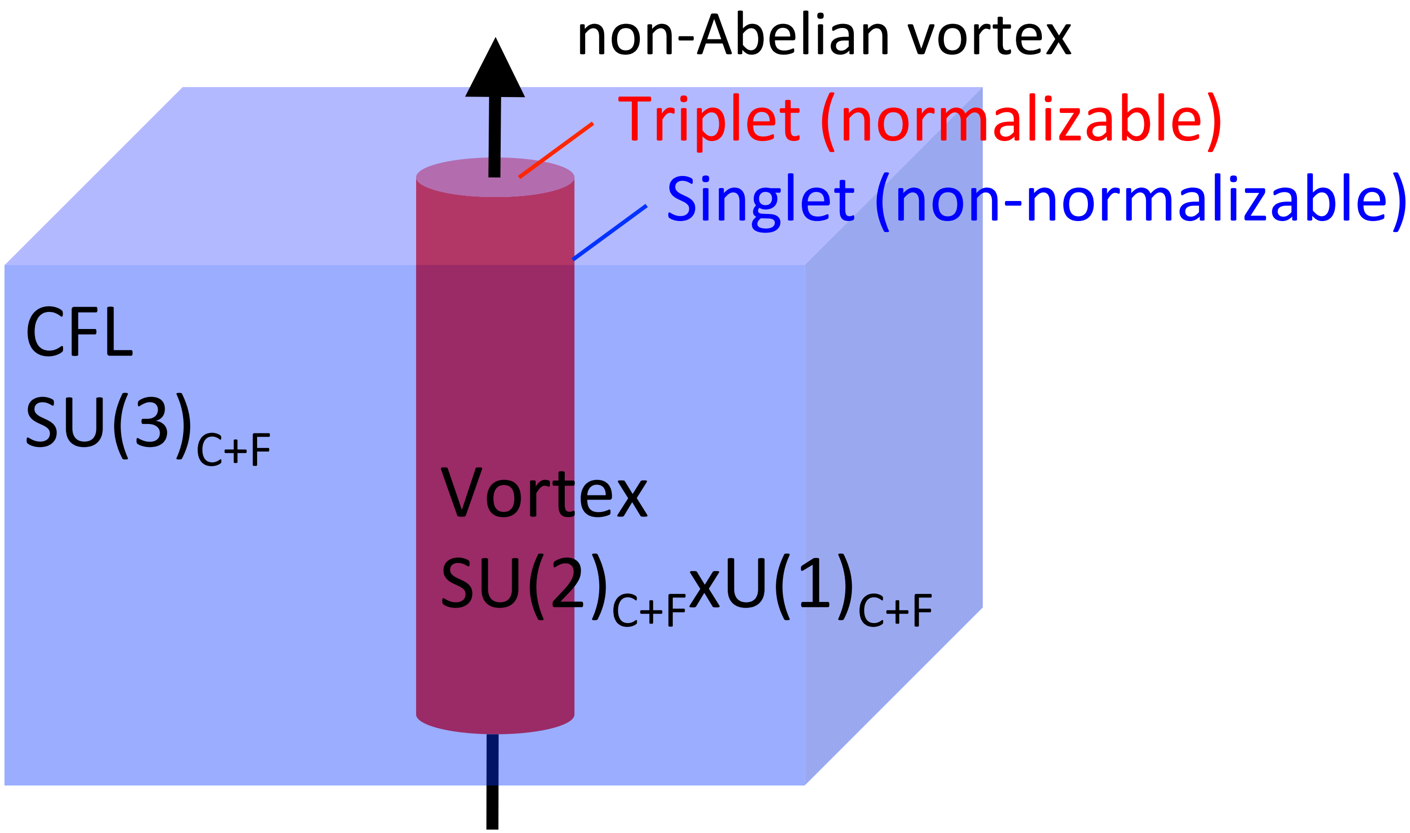
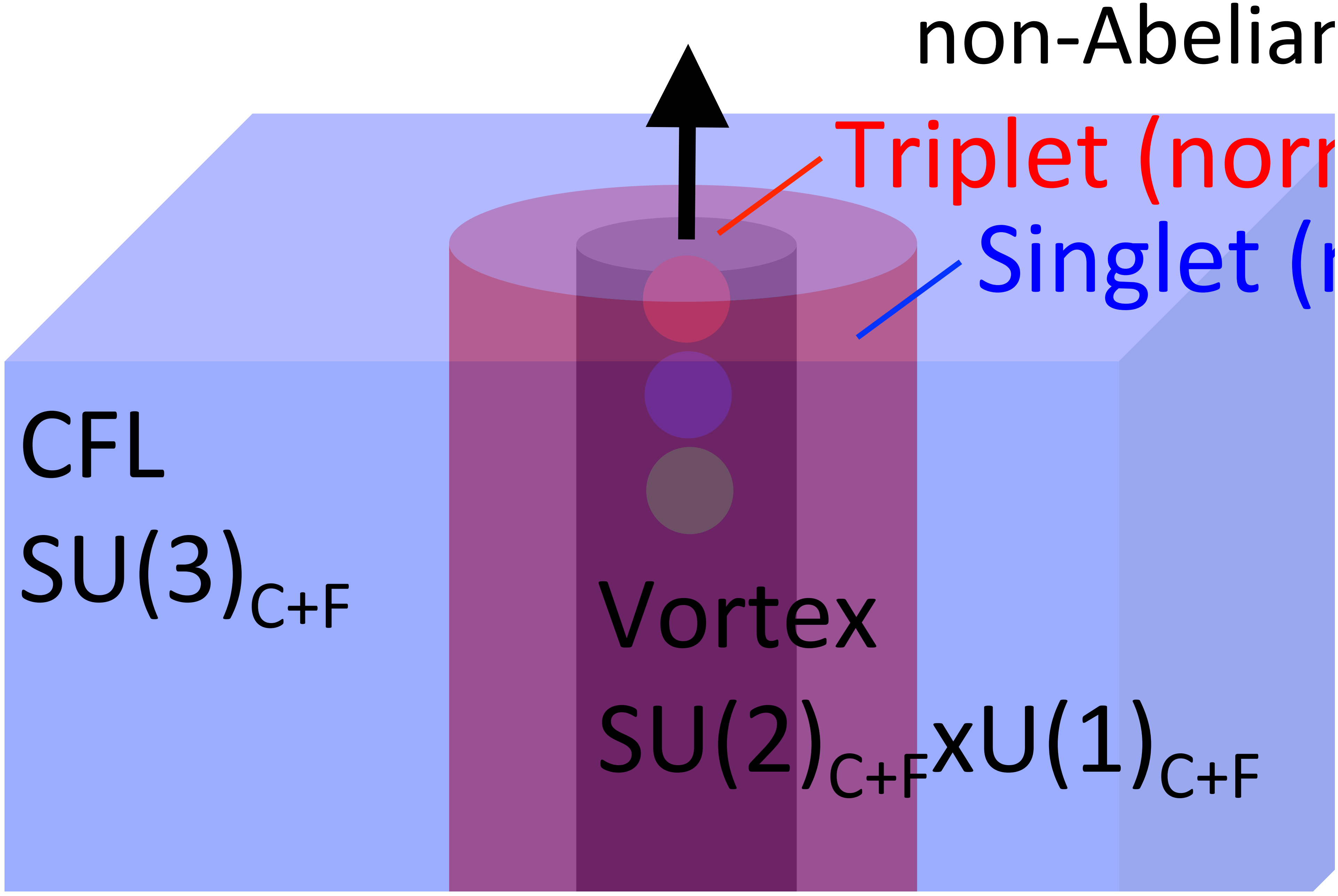
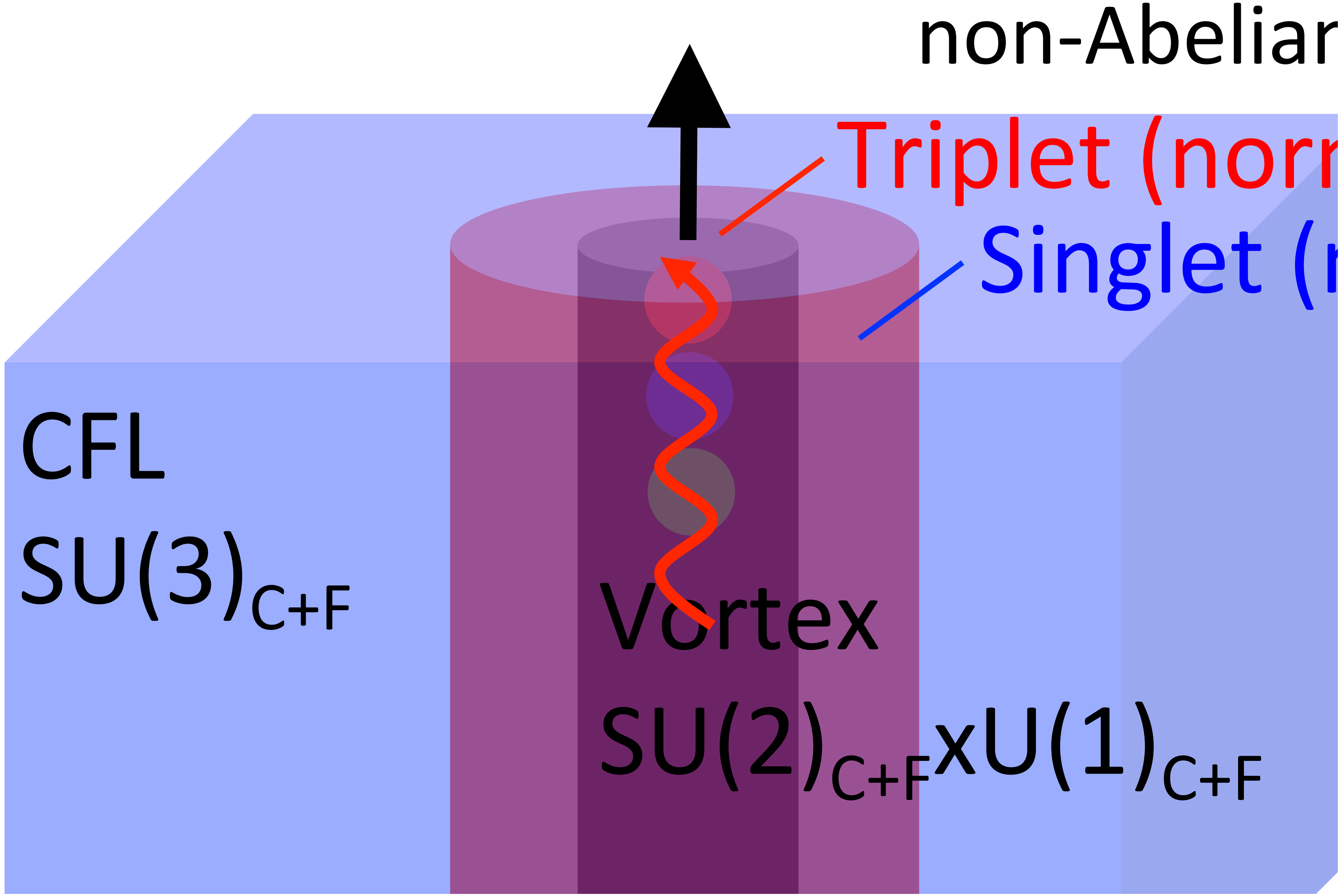



non-Abeliar
Triplet (norr
Singlet (r
CFL
$SU(3)_{C+F}$
Vortex
$SU(2)_{C+F} \times U(1)_{C+F}$


non-Abeliar
Triplet (norr
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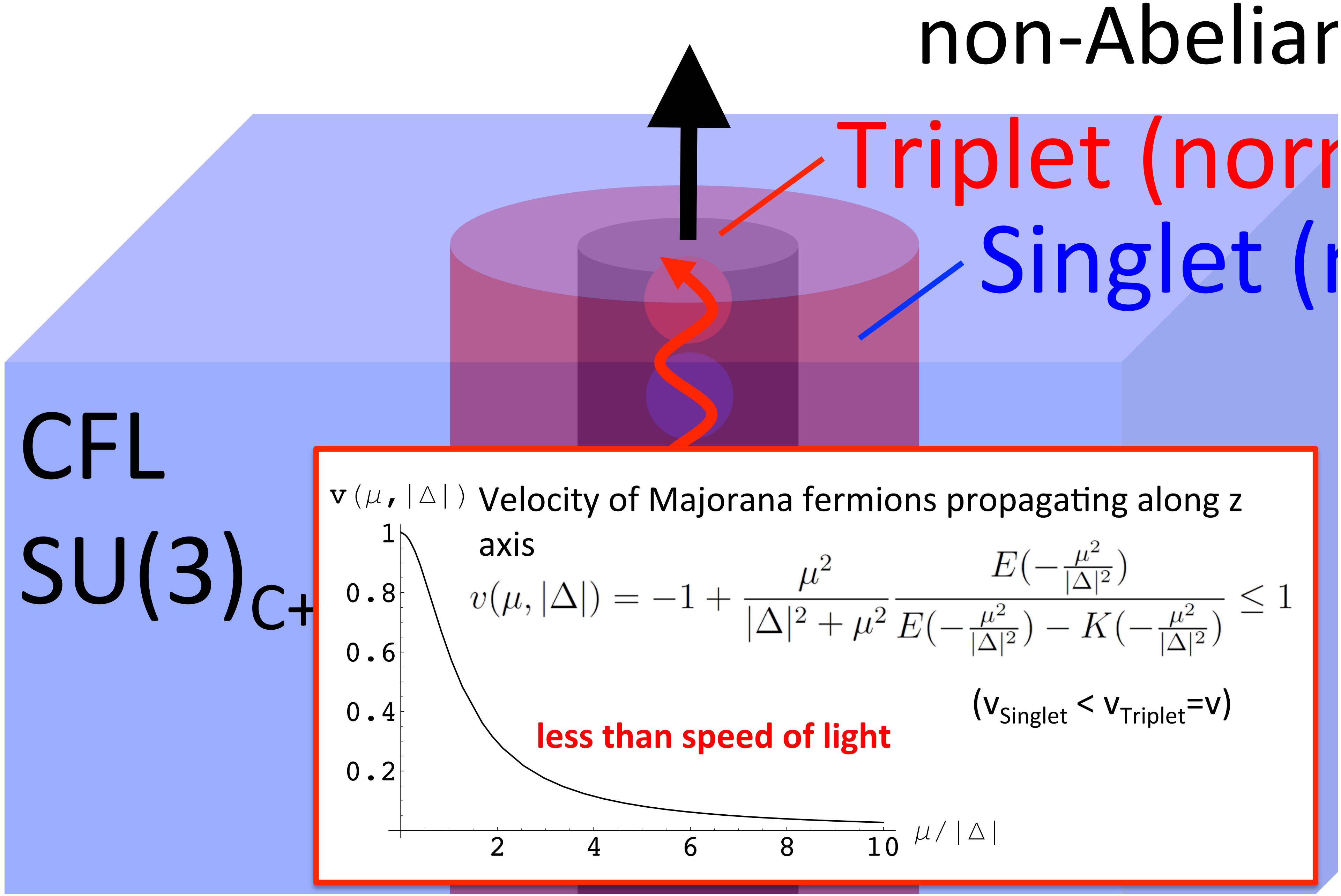
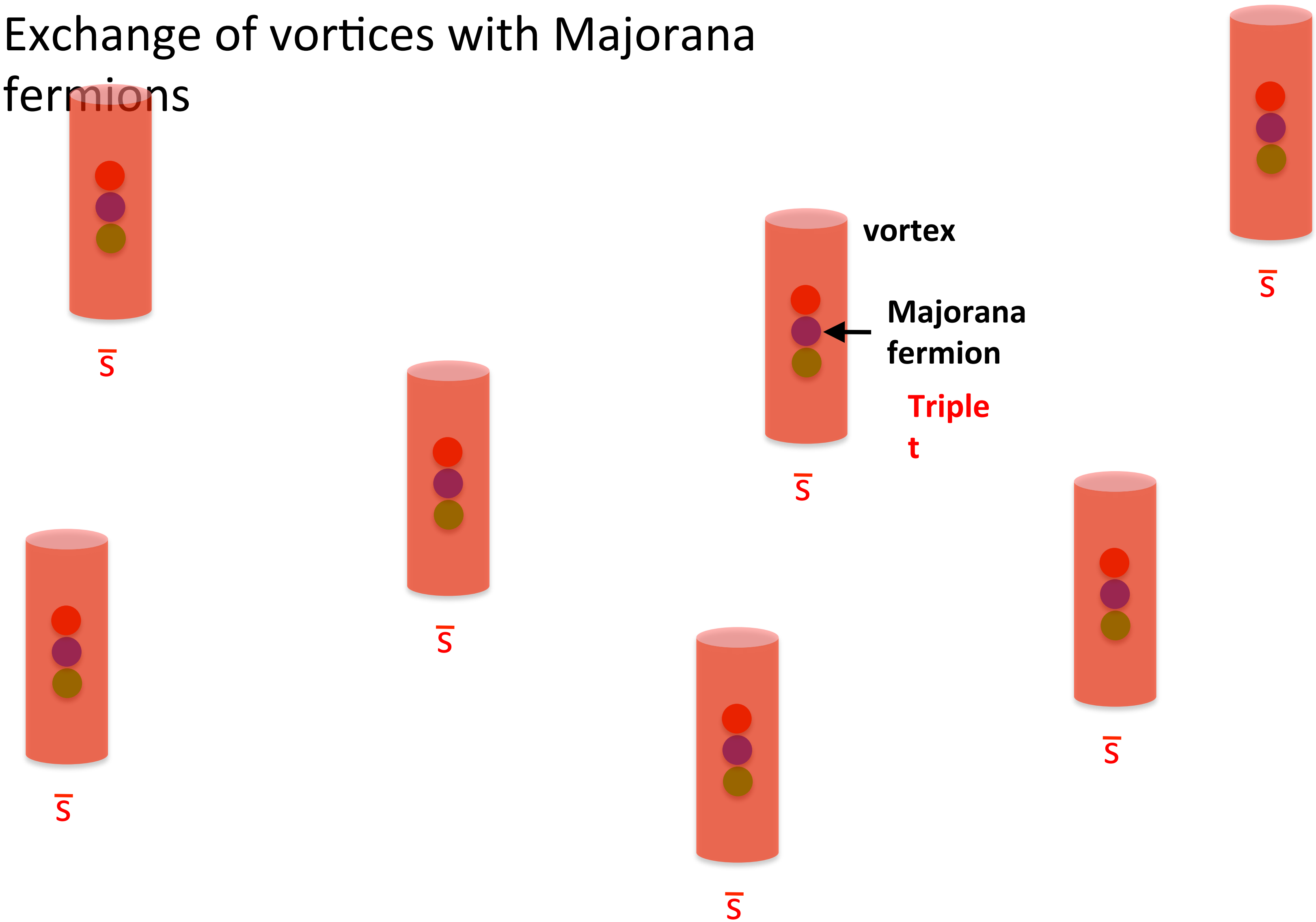


non-Abelia
Triplet (nor
Singlet (
CFL
$SU(3)_{C+}$
$v(\mu, |\Delta|)$
Velocity of Majorana fermions propagating along z axis
$$v(\mu,|\Delta|) = -1 + \frac{\mu^2}{|\Delta|^2+\mu^2}\frac{E(-\frac{\mu^2}{|\Delta|^2})}{E(-\frac{\mu^2}{|\Delta|^2}) - K(-\frac{\mu^2}{|\Delta|^2})} \leq 1$$
($v_{Singlet} < v_{Triplet} = v$)
less than speed of light
1
0.8
0.6
0.4
0.2
2
4
6
8
10
$\mu/|\Delta|$

1. Introduction
2. Majorana fermions in non-Abelian vortices
3. Non-Abelian statistics in non-Abelian voritces
4. Summary

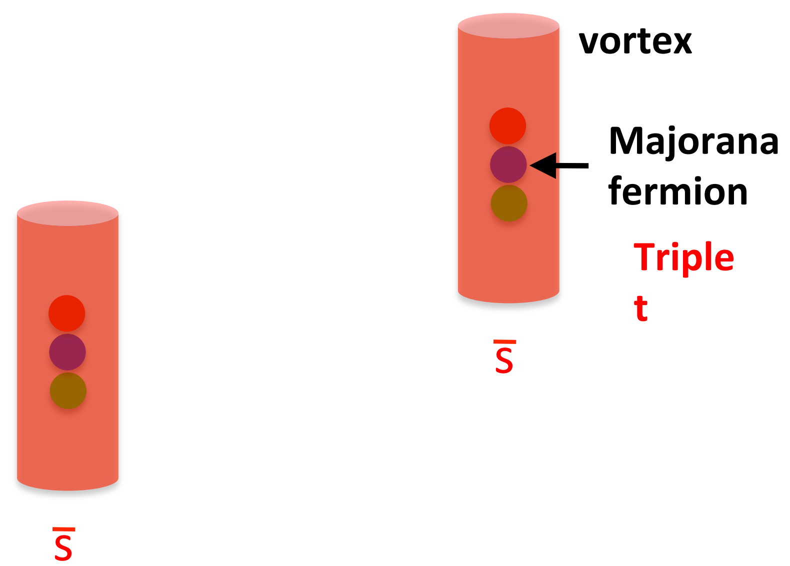
Non-Abelian statistics of non-Abelian vortices

Exchange of vortices with Majorana fermions



Non-Abelian statistics of non-Abelian vortices

Exchange of vortices with Majorana fermions



What is the quantum statistics?

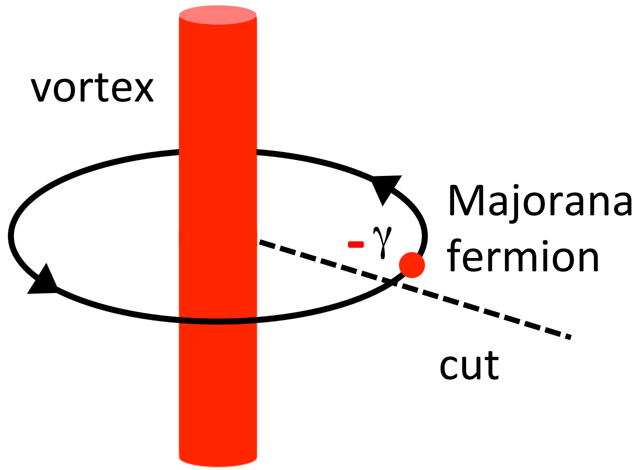
Non-Abelian statistics of non-Abelian vortices

Majorana operator γ

1) Majorana condition (particle=hole)

$$\gamma = \gamma^{\dagger}$$

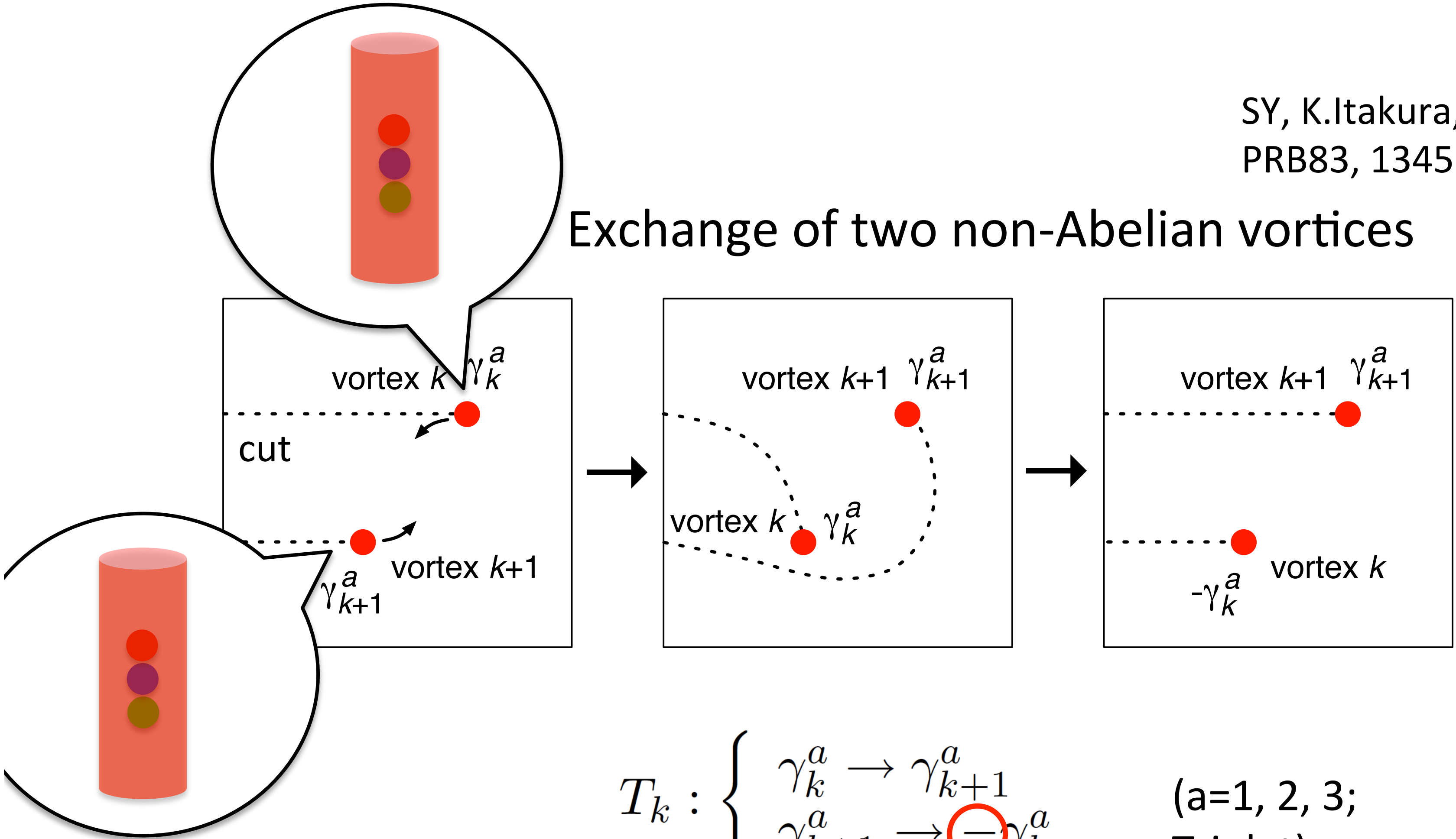
2) Phase changes by sign, when a Mojorana fermion turns around a vortex.



Non-Abelian statistics of non-Abelian vortices

SY, K.Itakura, M.Nitta,
PRB83, 134518 (2011)

Exchange of two non-Abelian vortices



$$T_k : \begin{cases} \gamma_k^a \longrightarrow \gamma_{k+1}^a \\ \gamma_{k+1}^a \longrightarrow -\gamma_k^a \end{cases}$$

(a=1, 2, 3; Triplet)

T_k is non-Abelian; $(T_k)^{-1} \neq T_k$

Cf. Abelian vortex: Ivanov, PRL86, 268 (2001)

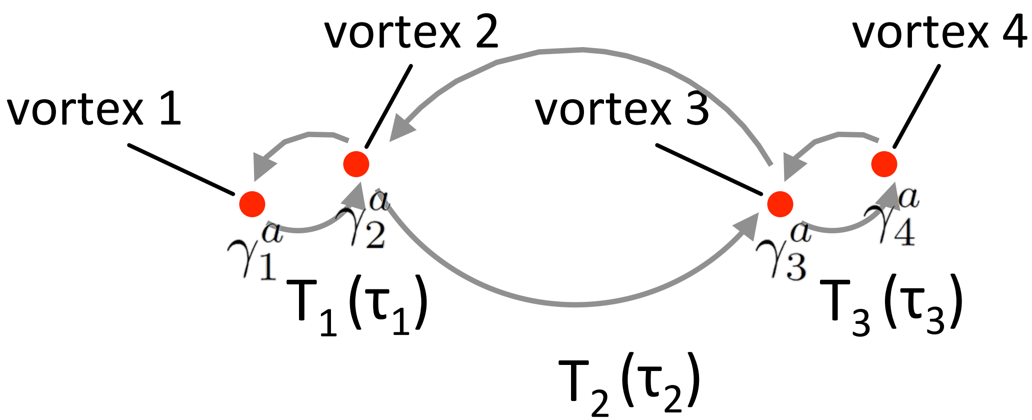
Cf. Bose/Fermi statistics

$T_1 \phi(1,2) = \phi(2,1)$

$= \pm \phi(1,2)$

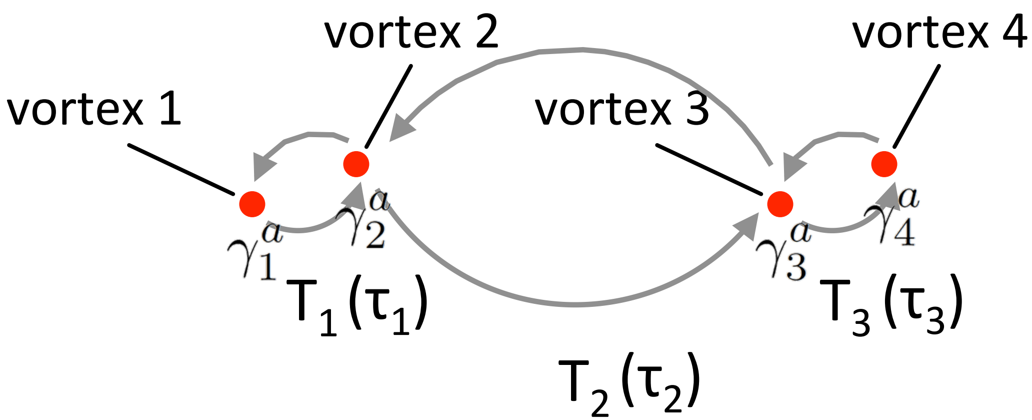
Non-Abelian statistics of non-Abelian vortices

Ex. n=4 non-Abelian vortices



Non-Abelian statistics of non-Abelian vortices

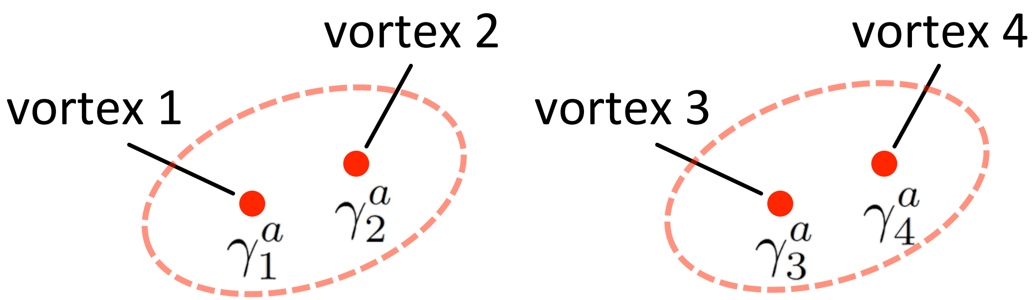
Ex. n=4 non-Abelian vortices



1) Exchange operator : $\tau_k = \prod_{a=1,2,3} \frac{1}{\sqrt{2}}(1 + \gamma_{k+1}^a \gamma_k^a) \qquad \gamma_l^a \to \tau_k \gamma_l^a \tau_k^{-1}$

Non-Abelian statistics of non-Abelian vortices

Ex. n=4 non-Abelian vortices



Dirac fermion operator (*non-local*) $\Psi_1^a = \frac{1}{2}(\gamma_1^a + i\gamma_2^a)$ $\quad \Psi_2^a = \frac{1}{2}(\gamma_3^a + i\gamma_4^a)$ (Triplet)

1) Exchange operator : $\tau_k = \prod_{a=1,2,3} \frac{1}{\sqrt{2}}(1 + \gamma_{k+1}^a \gamma_k^a)$ $\quad \gamma_l^a \to \tau_k \gamma_l^a \tau_k^{-1}$

2) Hilbert space : SU(2) representations $\{\Psi_k^a, \Psi_l^{b\dagger}\} = \delta_{kl}\delta^{ab}$

$$\mathbf{1}_{00} + \mathbf{1}_{33} + \mathbf{1}_{11} + \mathbf{1}_{22} + \mathbf{1}_{03} + \mathbf{1}_{30} + \mathbf{1}_{21} + \mathbf{1}_{12}$$ 8 Singlets

$$+ \ \mathbf{3}_{02} + \mathbf{3}_{31} + \mathbf{3}_{22} + \mathbf{3}_{11} + \mathbf{3}_{20} + \mathbf{3}_{13} + \mathbf{3}_{01} + \mathbf{3}_{32} + \mathbf{3}_{21} + \mathbf{3}_{12} + \mathbf{3}_{23} + \mathbf{3}_{10}$$ 12 Triplets

$$+ \ \mathbf{5}_{22} + \mathbf{5}_{11} + \mathbf{5}_{21} + \mathbf{5}_{12}$$ 4 Quintets

Non-Abelian statistics of non-Abelian vortices

Ex. n=4 non-Abelian vortices

Singlet

SU(2) Representation

$$\mathbf{1}_{00} + \mathbf{1}_{33} + \mathbf{1}_{11} + \mathbf{1}_{22} + \mathbf{1}_{03} + \mathbf{1}_{30} + \mathbf{1}_{21} + \mathbf{1}_{12}$$

$n_1 n_2$: n_1 (n_2) is number of Dirac fermions in pairing vortex 1, 2 (3, 4)

Parity: Even/Odd for total fermion number (n_1+n_2)

Singlet-Even

$$|\mathbf{1}_{00}\rangle = |0\rangle$$

$$|\mathbf{1}_{33}\rangle = i\frac{1}{3!}\epsilon^{abc}\frac{1}{3!}\epsilon^{def}\hat{\Psi}_1^{a\dagger}\hat{\Psi}_1^{b\dagger}\hat{\Psi}_1^{c\dagger}\hat{\Psi}_2^{d\dagger}\hat{\Psi}_2^{e\dagger}\hat{\Psi}$$

$$|\mathbf{1}_{11}\rangle = i\frac{1}{\sqrt{3}}\hat{\Psi}_1^{a\dagger}\hat{\Psi}_2^{a\dagger}|0\rangle$$

$$|\mathbf{1}_{22}\rangle = \frac{1}{\sqrt{3}}\frac{1}{2!}\epsilon^{abc}\frac{1}{2!}\epsilon^{ade}\hat{\Psi}_1^{b\dagger}\hat{\Psi}_1^{c\dagger}\hat{\Psi}_2^{d\dagger}\hat{\Psi}_2^{e\dagger}|0\rangle$$

Singlet-Odd

$$|\mathbf{1}_{03}\rangle = \frac{1}{3!}\epsilon^{abc}\hat{\Psi}_2^{a\dagger}\hat{\Psi}_2^{b\dagger}\hat{\Psi}_2^{c\dagger}|0\rangle$$

$$|\mathbf{1}_{30}\rangle = -i\frac{1}{3!}\epsilon^{abc}\hat{\Psi}_1^{a\dagger}\hat{\Psi}_1^{b\dagger}\hat{\Psi}_1^{c\dagger}|0\rangle$$

$$|\mathbf{1}_{21}\rangle = -\frac{1}{\sqrt{3}}\frac{1}{2!}\epsilon^{abc}\hat{\Psi}_1^{a\dagger}\hat{\Psi}_1^{b\dagger}\hat{\Psi}_2^{c\dagger}|0\rangle$$

$$|\mathbf{1}_{12}\rangle = i\frac{1}{\sqrt{3}}\frac{1}{2!}\epsilon^{abc}\hat{\Psi}_1^{a\dagger}\hat{\Psi}_2^{b\dagger}\hat{\Psi}_2^{c\dagger}|0\rangle$$

Non-Abelian statistics of non-Abelian vortices

Ex. n=4 non-Abelian vortices

Triplet

$$\mathbf{3}_{02} + \mathbf{3}_{31} + \mathbf{3}_{22} + \mathbf{3}_{11} + \mathbf{3}_{20} + \mathbf{3}_{13} + \mathbf{3}_{01} + \mathbf{3}_{32} + \mathbf{3}_{21} + \mathbf{3}_{12} + \mathbf{3}_{23} + \mathbf{3}_{10}$$

Triplet-Even

$$|\mathbf{3}_{02}\rangle = \frac{1}{2!}\epsilon^{abc}\hat{\Psi}_2^{b\dagger}\hat{\Psi}_2^{c\dagger}|0\rangle$$

$$|\mathbf{3}_{31}\rangle = -i\frac{1}{3!}\epsilon^{bcd}\hat{\Psi}_1^{b\dagger}\hat{\Psi}_1^{c\dagger}\hat{\Psi}_1^{d\dagger}\hat{\Psi}_2^{a\dagger}|0\rangle$$

$$|\mathbf{3}_{22}\rangle = \frac{1}{\sqrt{2}}\epsilon^{abc}\frac{1}{2!}\epsilon^{bde}\frac{1}{2!}\epsilon^{cfg}\hat{\Psi}_1^{d\dagger}\hat{\Psi}_1^{e\dagger}\hat{\Psi}_2^{f\dagger}\hat{\Psi}_2^{g\dagger}|0\rangle$$

$$|\mathbf{3}_{11}\rangle = i\frac{1}{\sqrt{2}}\epsilon^{abc}\hat{\Psi}_1^{b\dagger}\hat{\Psi}_2^{c\dagger}|0\rangle$$

$$|\mathbf{3}_{20}\rangle = -\frac{1}{2!}\epsilon^{abc}\hat{\Psi}_1^{b\dagger}\hat{\Psi}_1^{c\dagger}|0\rangle$$

$$|\mathbf{3}_{13}\rangle = i\frac{1}{3!}\epsilon^{bcd}\hat{\Psi}_1^{a\dagger}\hat{\Psi}_2^{b\dagger}\hat{\Psi}_2^{c\dagger}\hat{\Psi}_2^{d\dagger}|0\rangle$$

Triplet-Odd

$$|\mathbf{3}_{01}\rangle = \hat{\Psi}_2^{a\dagger}|0\rangle$$

$$|\mathbf{3}_{32}\rangle = i\frac{1}{3!}\epsilon^{bcd}\hat{\Psi}_1^{b\dagger}\hat{\Psi}_1^{c\dagger}\hat{\Psi}_1^{d\dagger}\frac{1}{2!}\epsilon^{aef}\hat{\Psi}_2^{e\dagger}\hat{\Psi}_2^{f\dagger}|0\rangle$$

$$|\mathbf{3}_{21}\rangle = \frac{1}{\sqrt{2}}\epsilon^{abc}\frac{1}{2!}\epsilon^{bde}\hat{\Psi}_1^{d\dagger}\hat{\Psi}_1^{e\dagger}\hat{\Psi}_2^{c\dagger}|0\rangle$$

$$|\mathbf{3}_{12}\rangle = -i\frac{1}{\sqrt{3}}\epsilon^{abc}\frac{1}{2!}\epsilon^{cde}\hat{\Psi}_1^{b\dagger}\hat{\Psi}_2^{d\dagger}\hat{\Psi}_2^{e\dagger}|0\rangle$$

$$|\mathbf{3}_{23}\rangle = \frac{1}{2!}\epsilon^{abc}\hat{\Psi}_1^{b\dagger}\hat{\Psi}_1^{c\dagger}\frac{1}{3!}\epsilon^{def}\hat{\Psi}_2^{d\dagger}\hat{\Psi}_2^{e\dagger}\hat{\Psi}_2^{f\dagger}|0\rangle$$

$$|\mathbf{3}_{10}\rangle = i\hat{\Psi}_1^{a\dagger}|0\rangle$$

Non-Abelian statistics of non-Abelian vortices

Ex. n=4 non-Abelian vortices

Quintet

$$\mathbf{5}_{22} + \mathbf{5}_{11} + \mathbf{5}_{21} + \mathbf{5}_{12}$$

Quintet-Even

$$|\mathbf{5}_{22}\rangle = i\mathcal{N}\left[\frac{1}{2}\left\{\frac{1}{2!}\epsilon^{acd}\hat{\Psi}_1^{c\dagger}\hat{\Psi}_1^{d\dagger}\frac{1}{2!}\epsilon^{bef}\hat{\Psi}_2^{e\dagger}\hat{\Psi}_2^{f\dagger} + \frac{1}{2!}\epsilon^{bcd}\hat{\Psi}_1^{c\dagger}\hat{\Psi}_1^{d\dagger}\frac{1}{2!}\epsilon^{aef}\hat{\Psi}_2^{e\dagger}\hat{\Psi}_2^{f\dagger}\right\}\right.$$
$$\left. - \frac{\delta^{ab}}{3}\frac{1}{2!}\epsilon^{cde}\hat{\Psi}_1^{d\dagger}\hat{\Psi}_1^{e\dagger}\frac{1}{2!}\epsilon^{cfg}\hat{\Psi}_2^{f\dagger}\hat{\Psi}_2^{g\dagger}\right]|0\rangle$$

$$|\mathbf{5}_{11}\rangle = -\mathcal{N}\left[\frac{1}{2}\left\{\hat{\Psi}_1^{a\dagger}\hat{\Psi}_2^{b\dagger} + \hat{\Psi}_1^{b\dagger}\hat{\Psi}_2^{a\dagger}\right\} - \frac{\delta^{ab}}{3}\hat{\Psi}_1^{c\dagger}\hat{\Psi}_2^{c\dagger}\right]|0\rangle$$

Quintet-Odd

$$|\mathbf{5}_{21}\rangle = -i\mathcal{N}\left[\frac{1}{2}\left\{\frac{1}{2!}\epsilon^{acd}\hat{\Psi}_1^{c\dagger}\hat{\Psi}_1^{d\dagger}\hat{\Psi}_2^{b\dagger} + \frac{1}{2!}\epsilon^{bcd}\hat{\Psi}_1^{c\dagger}\hat{\Psi}_1^{d\dagger}\hat{\Psi}_2^{a\dagger}\right\} - \frac{\delta^{ab}}{3}\frac{1}{2!}\epsilon^{cde}\hat{\Psi}_1^{c\dagger}\hat{\Psi}_1^{d\dagger}\hat{\Psi}_2^{e\dagger}\right]|0\rangle$$

$$|\mathbf{5}_{12}\rangle = -\mathcal{N}\left[\frac{1}{2}\left\{\hat{\Psi}_1^{a\dagger}\frac{1}{2!}\epsilon^{bcd}\hat{\Psi}_2^{c\dagger}\hat{\Psi}_2^{d\dagger} + \hat{\Psi}_1^{b\dagger}\frac{1}{2!}\epsilon^{acd}\hat{\Psi}_2^{c\dagger}\hat{\Psi}_2^{d\dagger}\right\} - \frac{\delta^{ab}}{3}\frac{1}{2!}\epsilon^{cde}\hat{\Psi}_1^{c\dagger}\hat{\Psi}_2^{d\dagger}\hat{\Psi}_2^{e\dagger}\right]|0\rangle$$

Non-Abelian statistics of non-Abelian vortices

Ex. n=4 non-Abelian vortices

k=1, 2, 3

Matrix of τ_k $$\boxed{\tau_k^{\mathcal{M},\mathcal{P}} = \sigma_k^{\mathcal{M}} \otimes h_k^{\mathcal{P}}}$$

$\mathcal{M} = \mathbf{1}, \mathbf{3}, \mathbf{5}$ **SU(2)**

$\mathcal{P} = \mathcal{E}(\mathrm{even}), \mathcal{O}(\mathrm{odd})$ **Parit y**

Non-Abelian statistics of non-Abelian vortices

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$\mathcal{P} = \mathcal{E}(\mathrm{even}), \mathcal{O}(\mathrm{odd})$ **Parity**

h_k^P : Ivanov's matrices (*Abelian* vortices) Cf. Ivanov, PRL86, 268 (2011)

$$h_1^{\mathcal{E}} = h_1^{\mathcal{O}} = \begin{pmatrix} e^{i\frac{\pi}{4}} & 0 \\ 0 & e^{-i\frac{\pi}{4}} \end{pmatrix} \qquad h_2^{\mathcal{E}} = h_2^{\mathcal{O}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \qquad h_3^{\mathcal{E}} = h_3^{\mathcal{O}\dagger} = h_1^{\mathcal{E}}$$

Non-Abelian statistics of non-Abelian vortices

Ex. n=4 non-Abelian vortices

k=1, 2, 3

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σ_k^M : Color-Flavor SU(2) **NEW for *non-Abelian* vortices !!**

Singlet

$$\sigma_1^{\mathbf{1}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_2^{\mathbf{1}} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \quad \sigma_3^{\mathbf{1}} = \sigma_1^{\mathbf{1}}$$

Triplet

$$\sigma_1^{\mathbf{3}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \sigma_2^{\mathbf{3}} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix} \quad \sigma_3^{\mathbf{3}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Quintet

$$\sigma_1^{\mathbf{5}} = \sigma_2^{\mathbf{5}} = \sigma_3^{\mathbf{5}} = 1$$

Non-Abelian statistics of non-Abelian vortices

Ex. n=4 non-Abelian vortices

k=1, 2, 3

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Triplet

$$\sigma_1^{\mathbf{3}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \sigma_2^{\mathbf{3}} = \frac{1}{2} \begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix} \quad \sigma_3^{\mathbf{3}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Quintet

$$\sigma_1^{\mathbf{5}} = \sigma_2^{\mathbf{5}} = \sigma_3^{\mathbf{5}} = 1$$

SU(2) *independent* (singlet) $h_k^P \otimes$ SU(2) *dependent* σ_k^M !!!

$\sigma_k{}^M$: Color-Flavor SU(2) **NEW for *non-Abelian* vortices !!**

Singlet $\sigma_1^{\mathbf{1}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $\sigma_2^{\mathbf{1}} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$ $\sigma_3^{\mathbf{1}} = \sigma_1^{\mathbf{1}}$

Triplet

$$\sigma_1^{\mathbf{3}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \sigma_2^{\mathbf{3}} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix} \quad \sigma_3^{\mathbf{3}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Quintet

$$\sigma_1^{\mathbf{5}} = \sigma_2^{\mathbf{5}} = \sigma_3^{\mathbf{5}} = 1$$

$\sigma_k{}^M$: Color-Flavor SU(2) **NEW for *non-Abelian* vortices !!**

Singlet

$$\sigma_1^{\mathbf{1}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_2^{\mathbf{1}} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \quad \sigma_3^{\mathbf{1}} = \sigma_1^{\mathbf{1}}$$

Triplet

$$\sigma_1^{\mathbf{3}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \sigma_2^{\mathbf{3}} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix} \quad \sigma_3^{\mathbf{3}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Quintet

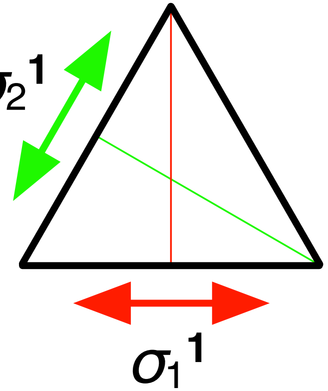
$$\sigma_1^{\mathbf{5}} = \sigma_2^{\mathbf{5}} = \sigma_3^{\mathbf{5}} = 1$$

H. S. M. Coxeter
(1907-2003)

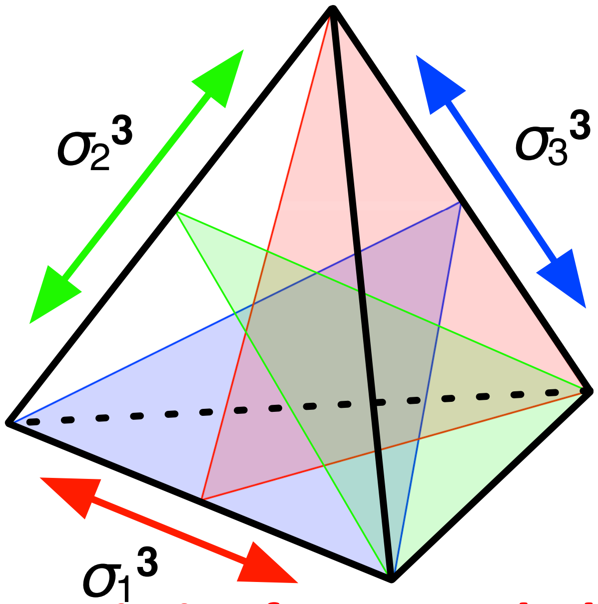
"Coxeter group"

In the Hilbert space ...

singlet (**1**)



triplet (**3**)



Polytopes appear in non-Abelian statistics for *non-Abelian* vortices !!

2-simplex (triangle) 3-simplex (tetrahedron)

Summary

- **Zero-mode Majorana fermions exist in CFL non-Abelian vortices.**
- **Non-Abelian statistics appears in non-Abelian vortices.**

M_1 : Δ=diag($e^{i\theta}$, 1, 1)

- **Majorana vortices (Majorana fermion; *non-local* Hilbert space)**

Abelian vortex → Ivanov's matrices Ivanov, PRL86, 268 (2001)

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Abelian vortex → Non-Abelian statistics SY, Itakura, Nitta, NPB859, 261 (2012)

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- **Applications to compact stars?**

The End