Numerical relativity and boost-invariant plasma thermalization

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M. Heller, RJ, P. Witaszczyk, work in progress
M. Heller, RJ, P. Witaszczyk, 1103.3452 (physics)
M. Heller, RJ, P. Witaszczyk, 1203.0755 (technical details)

Outline

Motivation

Point of reference: heavy-ion collision at RHIC/LHC:

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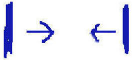

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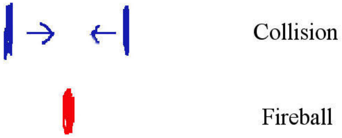

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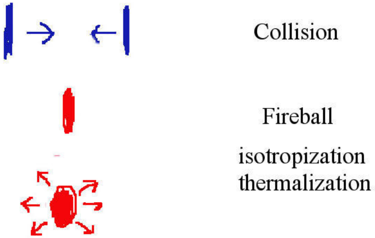

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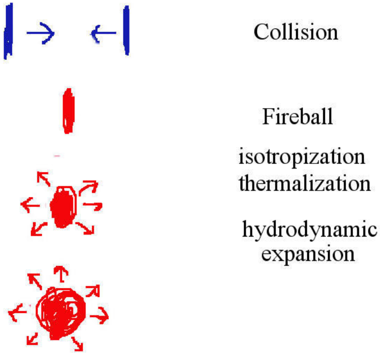

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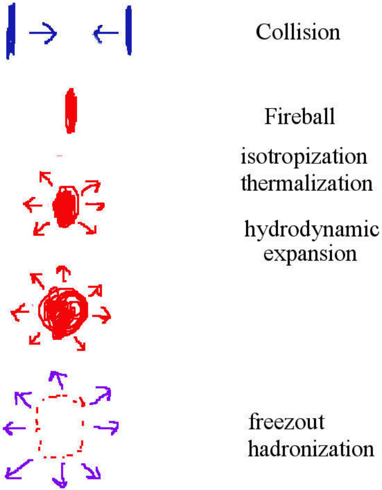

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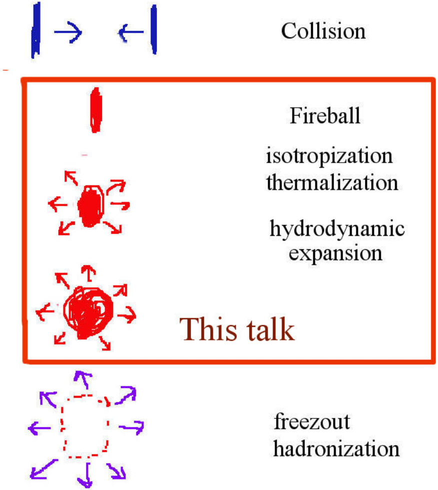

Key question:

Why can we apply a hydrodynamic description so early after the collision?

This problem is commonly reformulated as the problem of early thermalization (since local thermal equilibrium is commonly assumed to be a prerequisite of thermalization)

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Thermalization

- At weak coupling the obvious definition would be to require thermal momentum distributions for quarks and gluons...
- At strong coupling, the picture of a gas of gluons is not really valid — alternatively require that observables such as 2-point functions/spatial Wilson loops/ entanglement entropy are the same as for a thermal system...

  explored in the AdS/CFT context
- This is very good for studying relaxation processes where the final state is some uniform static plasma system — this is not so for the plasma undergoing expansion
- For an expanding plasma fireball we need *local* equilibrium — bilocal probes get contaminated by collective flow
- We adopt an *operational* definition of effective thermalization — the point when plasma starts being describable by (viscous) hydrodynamics.

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- **Hydrodynamics** isolates long wavelength effective degrees of freedom of a theory
- The energy-momentum tensor $T_{\mu\nu}$ is expressed in terms of a local temperature T and flow velocity u^μ
- $T_{\mu\nu}$ is expressed as an expansion in the gradients of the flow velocities (shown here for $\mathcal{N}=4$ SYM)

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu}+4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

$$+ \underbrace{(\pi T^2)\left(\log 2\, T^{\mu\nu}_{2a} + 2\, T^{\mu\nu}_{2b} + (2-\log 2)\left(\frac{1}{3}T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

- The coefficients of the various tensor structures are the transport coefficients. In a conformal theory these are pure numbers times powers of T.
- Full nonlinear hydrodynamic equations follow now from $\partial_\mu T^{\mu\nu}=0$
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The approach of [Bhattacharyya, Hubeny, Minwalla, Rangamani]

- Start from a static black hole with fixed temperature T which describes a fluid at rest, $u^\mu = (1, 0, 0, 0)$ with constant energy density
- Perform a boost to obtain a uniform fluid moving with constant velocity u^μ
- The resulting metric (in Eddington-Finkelstein coordinates) is

$$ds^2 = -2u_\mu dx^\mu dr - r^2 \left(1 - \frac{T^4}{\pi^4 r^4}\right) u_\mu u_\nu dx^\mu dx^\nu + r^2(\eta_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

where $r = \infty$ corresponds to the boundary, $r = T/\pi$ is the horizon while $r = 0$ is the position of the singularity.

$\boxed{\text{Promote } T \text{ and } u^\mu \text{ to (slowly-varying) functions of } x^\mu}$

Caveat: The metric is no longer an exact solution of Einstein's equations

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- Perform an expansion of the Einstein equations in gradients of spacetime fields.
- Find corrections to the metric at first and second order
- Require nonsingularity to fix integration constants
- Read off the resulting energy-momentum tensor $T_{\mu\nu}$
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Fluid/gravity duality versus nonequilibrium physics

- Perform an expansion of the Einstein equations in gradients of spacetime fields.
- Find corrections to the metric at first and second order
- Require nonsingularity to fix integration constants
- Read off the resulting energy-momentum tensor $T_{\mu\nu}$
- $T_{\mu\nu}$ is expressed in terms u^μ and T and their derivatives

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

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- Fluid/gravity duality is an expansion around some specific 0^{th} order geometry — this 0^{th} order geometry need not be relevant for the appropriate physics
- There exist interesting examples which are 'orthogonal' to hydrodynamics — cannot be described at all within this framework
  Example: isotropisation of uniform anisotropic plasma

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p_{\|}(t) & 0 & 0 \\ 0 & 0 & p_{\perp}(t) & 0 \\ 0 & 0 & 0 & p_{\perp}(t) \end{pmatrix}$$

- Plasma equilibration in heavy-ion collisions is a mixture of both types of physics...
- In the boost-invariant setting we may unambigously determine when deviations from (even all-order) viscous fluid dynamics start to be important
- Physically this means that then nonhydrodynamic degrees of freedom become relevant...

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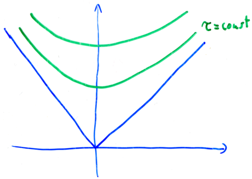
Boost-invariant flow

Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- In a conformal theory, $T^{\mu}_{\mu} = 0$ and $\partial_\mu T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- The longitudinal and transverse pressures are then given by

$$p_L = -\varepsilon - \tau\frac{d}{d\tau}\varepsilon \quad \text{and} \quad p_T = \varepsilon + \frac{1}{2}\tau\frac{d}{d\tau}\varepsilon\,.$$

- In this setting we may determine whether all-order viscous hydrodynamics is applicable (even without knowing its explicit form)
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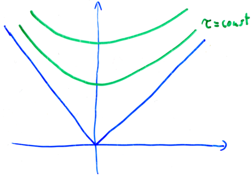

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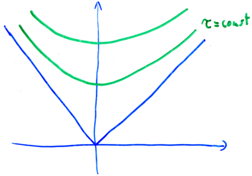

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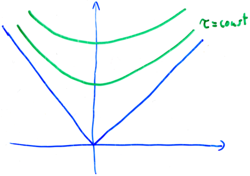

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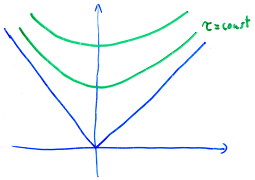

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Large τ behaviour of $\varepsilon(\tau)$

- Current result for large τ: RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}}\frac{1}{\tau^2} + \frac{1+2\log 2}{12\sqrt{3}}\frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}}\frac{1}{\tau^{\frac{10}{3}}} + \ldots$$

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Question 1: If we start from various initial conditions at $\tau = 0$ when does the above hydrodynamic form of $\varepsilon(\tau)$ starts being applicable?

Question 2: When are nonhydrodynamic degrees of freedom relevant for the plasma evolution?

Large τ behaviour of $\varepsilon(\tau)$

- New results for large τ: Work in progress

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - 0.6204\frac{1}{\tau^2} + 0.1148\frac{1}{\tau^{\frac{8}{3}}} + 0.03622\frac{1}{\tau^{\frac{10}{3}}} + 0.009934\frac{1}{\tau^{\frac{12}{3}}} + +0.0007284\frac{1}{\tau^{\frac{14}{3}}} + \ldots + \mathcal{O}\left(\tau^{-\frac{24}{3}}\right)$$

- Obtained by iteratively solving numerically equations within the fluid-gravity duality
- We can explore convergence properties of the hydrodynamic description
- Relevant for 'small initial data'
- Phenomenological 'all-order' proposals were put forward by Lublinski and Shuryak

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Aim: Study the evolution of $\varepsilon(\tau)$ all the way from $\tau = 0$ to large τ starting from various initial conditions and investigate the transition to hydrodynamic behaviour...

Method: Describe the time dependent evolving strongly coupled plasma system through a dual 5D geometry — given e.g. by

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z) dx^\mu dx^\nu + dz^2}{z^2} \equiv g^{5D}_{\alpha\beta} dx^\alpha dx^\beta$$

i) use Einstein's equations for the time evolution

$$R_{\alpha\beta} - \frac{1}{2} g^{5D}_{\alpha\beta} R - 6\, g^{5D}_{\alpha\beta} = 0$$

ii) read off $\langle T_{\mu\nu}(x^\rho) \rangle$ from the numerical metric $g_{\mu\nu}(x^\rho, z)$

$$g_{\mu\nu}(x^\rho, z) = \eta_{\mu\nu} + z^4 g^{(4)}_{\mu\nu}(x^\rho) + \ldots \qquad \langle T_{\mu\nu}(x^\rho) \rangle = \frac{N_c^2}{2\pi^2} \cdot g^{(4)}_{\mu\nu}(x^\rho)$$

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Initial conditions for the evolution of the plasma system

- Our point of departure — start with *arbitrary* initial conditions and look for common features/regularities
- In weakly coupled gauge theory, the analog would be to start from arbitrary momentum distributions of gluons and follow the evolution until equilibration
- At strong coupling the analog is a specific initial geometry in the bulk
- However, not unexpectedly, there is no direct quantitative interpretation in terms of e.g. gluon momenta distributions

We developed a quite different approach from Chesler's and Yaffe's since

1. We want to study the evolution right from $\tau = 0$ with energy-momentum conservation satisified throughout the evolution
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$$ds^2 = \frac{1}{z^2}\left(-e^{a(z,\tau)}d\tau^2 + e^{b(z,\tau)}\tau^2 dy^2 + e^{c(z,\tau)}dx_\perp^2\right) + \frac{dz^2}{z^2}$$

- The initial conditions are determined in terms of a *single* function, say $c_0(z)$. $a_0(z) = b_0(z)$ are determined through a constraint equation.
- A typical solution of the constraint equations is

$$a_0(z) = b_0(z) = 2\log\cos z^2 \qquad c_0(z) = 2\log\cosh z^2$$

- There is a *coordinate* singularity at $z = \sqrt{\pi/2}$ where

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The metric ansatz and numerical formalism

- **The key problem is what boundary conditions to impose in the bulk.** For a sample initial profile $c_0(u) = \cosh u$ ($u \equiv z^2$), there is a curvature singularity at $u = \infty$.
- *A-priori* we do not know where is the event horizon!
- We use the ADM freedom of foliation to ensure that all hypersurfaces end on a single spacetime point in the bulk — this ensures that we will control the boundary conditions even though they may be in a strongly curved part of the spacetime
- This also ensures that no information flows from outside our region of integration...
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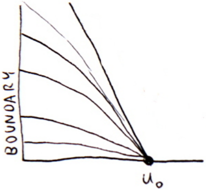

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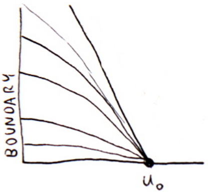

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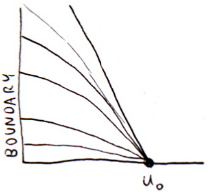

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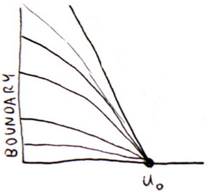

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- Depending on the relation of u_0 to the event horizon we can get quite different behaviours of the numerical simulation

- In order to extend the simulation to large values of τ neccessary for observing the transition to hydrodynamics we need to tune u_0 to be close to the event horizon.
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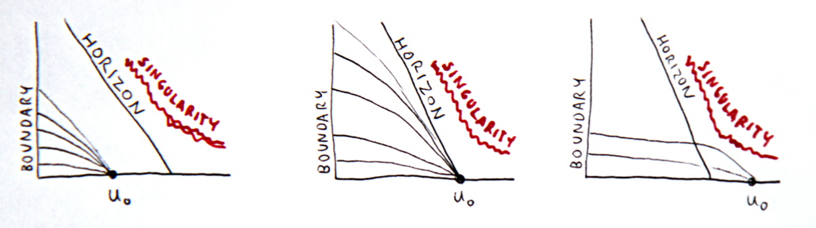

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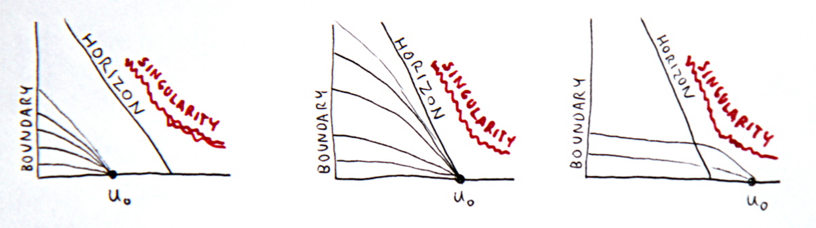

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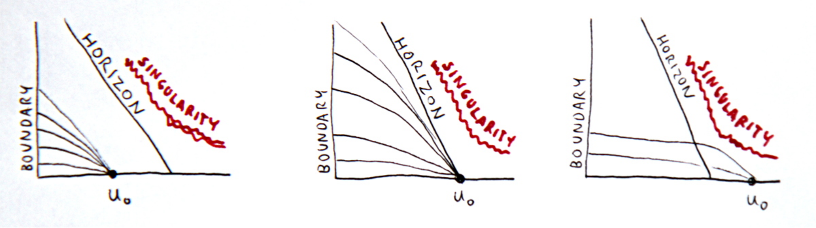

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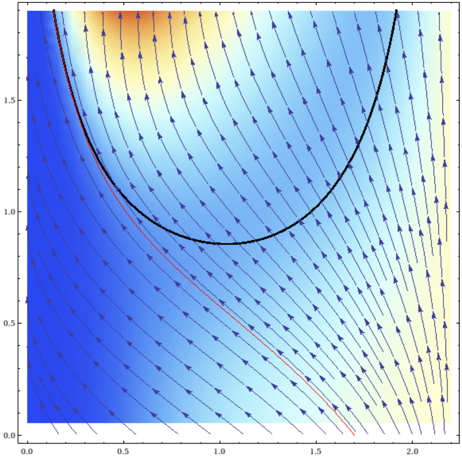

black line – dynamical horizon, arrows – null geodesics, colors represent curvature

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- Consequently, we set the (nondynamical) function $a(u)$ to

$$a(u) = \cos\left(\frac{\pi}{2}\frac{u}{u_0}\right)$$

- The remaining part of the lapse, $\alpha(t,u)$ is chosen to be a function of the metric coefficients

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- We have used 29 initial geometries at $\tau = 0$ which encode the initial conditions for the boost-invariant plasma system
- Technically each geometry is determined by a choice of the metric coefficient $c(\tau = 0, u)$.
- We have chosen quite different looking profiles e.g.

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c_1(u) &= \cosh u \\
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- We have chosen quite different looking profiles e.g.

$$
\begin{aligned}
c_1(u) &= \cosh u \\
c_3(u) &= 1 + \frac{1}{2} u^2 \\
c_7(u) &= 1 + \frac{\frac{1}{2} u^2}{1 + \frac{3}{2} u^2} \\
c_{10}(u) &= 1 + \frac{1}{2} u^2 e^{-\frac{u}{2}} \\
c_{15}(u) &= 1 + \frac{1}{2} u^2 e^{u} \\
c_{19}(u) &= 1 + \frac{1}{2} \tanh^2 \left(u + \frac{1}{25} u^2 \right)
\end{aligned}
$$

Some kinematic variables

It is convenient to eliminate explicit dependence on the number of degrees of freedom and use an *effective* temperature T_{eff} instead of $\varepsilon(\tau)$

$$\langle T_{\tau\tau} \rangle \equiv \varepsilon(\tau) \equiv N_c^2 \cdot \frac{3}{8}\pi^2 \cdot {T_{eff}}^4$$

Introduce the dimensionless quantity

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Nonequilibrium vs. hydrodynamic behaviour

Question: Can we describe the plasma system using just a flow velocity u^μ and (arbitrary number of) transport coefficients?

- Viscous hydrodynamics (up to any order in the gradient expansion) leads to equations of motion of the form

$$\frac{\tau}{w}\frac{d}{d\tau}w = \frac{F_{hydro}(w)}{w}$$

where $F_{hydro}(w)$ is a *universal function* completely determined in terms of the hydrodynamic transport coefficients e.g.

$$\frac{F^{\mathcal{N}=4}_{hydro}(w)}{w} = \frac{2}{3} + \frac{1}{9\pi w} + \frac{1-\log 2}{27\pi^2 w^2} + \frac{15 - 2\pi^2 - 45\log 2 + 24\log^2 2}{972\pi^3 w^3} + \ldots$$

- Therefore if plasma dynamics would be given by viscous hydrodynamics (even of arbitrary high order) a plot of $F(w) \equiv \tau \frac{d}{d\tau} w$ as a function of w would be a single curve for all the initial conditions
- Genuine nonequilibrium dynamics would, in contrast, lead to several curves...

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A plot of $F(w)/w$ versus w for various initial data

Questions:

i) How good is the agreement with hydrodynamics?

ii) To what extent is the plasma there truly thermalized?

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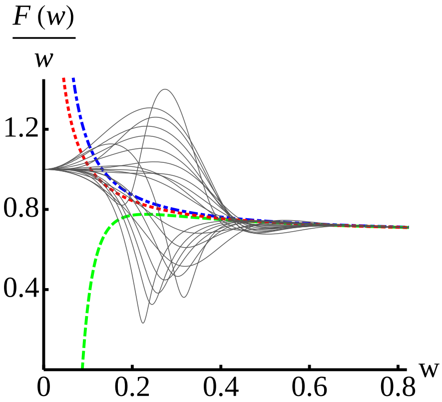

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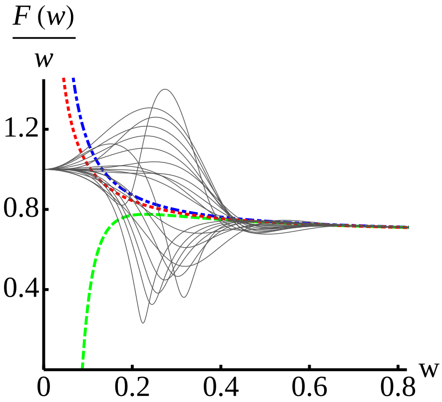

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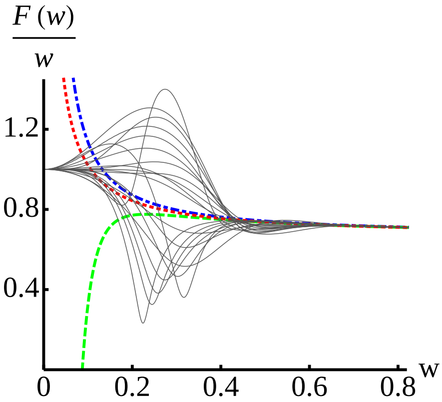

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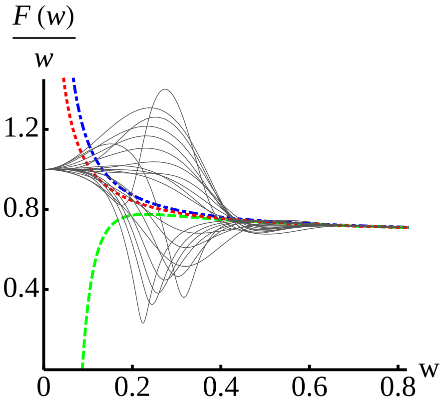

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- An observable sensitive to the details of the dissipative dynamics (e.g. hydrodynamics) is the pressure anisotropy

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- For a perfect fluid $\Delta p_L \equiv 0$. For a sample initial profile we get

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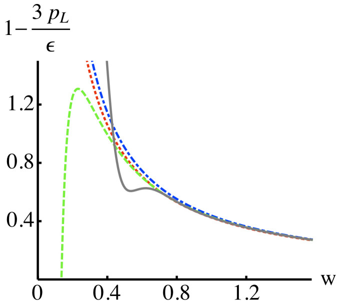

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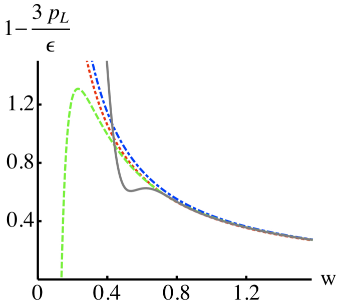

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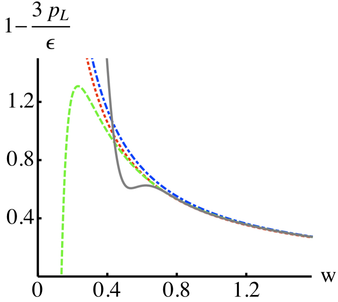

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- It is even not clear whether an exact *local* notion makes sense on the QFT side...
- However, *phenomenological* notion of local entropy density is widely used in (dissipative) hydrodynamics
- On the AdS side entropy is obtained from the area element of **an apparent horizon** joined to the point on the boundary by **a null geodesic**
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$$s = \frac{S}{\frac{1}{2} N_c^2 \pi^2 T_{eff}^2(0)}$$

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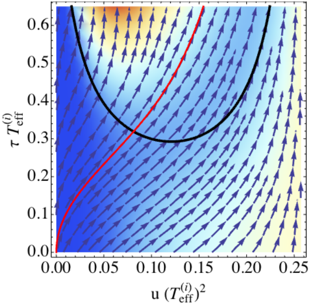

Final entropy

- For large proper-time, the dynamics is given by hydrodynamics, leading to the large τ expansion

$$T_{eff}(\tau)=\frac{\Lambda}{(\Lambda\tau)^{1/3}}\left\{1-\frac{1}{6\pi(\Lambda\tau)^{2/3}}+\frac{-1+\log 2}{36\pi^2(\Lambda\tau)^{4/3}}+\frac{-21+2\pi^2+51\log 2-24\log^2 2}{1944\pi^3(\Lambda\tau)^2+\ldots}\right\}$$

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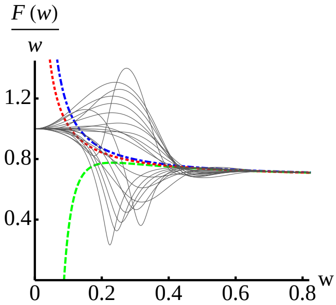

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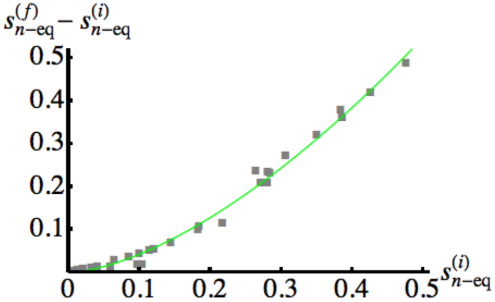

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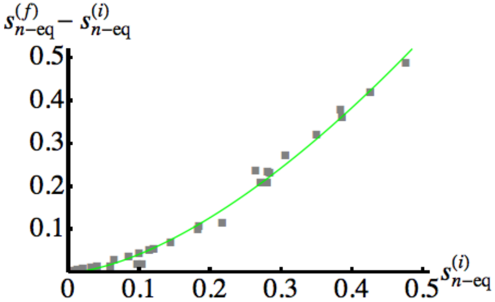

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Properties of (effective) thermalization

- We want to study systematically the properties of the plasma at the point when the dynamics becomes describable by viscous hydrodynamics...
- We adopted a numerical criterion for (effective) thermalization

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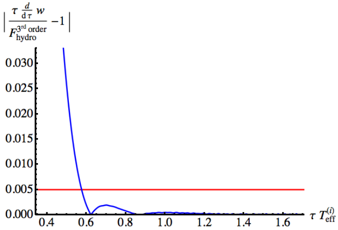

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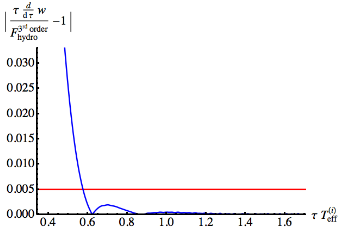

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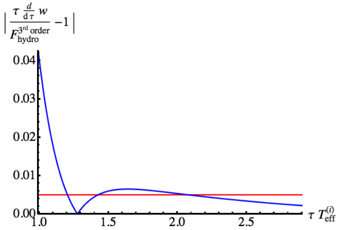

small entropy initial data

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- N.B. sample initial conditions for hydrodynamics at RHIC ($\tau_0 = 0.25\, fm$, $T_0 = 500\, MeV$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to $w = 0.63$
- The pressure anisotropy at thermalization is still sizable

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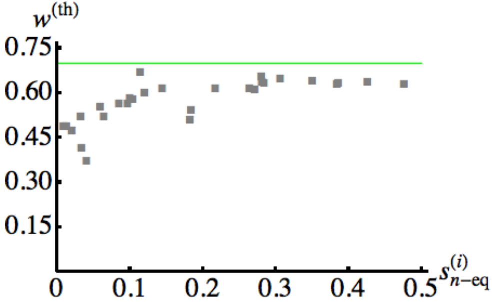

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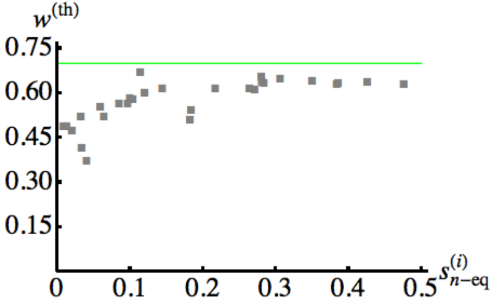

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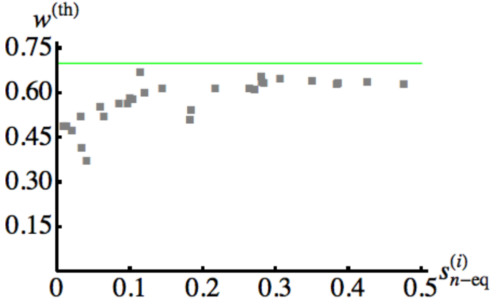

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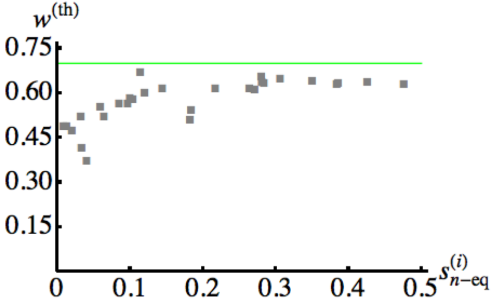

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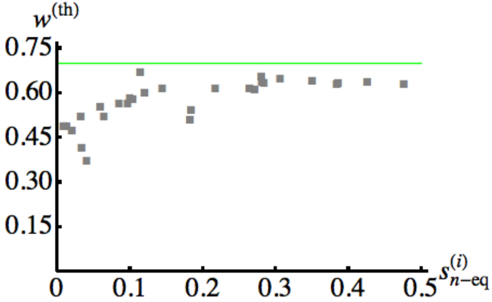

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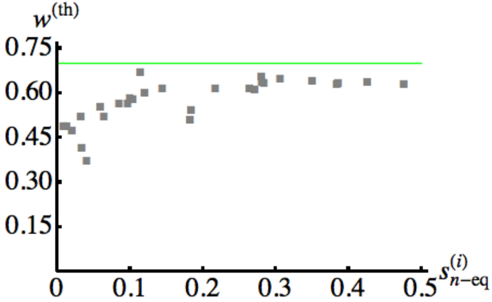

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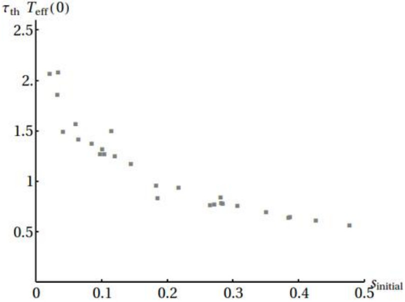

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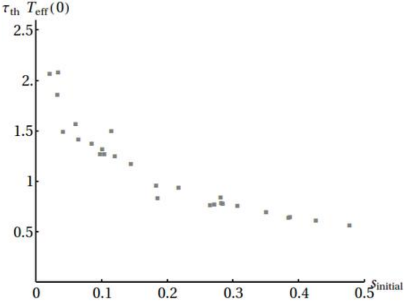

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- For $w = T_{th} \cdot \tau_{th} > 0.7$ we observe hydrodynamic behaviour but with sizeable pressure anisotropy (described wholly by viscous hydrodynamics)
- The plasma system at the transition to hydrodynamic description is still quite far from true thermal equilibrium (in agreement with results obtained by Chesler, Yaffe for their specific initial states)
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