Can We Constrain or Even Rule Out Multifield Inflation Models?

Godfrey Leung[1]

Supervisor - Prof Ed Copeland
[1]School of Physics and Astronomy, University of Nottingham

[2]Astronomy Centre, University of Sussex

collaboration work with - E.R.M.Tarrant[2] and C.T.Byrnes[2]
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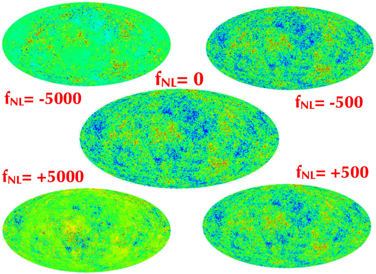
Overview

- Basic Background
  - Primordial Observables, Non–Gaussianity
  - Current Constraints
- Why Consider Multifield Inflation?
- Motivation - why study the effect of (p)reheating on primordial observables?
- Models Considered, Recap of δN formalism
- Results
  - Beyond Slow-roll
    - Bispectrum, $f_{\rm NL}$
    - Trispectrum, $\tau_{\rm NL}$
    - Scale Dependence of NonG
  - Consistency Relations
- Future Directions and Conclusion

Primordial Observables

- At level of $\langle\zeta\zeta\rangle$, spectral index, $n_\zeta - 1$ and tensor-to-scalar ratio r
- Beyond Power Spectrum, Non-Gaussianity
  - Amplitude, measured by non-linearity parameters. For instance, the bispectrum $\langle\zeta\zeta\zeta\rangle$, by $f_{\rm NL}$ and the trispectrum $\langle\zeta\zeta\zeta\zeta\rangle$, by $\tau_{\rm NL}$ and $g_{\rm NL}$

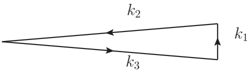


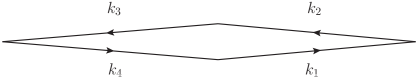
Credits: KICP, Uni of Chicago

  - scale–dependence, $n_{f_{\rm NL}}$, $n_{\tau_{\rm NL}}$ and $n_{g_{\rm NL}}$, defined by $\frac{d\ln f_{\rm NL}(k)}{d\ln k}$ for example

Local–type NonG

- In general, $f_{\rm NL}$, $\tau_{\rm NL}$ and $g_{\rm NL}$ are k_i (shape) dependent
- focus on local–shape, which could be generated in multifield models for superhorizon modes
- Definition, $\zeta = \zeta_G + (3/5) f_{\rm NL} (\zeta_G^2 - <\zeta_G^2>) + (9/24) g_{\rm NL} \zeta_G^3$
- $\tau_{\rm NL}$ related to terms of the same form as $f_{\rm NL}^2$
- peaks in the squeezed limit for $f_{\rm NL}$ ($k_1 \to 0$), collapsed limit for $\tau_{\rm NL}$ ($k_1 + k_2 \to 0$)





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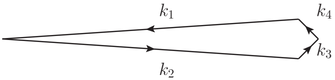
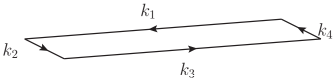
$g_{\rm NL}$ peaks in the squeezed limit ($k_i, k_j \to 0$)





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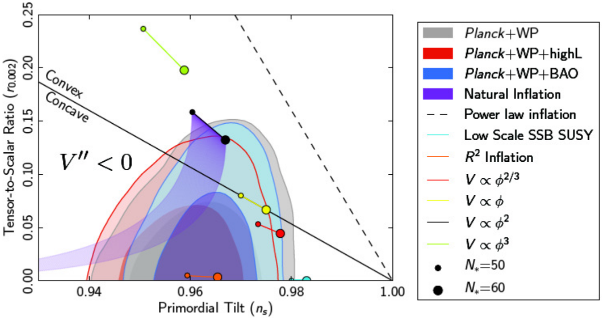
Current Constraints, Planck2013

- Power spectrum is nearly scale invariant, with red tilt $n_\zeta = 0.9603 \pm 0.0073$
- negligible tensor fluctuations, $r < 0.12$ $(95\% CL)$
- no primordial non-gaussianity, $f_{\rm NL}$ consistent with $O(1)$ in all shapes



Credits: Planck Collaboration 2013

- Conclusion: Consistent with simple single field inflation
- End of story for more complicated model setup like multifield inflation?

Why Consider Multifield Inflation?

Why not? "Absence of Evidence is not the Evidence of Absence"!

Why Consider Multifield Inflation?

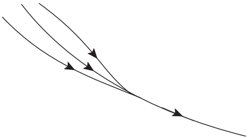
- Multifield models are more natural in the particle physics setup of inflation models
- For example, in string theory, other scalar degrees of freedom also become light in general if one scalar field is made light
- They offer richer phenomenology, e.g. hybrid inflation [Linde 91] and curvaton models [Lyth 02]
- Though they could give potential large nonG, $f_{\rm NL} > O(5)$, they generically gives Gaussian statistics just as single-field model
- They are not ruled out!

Difference Between Single and Multifield Models

- One significant difference between single and multifield is the presence of isocurvature modes
- isocurvature mode = perturbations in direction orthogonal to background inflationary trajectories
- curvature perturbation, ζ, is not conserved after horizon-crossing till adiabaticity condition is reached
- Adiabaticity condition

$$\frac{\delta\rho}{\delta P} = \frac{\dot{\rho}}{\dot{P}}$$

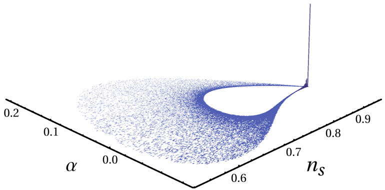
- or equivalently, the classical trajectories in the phase space of H converge

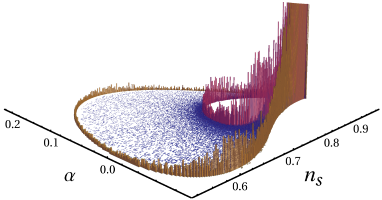


- Thus ζ and related observables can continue to evolve

Motivation

- Single-field inflation models are well classified and tested with Planck data [Planck collaboration 13]
- Yet a similar picture is missing for multifield models...
- Technical difficulties: even for same models, different initial conditions at horizon-crossing could lead to very different predictions
- Recently, by studying N-quadratic models, multifield models could still be predictive despite the issue of initial conditions [R.Easther et.al.13]

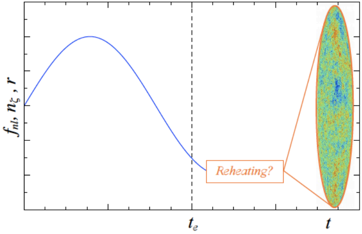




- Yet (p)reheating is not taken into account in the analysis...

Why Consider (P)reheating?

- as discussed, isocurvature mode can source curvature perturbation
- to compare with observations, observables related to ζ should be evaluated at the point where ζ is conserved
- isocurvature modes may not be exhausted at the end of inflation
- reheating thus could play a role in evolution of ζ



- Questions:
  1. Predictions evaluated during slow-roll stage reliable?
  2. If observables do evolve during (p)reheating, can we use this fact to constrain physics of (p)reheating?

Models Considered

- We study the class of multifield models

$$S_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - W(\varphi,\chi)\right]$$

  In particular, we study the two-field models with canonical KE terms

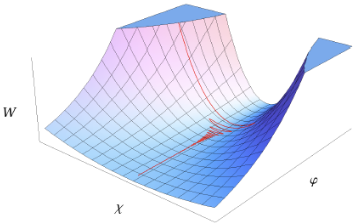
- We consider simple perturbative reheating and model it by adding a friction term in EOM [Kofman 96]

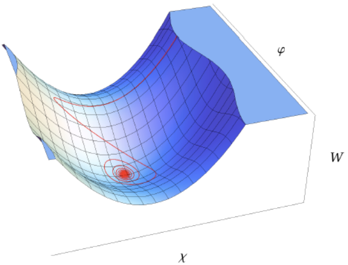
$$\ddot{\phi}_I + (3H + \Gamma_I)\dot{\phi}_I + W_{,\phi_I} = 0$$

- Γ is switched on after first passage through the minima after slow-roll
- For simplicity, we take Γ to be constant

Examples

- We study a number of two–field models, where minima exist in one or both field directions
- In particular, potentials with separable form





1. one min, quadratic exponential potential, $W = W_0 \chi^2 e^{-\lambda\varphi^2/M_P}$ (left)
2. two min, effective N-flation model, $W = W_0 \left[\frac{1}{2} m^2 \chi^2 + \Lambda^4 \left(1 - \cos\left(\frac{2\pi}{f}\varphi\right)\right)\right]$ (right)

Recap of δN formalism

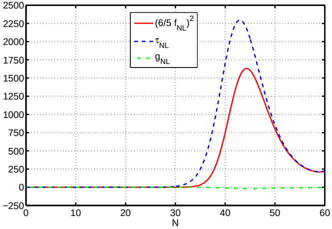
- We apply the δN formalism to find ζ and its correlators
- Written in terms of the δN derivatives, the key primordial observables $f_{\rm NL}$, n_ζ, r are

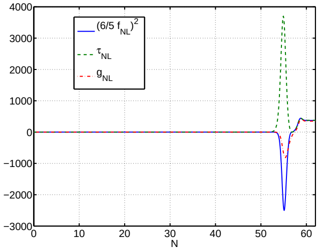
$$\begin{aligned} r &= \frac{8}{\sum_I N_{,I}^2}, \\ n_\zeta - 1 &= -2\epsilon_* + \frac{2}{H_*}\frac{\sum_{IJ}\dot{\varphi}_{*J} N_{,JI} N_{,I}}{\sum_K N_{,K}^2}, \\ f_{\rm NL}^{(4)} &= \frac{5}{6}\frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{\left(\sum_I N_{,I}^2\right)^2}. \end{aligned}$$

  where $f_{\rm NL}^{(4)}$ is the shape–independent part of $f_{\rm NL}$
- For canonical models, $f_{\rm NL}^{local}$ is dominated by $f_{\rm NL}^{(4)}$ [Vernizzi & Wands 06]

Slow-roll Predictions

- Analytic formulae of δN derivatives exist for separable potentials in the slow-roll limit [Vernizzi & Wands 06, Elliston et.al. 12]
- From analytic formulae, one can see large nonG only possible if the fields start close to a ridge/valley at horizon-crossing
- Evolution during slow-roll [Elliston et.al. 12 & Anderson et.al. 12]

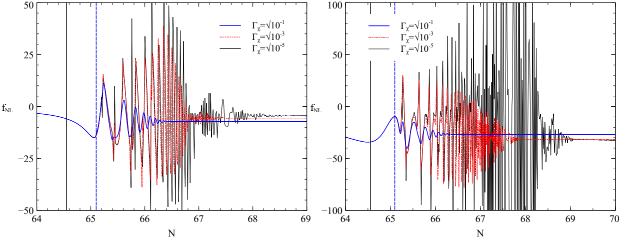




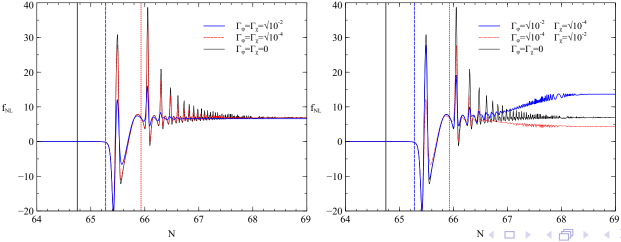
- quadratic exponential potential (left), N-flation (right)
- in general, adiabaticity condition is not reached by the end of inflation (except in N-flation for some model parameters)
- No analytic formulae beyond slow-roll, but can be found numerically

Beyond Slow-Roll, Bispectrum, $f_{\rm NL}$

- One min case, quadratic exponential potential (left: $\lambda = 0.06$; right: $\lambda = 0.05$, $\varphi_* = 10^{-3}M_{\rm p}$, $\chi_* = 16.0M_{\rm p}$)

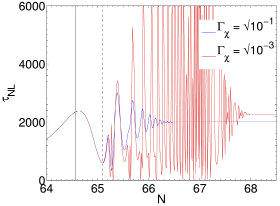


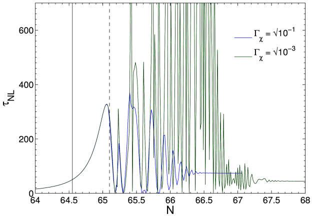
- Two min case, effective N-flation ($\Lambda^4 = 1/4\pi^2$, $\varphi_* = (\frac{1}{2} - 0.001)M_{\rm p}$ and $\chi_* = 16.0M_{\rm p}$)



Beyond Slow-Roll, Trispectrum, $\tau_{\rm NL}$

- One min case, quadratic exponential potential (left: $\lambda = 0.06$; right: $\lambda = 0.05$)





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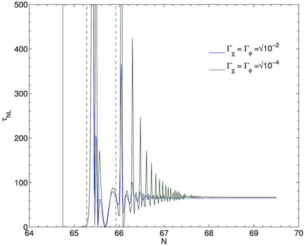
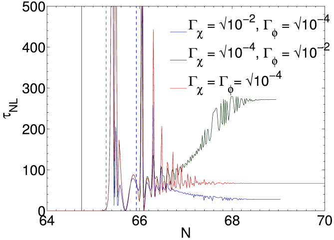




Other Observables, n_ζ and r

- No generic trend for $f_{\rm NL}$ and $\tau_{\rm NL}$, the changes are model-dependent
- Similar results for n_ζ and r, with n_ζ being more robust and less sensitive to reheating in general

χ^2 minimum: $f_{\rm NL}(t_e) = -5.93$, $n_\zeta(t_e) = 0.763$, $r(t_e) = 2.8 \times 10^{-4}$

Γ_χ	$f_{\rm NL}^{\rm final}$	$n_s^{\rm final}$	$r^{\rm final}$
$\sqrt{10^{-5}}$	-4.35	0.761	2.4×10^{-4}
$\sqrt{10^{-3}}$	-5.54	0.762	3.9×10^{-4}
$\sqrt{10^{-1}}$	-7.14	0.762	6.3×10^{-4}

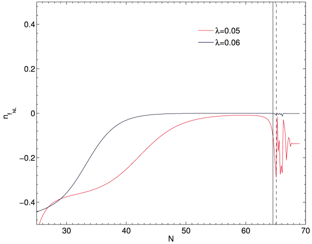
χ^2 minimum: $f_{\rm NL}(t_e) \approx 0$, $n_s(t_e) = 0.969$, $r(t_e) = 0.124$

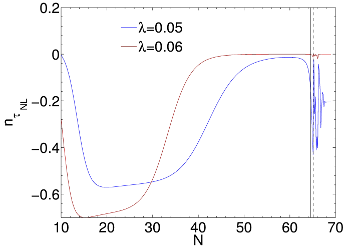
Γ_φ	Γ_χ	$f_{\rm NL}^{\rm final}$	$n_s^{\rm final}$	$r^{\rm final}$
0	0	6.88	0.935	4.6×10^{-4}
$\sqrt{10^{-2}}$	$\sqrt{10^{-2}}$	6.59	0.969	4.3×10^{-4}
$\sqrt{10^{-4}}$	$\sqrt{10^{-4}}$	6.83	0.965	4.6×10^{-4}
$\sqrt{10^{-2}}$	$\sqrt{10^{-4}}$	13.66	0.963	1.0×10^{-3}
$\sqrt{10^{-4}}$	$\sqrt{10^{-2}}$	4.37	0.974	2.7×10^{-4}

- quartic minima also give same qualitative results
- For two minima N-flation case, observables seem only sensitive to the ratio $R \equiv \Gamma_\chi/\Gamma_\varphi$
- Trispectrum, $g_{\rm NL}$
  1. sensitive to reheating as well in general
  2. beyond detectable level in general even if $f_{\rm NL}$ and $\tau_{\rm NL}$ are large

Scale Dependence, $n_{f_{\rm NL}}$ and $n_{\tau_{\rm NL}}$

- in general small $< O(0.1)$, but could be made large for the quadratic exponential model $W = W_0 \chi^2 e^{-\lambda\varphi^2/M_{\rm P}}$ in some cases





- eg.

End of Reheating, $\lambda = 0.05$

Γ_χ	$f_{\rm NL}$	$\tau_{\rm NL}$	$g_{\rm NL}$	$n_{f_{\rm NL}}$	$n_{\tau_{\rm NL}}$
$\sqrt{10^{-5}}$	-33.4	2.25×10^3	-13	-0.105	-0.157
$\sqrt{10^{-3}}$	-31.5	2.27×10^3	-11.6	-0.137	-0.205
$\sqrt{10^{-1}}$	-26.9	2.01×10^3	-9.96	-0.177	-0.266

Multifield Consistency Relations

- Like single-field models, there are also consistency relations in multifield models. For example

Non-vacuum dominated sum-separable potential, slow-roll limit

$$\frac{27}{25} g_{\rm NL} \approx \tau_{\rm NL}$$ [Elliston et.al. 12]

Two-field local type models

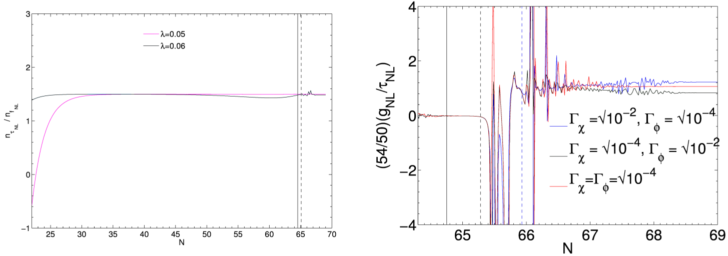
If $\zeta(k) = \zeta_k^{G,\varphi} + \zeta_k^{G,\chi} + f_\varphi(\zeta^{G,\varphi} \star \zeta^{G,\varphi})_k + g_\varphi(\zeta^{G,\varphi} \star \zeta^{G,\varphi} \star \zeta^{G,\varphi})_k$, then

$$3n_{f_{\rm NL}} = 2n_{\tau_{\rm NL}}$$ [Byrnes et.al. 12]

- Suyama-Yamaguchi inequality, $\tau_{\rm NL} \geq (\frac{6}{5} f_{\rm NL})^2$. A strong violation of equality seems rather implausible in the slow-roll limit, requires extreme fine-tuned conditions [Tegmark et.al.12]

Consistency Relations Beyond Slow-roll

- Consistency relations are much more robust to physics of reheating. For example
  1. $n_{\tau_{\rm NL}} = (3/2)n_{f_{\rm NL}}$, two–field local type [Byrnes et.al.10]
  2. $(27/25)g_{\rm NL} \approx \tau_{\rm NL}$, non–vacuum dominated sum–separable potential [Elliston et.al. 12]



- the quadratic exponential model (left) and the effective N-flation model (right)
- the Suyama-Yamaguchi inequality remains mildly violated for large range of reheating timescales

Future Directions and Conclusion

Future Directions

- Sudden decay approximation is a good approximation for sum-separable models [Meyers et.al. 13]
- Apply the sudden decay approximation beyond bispectrum to models with non-sum-separable potentials, where there are couplings between the fields?
- Study other classes of multifield models, such as non-minimal couplings and non-canonical KE terms

Take home message

- Evolution during reheating is important if isocurvature modes are still present
- Observables evaluated at end of inflation in general are different from that after reheating
- $g_{\rm NL}$ in general would be too small to be observed in multifield models
- it may still be possible to test different classes of models by studying consistency relations between observables
- More work to be done

Thank you!