Updates on Horava-Lifshitz gravity

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ref. arXiv: 1007.5199 (review of HL gravity)
arXiv: 1206.1338 w/ K.Lin, A.Wang
arXiv: 1310.6666 w/ K.Lin, A.Wang, T.Zhu

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- **Scaling dim of ϕ**
  t → b t (E → b^{-1}E)
  x → b x
  ϕ → b^s ϕ
  1+3-2+2s = 0
  s = -1

- Renormalizability

$$n \leq 4$$

- Gravity is highly non-linear and thus non-renormalizable

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Anisotropic scaling

  t → b^z t (E→b^{-z}E)

  x → b x

  ϕ → b^s ϕ

  z+3-2z+2s = 0

  s = -(3-z)/2

- s = 0 if z = 3

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- For z = 3, any nonlinear interactions are renormalizable!
- **Gravity becomes renormalizable!?**

Cosmological implications

Horava-Lifshitz Cosmology: A Review, arXiv: 1007.5199

- The z=3 scaling solves the horizon problem and leads to scale-invariant cosmological perturbations without inflation (Mukohyama 2009).
- New mechanism for generation of primordial magnetic seed field (S.Maeda, Mukohyama, Shiromizu 2009).
- Higher curvature terms lead to regular bounce (Calcagni 2009, Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- Absence of local Hamiltonian constraint leads to DM as integration “constant” (Mukohyama 2009).

Usual story

- $\omega^2 >> H^2$: oscillate H = (da/dt) / a

  $\omega^2 << H^2$: freeze a : scale factor

  oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

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  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$
  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.
- Scaling law
  t → b t (E → b^{-1}E)
  x → b x
  $\phi \to b^{-1} \phi$
  Scale-invariance requires almost const. H, i.e. inflation.

⇒ $$\delta\phi \propto E \sim H$$

New story with z=3

Mukohyama 2009

- oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$

  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$

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- Scaling law
  $t \to b^3 t$ ($E \to b^{-3}E$)
  $x \to b\, x$
  $\phi \to b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!

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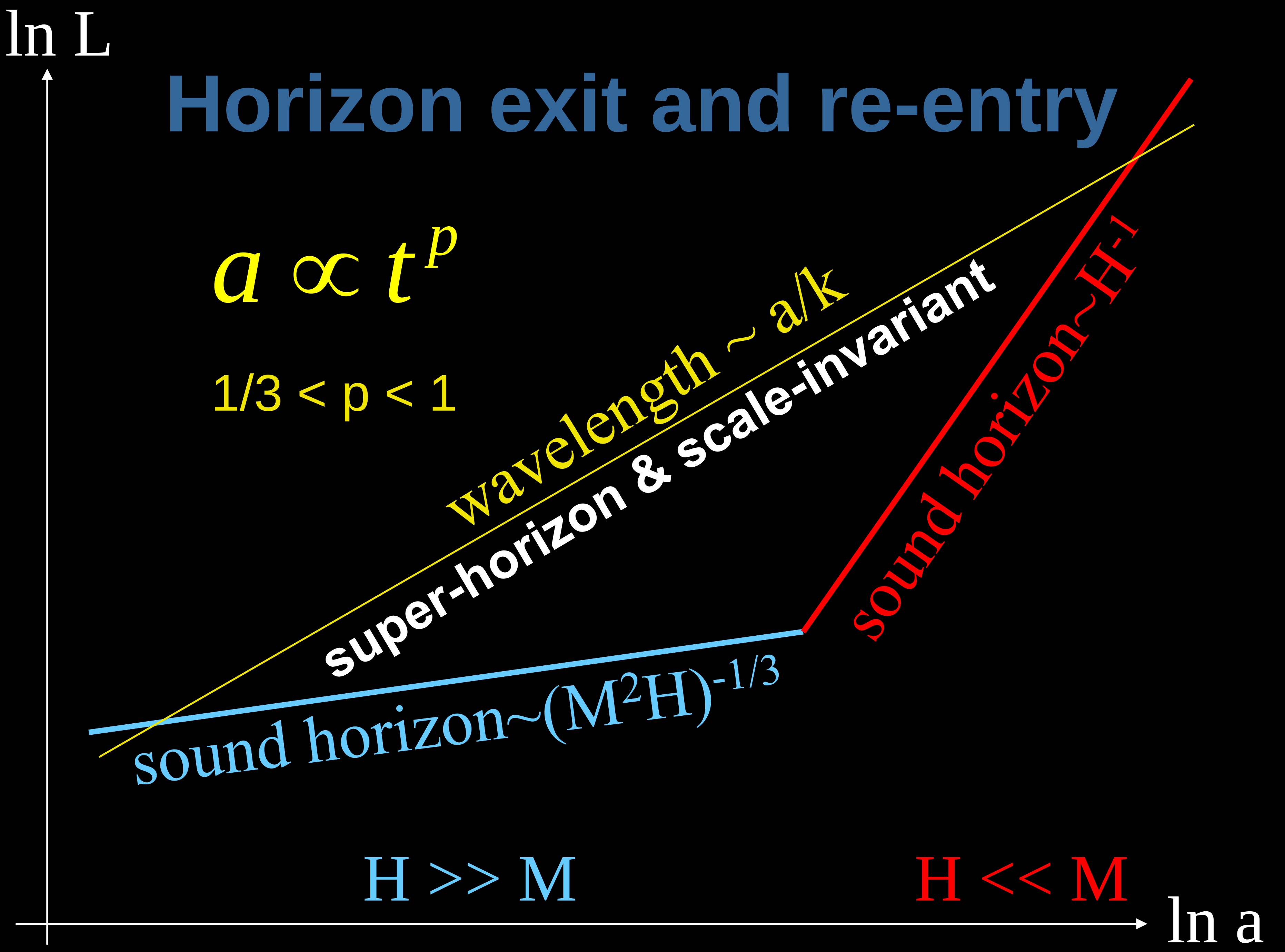
  Scale-invariant fluctuations!

- Tensor perturbation $P_h \sim M^2/M_{Pl}^2$

Horizon exit and re-entry



Minimal Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2dt^2 + g_{ij}\,(dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \rightarrow t'(t), \quad x^i \rightarrow x'^i(t,x^j)$
- Anisotropic scaling with z=3 in UV
  $t \rightarrow b^z\, t, \quad x^i \rightarrow b\, x^i$
- Ingredients in the action

$$N dt \qquad \sqrt{g}\,d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$$

($C_{ijkl} = 0$ in 3d)

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad D_i R_{jk} D^i R^{jk} \quad D_i R D^i R \right.$$

$$\left. R_i^j R_j^k R_k^i \quad R R_i^j R_j^i \quad R^3 \quad \right]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other terms

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R_i^j R_j^i \quad R^2 \quad]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R \quad]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x [\quad 1 \quad]$$

Physical d.o.f.

- (6 + 3) – 3 – 3 = 3
  g_{ij} : 6 components
  N^i : 3 components
  $x^i \rightarrow x'^i(t,x)$: 3 gauge d.o.f.
  $\delta I/\delta N^i = 0$: 3 constraints
- 3 = 2 + 1
  tensor graviton: 2 d.o.f.
  scalar graviton: 1 d.o.f.

Different versions of HL gravity

- There are versions w/wo the projectability condition.
- Horava's original proposal was **with the projectability condition, N=N(t).**
- **Naïve non-projectable extension is inconsistent** [c.f. Henneaux, et.al. 2009].
- Inclusion of $a_i = (\ln N)_{,i}$ (and thus more terms) in the action can cure the non-projectable extension [Blas, Pujolas and Sibiryakov 2009].
- **U(1) extension [Horava & Melby-Thompson 2010]. String theory embedding has been discussed in this context [Janiszewski&Karch 2013]**

HL gravity with extra U(1)

- Existence of scalar graviton is not necessarily a problem but is at least a source of technical complications.
- In order to get rid of the scalar graviton, Horava & Melby-Thompson (2010) introduced an **extra local U(1) symmetry.**
- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$, "gauge field" $A(t,x)$, "Newtonian pre-potential" $\nu(t,x)$
- **A/N and ν transform as scalars**

U(1) extension of HL gravity

- Local U(1)

$$\begin{aligned} \delta N &= 0 \\ \delta N^i &= N g^{ij}\partial_j \alpha \\ \delta g_{ij} &= 0 \\ \delta A &= N\partial_\perp \alpha \\ \delta \nu &= \alpha \end{aligned}$$

K_{ij} , A/N : not invariant

$\tilde{K}_{ij}$, σ : invariant

- Ingredients in the action

$$Ndt \quad \sqrt{g}\,d^3x \quad g_{ij} \quad D_i \quad R_{ij}$$

$$\tilde{K}_{ij} \equiv K_{ij} + D_i D_j \nu \qquad \sigma \equiv \frac{A}{N} - \partial_\perp \nu - \frac{1}{2} g^{ij}\partial_i \nu \partial_j \nu$$

- Scaling dimensions

$$[\partial_i] = 1, \quad [\partial_t] = z, \quad [dt d^3\vec{x}] = -z-3, \quad [\partial_\perp] = z,$$

$$[g_{ij}] = 0, \quad [N_i] = [N^i] = z-1, \quad [N] = 0,$$

$$[\alpha] = z-2, \quad [A] = 2z-2, \quad [\nu] = z-2.$$

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(\tilde{K}_{ij}\tilde{K}^{ij} - \lambda \tilde{K}^2 \right)$$

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x \Big[\quad D_i R_{jk} D^i R^{jk} \qquad D_i R D^i R$$

$$R_i^j R_j^k R_k^i \qquad R R_i^j R_j^i \qquad R^3 \qquad \Big]$$

- New term with σ ([σ]=2z-2=4)

$$\int N dt \sqrt{g} d^3x \Big[\quad R\sigma \quad \Big]$$

Relevant deformations (with parity)

- New term with σ

$$\int N dt \sqrt{g} d^3x [\quad \sigma \quad]$$

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R_i^j R_j^i \quad R^2 \quad]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R \quad]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x [\quad 1 \quad]$$

cf. This construction is based on da Silva (2012).

Absence of scalar graviton

- Background eom for N = 1, $N^i = 0$, $g_{ij} = \delta_{ij}$, A = 0 , $\nu = 0$ → $\Lambda = \Omega = 0$
- Scalar perturbation

$$N = 1 \qquad N_i = \partial_i \beta \qquad g_{ij} = (1 + 2\zeta)\delta_{ij} \qquad A \qquad \nu = 0$$

- Quadratic action

$$I_{\vec{k}} = \int dt \left[-\frac{3}{2}(3\lambda - 1)\dot{\zeta}^2 - (3\lambda - 1)\vec{k}^2 \beta \dot{\zeta} + \vec{k}^2 \zeta^2 - 2\vec{k}^2 A \zeta - \frac{1}{2}(\lambda - 1)\vec{k}^4 \beta^2 \right]$$

- A-eom & β-eom & ζ-eom → $\zeta = \beta = A = 0$
- **Extra U(1) eliminates scalar graviton!**
- This result extends to FLRW background

Coupling to matter at low-E

- Among (N, N^i, g_{ij}), N^i is not U(1) invariant but $\tilde{N}^i \equiv N^i - N g^{ij} \partial_j \nu$ is **U(1) invariant**.
- In addition to $(N, \tilde{N}^i, g_{ij})$, there is a **U(1) invariant** scalar **σ** and it can also couple to matter at low-E.
- The equivalence principle requires that coupling to matter should be universal.
- A proposal: $(\tilde{N}, \tilde{N}^i, \tilde{g}_{ij})$ couple to matter universally, where

$$\tilde{N} \equiv F(\sigma) N \qquad \tilde{g}_{ij} \equiv \Omega^2(\sigma) g_{ij}$$

A possible scenario

- Consider a **heavy scalar field** χ neutral under U(1) with potential V(χ) + σ U(χ)
- Suppose that $\tilde{N} \equiv f(\chi) N$ and $\tilde{g}_{ij} \equiv \omega^2(\chi) g_{ij}$ couple to matter. After integrating out χ, we obtain f(χ) → F(σ), ω(χ) → Ω(σ).
- In general (F, Ω) depend on matter species, but universality may emerge at low-E. It is worthwhile trying to see if this is possible.
- c.f. Emergent Lorentz symmetry: Lorentz-invariant IR fixed point (Chadha and Nielsen 1983) & SUSY or/and strong dynamics to speed-up the RG flow

Solar system tests

- Matter propagates on the 4d metric
  $$\gamma_{\mu\nu}dx^{\mu}dx^{\nu} = -\tilde{N}^2 dt^2 + \tilde{g}_{ij}(dx^i + \tilde{N}^i dt)(dx^j + \tilde{N}^j dt)$$
- Define $T^{\mu\nu}$ by varying matter action w.r.t. $\gamma_{\mu\nu}$.
- Introduce PPN parameters for $\gamma_{\mu\nu}$.
- By using gravity equations of motion with $\Lambda = \Omega = 0$, express PPN parameters in terms of other parameters of the theory.
- All solar system tests are passed if
  $|c_g^2 - 1|$, $|F'(\sigma=0) - 1|$, $|\Omega'(\sigma=0)| < 10^{-5}$
  Here, $F(\sigma=0)$ and $\Omega(\sigma=0)$ are set to 1.
- This condition is insensitive to λ.

PPN parameters

$$G = \frac{1}{8}\,\frac{a1^2\,\gamma 1}{Mp^2\,\pi}$$

$$beta\_ = \frac{1}{2}\,\frac{\gamma 1\; a1 + 1}{\gamma 1\; a1}$$

$$gamma\_ = -\frac{-1 + \gamma 1\; a2}{\gamma 1\; a1}$$

$$\alpha 1 = -\frac{4\left(-a1\;\gamma 1\; a2 + a1^2\,\gamma 1 - 2 + a1\right)}{a1^2\,\gamma 1}$$

$$\alpha 3 = 0$$

$$\alpha 2 = -\frac{\lambda\, a1^2\,\gamma 1 + 6\,\lambda\, a1 - 4\,\lambda - 3\,\lambda\, a1^2 + a1^2 + 2 - a1^2\,\gamma 1 - 2\, a1}{a1^2\,\gamma 1\,(\lambda - 1)}$$

$$\zeta 1 = \frac{(-1 + 3\,\lambda)\,(-1 + a1)}{\gamma 1\; a1\,(\lambda - 1)}$$

$$zetaB = -\frac{(-1 + 3\,\lambda)\,(-1 + a1)}{\gamma 1\; a1\,(\lambda - 1)}$$

$$\zeta 2 = 0 \qquad \zeta 3 = 0 \qquad \zeta 4 = 0 \qquad xi\_ = 0$$

$\gamma_1 = -c_g^2$ $\quad a_1 = F'(\sigma=0)$ $\quad a_2 = \Omega'(\sigma=0)$

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- The original theory has an additional d.o.f. called scalar graviton. This is not necessarily a problem but leads to a lot of technical complications. [See the review for discussions.]
- In order to get rid of the scalar graviton, Horava & Melby-Thompson (2010) introduced an **extra local U(1) symmetry.**
- **The U(1) extension (with projectability condition) indeed removes the scalar graviton.**
- **We proposed a universal coupling to matter.**
- **We calculated all PPN parameters.**
- **All solar-system constraints are satisfied under a certain condition.**
- **Cosmology (e.g. flatness problem) is under study.**