Inflationary Gravitational Waves with Unusual Properties

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MiniWorkshop on Gravitation and Cosmology, Kyoto, 06/02/14

LS, 1101.1525, JCAP
J. Cook and LS, 1109.0022, PRD
J. Cook and LS, 1307.7077, JCAP

Scalar perturbations during inflation ☛ rich phenomenology:

- Features
- Isocurvature
- Non vacuum states
- Nongaussianities
- Oscillations
- ...

Tensors typically assumed to be boring....

$$\mathcal{P}_t \propto \frac{H^2}{M_P^2}$$

H ↘ during inflation ☛ slightly red spectrum

This talk:

Non-boring
tensors

The system

.Barnaby et al 2012

A rolling pseudoscalar ϕ (*not* the inflaton) interacting with a *U(1)* gauge field via

$$\mathcal{L}_{\phi FF} = \frac{\phi}{f}\,\epsilon_{\alpha\beta\gamma\delta}\,F^{\alpha\beta}\,F^{\gamma\delta}$$

(f=constant with dimensions of a mass)

The gauge field is decomposed into helicity-λ modes

$$\mathbf{A}(\mathbf{x},\tau) = \sum_{\lambda=\pm}\int\frac{d^3\mathbf{k}}{(2\,\pi)^{3/2}}\left[a_{\mathbf{k}}^{\lambda}\,A_{\lambda}^{\mathbf{k}}(\tau)\;\mathbf{e}^{\lambda}(\mathbf{k})\,e^{i\mathbf{k}\,\mathbf{x}} + a_{\mathbf{k}}^{\lambda\,\dagger}\,A_{\lambda}^{*\,\mathbf{k}}(\tau)\;\mathbf{e}^{\lambda\,*}(\mathbf{k})\,e^{-i\mathbf{k}\,\mathbf{x}}\right]$$

The mode functions $A_\lambda^k(\tau)$ are sourced by the rolling ϕ

(assume $d\phi/dt$ constant):

$$A''_\lambda + \left(\mathbf{k}^2 + \lambda \frac{\phi'}{f} |\mathbf{k}|\right) A_\lambda = 0$$

for $\lambda = -$, the "mass term" is negative and large for ~ 1 Hubble time:

.Anber and LS 06

Exponential amplification of *left handed modes only*!

parity violation!

$$A_L \propto \exp\left\{\frac{\pi}{2}\frac{\dot{\phi}}{f\,H}\right\}$$

Generation of parity violating, large amplitude gravitational waves

The energy of the electromagnetic field sources gravitational waves of helicity-λ h_λ:

(note: this is an operator equation)

$$h''_\lambda + 2\,\frac{a'}{a}\,h'_\lambda + \mathbf{k}^2\,h_\lambda = \frac{2}{M_P^2}\,\Pi_\lambda^{ij}\,T_{ij}^{\mathrm{EM}}$$

Projector on helicity-λ components

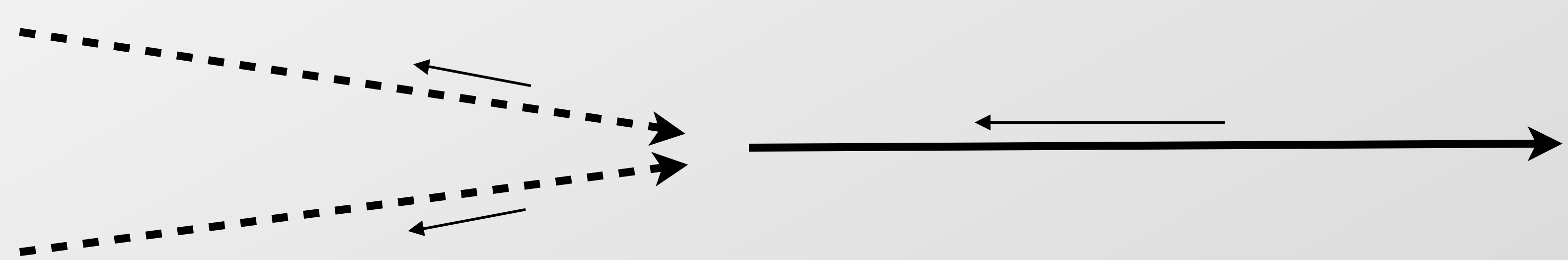
Spatial components of gauge field stress-energy tensor

Parity violating gravitational waves

A_L and A_R have different amplitudes

$$\langle h_L h_L \rangle \neq \langle h_R h_R \rangle$$

Physics: in the limit of small transverse momentum two LH photons cannot create a RH graviton



The parity-violating power spectrum

$$\mathcal{P}_L(\mathbf{k}) = \frac{H^2}{\pi^2\, M_P^2}\left(1 + 9 \times 10^{-7}\, \frac{H^2}{M_P^2}\, \frac{e^{4\pi\xi}}{\xi^6}\right)$$

$$\mathcal{P}_R(\mathbf{k}) = \frac{H^2}{\pi^2\, M_P^2}\left(1 + 2 \times 10^{-9}\, \frac{H^2}{M_P^2}\, \frac{e^{4\pi\xi}}{\xi^6}\right)$$

"standard"
parity-invariant part

parity-violation

$$\xi \equiv \frac{\dot{\phi}}{2\, f\, H} \gtrsim 1$$

How do we see the effect of parity violating GWs?

While T and E modes are parity-even,
B is parity-odd

<TB> and <EB> power spectra should vanish in parity-invariant CMB

Detection prospects related to observability of nonzero <*EB*> and/or <*TB*>

Saito Ichicki Taruya 07,
Contaldi Maguejio Smolin 08,
Gluscevic Kamionkowski 10

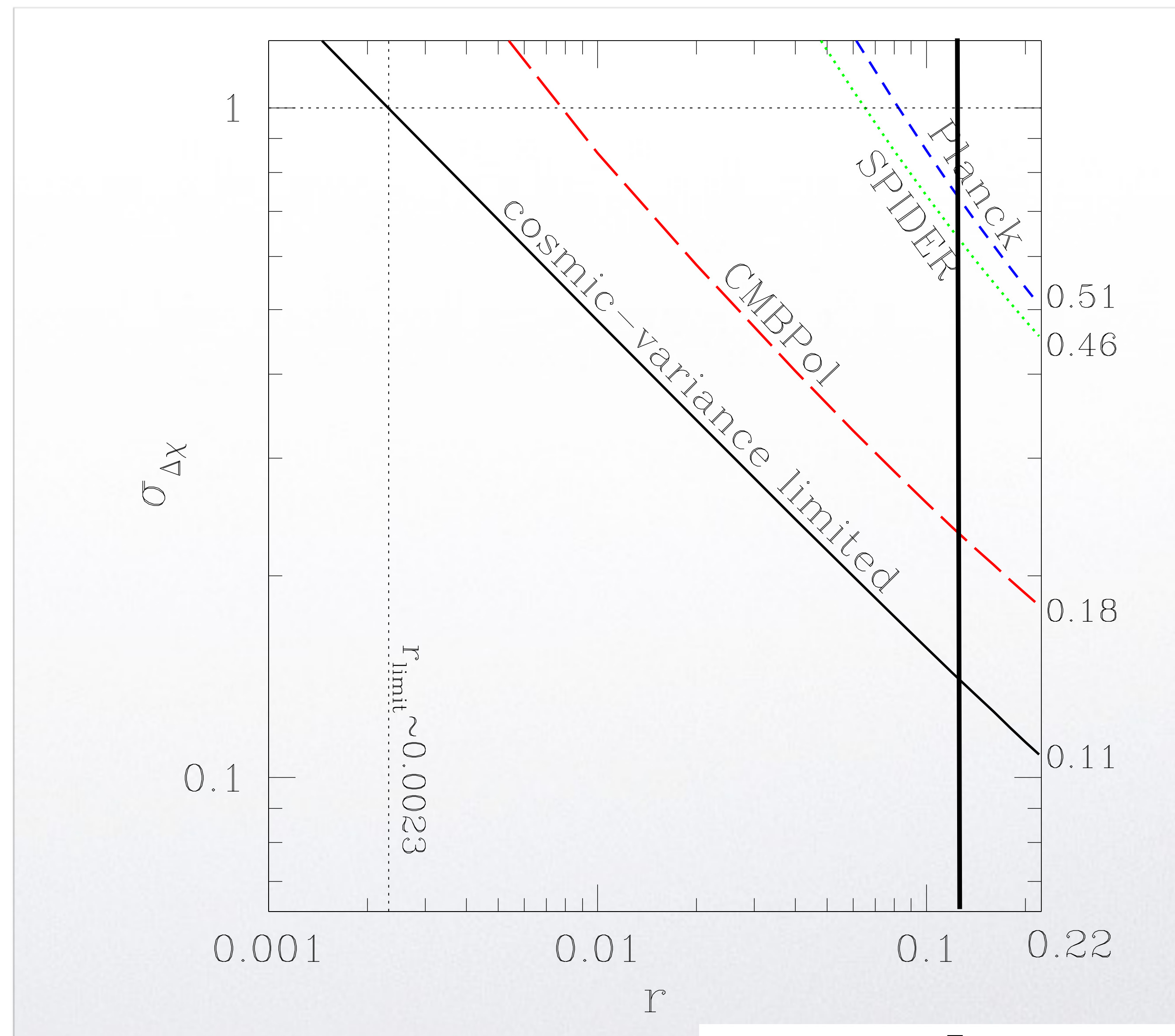
Depend on two parameters

$$r = \frac{\mathcal{P}_R + \mathcal{P}_L}{\mathcal{P}_T}$$

tensor-to-scalar ratio

$$\Delta\chi = \frac{\mathcal{P}_R - \mathcal{P}_L}{\mathcal{P}_R + \mathcal{P}_L}$$

chirality of primordial perturbations



From
Gluscevic Kamionkowski 10

For our system

$$\Delta\chi = \frac{4.3\times 10^{-7}\,\frac{e^{4\,\pi\,\xi}}{\xi^6}\,\frac{H^2}{M_P^2}}{1+4.3\times 10^{-7}\,\frac{e^{4\,\pi\,\xi}}{\xi^6}\,\frac{H^2}{M_P^2}}\,.$$

$$\xi \equiv \frac{\dot\phi}{2\,f\,H}$$

Exponential dependence on the coupling *1/f*

In principle sizable parity violation in large portion of parameter space.

Anything more?

Photons source metric perturbations
in a 2→1 process

(equilateral)

nongaussianities

Photons source metric perturbations in a 2→1 process

scalars and tensors produced (in principle) with same efficiency...

.Barnaby et al 2012

scalar

$$\hat{\zeta}(\mathbf{k}) = -\frac{H^2}{4\,M_P^2}\int d\tau'\,G_k(\tau,\tau')\,\tau'^2\int\frac{d^3\mathbf{q}}{(2\pi)^{3/2}}\left(-1+\frac{(q-|\mathbf{k}-\mathbf{q}|)^2}{k^2}\right)$$
$$\times\left[\hat{A}_i'(\mathbf{q},\tau')\,\hat{A}_i'(\mathbf{k}-\mathbf{q},\tau') - \varepsilon_{iab}\,q_a\,\hat{A}_b(\mathbf{q},\tau')\,\varepsilon_{icd}\,(k_c-q_c)\,\hat{A}_d(\mathbf{k}-\mathbf{q},\tau')\right]$$

Sorbo 10

tensor

$$\hat{h}_\pm(\mathbf{k}) = -\frac{2\,H^2}{M_P^2}\int d\tau'\,G_k(\tau,\tau')\,\tau'^2\int\frac{d^3\mathbf{q}}{(2\pi)^{3/2}}\,\Pi_\pm^{lm}(\mathbf{k})\,\times$$
$$\times\left[\hat{A}_l'(\mathbf{q},\tau')\,\hat{A}_m'(\mathbf{k}-\mathbf{q},\tau') - \epsilon_{lab}\,q_a\,\hat{A}_b(\mathbf{q},\tau')\,\varepsilon_{mcd}\,(k_c-q_c)\,\hat{A}_d(\mathbf{k}-\mathbf{q},\tau')\right]$$

...but phase space

⇓

LH tensors much more efficiently produced than RH tensors ***and scalars***

Three point functions...

Barnaby et al 12

$$\langle \hat{\zeta}(\mathbf{k}_1)\, \hat{\zeta}(\mathbf{k}_2)\, \hat{\zeta}(\mathbf{k}_3) \rangle_{\text{equil}} = 2.6 \times 10^{-13}\, \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\, \frac{H^6}{M_P^6}\, \frac{e^{6\pi\xi}}{\xi^9}$$

$$\langle \hat{h}_-(\mathbf{k}_1)\, \hat{h}_-(\mathbf{k}_2)\, \hat{h}_-(\mathbf{k}_3) \rangle_{\text{equil}} = 6.1 \times 10^{-10}\, \frac{\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)}{k^6}\, \frac{H^6}{M_P^6}\, \frac{e^{6\pi\xi}}{\xi^9}$$

Cook and LS 13

large *tensor* nongaussianities

with

small *scalar* nongaussianities

Small scalar nongaussianities
Large tensor nongaussianities

Cook and LS 13

Let us quantify the effect ☞ observables

Also tensors source temperature fluctuations!
(at *l≪100*)

Flat sky approximation (for *l≫1)*

$$a(\mathbf{l}) = \int \frac{d^2\mathbf{n}}{2\pi}\, e^{-i\,\mathbf{l}\,\mathbf{n}}\, \frac{\delta T}{T}(\mathbf{n}) \qquad B_{\mathbf{l}_1\mathbf{l}_2\mathbf{l}_3}\,\delta^{(2)}(\mathbf{l}_1+\mathbf{l}_2+\mathbf{l}_3) \equiv \langle a(\mathbf{l}_1)\,a(\mathbf{l}_2)\,a(\mathbf{l}_3)\rangle$$

Small scalar nongaussianities
Large tensor nongaussianities

Cook and LS 13

Let us quantify the effect ☞ observables

Also tensors source temperature fluctuations!
(at $l \ll 100$)

Flat sky approximation (for $l \gg 1$)

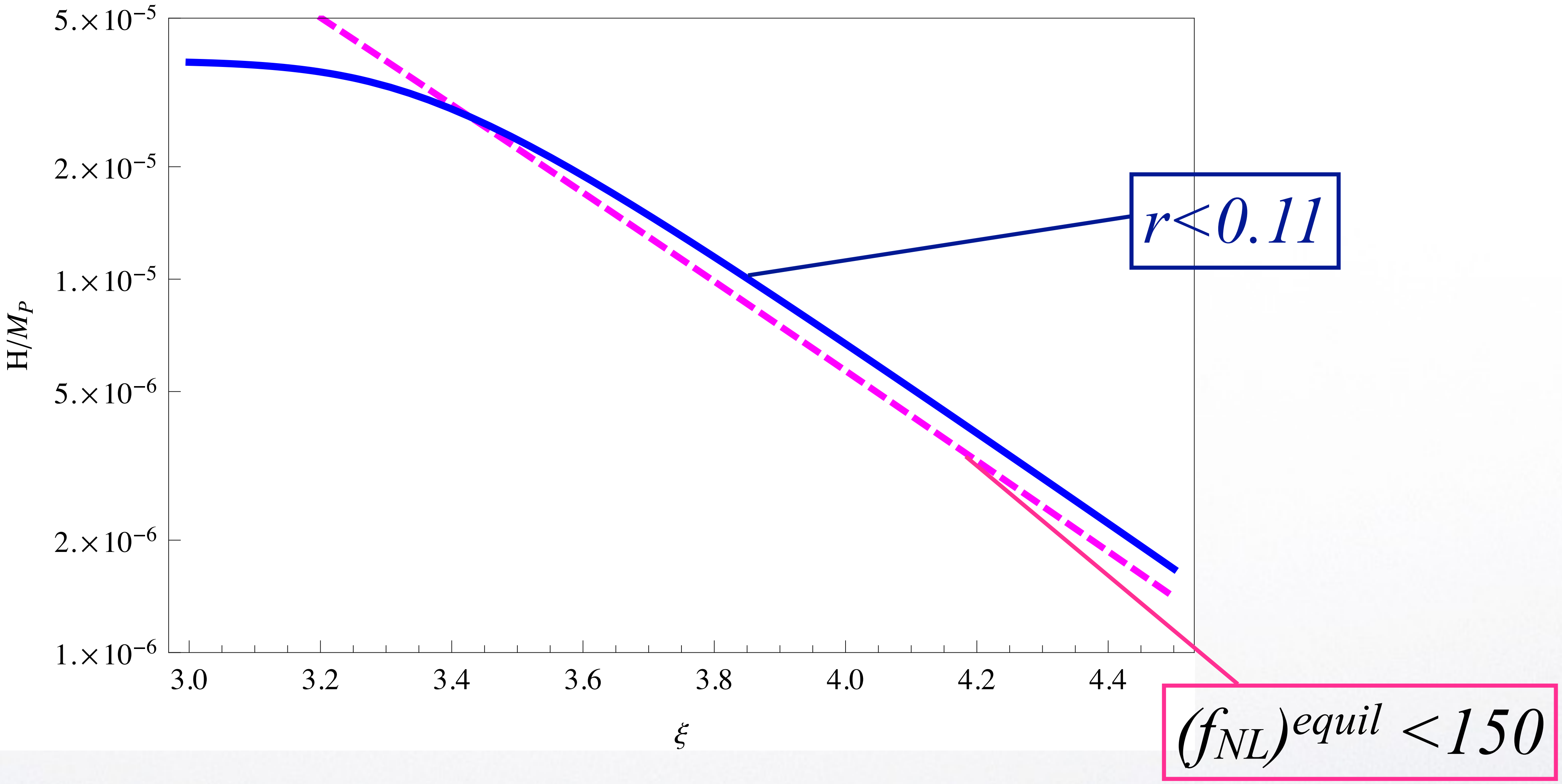
$$l^4 \left(B^{\rm scalar}_{\mathbf{l}_1\mathbf{l}_2\mathbf{l}_3}\right)^{\rm equil} \simeq 4.0 \times 10^{-16}\, \frac{H^6}{M_P^6}\, \frac{e^{4\,\pi\,\xi}}{\xi^9}$$

$$l^4 \left(B^{\rm tensor}_{\mathbf{l}_1\mathbf{l}_2\mathbf{l}_3}\right)^{\rm equil} \simeq 1.8 \times 10^{-12}\, \frac{H^6}{M_P^6}\, \frac{e^{4\,\pi\,\xi}}{\xi^9}$$

Effect of tensor on $<\delta T^3>$
is ~4500 times
stronger than scalar!

negligible

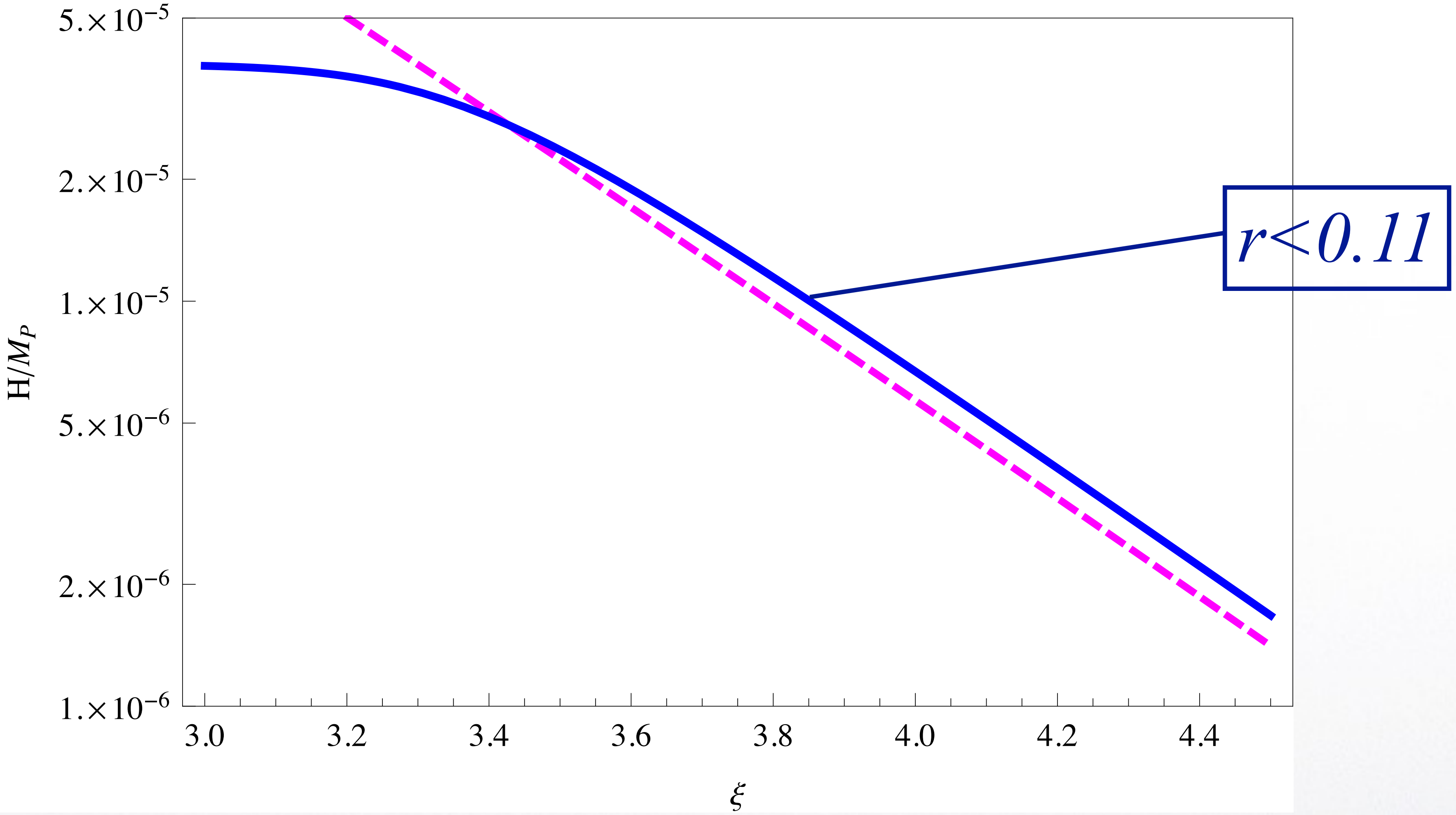
Constraints on parameter space



(note this is scale dependent f_{NL}, vanishes as $l \gtrsim 100$)

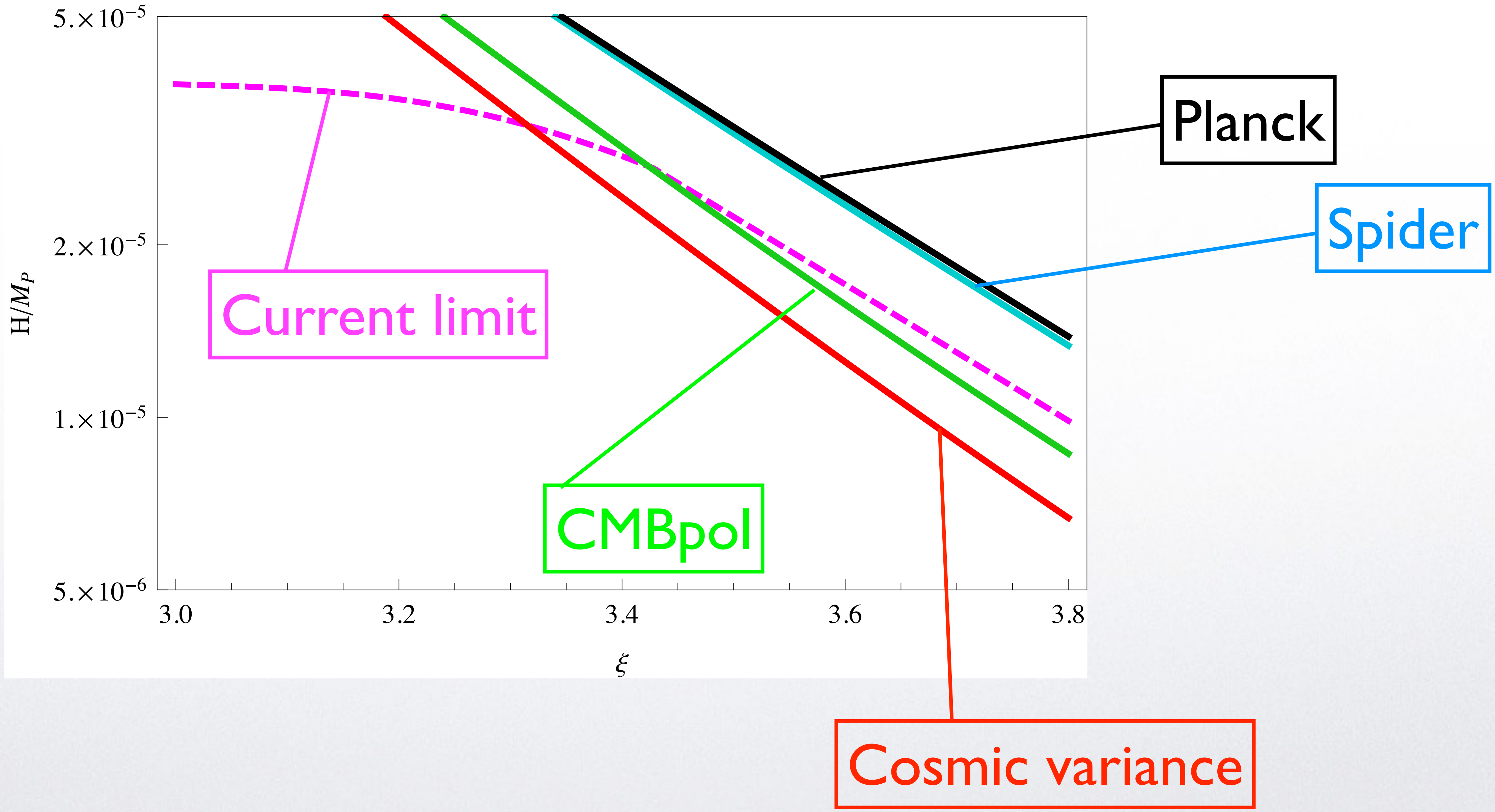
(⇒ magenta line is too restrictive, see below)

BTW…



…the standard relation $r \propto H^2$ does not hold in this model!

Detectable chiral tensors?



More detailed analysis

Shiraishi, Ricciardone, Saga 13

Spectra of equilateral f_{NL}
for T, E and B modes

$l_1+l_2+l_3=even$ &
$l_1+l_2+l_3=odd$
both nonvanishing

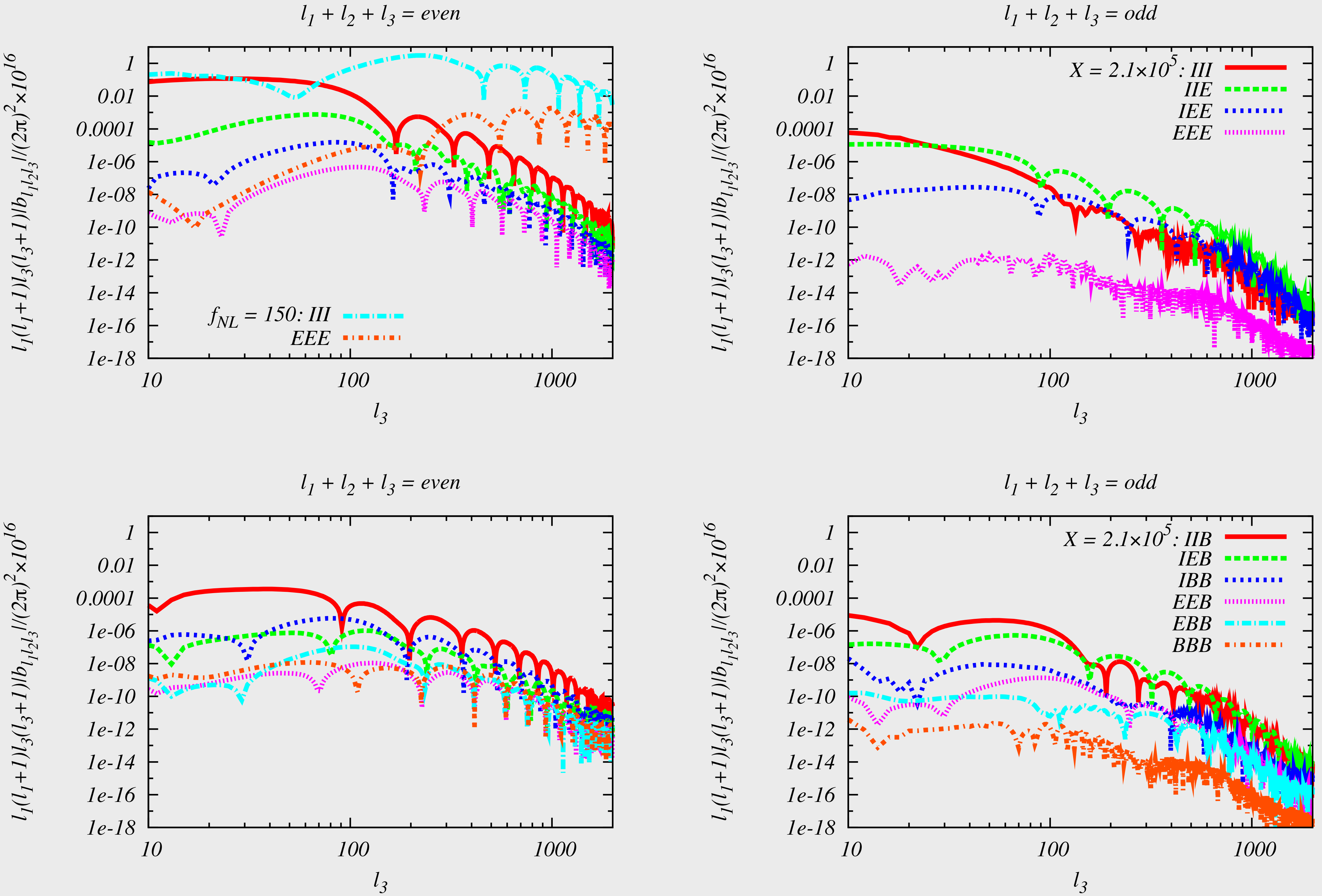


Figure 1. All possible CMB bispectra, i.e., $\langle III\rangle$, $\langle IIE\rangle$, $\langle IEE\rangle$ and $\langle EEE\rangle$ (top two panels), and $\langle IIB\rangle$, $\langle IEB\rangle$, $\langle IBB\rangle$, $\langle EEB\rangle$, $\langle EBB\rangle$ and $\langle BBB\rangle$ (bottom two panels), induced by the tensor non-Gaussianity with $X = 2.1 \times 10^5$ and $\mathcal{P} = 2.5 \times 10^{-9}$ for $\ell_1 + 2 = \ell_2 + 1 = \ell_3$. Left and right two panels describe the parity-even ($\ell_1 + \ell_2 + \ell_3 =$ even) and parity-odd ($\ell_1 + \ell_2 + \ell_3 =$ odd) components, respectively. For comparison, we also plot $\langle III\rangle$ and $\langle EEE\rangle$ from the equilateral non-Gaussianity with $f_{\rm NL} = 150$. Other cosmological parameters are fixed using the *Planck* results [29]. The parity-odd bispectra seem to oscillate rapidly since they hate symmetric signals as $\ell_1 \sim \ell_2 \sim \ell_3$.

Shiraishi, Ricciardone, Saga 13

Expected errors on $(\varepsilon\, e^{2\pi\xi}/\xi^3)^3/10^{15}$

Shiraishi, Ricciardone, Saga 13

	III	EEE	all $I+E$	BBB $(r=0.05)$	BBB $(r=5\times10^{-4})$
Planck	127 (129)	232 (233)	56 (65)	17 (19)	2.1 (2.1)
PRISM	127 (129)	83 (84)	25 (30)	0.87 (1.0)	0.015 (0.017)
ideal	127 (129)	82 (83)	25 (29)	0.12 (0.20)	$1.2\ (2.0)\times10^{-4}$

Table 1. Expected 1σ errors of X^3 normalized by 10^{15} in the III, EEE, all $I+E$ cases ($\ell_{\rm max}=1000$) and the BBB case ($\ell_{\rm max}=500$) for each experiment. The tensor-to-scalar ratio r determines the amplitude of the B-mode cosmic variance spectrum. Here we summarize the results estimated from both the parity-even and parity-odd signals. In addition, for comparison, the errors from the parity-even signals alone are written in parentheses.

Room for improvement with Planck polarization

But NOTE!
Planck+WP constraint on r gives 10 in the above table

Conclusion
(partial)

Existence proof of

Large tensor nongaussianities
with small scalar nongaussianities
+
Parity violation
=
Interesting phenomenology in the CMB.

More fun with GWs...

…now in the case where the inflaton is directly coupled to the gauge field...

Inflationary GWs for LIGO

$$\mathcal{P}_R(\mathbf{k}) = \frac{H^2}{\pi^2 M_P^2}\left(1 + 9 \times 10^{-7}\,\frac{H^2}{M_P^2}\,\frac{e^{4\pi\xi}}{\xi^6}\right)$$

$$\mathcal{P}_L(\mathbf{k}) = \frac{H^2}{\pi^2 M_P^2}\left(1 + 2 \times 10^{-9}\,\frac{H^2}{M_P^2}\,\frac{e^{4\pi\xi}}{\xi^6}\right)$$

Cook, LS 1109.0022

ξ *increases* during inflation

$$\xi \equiv \frac{\dot{\phi}}{2\,f\,H} \gtrsim 1$$

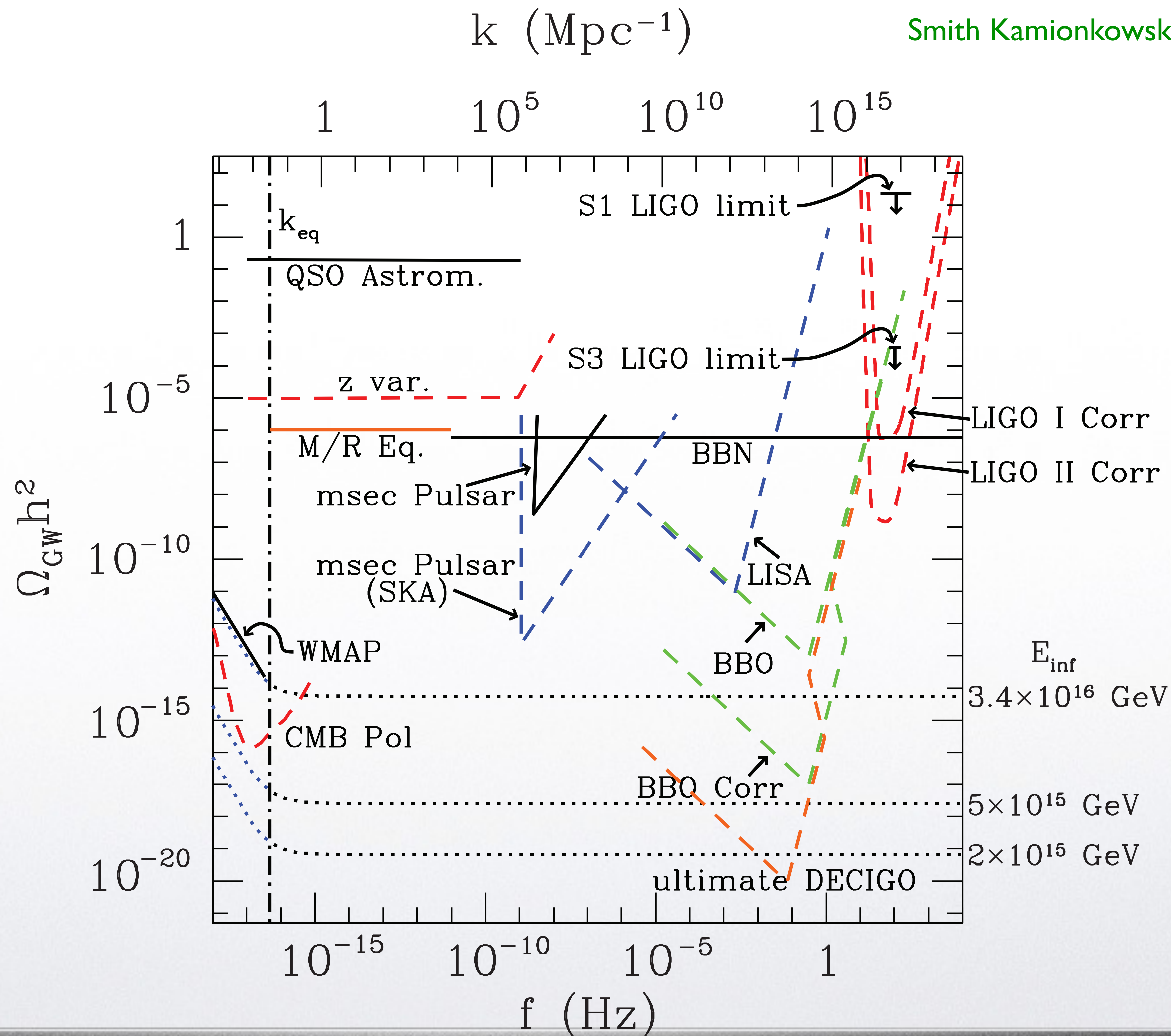
GWs produced towards the end of inflation (i.e. at smaller scales) have larger amplitude

might be detected by advanced LIGO!

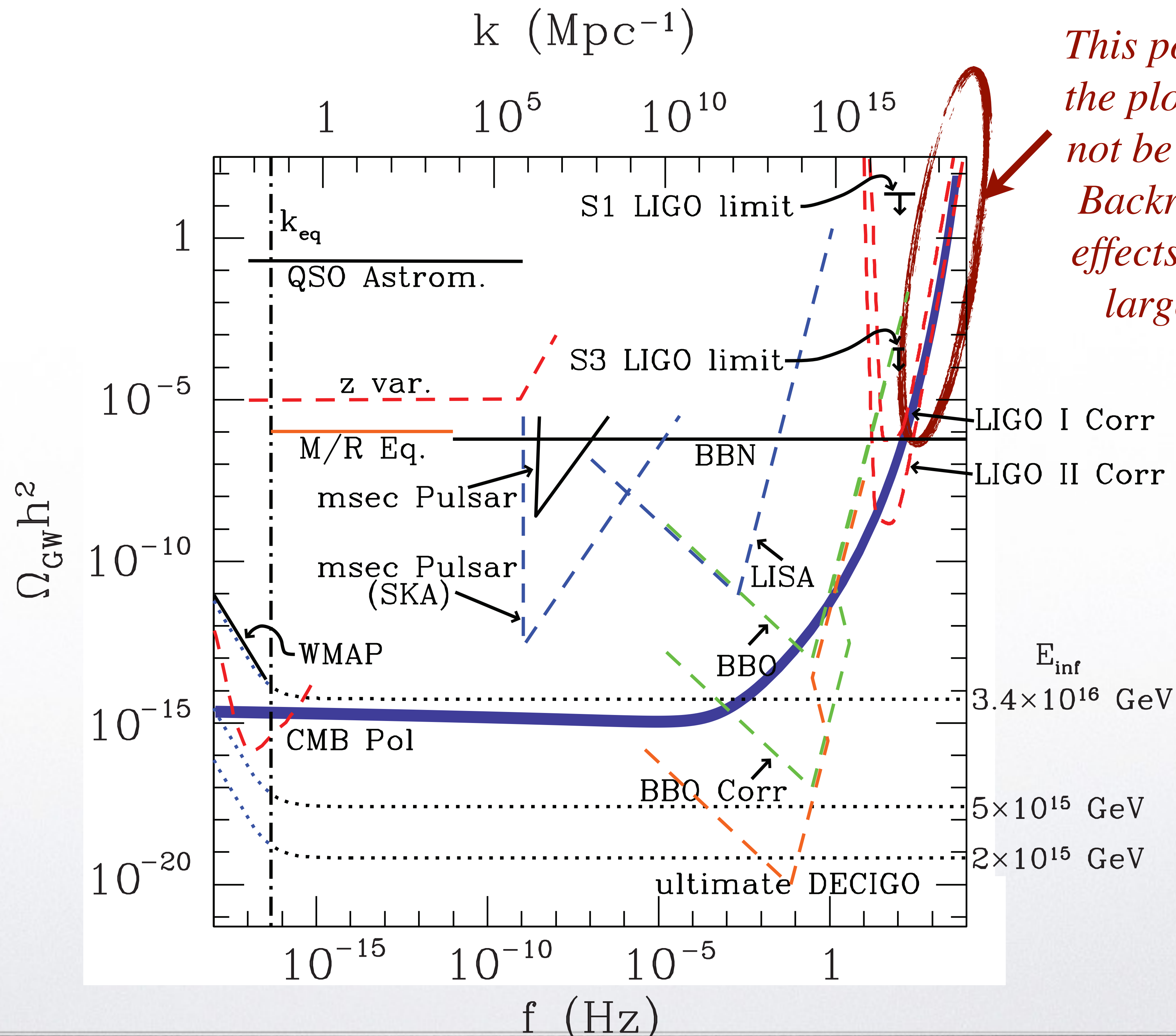
Note: constraints from f_{NL} do not apply at LIGO scales!

Prospects of direct detection of GWs of inflationary origin

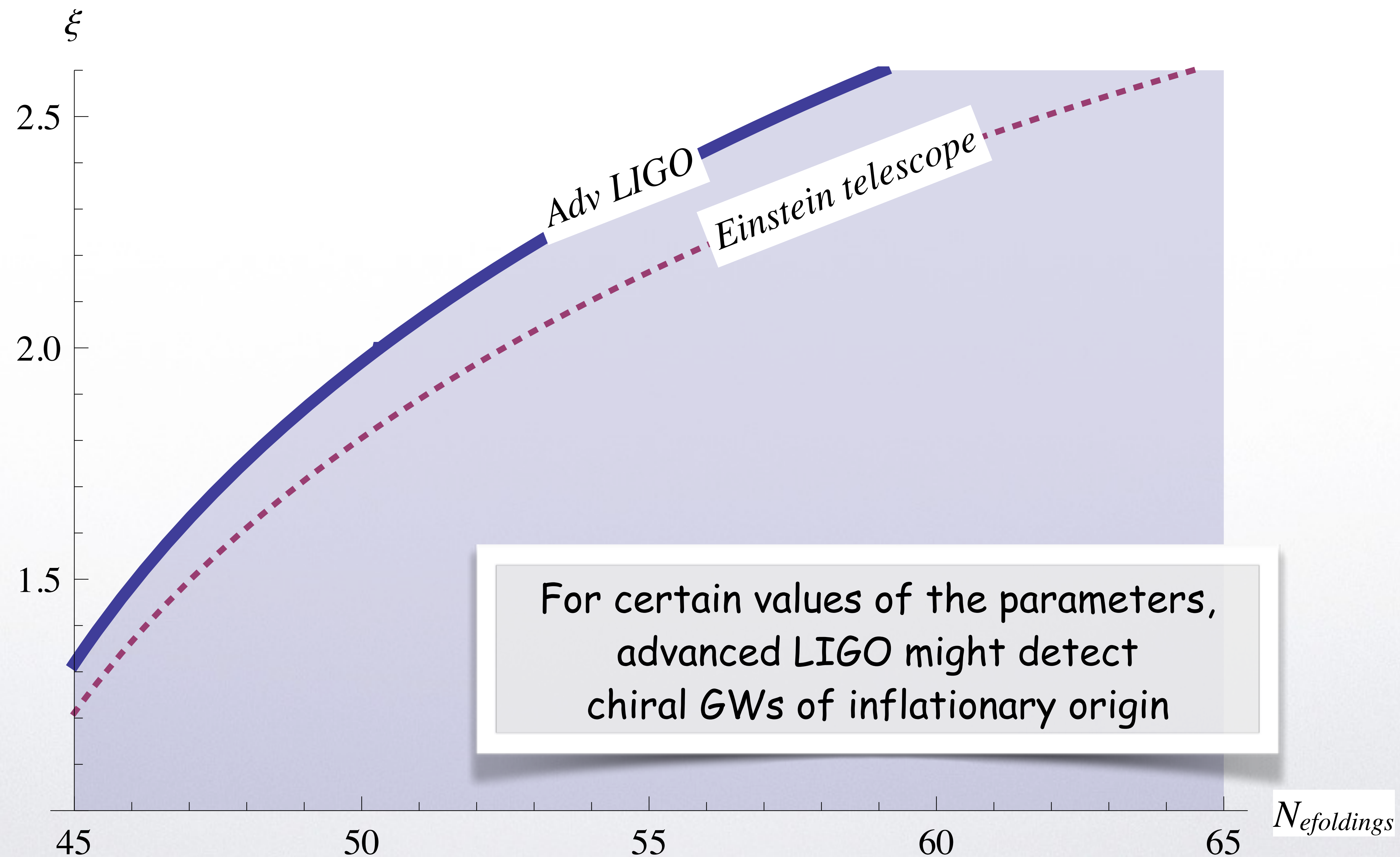
Smith Kamionkowski Cooray 06



N=50 efoldings
$V(\phi)=m^2 \phi^2/2$,
$\xi_{COBE}=2.1$



This portion of the plot should not be trusted! Backreaction effects are too large here



For certain values of the parameters, advanced LIGO might detect chiral GWs of inflationary origin

A few comments

These tensor modes would be chiral! Crowder et al 12

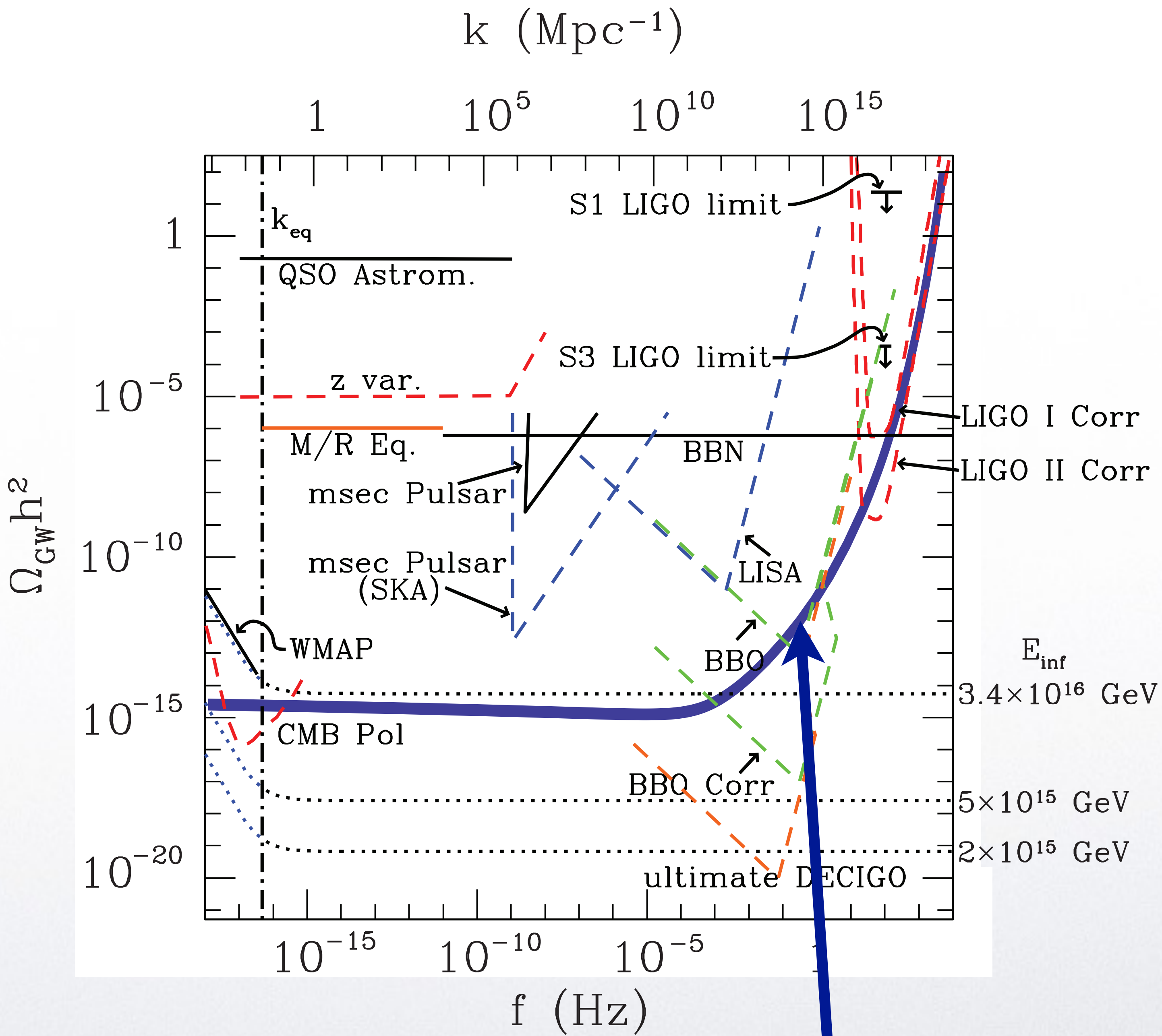
The GWs produced this way should be strongly nongaussian Thrane 12

Signal might correlate with nongaussianities at CMB/ LSS scales

Large and nongaussian fluctuations at the end of inflation might generate primordial BHs

Linde and Pajer 13

And by the way....



Example of **BLUE** tensor spectrum
without violation of energy conditions

Conclusion

Even if tensors usually look boring,
they can have a rich phenomenology
(and all we have seen originates from a single operator)

Nonvanishing <*EB*> and <*TB*> could also be produced by some late-Universe effect (e.g. pseudoscalar quintessence)

Gluscevic and Kamionkowski 2010 have however shown that it is possible to distinguish a primordial <*EB*> and <*TB*> from a late one

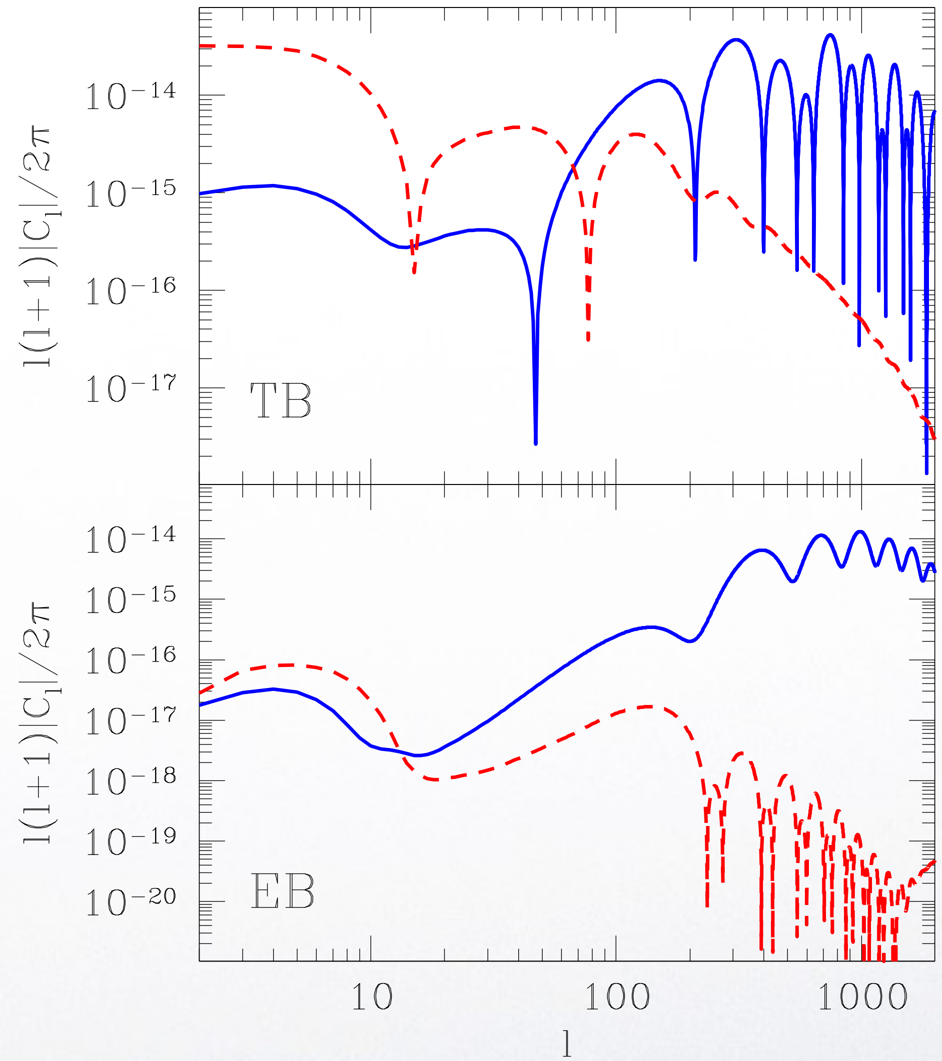


FIG. 5: We show TB and EB power spectra from chiral GWs for $\Delta\chi = 0.2$ and $r = 0.22$ (dashed red curves) and from cosmological birefringence for $\Delta\alpha = 5'$ (solid blue curves).

Also note

A “natural” coupling that might lead to nonvanishing <*EB*> and <*TB*> is

$$\delta\mathcal{L} = \frac{\phi}{f'}\epsilon_{\alpha\beta\gamma\delta}\, R^{\alpha\beta}{}_{\mu\nu}\, R_{\gamma\delta}{}^{\mu\nu}$$

however...

Lyth Quinbay Rondriguez 05
Satoh 10

Action for tensor modes in theory with $\phi\, R\, \tilde{R}$

$$\mathcal{S} = \sum \frac{1}{2} \int d\tau \, \frac{d^3 k}{(2\,\pi)^3} A_\lambda \left(|h'_\lambda|^2 - k^2 \, |h_\lambda|^2 \right)$$

$$A_\lambda = 1 - \lambda \frac{k}{a} \, \frac{\dot{\phi}}{2\, f' \, M_P^2}$$

for k too large one of the modes is strongly coupled and/or a ghost

if we choose parameters so to stay away from strongly coupled regime, then effect on tensor modes is too weak

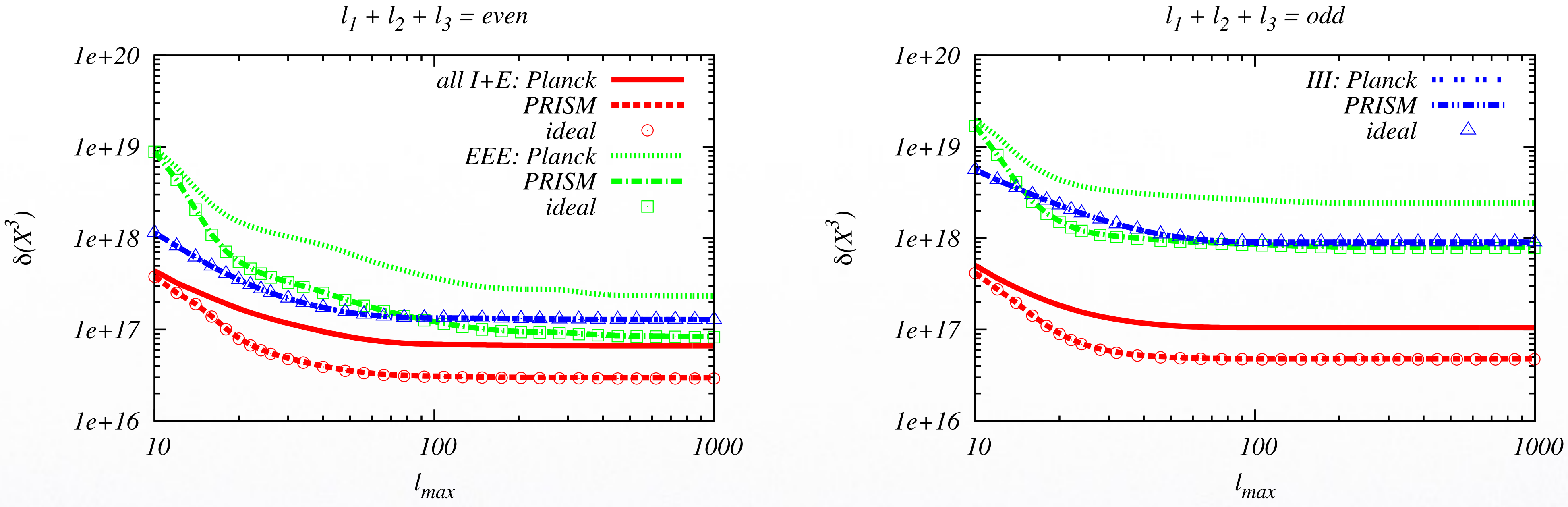


Figure 2. Expected 1σ errors of X^3 (5.5) obtained by using the parity-even (left panel) and parity-odd (right panel) signals in all types of the temperature and E-mode bispectra (red lines), the E-mode auto-bispectrum alone (green lines) and the temperature auto-bispectrum alone (blue lines). Here we assume the *Planck*, PRISM, and cosmic-variance-limited ideal experiments.