

# Halo approach and Test of Gravity with Redshift-Space Distortions

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C. Hikage and K.Y. JCAP 08(2013)

and collaboration with G. Huetsi, A. Terukina, T. Kanemaru,  
C. Hikage, A. Oka, et al.

- 1. Introduction of RSD
- 2. Halo approach
- 3. New test of gravity
- 4. Summary and conclusions

# 1. Introduction

## Redshift-space distortions

## Test of gravity theory

peculiar velocity of galaxy  $\vec{V}$   
shifts the redshift by the Doppler effect

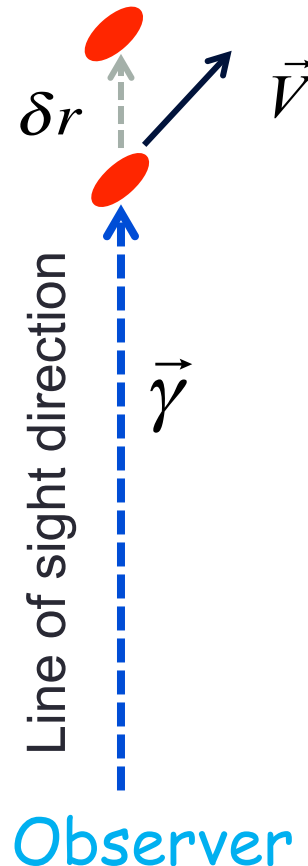
$$\delta z = (1+z)\vec{\gamma} \cdot \vec{V}$$

shift of the radial position in redshift space

$$s = r + \delta r$$

$$\delta r = \frac{\delta z}{H(z)} = \frac{(1+z)\vec{\gamma} \cdot \vec{V}}{H(z)}$$

Galaxies distribution in the redshift-space  
is different from that in the real-space



## Linear (redshift-space) distortion (Kaiser 1987)

Within the linear theory of density perturbations,  $V$  is connected with  $\delta$ , and this RSD reflects the growth rate of density perturbation

galaxy power spectrum with linear distortion

$$P^s(k, \mu) = (b + f\mu^2)^2 P_{\text{linear}}(k)$$

$\mu$  is the directional cosine between the line of sight direction and the wave number vector

$$f = \frac{d \ln \delta(a)}{d \ln a} \quad \mu = \frac{\vec{k} \cdot \vec{\gamma}}{|\vec{k}|} = \cos \theta$$

Correlation becomes anisotropic !

galaxy density contrast in redshift-space

$$\tilde{\delta}_g^s = \tilde{\delta}_g^r + \mu^2 f \tilde{\delta}$$

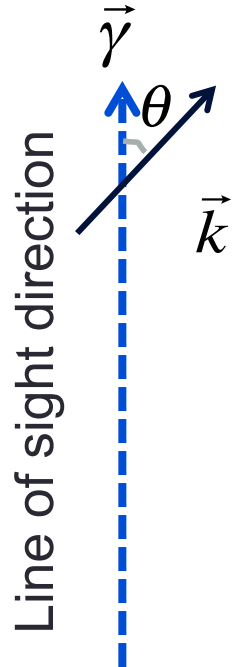
$$\frac{\partial \delta(t, x)}{\partial t} + \frac{1}{a} \nabla \cdot \vec{V}(t, x) = 0$$

$$s = r + \delta r = r + \frac{\vec{\gamma} \cdot \vec{V}}{aH}$$

$$\bar{n}_g (1 + \delta_g^s) ds^2 = \bar{n}_g (1 + \delta_g^r) dr^2$$

$$\tilde{\delta}_g^r = b \tilde{\delta}$$

galaxy density contrast in real-space



Observer

# Multipole power spectrum $\rightarrow$ anisotropy

$L_\ell(\mu)$  Legendre Polynomial

$$P^s(k, \mu) = \sum_{\ell=0,2,4,\dots} P_\ell(k) L_\ell(\mu) (2\ell + 1)$$

Within linear theory

$$P(k, \mu) = (b + f\mu^2)^2 P_{linear}(k)$$

Kaiser (87)

$$P_0(k) \text{ monopole} \longrightarrow P_0(k) = \left(b^2 + \frac{2}{3}bf + \frac{1}{5}f^2\right) P_{linear}(k)$$

$$P_2(k) \text{ quadrupole} \longrightarrow P_2(k) = \left(\frac{4}{15}bf + \frac{4}{35}f^2\right) P_{linear}(k)$$

$$P_4(k) \text{ hexadecapole} \longrightarrow P_4(k) = \frac{8f^2}{315} P_{linear}(k)$$

$$P_6(k) \text{ tetrahexadecapole} \longrightarrow P_6(k) = 0$$

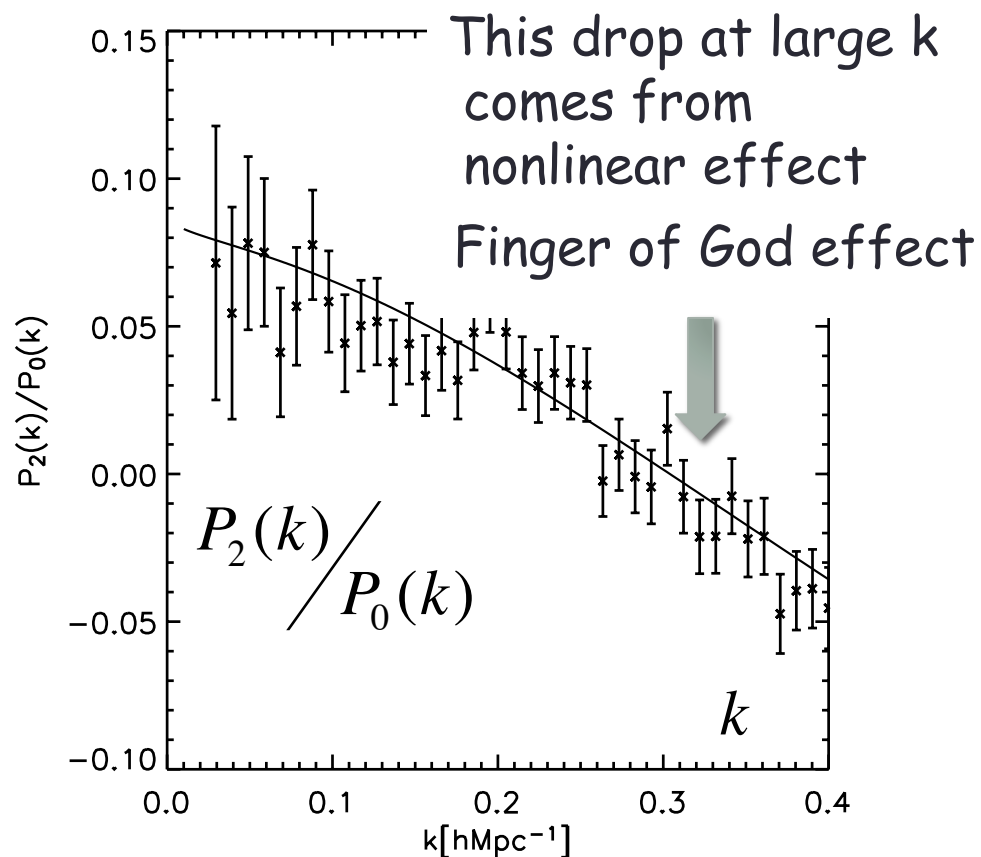
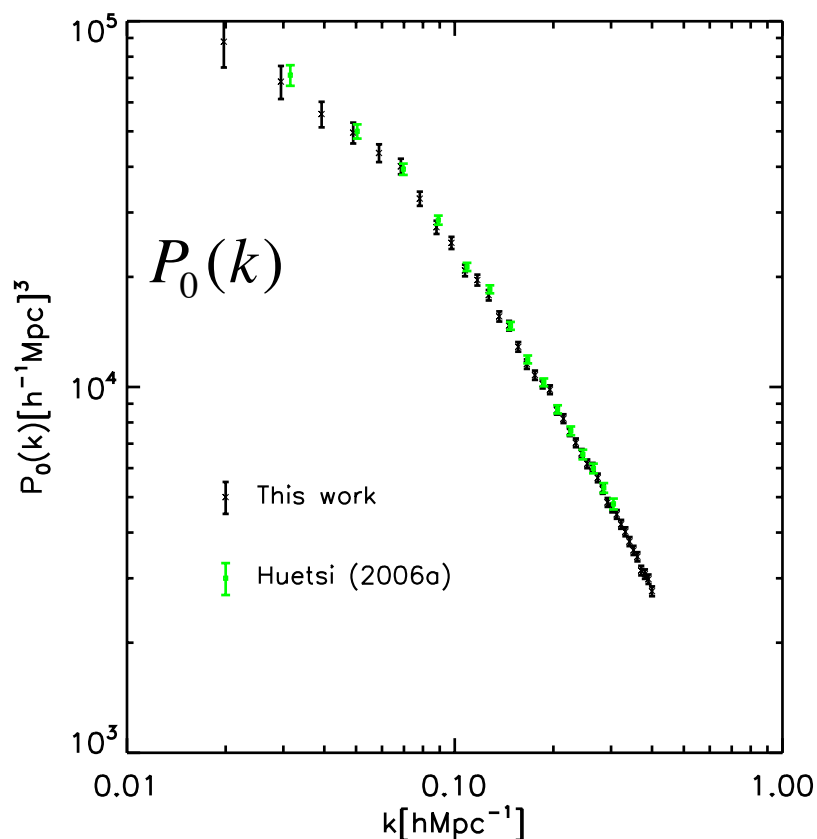
Growth rate  $f$  can be measured using multipole spectrum

$$\text{Linear theory predicts } \frac{P_2(k)}{P_0(k)} = \frac{\frac{4}{15}bf + \frac{4}{35}f^2}{b^2 + \frac{2}{3}bf + \frac{1}{5}f^2} = \text{const.}$$

# Test of gravity with $P_\ell(k)$

Yamamoto, Sato, Huetsi (08)  
(cf. Gozzo et al., 08)

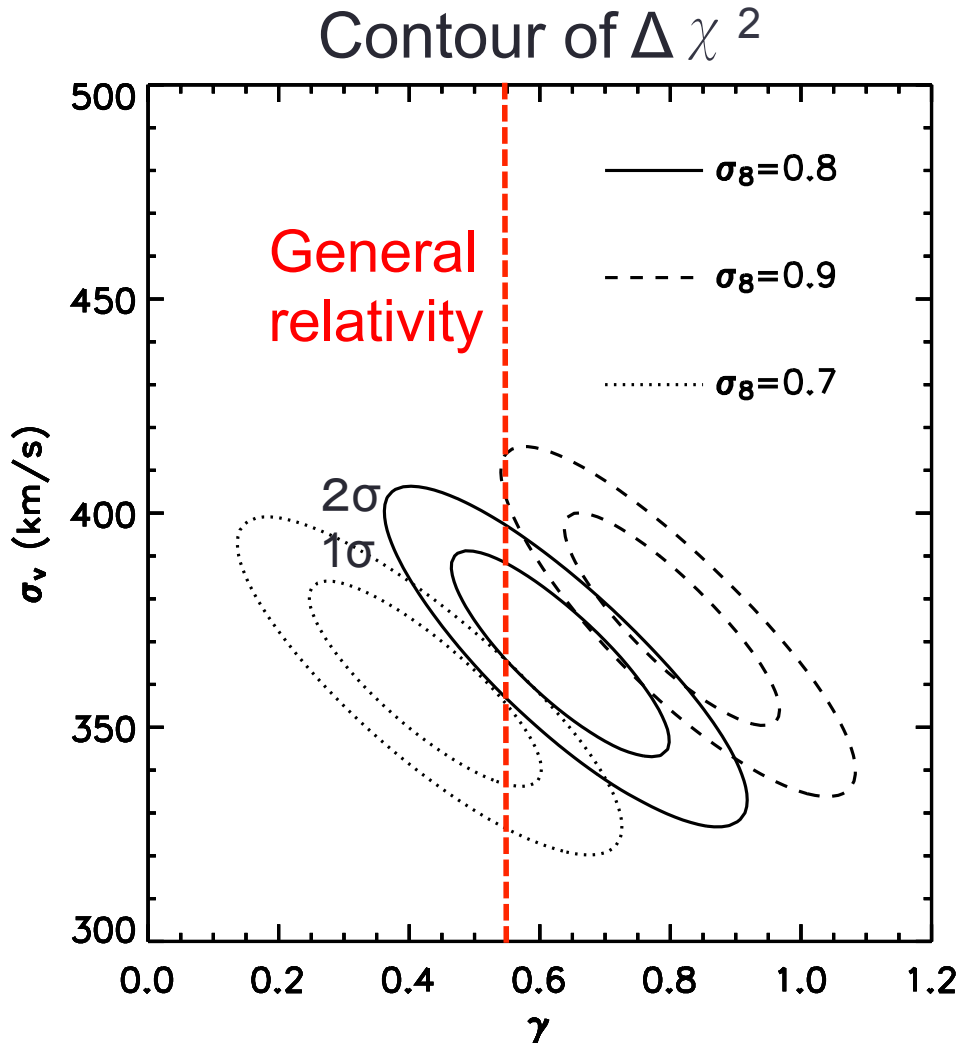
SDSS II DR5 luminous red galaxy (LRG) sample power spectrum



Test of general relativity

# Test of general relativity KY, Sato, Huetsi (2008)

random velocity dispersion



$$f(a) = \frac{d \ln \delta(a)}{d \ln a}$$

$$= [\Omega_m(a)]^\gamma$$

Within general relativity

$$\gamma = 0.55 - 0.56$$

reproduces the exact formula well.

# Test of $f(R)$ gravity

(Yamamoto, Nakamura, Huetsi, Narikawa, Sato, 2010)

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R + f(R)] + \int d^4x \sqrt{-g} L_m$$

A parameter of the theory is the effective mass of the new degree of freedom,

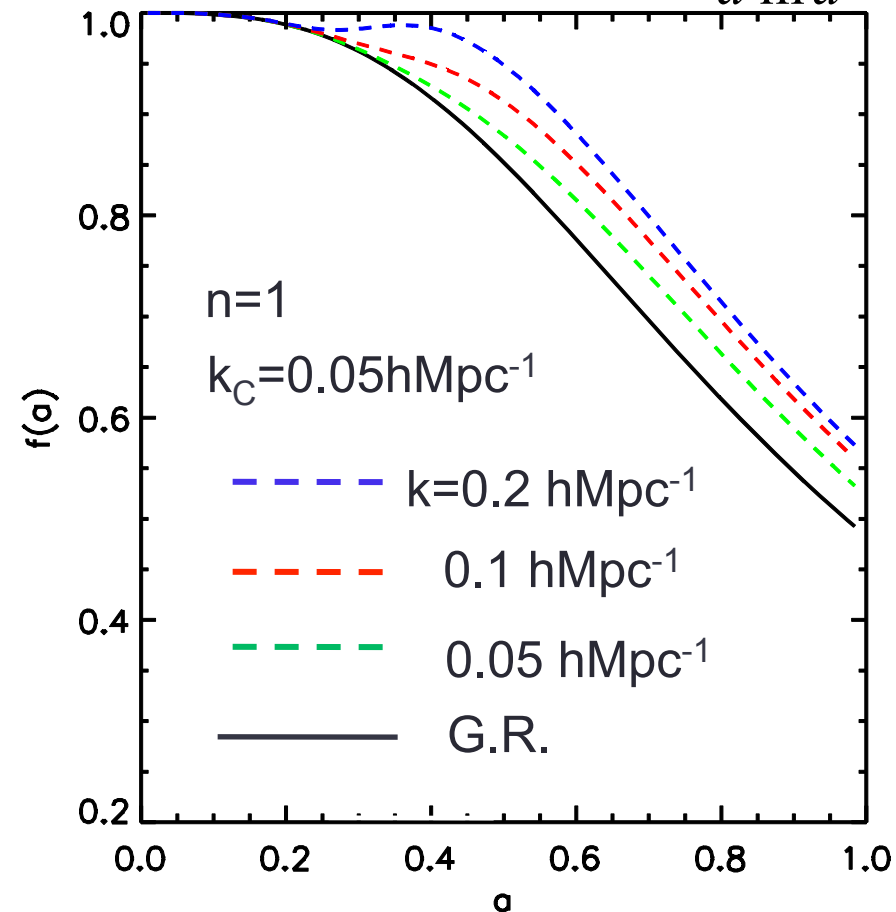
The effective mass determines a characteristic Compton wavenumber

$(k \ll k_c = m_{eff}^{-1})$  general relativity

$(k \gg k_c = m_{eff}^{-1})$  Modified gravity

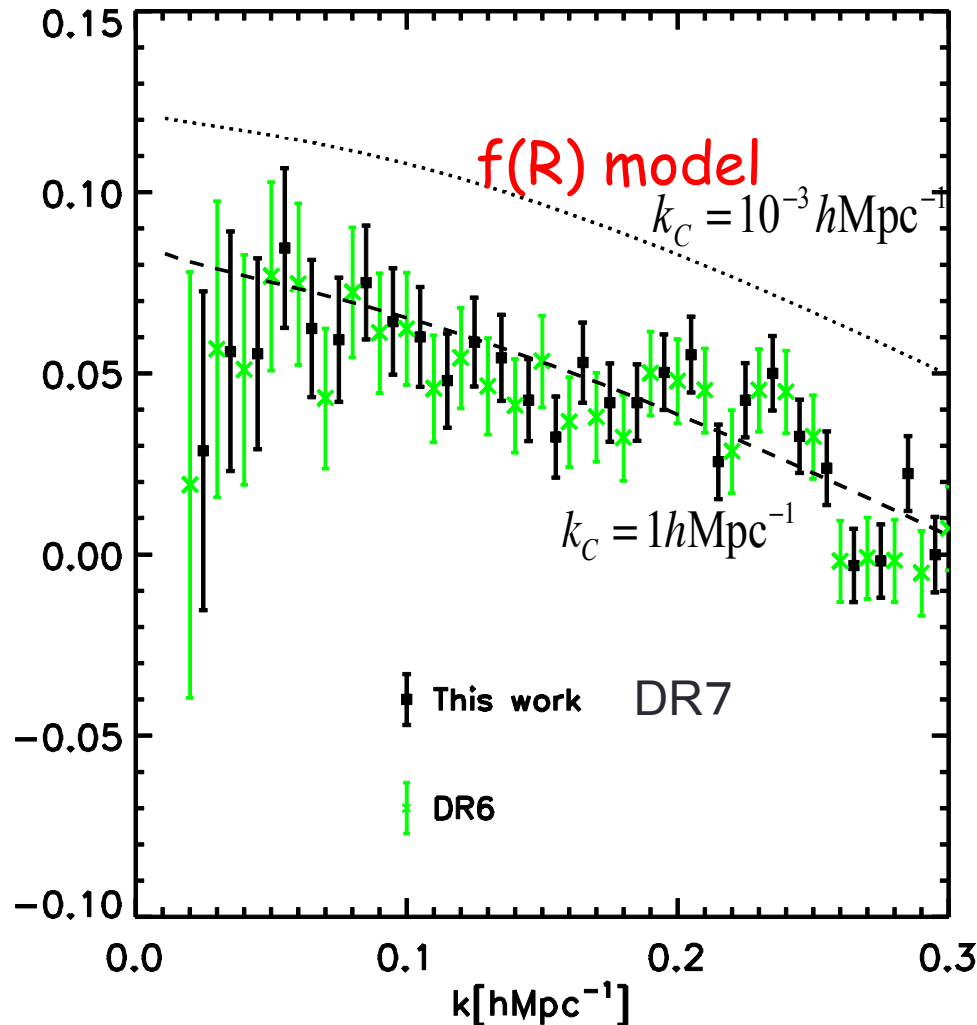
$$\frac{G_{eff}}{G_N} = \frac{4}{3}$$

growth rate  $f = \frac{d \ln D_1}{d \ln a}$



# Constraint of $f(R)$ gravity

(Yamamoto, Nakamura, Huetsi,  
Narikawa, Sato, 2010)



$$k_C \leq 0.04 h\text{Mpc}^{-1}$$

$$(|f_{R0}| \leq 10^{-4})$$

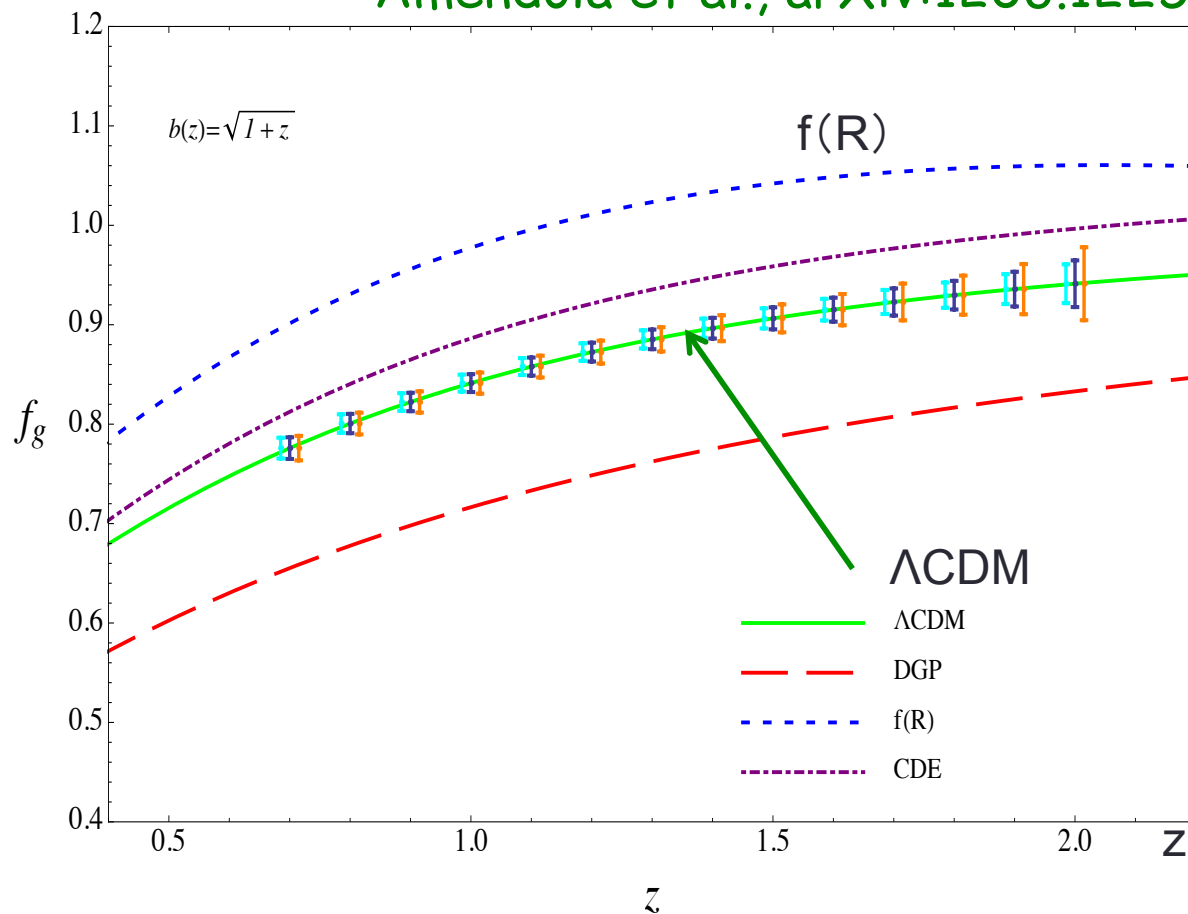
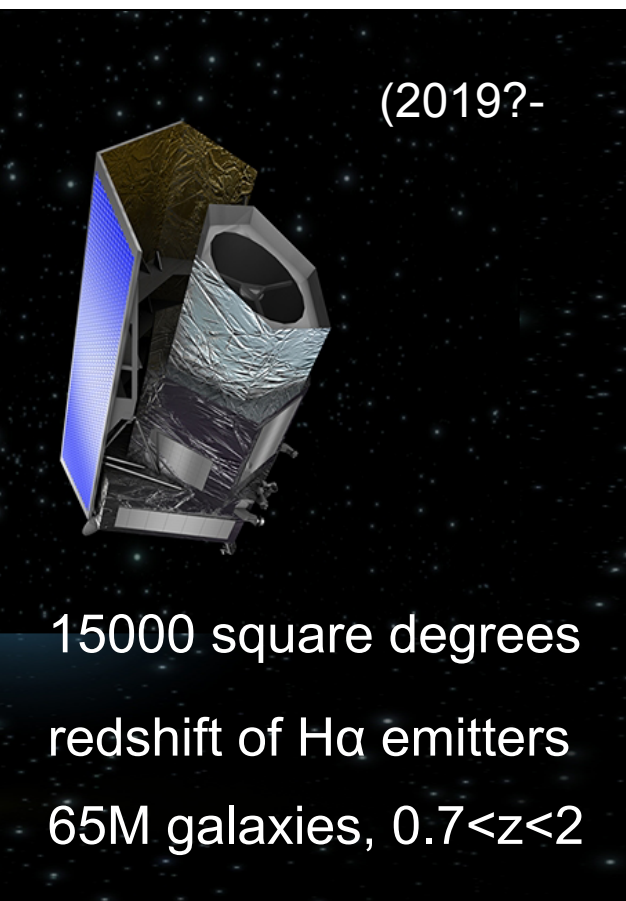
More robust constraint  
Influences of systematics

(cf. Taruya-san's talk,  
Oka et al., 2014)



# Future prospect with Euclid satellite

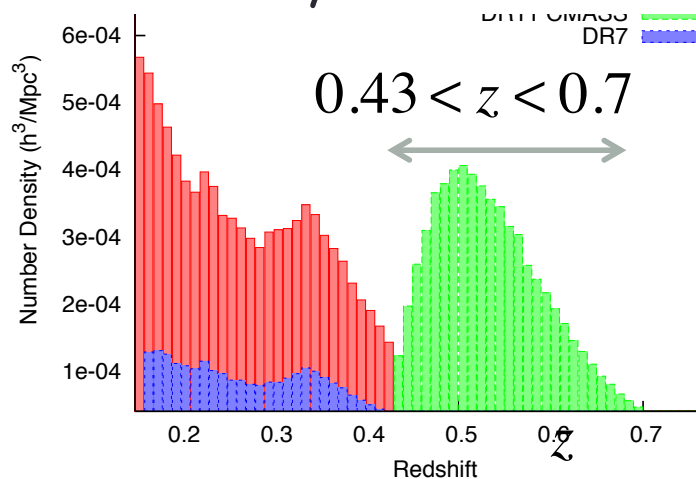
Amendola et al., arXiv:1206.1225



Measurements of the redshift-space distortions is expected to be a promising way for testing gravity.

# SDSS BOSS DR11 CMASS sample

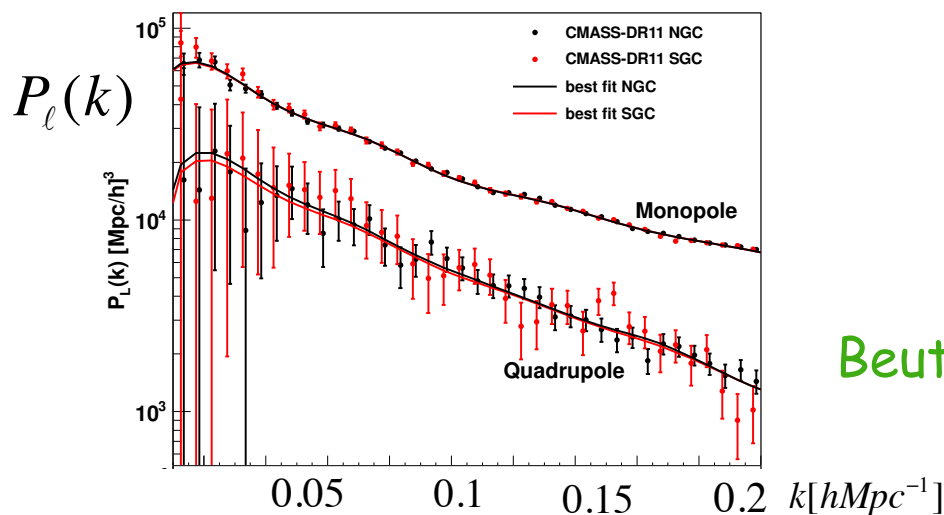
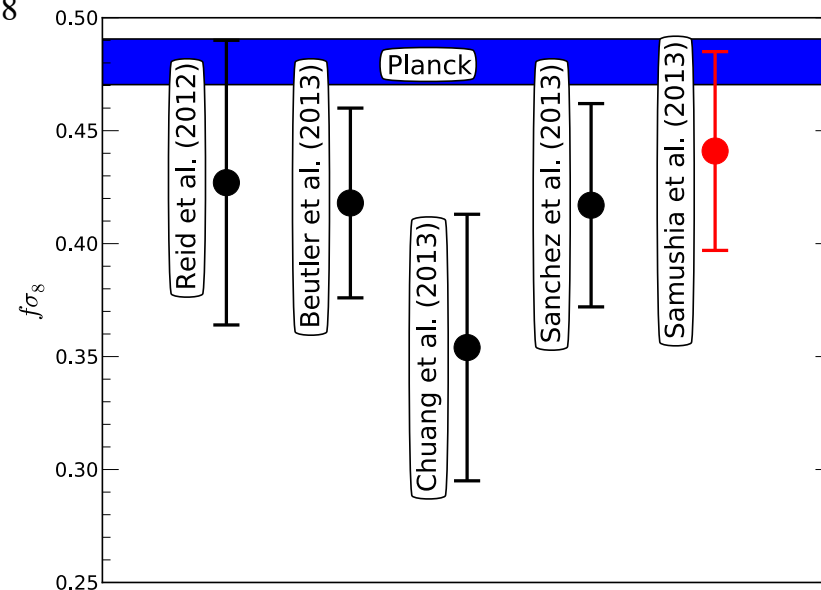
Number density



Chuang et al. (2013)

Samushia et al (2013)

$f\sigma_8$

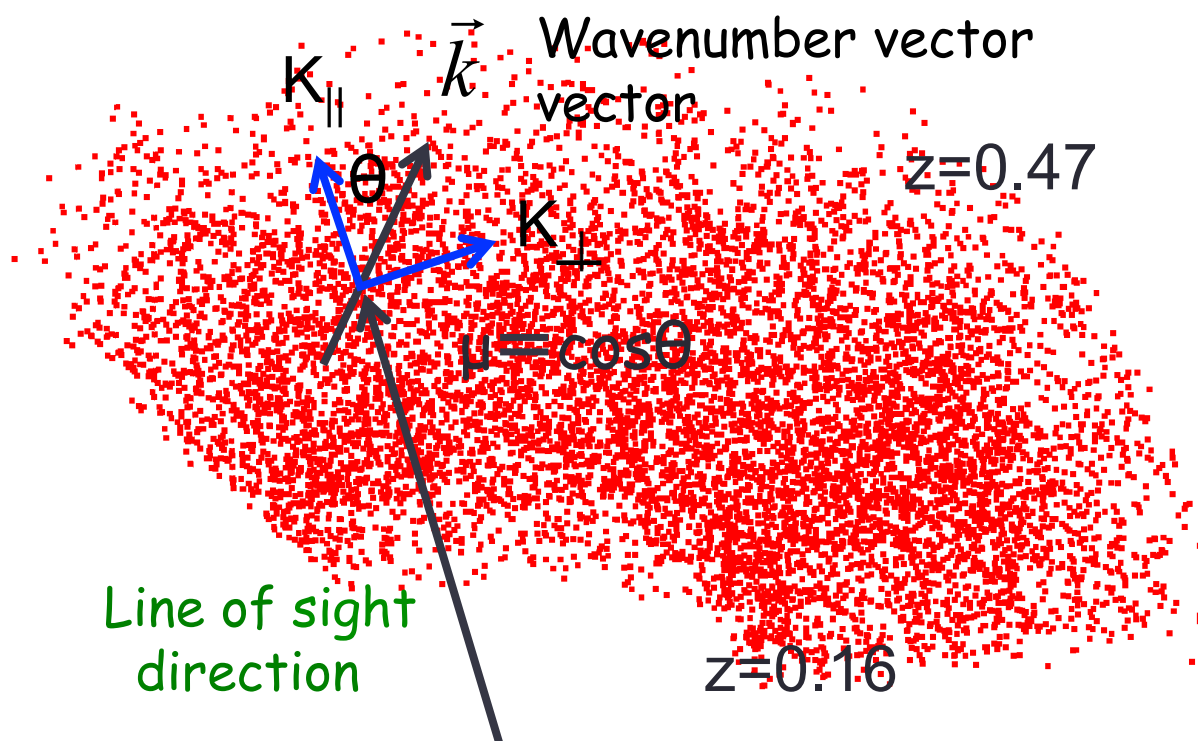


Constraint on  $f\sigma_8$   
smaller than Planck prediction

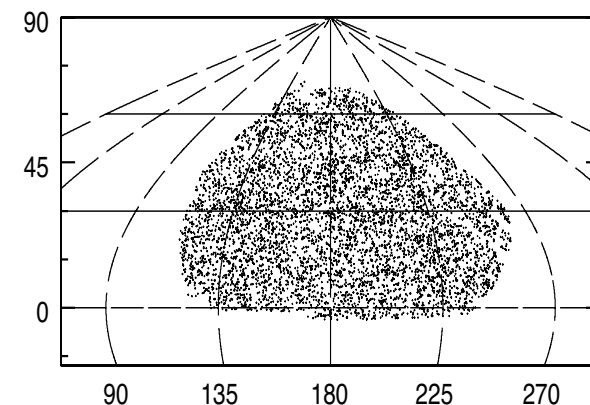
Beutler et al.(2013)

Tension between Planck and Galaxy

Multipole power spectrum analysis of SDSS luminous red galaxy (LRG) sample (K.Y., Sato, Huesti 2008-)  
(K.Y.+ 2010)

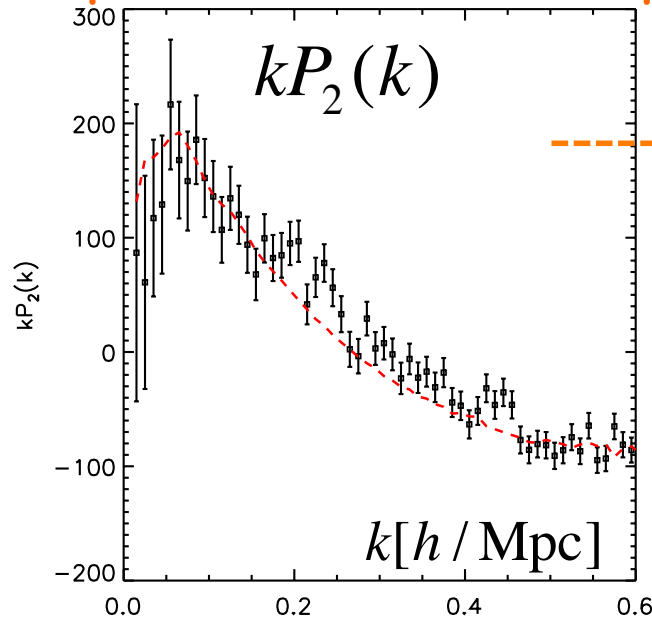
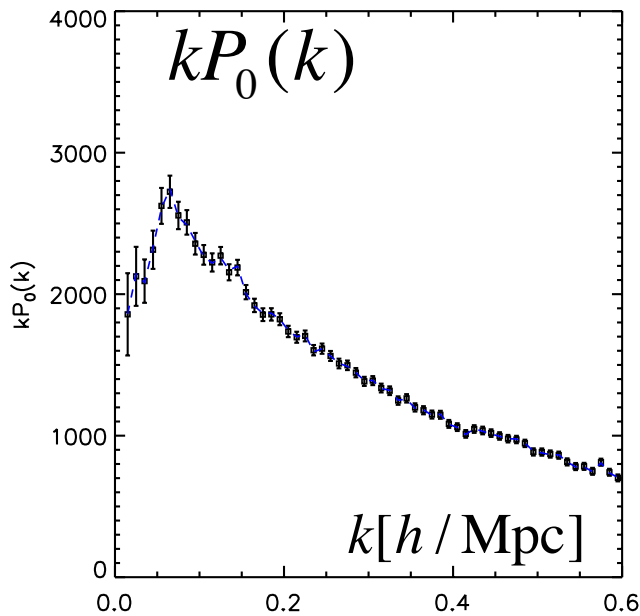


$10^5$  galaxies  
7150 degree<sup>2</sup>



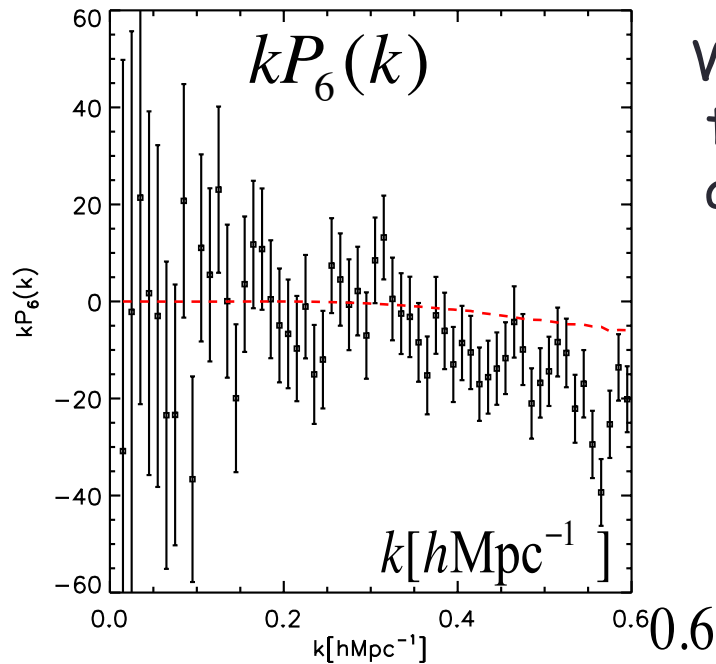
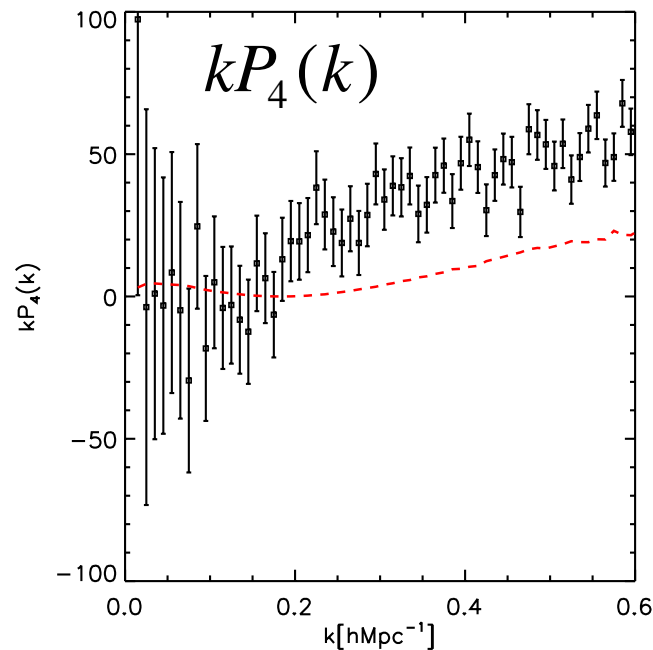
An algorithm to measure the multipole power spectrum. (K.Y., et al, 2006)

# Problem in multipole power spectrum of LRG sample (SDSS DR7)



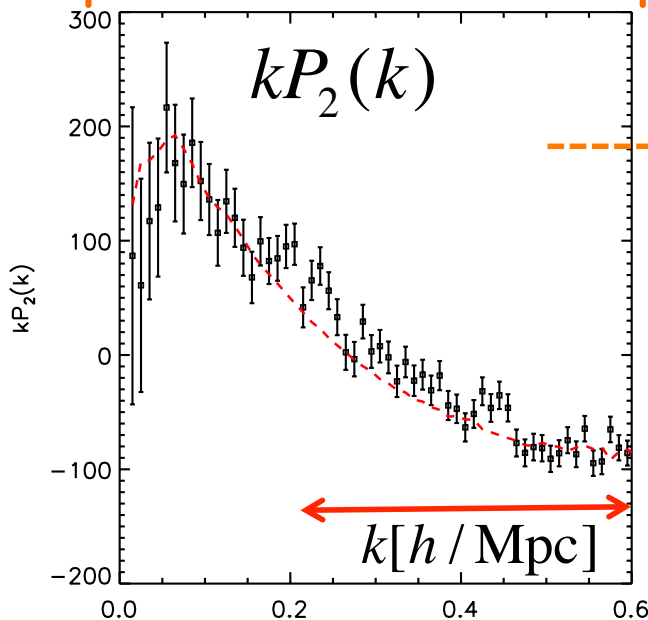
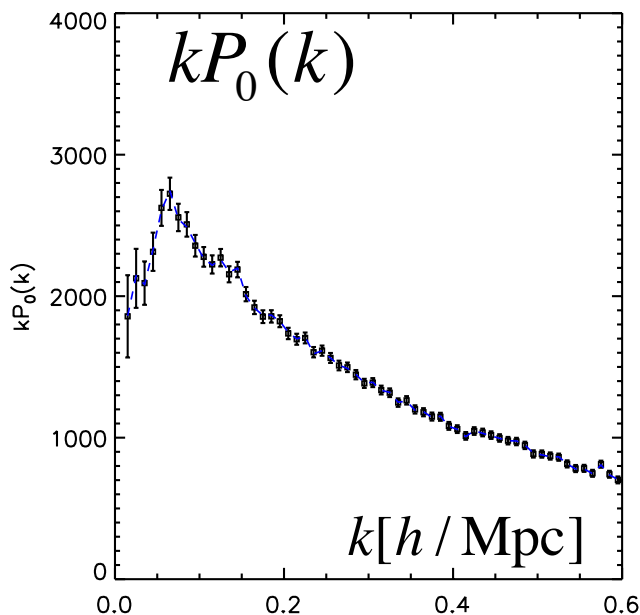
----- Theoretical model does not fit all the spectra at the same time.

In the linear theory,  $P_6(k)=0$



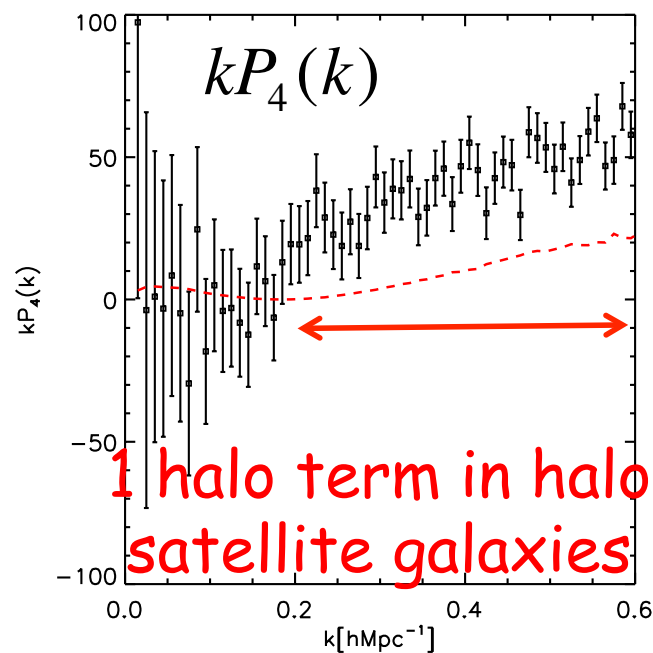
What determines the feature at large  $k$ ?

# Problem in multipole power spectrum of LRG sample (SDSS DR7)

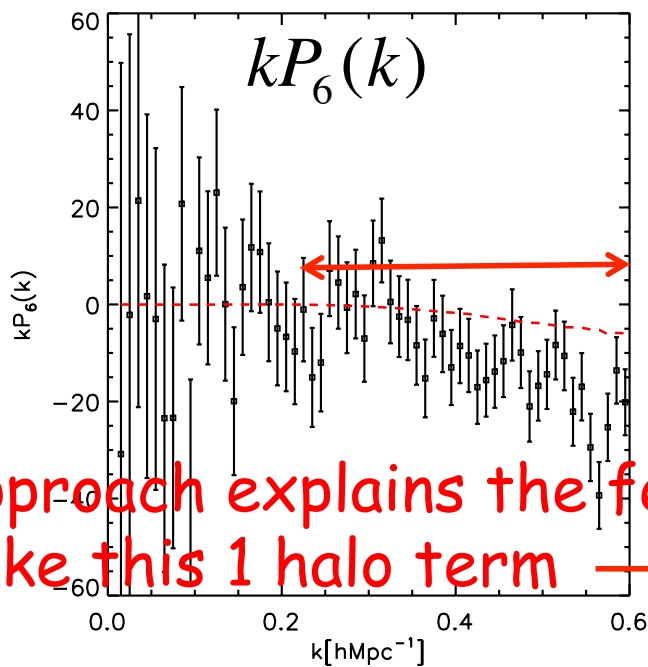


----- Theoretical model does not fit all the spectra at the same time.

In the linear theory,  $P_6(k)=0$



What determines the feature at large  $k$ ?



(Hikage and KY. 2013)

1 halo term in halo approach explains the feature at large  $k$   
 satellite galaxies make this 1 halo term → gravity test

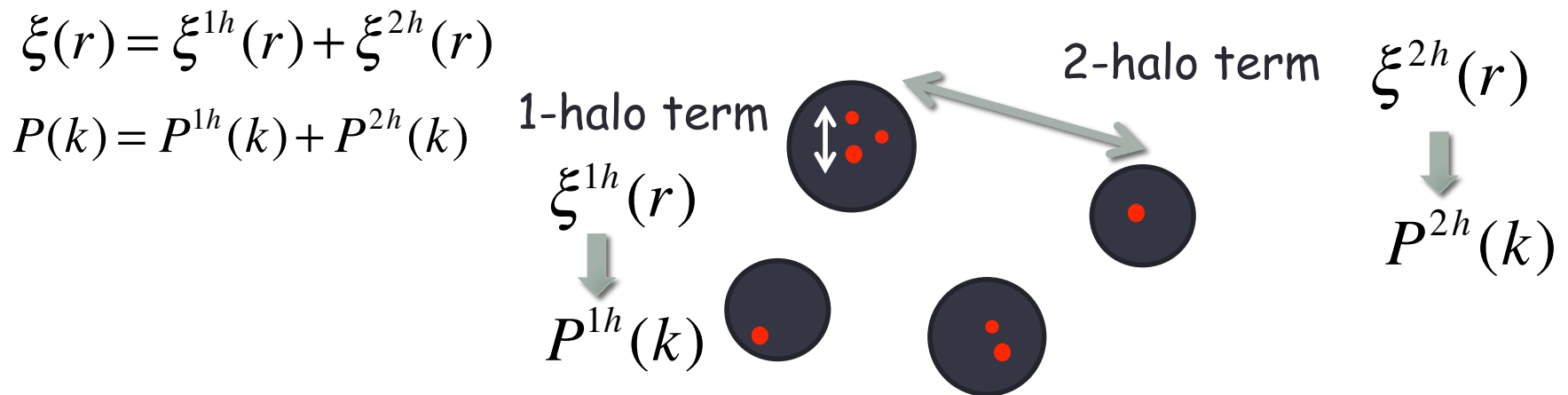
## 2. Halo approach

Every DM and galaxy reside in a dark matter halo, their distribution is described on the basis of the halo density profile  $\rho(r, M)$  with mass  $M$  and halo's correlation

Two point correlation is modeled as the sum of the contribution from **the pairs in the same halo (1 halo term)**, and from **the pairs in the different halos (2 halo term)**

$$\xi(r) = \xi^{1h}(r) + \xi^{2h}(r)$$

$$P(k) = P^{1h}(k) + P^{2h}(k)$$



1-halo term is the convolution of the density profile of DM or galaxy

$$\xi^{1h}(\vec{r}) = \frac{1}{\bar{\rho}^2} \int dM \frac{dn(M)}{dM} \int d\vec{y} \rho(|\vec{y}|, M) \rho(|\vec{y} + \vec{r}|, M)$$

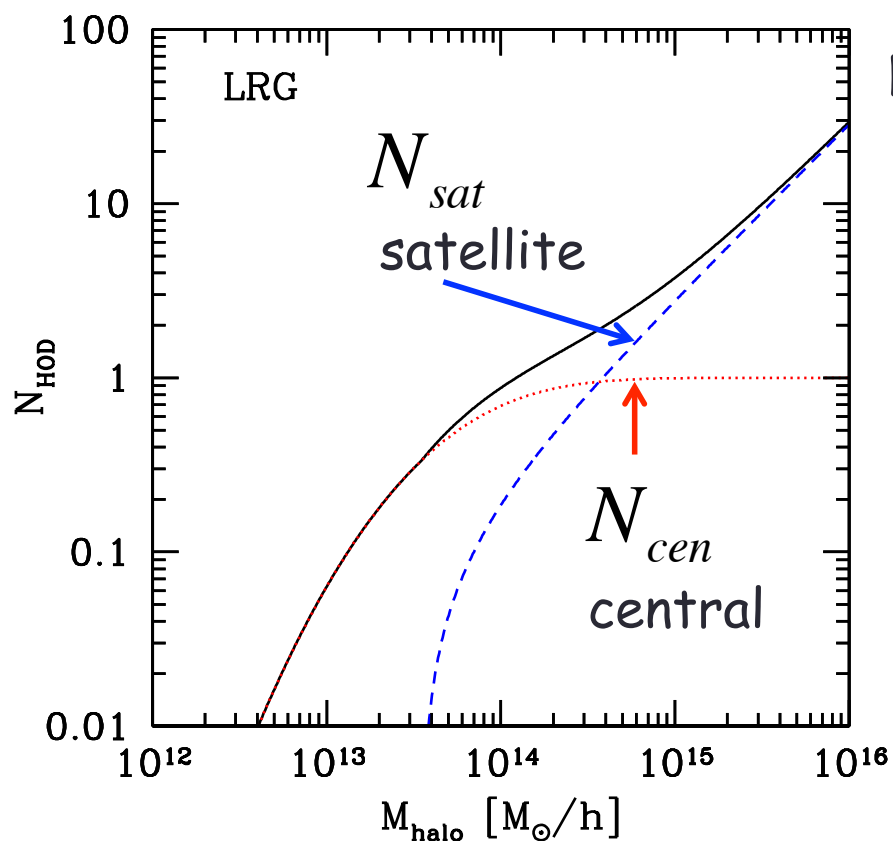
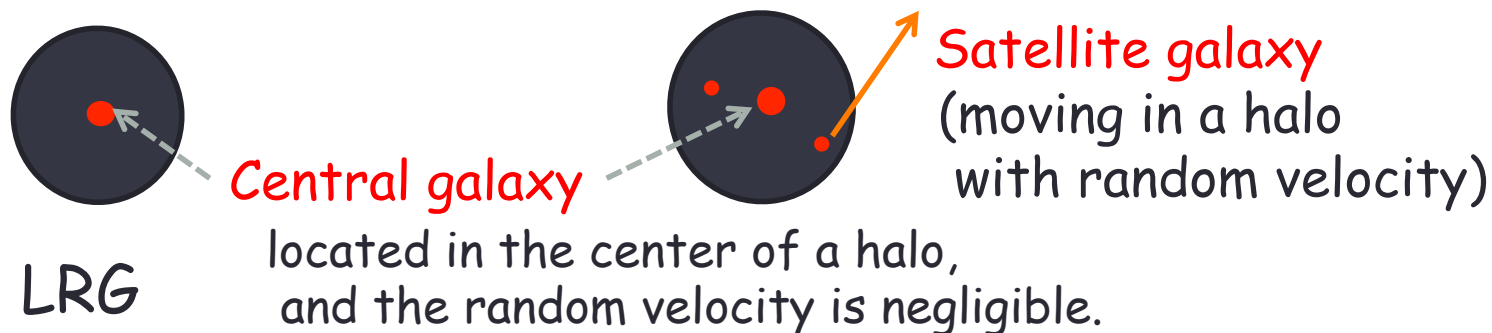
$$P^{1h}(k) = \frac{1}{\bar{\rho}^2} \int dM \frac{dn(M)}{dM} |u(k, M)|^2$$

Fourier Transform

$$u(k, M) = \frac{1}{(2\pi)^{3/2}} \int d\vec{y} \rho(|y|, M) e^{i\vec{k} \cdot \vec{y}}$$

galaxy correlation in halo model needs  
**Halo occupation distribution (HOD)**

Probability of galaxy number  
 in a halo with mass  $M$



Number of LRGs in a halo with mass  $M$   
 (Zheng, et al. 05, Zehavi, et al  
 Reid, Spergel 09)

$$N_{\text{cen}} = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\log_{10} M - \log_{10} M_{\text{min}}}{\sigma_{\log M}} \right) \right]$$

$$N_{\text{sat}} = \left( \frac{M - M_{\text{cut}}}{M_1} \right)^{\alpha}$$

Satellite fraction is 5% in the total LRGs

# Power spectrum in redshift-space $P(k, \mu) = P^{1h}(k, \mu) + P^{2h}(k, \mu)$

## 1-halo term

$$P^{1h}(k, \mu) = \frac{1}{\bar{n}^2} \int dM \frac{dn(M)}{dM} \left( \overbrace{2N_{cen} N_{sat}}^{\text{Central-Satellite}} p(k, \sigma_V, M) + \overbrace{N_{sat}^2}^{\text{Satellite-Satellite}} p(k, \sigma_V, M)^2 \right)$$

## 2-halo term

$$P^{2h}(k, \mu) \cong \frac{1}{\bar{n}^2} \left[ \int dM \frac{dn(M)}{dM} \langle N_{cen} \rangle (1 + \langle N_{sat} \rangle p(k, \sigma_V, M)) (b(M) + f\mu^2) \right]^2 P_m(k)$$

Linear distortion

Finger of God effect

Linear distortion

satellite galaxies have random velocity

Probability distribution function of random velocity  $V$  of satellite galaxy in a halo with mass  $M$

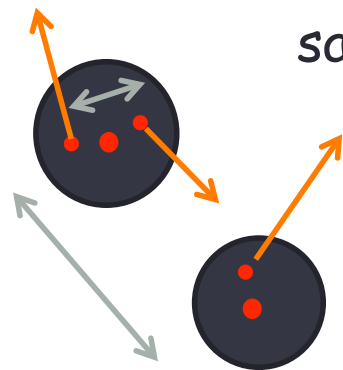
$$F(V_{\parallel}) = \frac{1}{\sqrt{2\pi}\sigma_V(M)} \exp\left[-\frac{V_{\parallel}^2}{2\sigma_V^2(M)}\right]$$

Finger of God RSD

Randomly mapped

$$s_{\parallel} = r_{\parallel} + \frac{V_{\parallel}}{aH}$$

$$p(k, \sigma_V, M) \cong u(k, M) \exp\left[-\frac{\sigma_V^2(M) k^2 \mu^2}{2a^2 H^2}\right]$$





# Power spectrum in redshift-space $P(k, \mu) = P^{1h}(k, \mu) + P^{2h}(k, \mu)$

## 1-halo term

$$P^{1h}(k, \mu) = \frac{1}{\bar{n}^2} \int dM \frac{dn(M)}{dM} \left( \overbrace{2N_{cen} N_{sat}}^{\text{Central-Satellite}} p(k, \sigma_V, M) + \overbrace{N_{sat}^2}^{\text{Satellite-Satellite}} p(k, \sigma_V, M)^2 \right)$$

## 2-halo term

$$P^{2h}(k, \mu) \cong \frac{1}{\bar{n}^2} \left[ \int dM \frac{dn(M)}{dM} \langle N_{cen} \rangle \left( 1 + \langle N_{sat} \rangle p(k, \sigma_V, M) \right) (b(M) + f\mu^2) \right]^2 P_m(k)$$

Linear distortion

Finger of God effect

satellite galaxies have random velocity

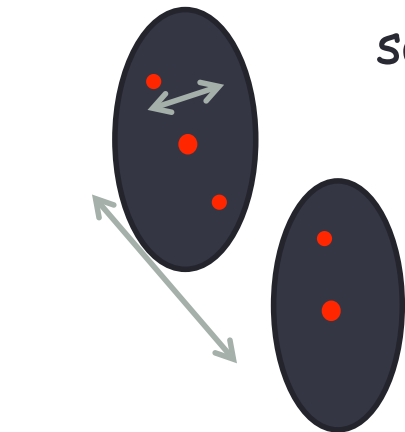
Probability distribution function  
of random velocity  $V$  of satellite galaxy  
in a halo with mass  $M$

$$F(V_{\parallel}) = \frac{1}{\sqrt{2\pi}\sigma_V(M)} \exp\left[-\frac{V_{\parallel}^2}{2\sigma_V^2(M)}\right]$$

Fourier transform of elongated satellite distribution

$$p(k, \sigma_V, M) \cong u(k, M) \exp\left[-\frac{\sigma_V^2(M) k^2 \mu^2}{2a^2 H^2}\right]$$

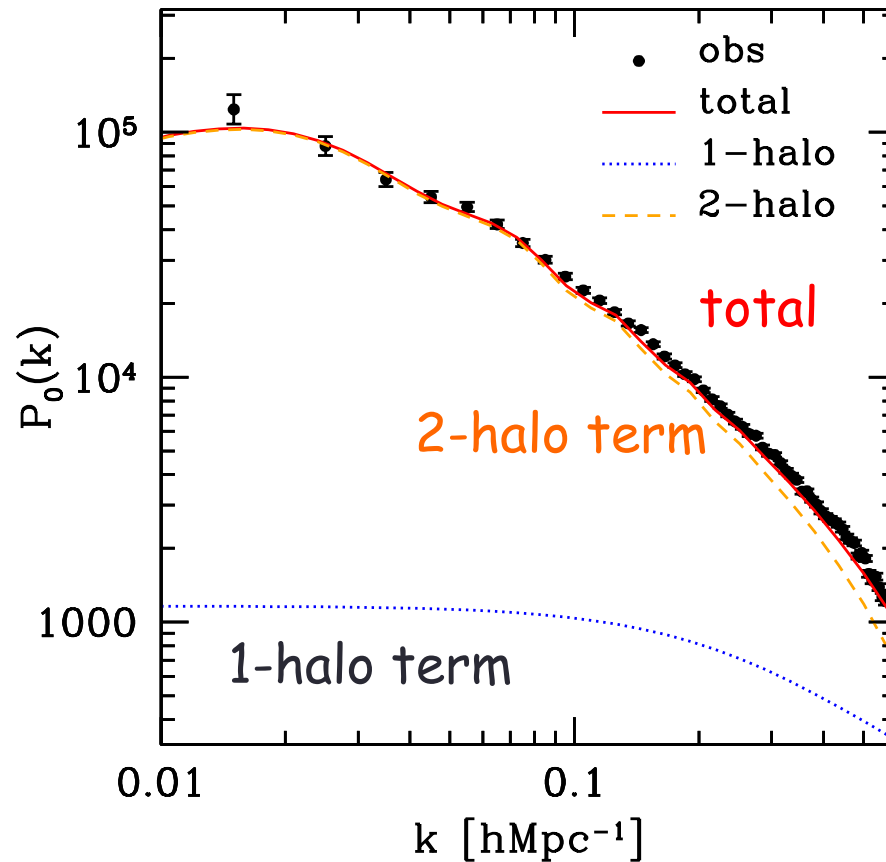
Finger of God  
RSD



$$s_{\parallel} = r_{\parallel} + \frac{V_{\parallel}}{aH}$$

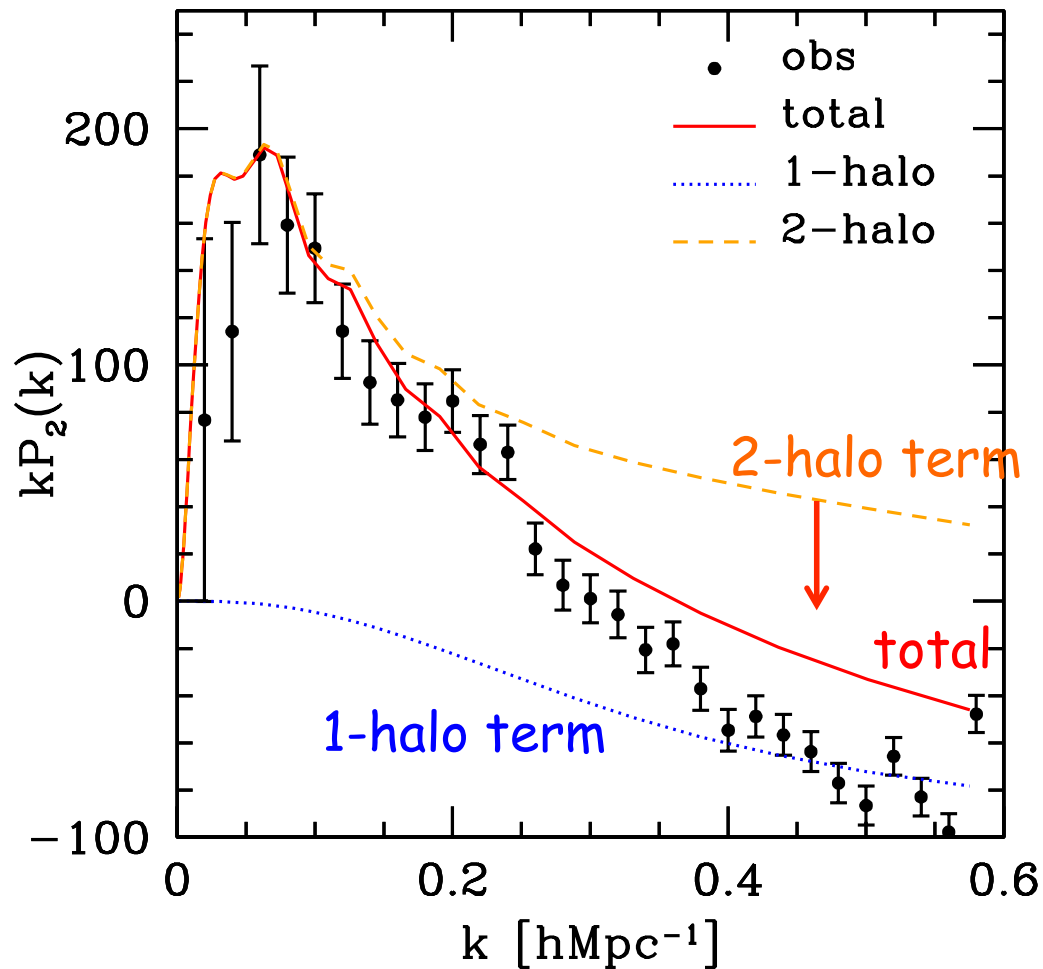
# Monopole power spectrum

$P_0(k)$  Comparison with LRG sample



No significant contribution of 1-halo term to the monopole spectrum (a few times 10 % level)

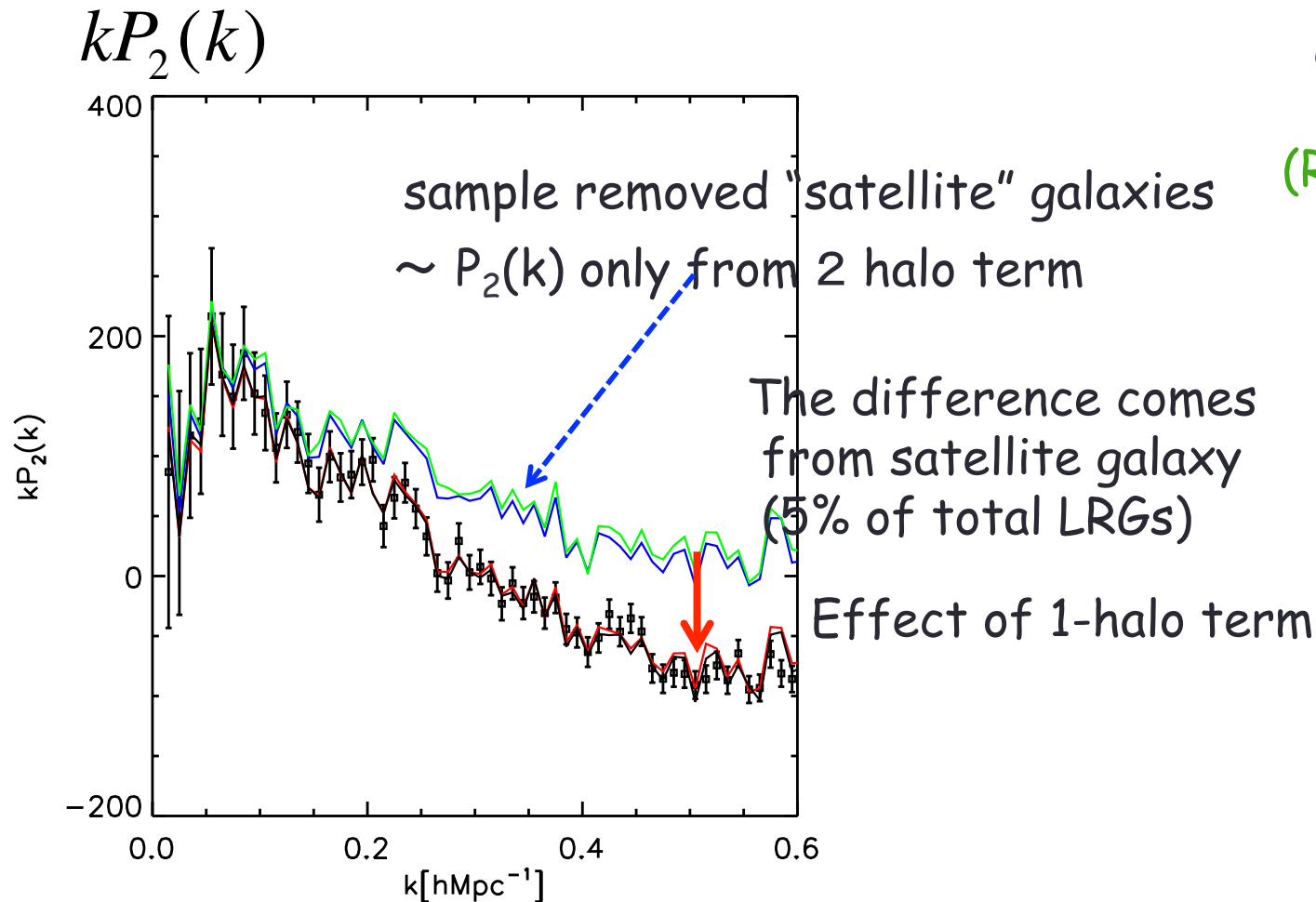
# $kP_2(k)$ Quadrupole spectrum



Significant contribution  
of 1-halo term  
to quadrupole spectrum  
at large  $k$  ( $k > 0.2 h\text{Mpc}^{-1}$ )

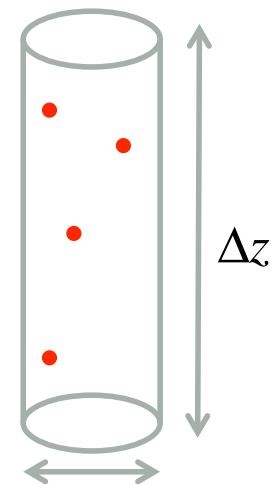
Possible contribution  
at quasi-linear regime  
( $k < 0.2 h\text{Mpc}^{-1}$ )

Significant contribution of 1-halo term to the quadrupole is confirmed by observational data



Count In Cylinder  
 technique  
 (Reid, Spergel 2008)

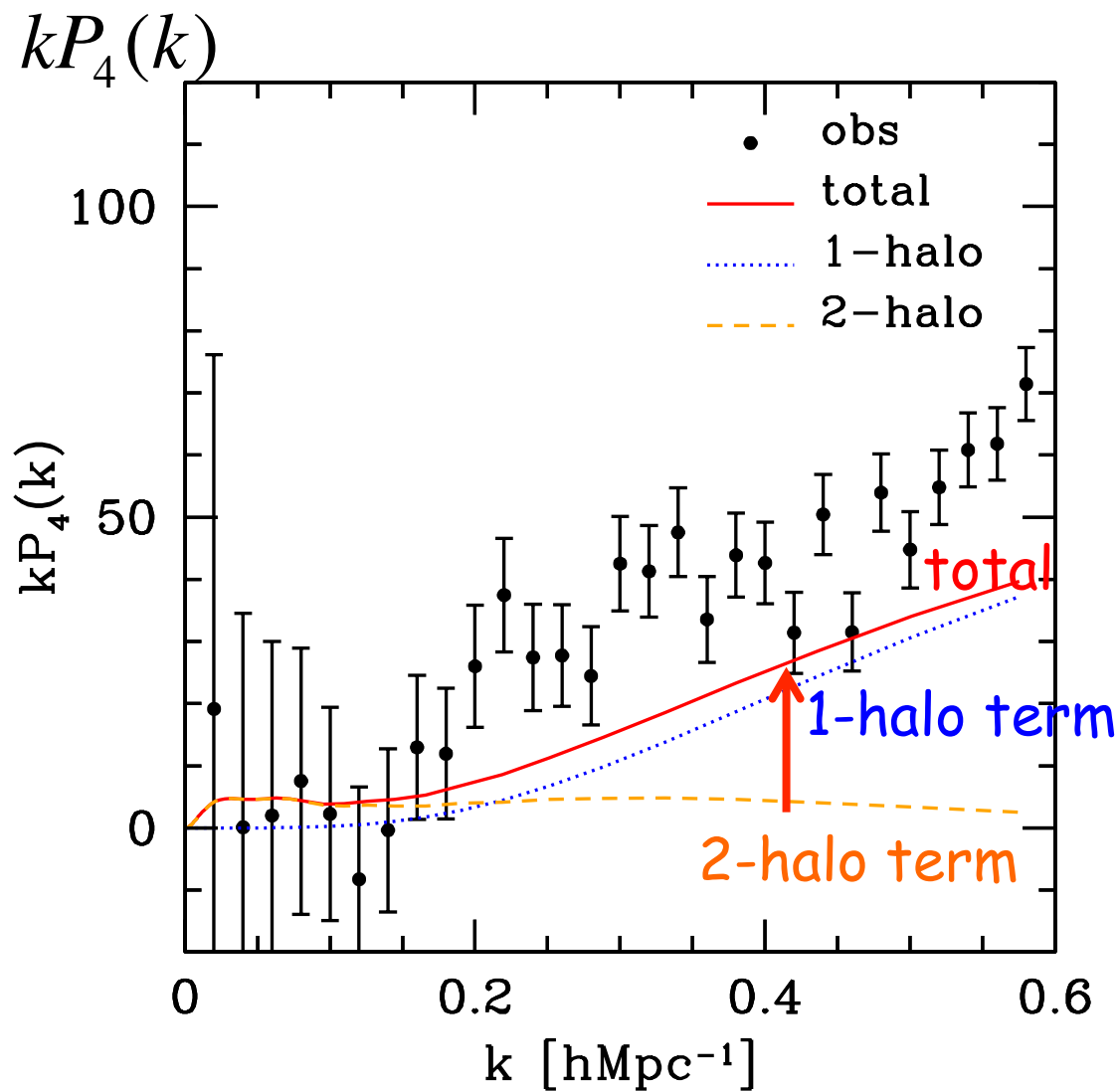
multiple system  
 definition



$$\Delta r_{\perp} < 0.8 h^{-1} \text{Mpc}$$

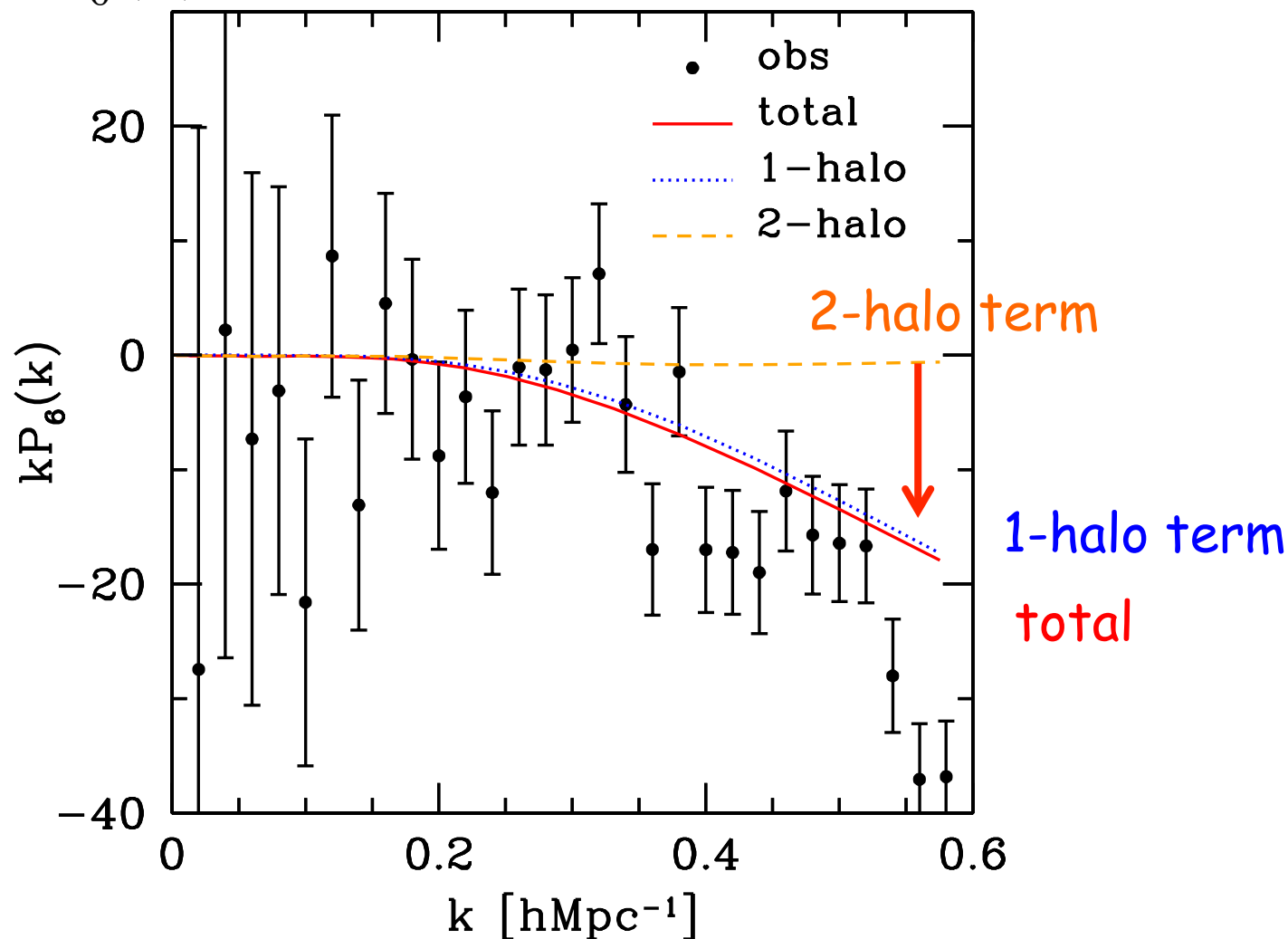
$$\frac{\Delta z}{1+z} < 0.006$$

1-halo term dominates  $P_\ell(k)$  for  $\ell \geq 4$



1-halo term dominates  $P_\ell(k)$  for  $\ell \geq 4$

$kP_6(k)$



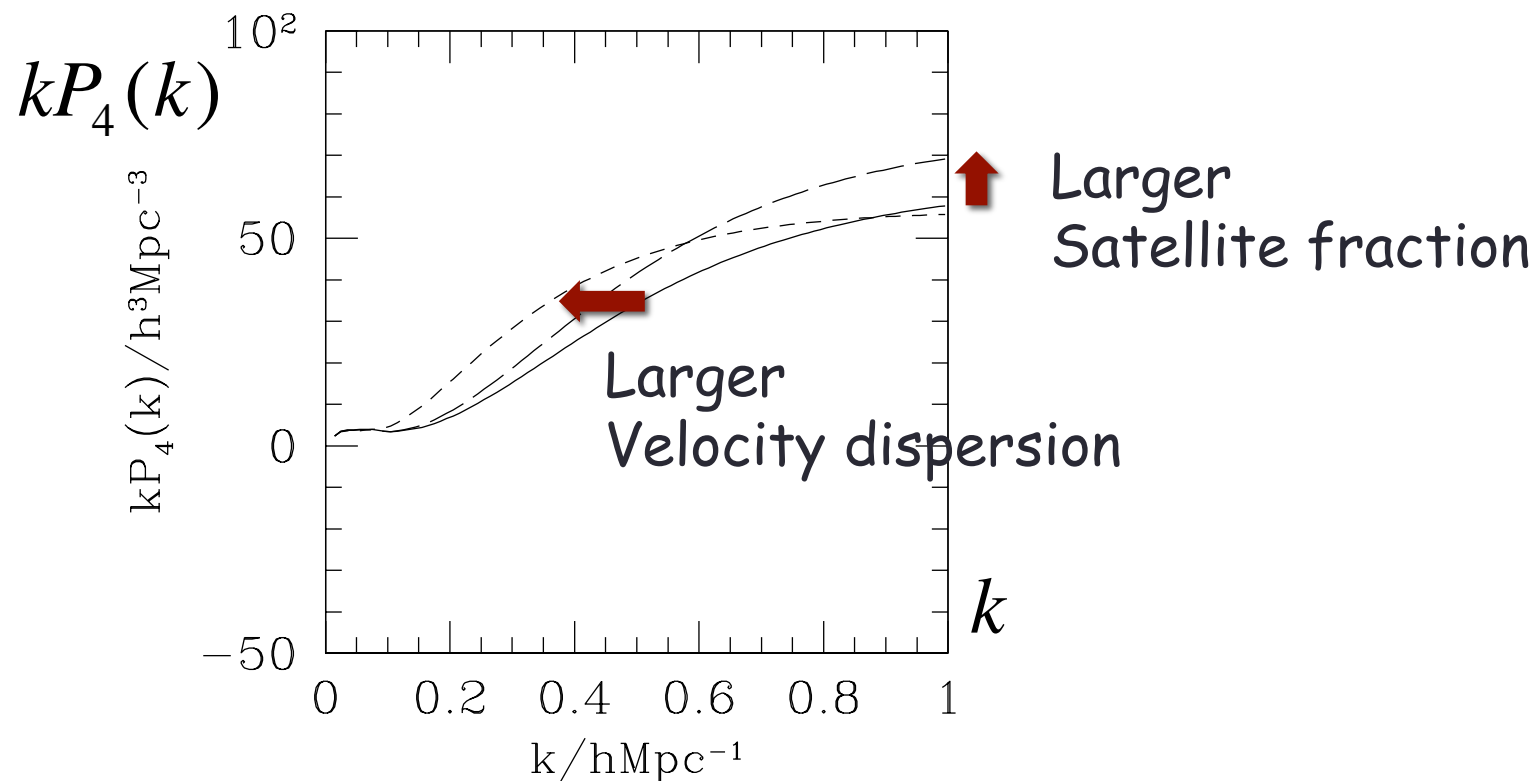
# Information from 1-halo term

$$P^{1h}(k, \mu) \cong \frac{1}{\bar{n}^2} \int dM \frac{dn(M)}{dM} 2 \langle N_{cen} \rangle \langle N_{sat} \rangle u(k, M) \exp \left[ - \frac{\sigma_v^2(M) k^2 \mu^2}{2a^2 H^2} \right]$$

Central - Satellite pair

Satellite fraction(HOD)

Random velocity dispersion  
of satellite galaxies



# Information from 1-halo term

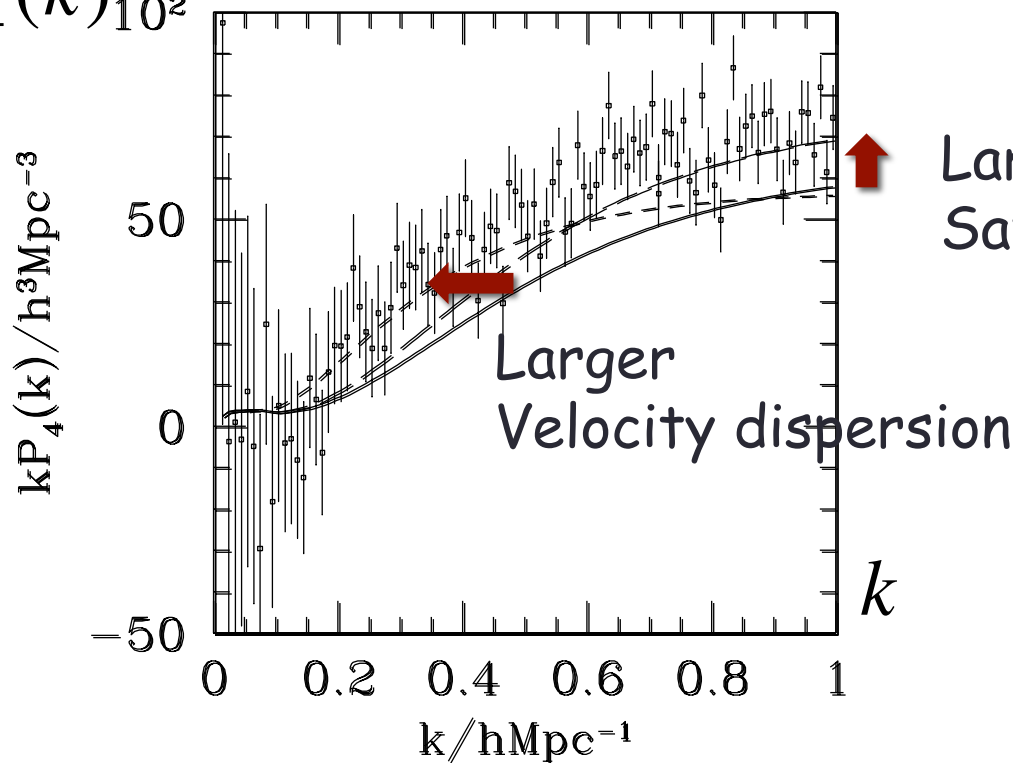
$$P^{1h}(k, \mu) \cong \frac{1}{\bar{n}^2} \int dM \frac{dn(M)}{dM} 2 \langle N_{cen} \rangle \langle N_{sat} \rangle u(k, M) \exp \left[ - \frac{\sigma_v^2(M) k^2 \mu^2}{2a^2 H^2} \right]$$

Satellite fraction(HOD)

(Reid Spergel, LRG)  
(White, CMASS)

Random velocity dispersion  
of satellite galaxies

$kP_4(k)$





### 3. New test of gravity with multipole spectrum

Test of gravity with random velocity of satellite galaxies

$k > 0.2h^{-1}Mpc$  Random velocity dispersion in a halo with mass  $M$

$$\sigma_V^2 = \sigma_V^2(M) \quad \sigma_V^2(M) \propto \frac{GM}{r_{vir}} \propto M^{2/3} \quad (\text{if random motion is virialized})$$



(if this relation is modeled precisely)

1-halo term and multipole power spectrum



Observations

Test of gravity with RSD uses (quasi)-linear regime

$k < 0.2h^{-1}Mpc$  Linear growth rate  $f = \frac{d \ln D_1(a)}{d \ln a}$


## Simple model of the random velocity of satellite galaxies

$$\sigma_v^2(M) \propto \frac{GM}{r_{vir}} \propto M^{2/3} \quad \text{Hikage, Takada, Spergel(2012)}$$

Assume the singular isothermal like velocity dispersion

$$\sigma^2(r) = \frac{GM(<r)}{2r}$$

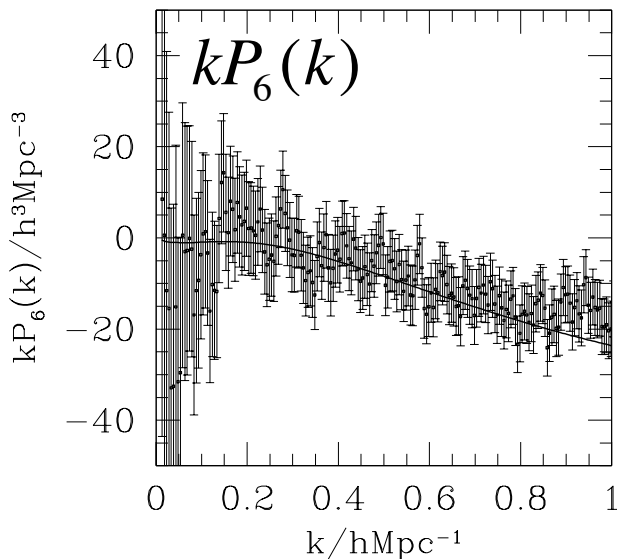
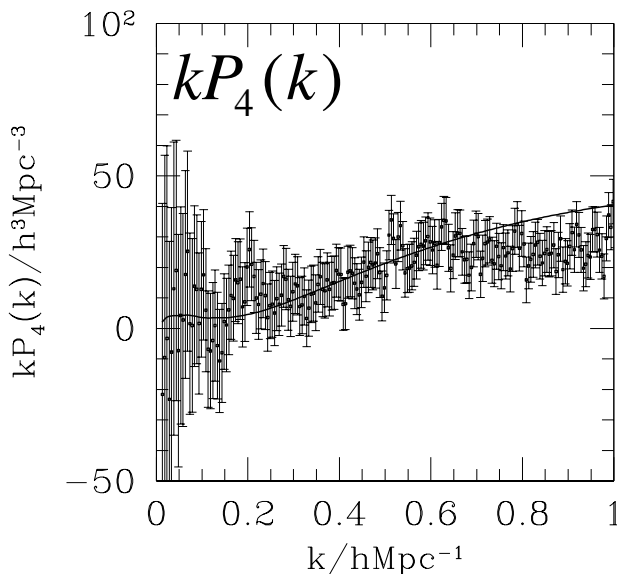
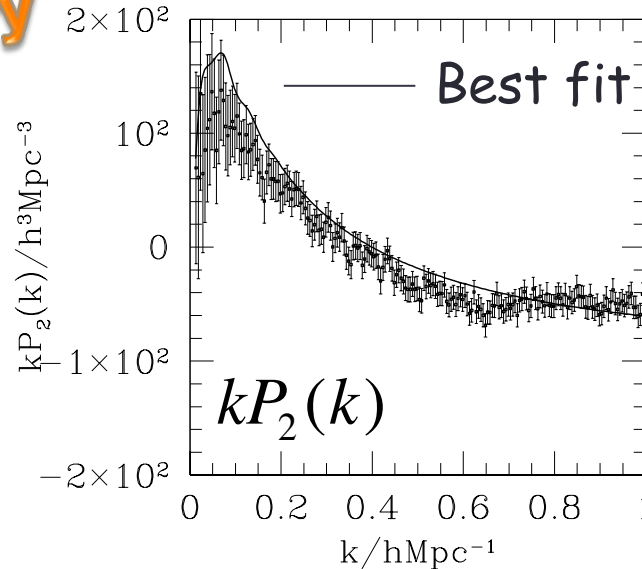
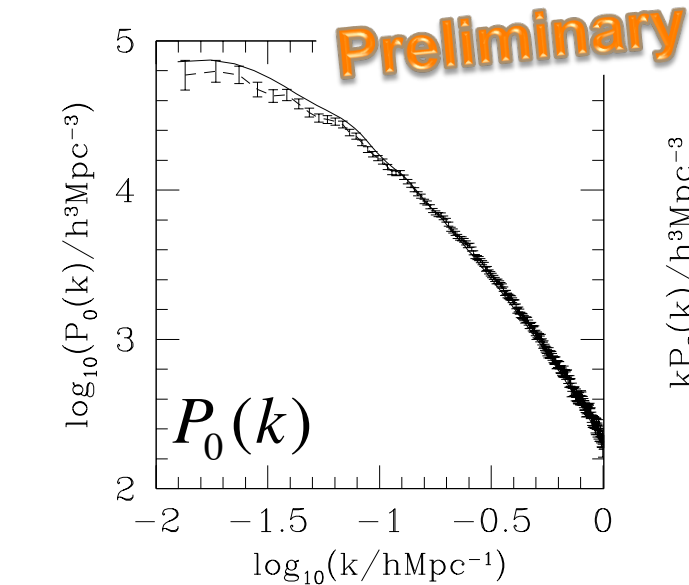
Average over the satellite probability distribution function  
(normalized NFW density profile  $\hat{\rho}(r)$ )

$$\begin{aligned} \sigma_v^2(M) &= \int_0^{r_{vir}} dr r^2 \hat{\rho}(r) \sigma^2(r) \\ &\cong \beta \frac{GM}{2r_{vir}} \quad \beta = \begin{cases} 0.7 & (z = 0.57) \\ 0.8 & (z = 0.35) \\ 1 & (z = 0) \end{cases} \end{aligned}$$


MCMC analysis comparing with  $P_L(k)$  of BOSS CMASS DR9

# CMASS DR9 power spectrum

(North, 2600 deg.<sup>2</sup>, 2×10<sup>5</sup> gals.)



MCMC analysis with  
 $0.15h\text{Mpc}^{-1} < k < 1h\text{Mpc}^{-1}$

$$b(k) = b_0 \frac{1 + A_2 k^2}{1 + A_1 k}$$

$$1 - f_{\text{coll}} \cong 0.12 \pm 0.02$$

$$\beta \cong 0.77 \pm 0.05$$



$$\beta = 0.71$$

Simple model's  
 theoretical prediction  
 the Lambda CDM

(cosmology, HOD fixed)

## Conclusion

1-halo term makes quite large contribution to the higher multipole  $P_l(k)$ .

$P_4(k)$ ,  $P_6(k)$  are dominated by 1-halo term,  $P_2(k)$  is significantly contaminated

1 halo term comes from the satellite (LRG case, satellite is 5%)

1-halo term can never be described by the fluid approximate equations.

## Impact and future work

$$k < 0.2h^{-1} \text{Mpc}$$

- There may be contamination of the satellite galaxies in ongoing and future redshift survey, (SDSSBOSS), Subaru PFS and EUCLID

$$k > 0.2h^{-1} \text{Mpc}$$

- New test of gravity on halo scale with multipole spectrum at high  $k$   
This method might be used as a test of dark matter properties.
- More precise modeling of the halo approach comparing with simulations