Hawking radiation as instantons

Some conceptual and practical issues

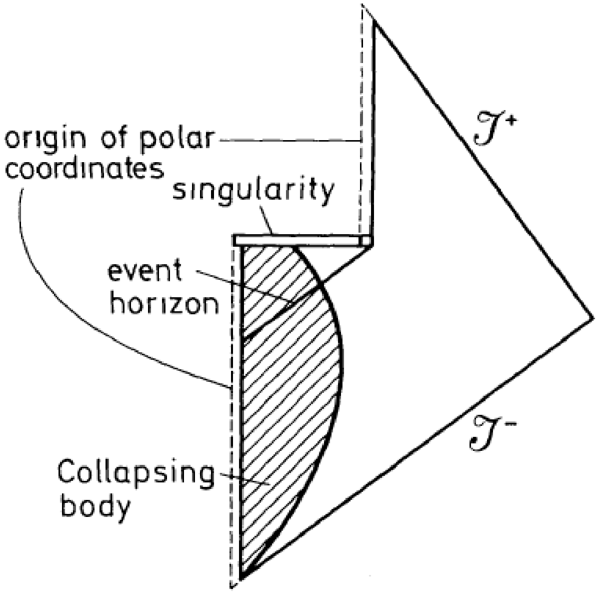
Dong-han Yeom

염동한

Yukawa Institute for Theoretical Physics, Kyoto University

京都大学基礎物理学研究所

Information loss problem



Since a few decades, there were progresses on the information loss problem.

The information loss problem integrates many ideas: general relativity, quantum field theory, holography, areal entropy formula, information theory, etc.

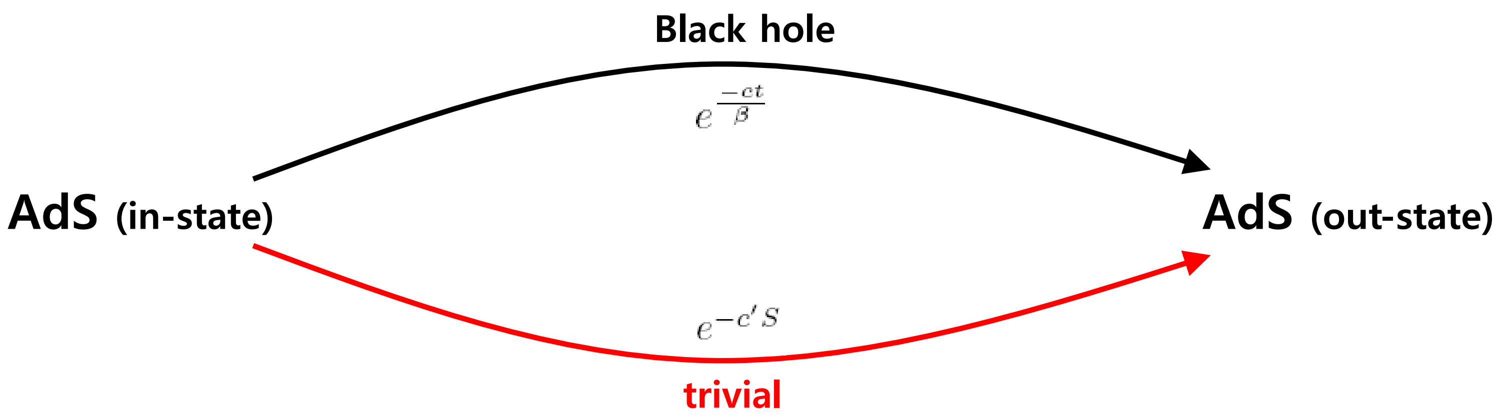
The **failure of black hole complementarity** reveals that at least one has to give up GR or local QFT to explain unitarity.

How can we go further for deeper understanding?

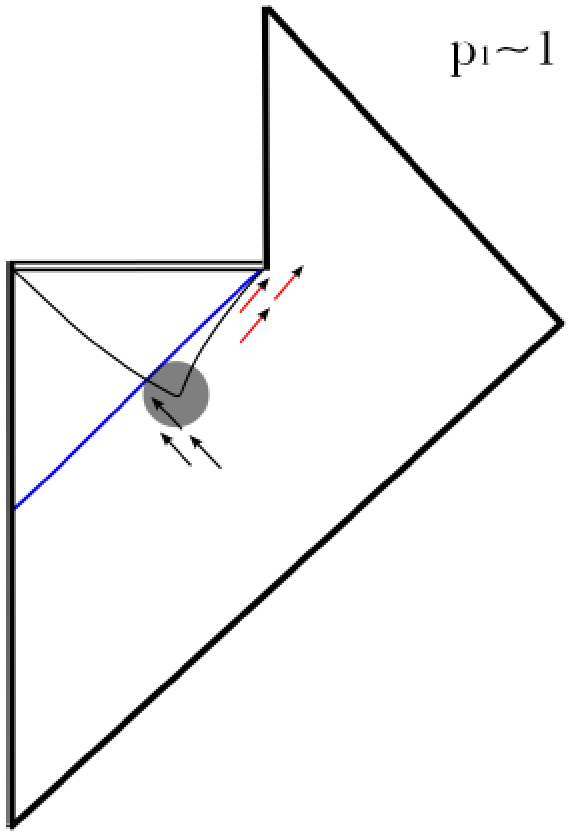
Brief conceptual picture

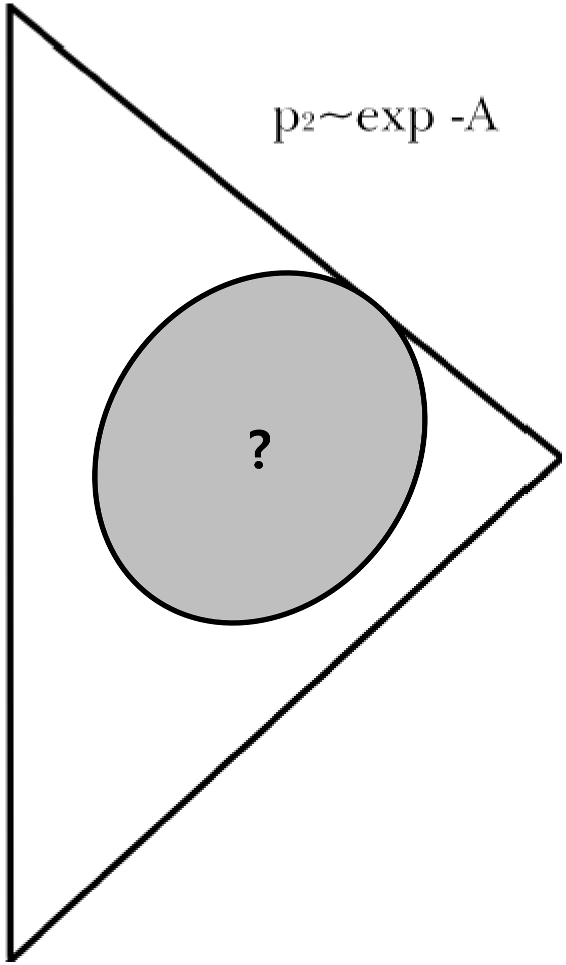
Idea of Maldacena and Hawking

Maldacena argument, revisited

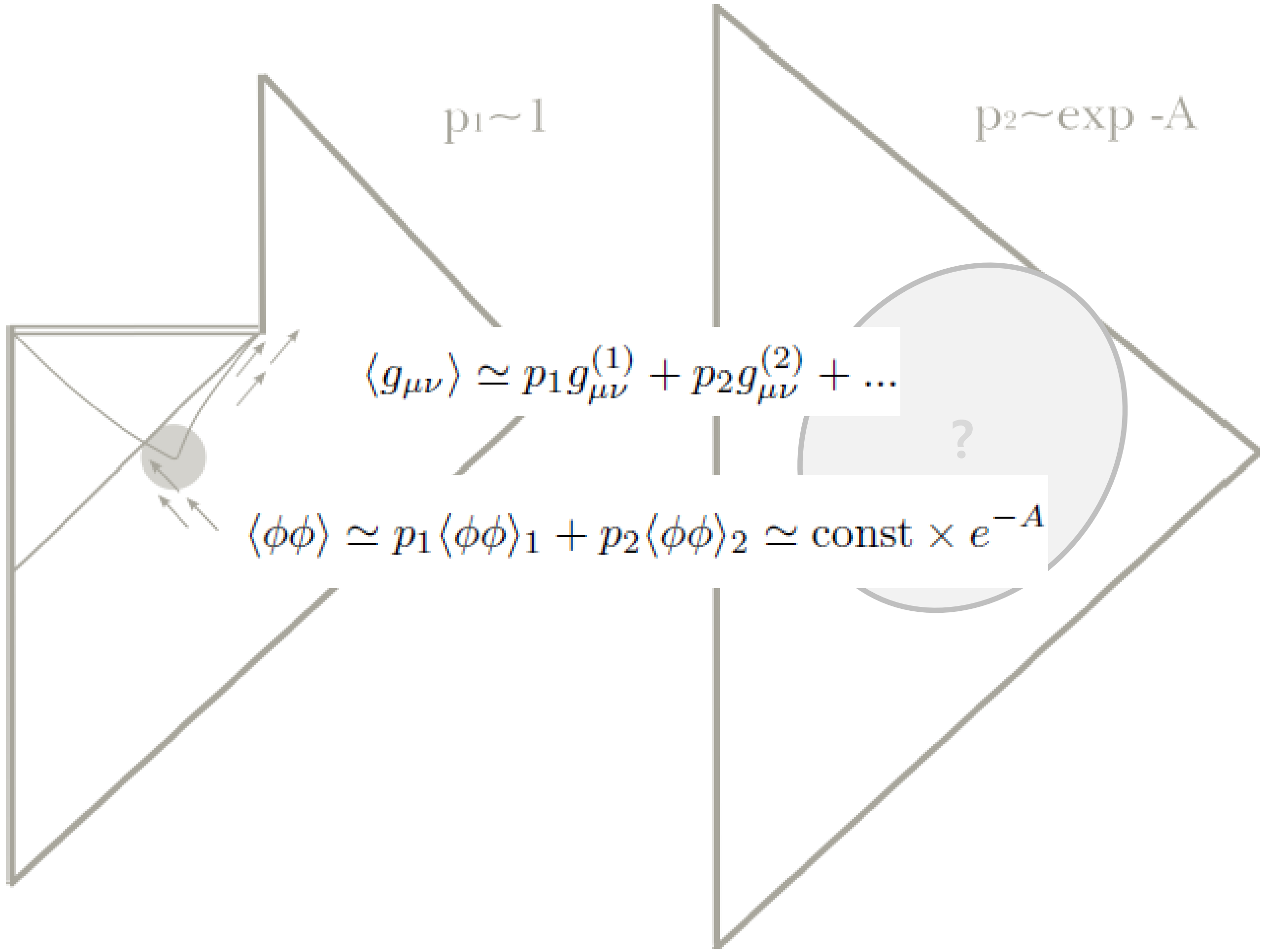


Non-trivial and trivial topology





Non-trivial and trivial topology



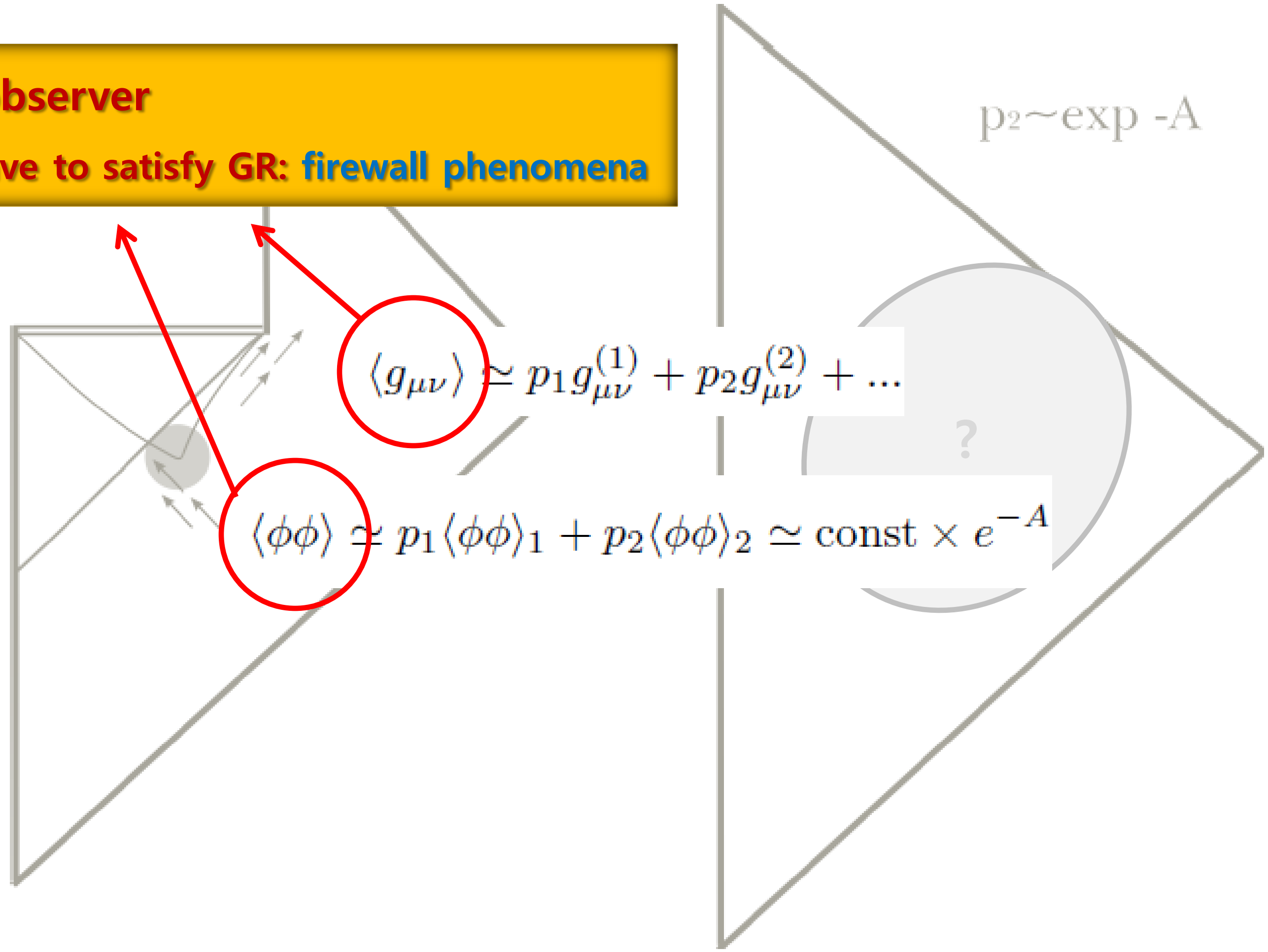
$$\langle g_{\mu\nu} \rangle \simeq p_1 g^{(1)}_{\mu\nu} + p_2 g^{(2)}_{\mu\nu} + ...$$

$$\langle \phi\phi \rangle \simeq p_1 \langle \phi\phi \rangle_1 + p_2 \langle \phi\phi \rangle_2 \simeq \text{const} \times e^{-A}$$

Firewall phenomena

Unitary observer

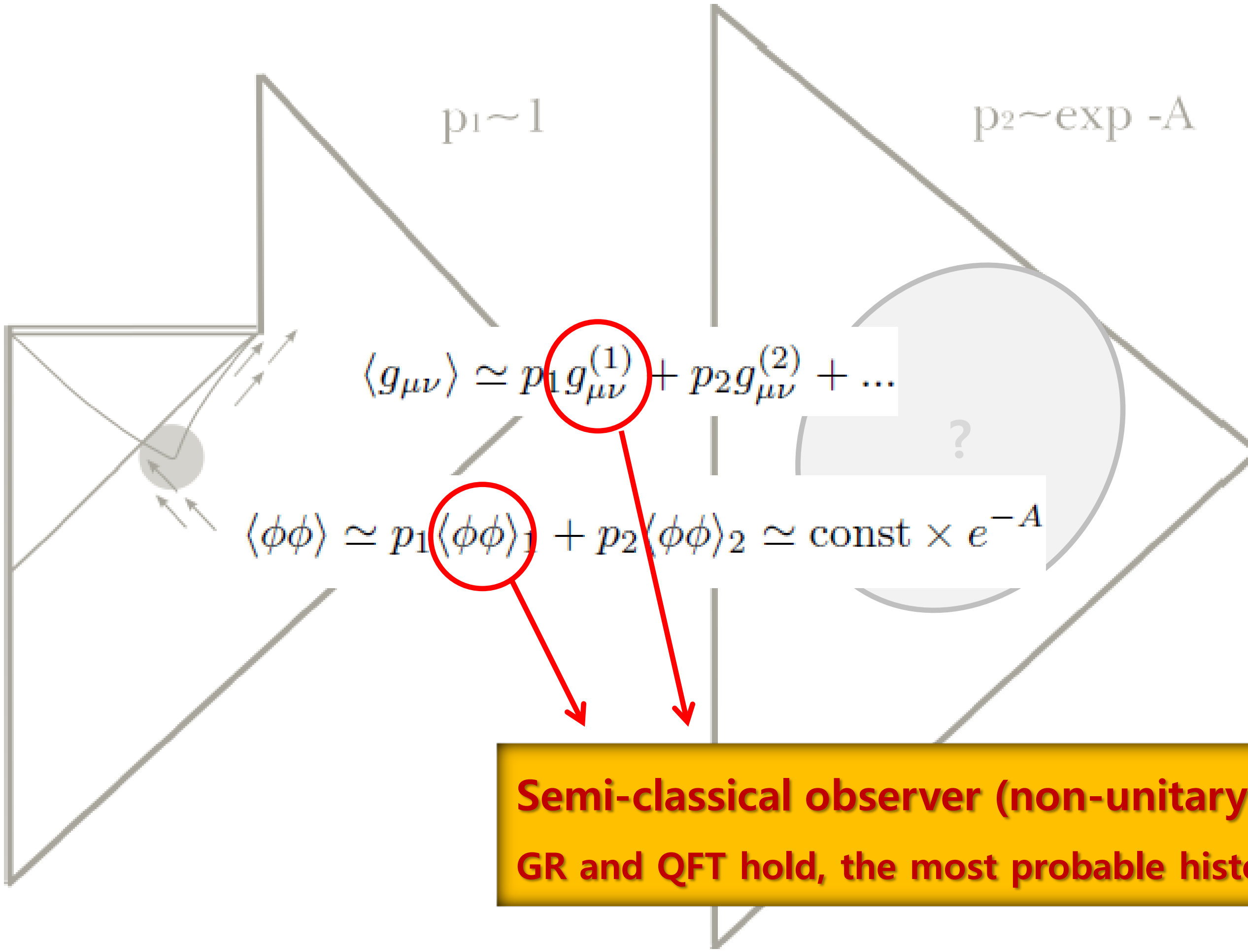
Doesn't have to satisfy GR: firewall phenomena



$$\langle g_{\mu\nu} \rangle \simeq p_1 g^{(1)}_{\mu\nu} + p_2 g^{(2)}_{\mu\nu} + \ldots$$

$$\langle \phi\phi \rangle \simeq p_1 \langle \phi\phi \rangle_1 + p_2 \langle \phi\phi \rangle_2 \simeq \mathrm{const} \times e^{-A}$$

Effective loss of information



$$\langle g_{\mu\nu} \rangle \simeq p_1 g^{(1)}_{\mu\nu} + p_2 g^{(2)}_{\mu\nu} + \ldots$$

$$\langle \phi\phi \rangle \simeq p_1 \langle \phi\phi \rangle_1 + p_2 \langle \phi\phi \rangle_2 \simeq \text{const} \times e^{-A}$$

Semi-classical observer (non-unitary)

GR and QFT hold, the most probable history

Weather Forecasting?

Information Preservation and Weather Forecasting for Black Holes*

S. W. Hawking[1]

It has been suggested [1] that the resolution of the information paradox for evaporating black holes is that the holes are surrounded by firewalls, bolts of outgoing radiation that would destroy any infalling observer. Such firewalls would break the CPT invariance of quantum gravity and seem to be ruled out on other grounds. A different resolution of the paradox is proposed, namely that gravitational collapse produces apparent horizons but no event horizons behind which information is lost. This proposal is supported by ADS-CFT and is the only resolution of the paradox compatible with CPT. The collapse to form a black hole will in general be chaotic and the dual CFT on the boundary of ADS will be turbulent. Thus, like weather forecasting on Earth, information will effectively be lost, although there would be no loss of unitarity.

An example of explicit construction

Power of non-perturbative effects

True vacuum bubble in AdS

Let us consider the true vacuum bubble in AdS.

$$ds^2 = -f(R)dT^2 + \frac{1}{f(R)}dR^2 + R^2 d\Omega^2,$$

outside: $$f_+(R) = 1 - \frac{2M_+}{R} + \frac{R^2}{l_+^2},$$

inside: $$f_-(R) = 1 - \frac{2M_-}{R} + \frac{R^2}{l_-^2}.$$

junction equation: $$\epsilon_- \sqrt{\dot{r}^2 + f_-(r)} - \epsilon_+ \sqrt{\dot{r}^2 + f_+(r)} = 4\pi r \sigma$$

$$\dot{r}^2 + V(r) = 0,$$

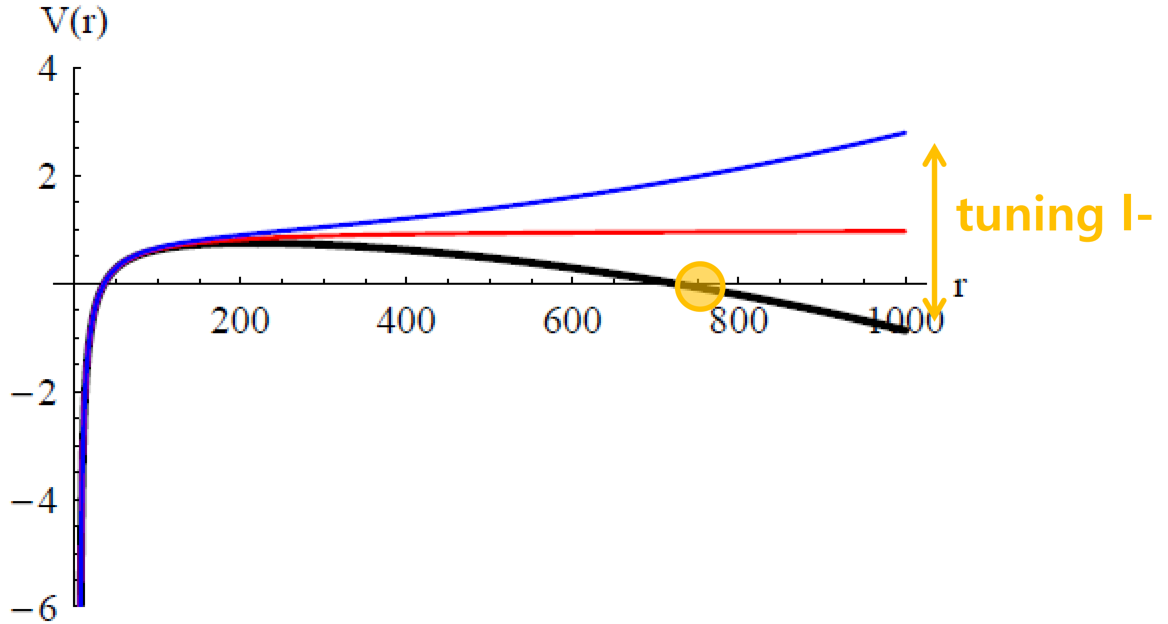
effective potential: $$V(r) = f_+(r) - \frac{\left(f_-(r) - f_+(r) - 16\pi^2\sigma^2 r^2\right)^2}{64\pi^2\sigma^2 r^2}$$

True vacuum bubble in AdS

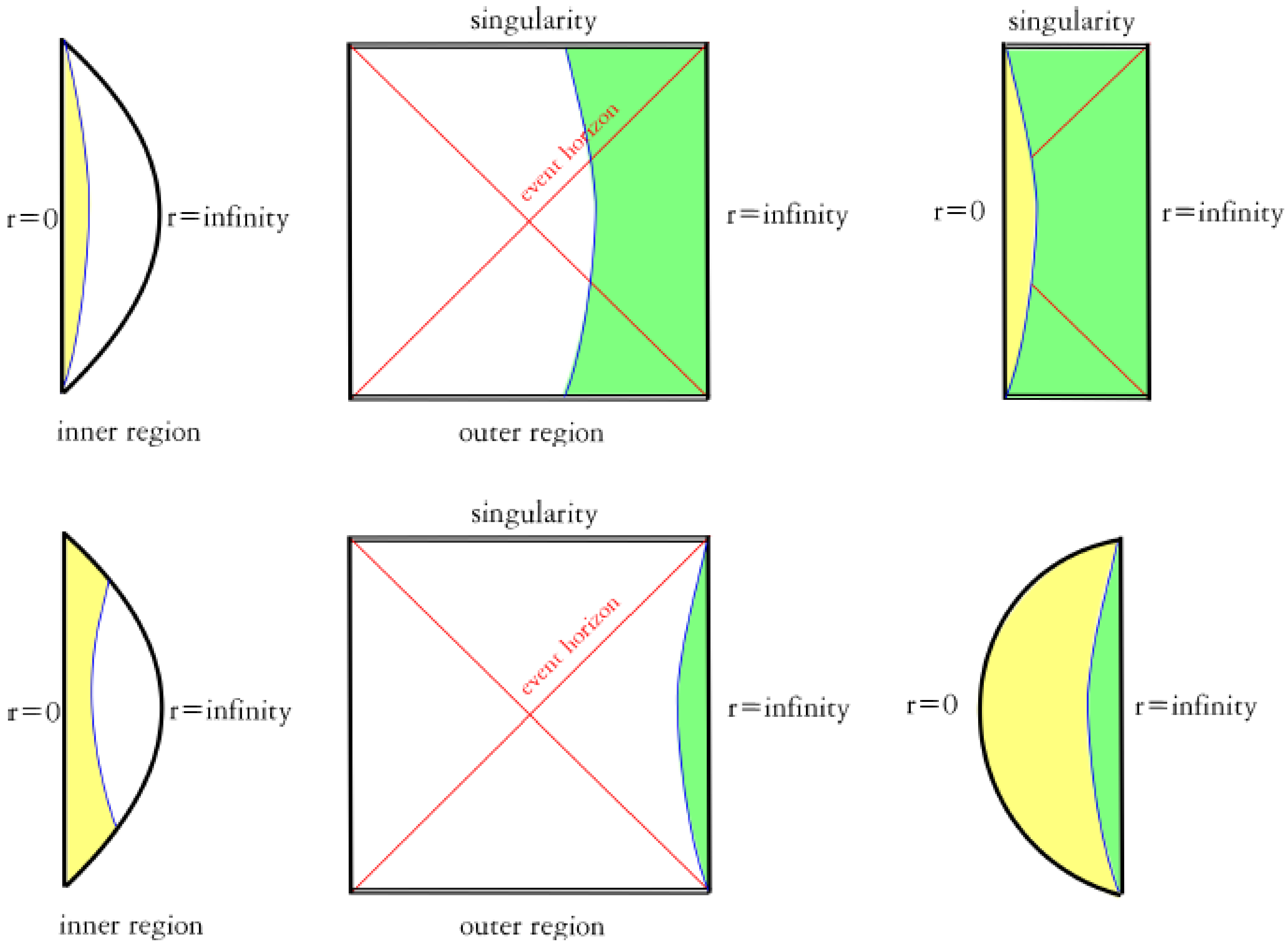
For given M_+ **,** l_+**, and** $M_- = 0$ **,**

there exists l_- **such that** $V(r)$ **allows a bouncing solution,**

even for a large (hence, eternal) **black hole in AdS.**

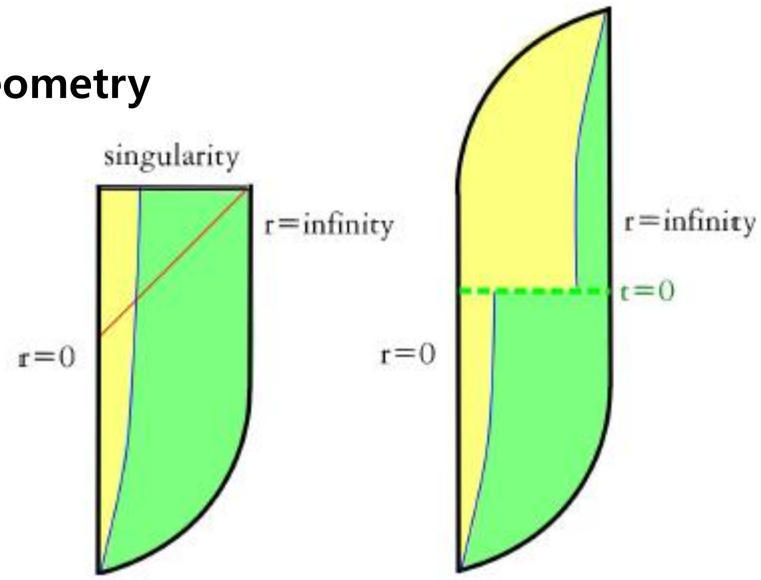


Classical trajectories



Power of non-perturbative effects

Semi-classical geometry



Fischler-Morgan-Polchinski tunneling

(non-perturbative process)

toward trivial topology

Where are we?

The consideration of **non-perturbative effects** is the correct way to understand the information loss problem.

Euclidean quantum gravity is the highway to study non-perturbative effects.

However, **the thin-shell model is not sufficient**, since we require a special vacuum structure to choose l_-.

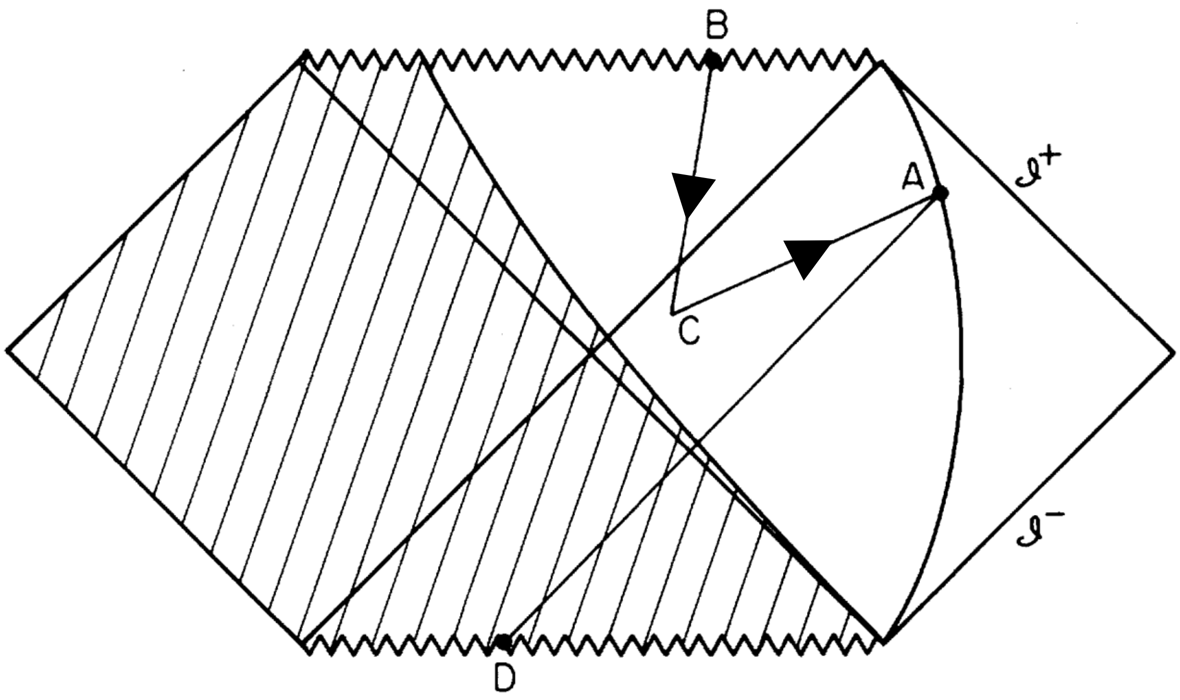
Can we construct **general instantons** that can be applied for general black holes that intermediate **trivial topology**?

Hawking radiation as instantons

Sasaki and DY, in preparation

Hartle and Hawking in 1976

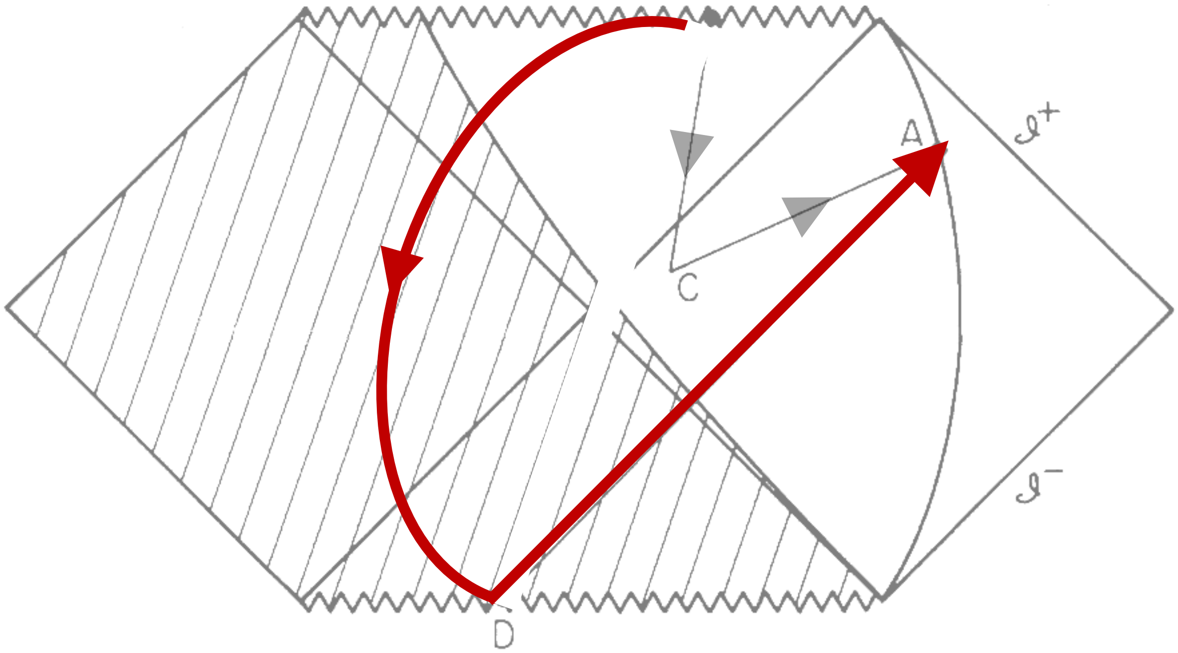
Hartle and Hawking calculated the probability of a particle that **moves from inside to outside** the horizon, using the **path integral method**.



Hartle and Hawking in 1976

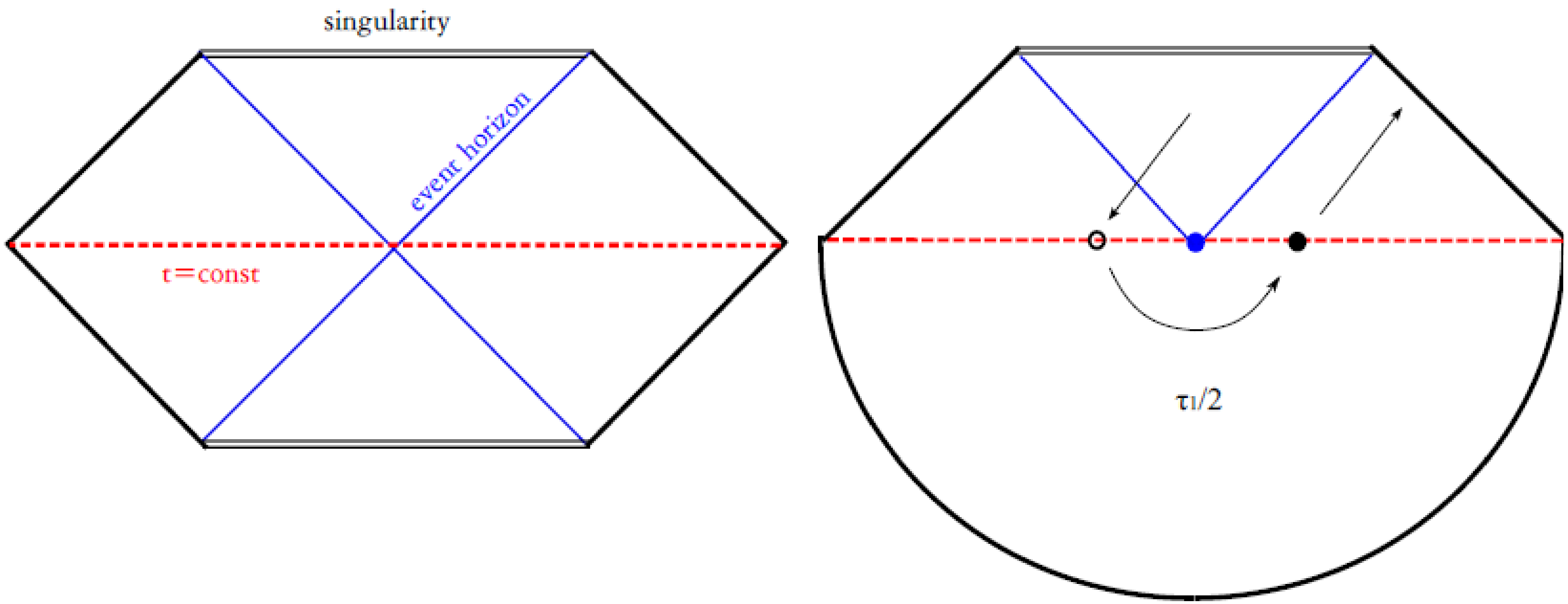
Due to analyticity, we can calculate by **Euclidean analytic continuation** that connects from upper to lower of the Schwarzschild solution.

(probability to emit a particle with energy E) $= e^{-2\pi E/\kappa} \times$ (probability to absorb a particle with energy E)



Analyticity

Due to analyticity, we can choose different paths that use the **Euclidean manifold outside the event horizon**.

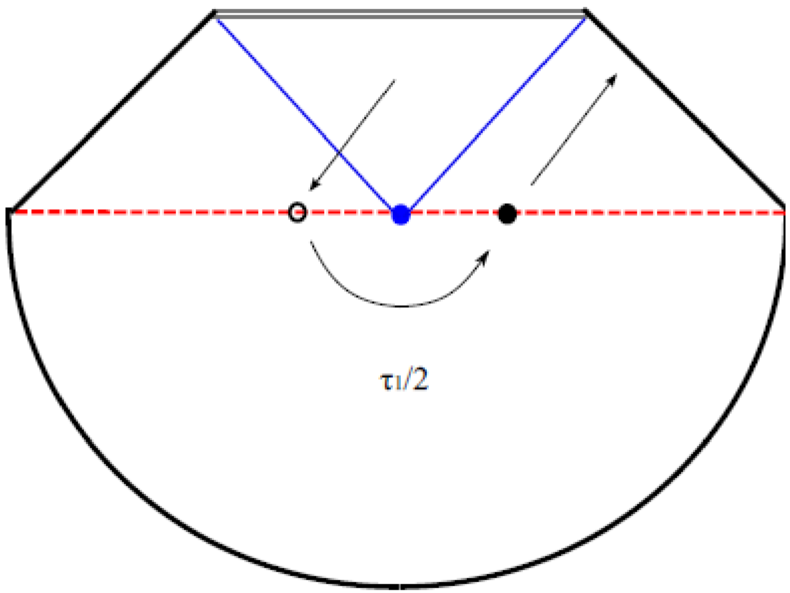


Parikh-Wilczek method

This gives the same result compared to the Parikh-Wilczek method. **Parik-Wilczek's Lorentzian imaginary action** is the same as the **Euclidean half-way action**.

$$\mathrm{Im}S = \mathrm{Im}\int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}} p_r dr \simeq \omega\frac{\tau_1}{2}$$

$$\omega\frac{\tau_1}{2} = \int_0^{\tau_1/2} H d\tau = S_{\mathrm{E}}$$



Also, equivalent path

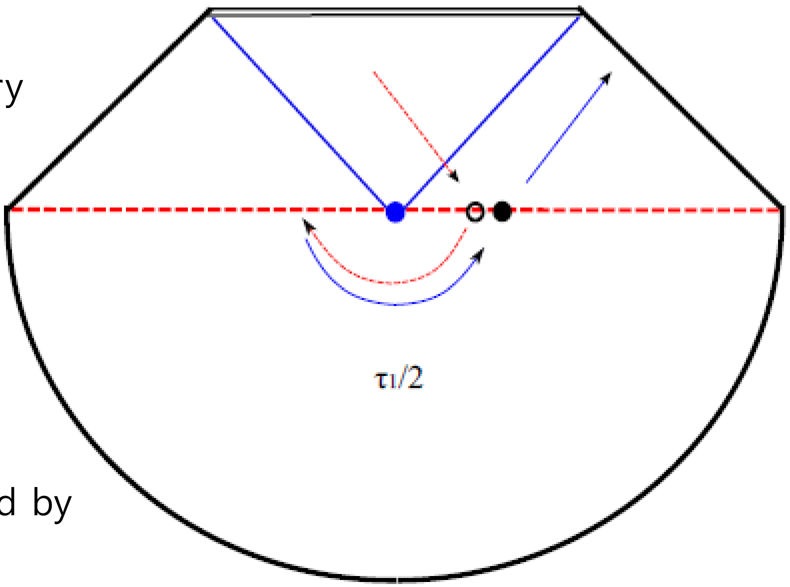
Due to analyticity, we can choose the other path that all things can happen on the **right patch of the Penrose diagram**. This gives still a consistent interpretation.

- An **asymptotic observer** measures the blue history that has the **entropy cost** $+\omega\frac{\tau_1}{2}$ x 2

- The **ghost particle** falls into the black hole and the entropy cost is $-\omega\frac{\tau_1}{2}$ x 2

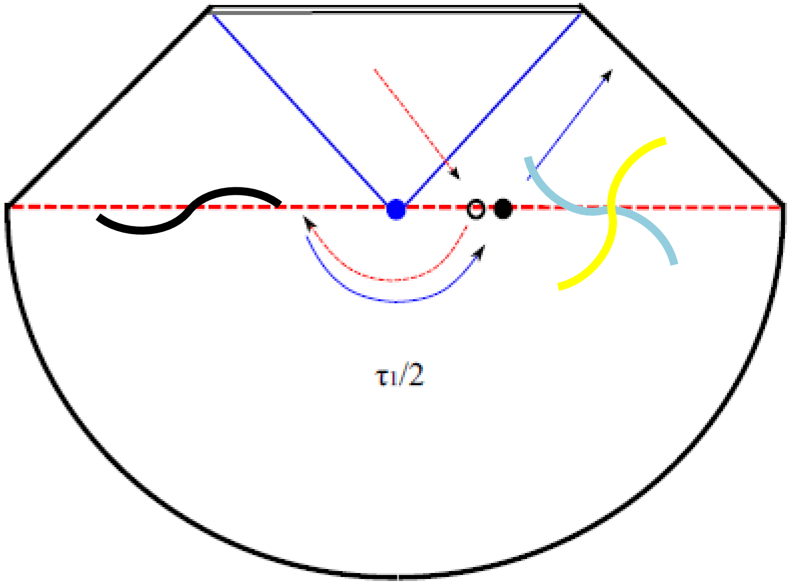
- Due to the ghost particle, **areal entropy** decreased by

$$-\delta M/T = -\omega/T$$



From particles to field

If there are many particles, it will be approximated by **instantons**.

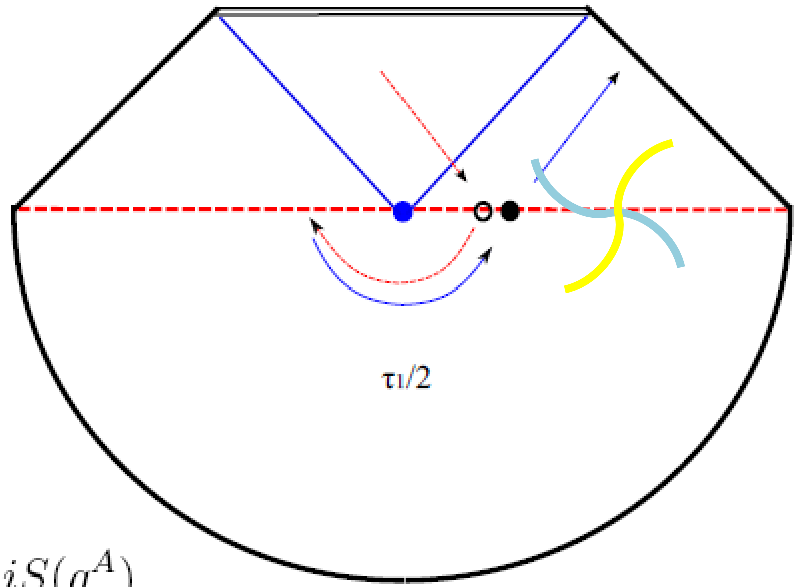


From particles to field

Such instantons should be restricted.

- We require the **energy conservation** for any time slice.

(real dof) = (imaginary dof)



Complexified action $$I(q^A) = I_R(q^A) - iS(q^A)$$

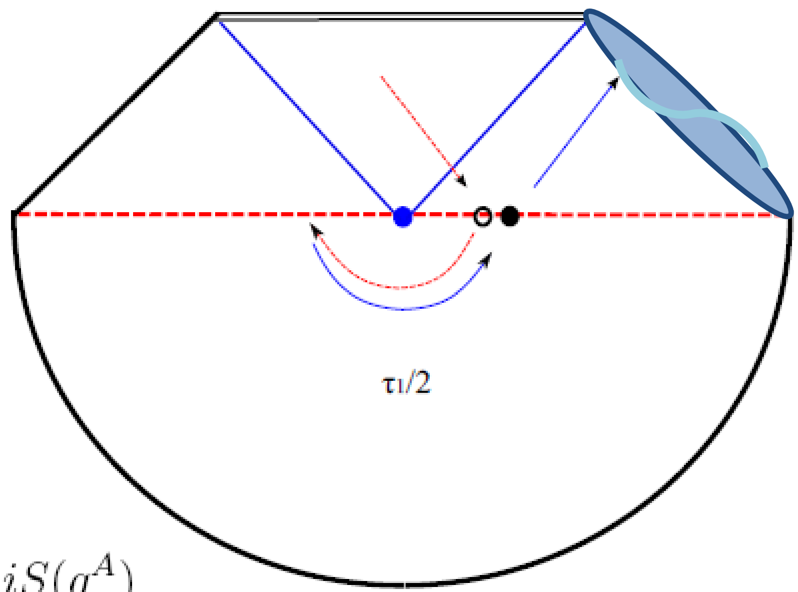
Classicality condition $$|(\nabla I_R)^2| \ll |(\nabla S)^2|$$

From particles to field

Such instantons should be restricted.

- We require the **energy conservation** for any time slice.
- We require the **asymptotic classicality**.

So, for the asymptotic boundary, **real parts** should be dominated by the imaginary parts.



Complexified action $I(q^A) = I_R(q^A) - iS(q^A)$

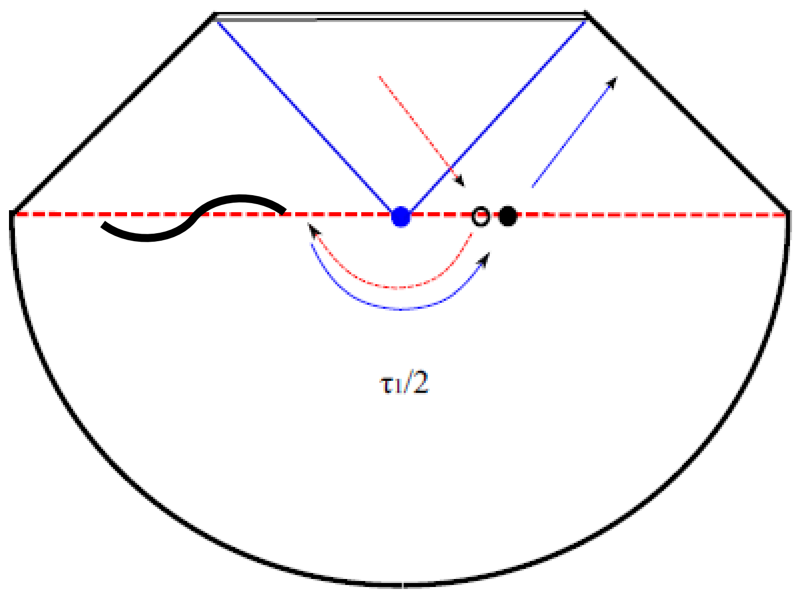
Classicality condition $|(\nabla I_R)^2| \ll |(\nabla S)^2|$

From particles to field

Such instantons should be restricted.

- We require the **energy conservation** for any time slice.
- We require the **asymptotic classicality**.
- At $\tau = 0$, we give the **vacuum condition**, i.e., quantum fluctuation is localized on the horizon.

$$\phi(r, \tau = 0) \sim \delta(r)$$



Step0: Field ansatz (2D CGHS model)

$$
\begin{aligned}
\delta f_q(t,r) &= \sqrt{\frac{2}{\pi}}\int_0^\infty \left[\left(g^+_{\rm re}(\omega)+ig^+_{\rm im}(\omega)\right)\cos\omega(t+r)+\left(h^+_{\rm re}(\omega)+ih^+_{\rm im}(\omega)\right)\sin\omega(t+r)\right] \\
&\quad +\left[\left(g^-_{\rm re}(\omega)+ig^-_{\rm im}(\omega)\right)\cos\omega(t-r)+\left(h^-_{\rm re}(\omega)+ih^-_{\rm im}(\omega)\right)\sin\omega(t-r)\right]d\omega \\
&= \frac{1}{\sqrt{2\pi}}\int_0^\infty \Big[\left(g^+_{\rm re}(\omega)+ig^+_{\rm im}(\omega)-i\left(h^+_{\rm re}(\omega)+ih^+_{\rm im}(\omega)\right)\right)e^{i\omega(t+r)} \\
&\quad +\left(g^+_{\rm re}(\omega)+ig^+_{\rm im}(\omega)+i\left(h^+_{\rm re}(\omega)+ih^+_{\rm im}(\omega)\right)\right)e^{-i\omega(t+r)}\Big] \\
&\quad +\Big[\left(g^-_{\rm re}(\omega)+ig^-_{\rm im}(\omega)-i\left(h^-_{\rm re}(\omega)+ih^-_{\rm im}(\omega)\right)\right)e^{i\omega(t-r)} \\
&\quad +\left(g^-_{\rm re}(\omega)+ig^-_{\rm im}(\omega)+i\left(h^-_{\rm re}(\omega)+ih^-_{\rm im}(\omega)\right)\right)e^{-i\omega(t-r)}\Big]d\omega.
\end{aligned}
$$

The most general field ansatz.

Step0: Field ansatz (2D CGHS model)

$$\begin{aligned}
\delta\tilde{\psi}_q(t,r) \quad = \quad & \frac{1}{\sqrt{2\pi}}\int_0^\infty \Big[e^{\omega\tau_{\mathrm{T}}/2}\left(g^+_{\mathrm{re}}(\omega)+ig^+_{\mathrm{im}}(\omega)-i\left(h^+_{\mathrm{re}}(\omega)+ih^+_{\mathrm{im}}(\omega)\right)\right)e^{i\omega(t+r)} \\
& +e^{-\omega\tau_{\mathrm{T}}/2}\left(g^+_{\mathrm{re}}(\omega)+ig^+_{\mathrm{im}}(\omega)+i\left(h^+_{\mathrm{re}}(\omega)+ih^+_{\mathrm{im}}(\omega)\right)\right)e^{-i\omega(t+r)}\Big] \\
& +\Big[e^{\omega\tau_{\mathrm{T}}/2}\left(g^-_{\mathrm{re}}(\omega)+ig^-_{\mathrm{im}}(\omega)-i\left(h^-_{\mathrm{re}}(\omega)+ih^-_{\mathrm{im}}(\omega)\right)\right)e^{i\omega(t-r)} \\
& +e^{-\omega\tau_{\mathrm{T}}/2}\left(g^-_{\mathrm{re}}(\omega)+ig^-_{\mathrm{im}}(\omega)+i\left(h^-_{\mathrm{re}}(\omega)+ih^-_{\mathrm{im}}(\omega)\right)\right)e^{-i\omega(t-r)}\Big]\,d\omega.
\end{aligned}$$

After the Wick rotations

Step1: Energy conservation

Redefine mode functions:

$$
\begin{aligned}
g^-_{\rm re} &= \frac{1}{2}\left((g_{\rm re}-h_{\rm im})+e^{-\omega\tau_{\rm T}}(g_{\rm re}+h_{\rm im})\right),\\
g^-_{\rm im} &= \frac{1}{2}\left((g_{\rm im}+h_{\rm re})+e^{-\omega\tau_{\rm T}}(g_{\rm im}-h_{\rm re})\right),\\
h^-_{\rm re} &= \frac{1}{2}\left((g_{\rm im}+h_{\rm re})-e^{-\omega\tau_{\rm T}}(g_{\rm im}-h_{\rm re})\right),\\
h^-_{\rm im} &= \frac{1}{2}\left((-g_{\rm re}+h_{\rm im})+e^{-\omega\tau_{\rm T}}(g_{\rm re}+h_{\rm im})\right).
\end{aligned}
$$

For energy conservation:

$$
\begin{aligned}
g^+_{\rm re} &= -g^-_{\rm im},\\
g^+_{\rm im} &= g^-_{\rm re},\\
h^+_{\rm re} &= -h^-_{\rm im},\\
h^+_{\rm im} &= h^-_{\rm re}.
\end{aligned}
$$

Step2: Classicality

For classicality:

$$|g_{\rm re}(\omega)| \gg |g_{\rm im}(\omega)|$$
$$|h_{\rm re}(\omega)| \gg |h_{\rm im}(\omega)|$$

The most general **asymptotic classical** instantons.

$$\delta\tilde{\psi}_q^{\rm out}(t,r) = \sqrt{\frac{2}{\pi}}\int_0^\infty e^{-\omega\tau_{\rm T}/2}\left(g(\omega)\cos\omega x^- + h(\omega)\sin\omega x^-\right)d\omega,$$
$$\delta\tilde{\psi}_q^{\rm in}(t,r) = \sqrt{\frac{2}{\pi}}\int_0^\infty e^{-\omega\tau_{\rm T}/2}i\left(g(\omega)\cos\omega x^+ + h(\omega)\sin\omega x^+\right)d\omega.$$

$$\delta N_\omega = \frac{1}{2}e^{-\omega\tau_{\rm T}}\left(g^2(\omega) + h^2(\omega)\right)$$

Step3: The most probable instanton

$$
\begin{aligned}
S_{\mathrm{E}} &= \frac{i}{4\pi}\int_0^\infty \left(g^2(\omega) - h^2(\omega)\right)\omega\left(1 - e^{-2\omega\tau_{\mathrm{T}}}\right)d\omega \\
&\quad -\frac{1}{2\pi}\int_0^\infty g(\omega)h(\omega)\omega\left(1 - e^{-\omega\tau_{\mathrm{T}}}\right)^2 d\omega + (\text{area change})
\end{aligned}
$$

This term should vanish,
compared to particle level description!

The partial entropy of the Euclidean manifold will be vanished. Hence, **the instanton should be even or odd**.

Step4: Initial vacuum condition

The natural vacuum condition at the beginning of the instanton is defined by

$$N_\omega^{\mathrm{p}} = -N_\omega^{\mathrm{g}} = 1$$

This means that

$$g^2 \ \text{or} \ h^2 = 2 - 2e^{-2\omega/T} + \ldots \simeq 2$$

is required for an instanton generated from the vacuum fluctuation. Therefore, we explain the **Boltzmann distribution**.

$$\delta N_\omega \simeq e^{-\omega/T}$$

Summarize

This ansatz has **8D** degree of freedom. (real/imaginary) x (out-going/in-going) x (left-moving/right-moving)

Step1: Due to the **energy conservation**, **4D** is reduced. (real part = imaginary part)

Step2: Due to the **asymptotic classicality**, the out-going real parts are dominated the imaginary parts. This reduces **2D**.

Step3: Due to the **most probable** condition, we choose even or odd mode. This reduces **1D**.

Step4: Finally, we should give the initial **vacuum** condition. This reduces **1D**.

In conclusion, up to the leading order, we obtain the correct **Boltzmann distribution**. For higher orders, we need to add further quantum corrections.

$$(\text{probability to emit a particle with energy } E) = e^{-2\pi E/\kappa} \times (\text{probability to absorb a particle with energy } E)$$

New1: Quantum particles?

There may be some possibility that non-classical particles are emitted. These may include some **quantum information**.

$$\begin{aligned}
\mathrm{Re}\, S_{\mathrm{E}} &= -\mathrm{Re}\left[\frac{1}{2\pi}\int_0^\infty g(\omega)h(\omega)\omega\left(1-e^{-\omega/T}\right)^2 d\omega\right] \\
&\quad -\mathrm{Im}\left[\frac{1}{4\pi}\int_0^\infty \left(g^2(\omega)-h^2(\omega)\right)\omega\left(1-e^{-2\omega/T}\right) d\omega\right] \\
&= -\frac{1}{2\pi}\int_0^\infty \left(g_{\mathrm{re}}h_{\mathrm{re}}-g_{\mathrm{im}}h_{\mathrm{im}}\right)\omega\left(1-e^{-\omega/T}\right)^2 d\omega \\
&\quad +\frac{1}{4\pi}\int_0^\infty \left(-2g_{\mathrm{re}}g_{\mathrm{im}}+2h_{\mathrm{re}}h_{\mathrm{im}}\right)\omega\left(1-e^{-2\omega/T}\right) d\omega.
\end{aligned}$$

New2: Thermal excitation?

We can also consider the effects of thermal excitations. Then the vacuum is shifted

$$N_\omega^{\mathrm{P}} = -N_\omega^{\mathrm{g}} = \frac{g^2}{2}\left(1 + e^{-2\omega/T}\right) = a_0 + a_1 e^{-\omega/T'} + a_2 e^{-2\omega/T'} + \ldots$$

Therefore, the mode function is in general

$$g^2(\omega) = \sum_{n,m=0}^{\infty} b_{n,m} e^{-n\omega/T' - m\omega/T}$$

Although the probability is highly suppressed, anyway there can exist a solution as follows.

Conclusion

- **Euclidean instantons** can be used to understand not only perturbative **Hawking radiation**, but also non-perturbative **new effects**.

- For **the omnipotent observer** in the superspace, uniterity can be restored and a **firewall-like object** can be observed.
- For **semi-classical observers**, information is **effectively lost**.