Coleman-de Luccia instantons in nonlinear Massive Gravity

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Based on:

YZ, Ryo Saito and Misao Sasaki, JCAP 02 (2013) 029 [1210.6224]
MS, DY and YZ, CQG 30 (2013) 232001 [1307.5948]
YZ, RS, YZ and MS, 1312.0709

Outline

1. Massive Gravity Theory and Motivation of work
2. Setup of model
3. Coleman-de Luccia solutions
4. Conclusion and Future Prospects

Massive Gravity theory

General Relativity (GR): $$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}R\,,$$

In 3+1 dim, for symmetric tensor $g_{\mu\nu}$, the propagating degrees of freedom (dof) can be counted as:

$$6 \quad -4 \qquad = 2$$

Lagrangian multiplier

Helicity ± 2

Such situation changes in the Massive Gravity Theory.

In Massive Gravity (MG), the mass of graviton is non-vanishing, which breaks the gauge invariance

$$S = \frac{1}{16\pi G}\int d^4x \, \sqrt{-g}[R(g) - m^2 V(g)]$$

$$\supset -\frac{m^2}{16\pi G}\int d^4x \sqrt{\gamma} N V(\gamma, N, N^i)$$

Generally speaking, the dof is

$$6 - 0 = 6$$

No Lagrangian multiplier…

Helicity $\pm 2, \ \pm 1, \ 0,$

(Boulware & Deser '72)

A non-linear construction of massive gravity theory (dRGT) was proposed in 2010, where the BD ghost is removed by specially designed non-linear terms, so that the lapse function N becomes a Lagrangian Multiplier, which removes the ghost degree of freedom.

Non-linear Massive Gravity (dRGT)

C. de Rham, G. Gabadadze, Phys. Rev. D 82, 044020 (2010);
C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett 106, 231101 (2011);
S. F. Hassan and R. A. Rosen, JHEP 1107, 009 (2011)

$$S_{MG} = \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right],$$

where $[\mathcal{K}] = tr\left(K^\nu_\mu\right)$

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right),$$

$$\mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right),$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right),$$

$$\mathcal{K}^\mu_\nu \equiv \delta^\mu_\nu - \sqrt{g^{\mu\sigma} G_{ab}(\phi) \partial_\nu \phi^a \partial_\sigma \phi^b}.$$

fiducial metric (→ G_{ab})

Stuckelberg field (→ ϕ^a)

Self-accelerating solution is found in context of non-linear massive gravity, where two branches exist with effective cosmological constant consists of a contribution from mass of graviton. A. E. Gumrukcuoglu, C. Lin and S. Mukohyama. JCAP 106, 231101(2011);

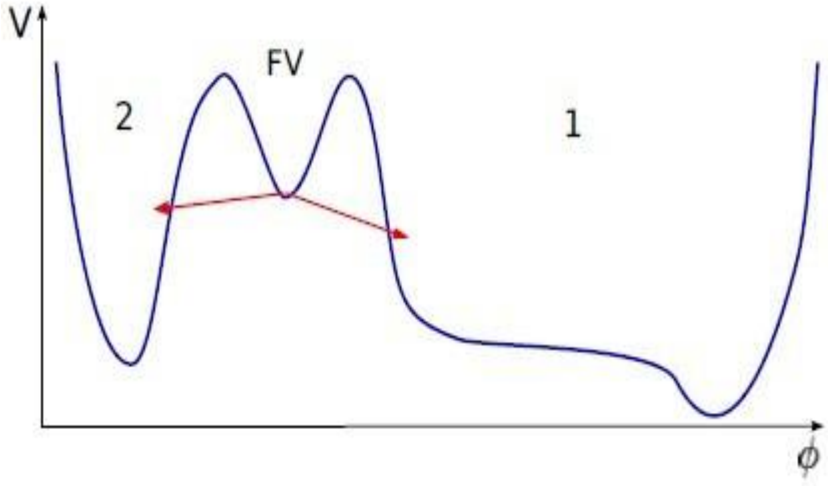
$$\Lambda_{\pm} = -\frac{m_g^2}{(\alpha_3 + \alpha_4)^2} \left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2} \right],$$

There seems to be some hope to explain the current acceleration, but…

Cosmological Constant Problem is not solved in this theory

A possible resolution: Landscape of Vacua

S. Weinberg, Rev. Mod. Phys. 61, 1 (1989)



S. Coleman and F. de Luccia, Phys.Rev. D21, 3305, (1980)

- the field can (and will) tunnel from a metastable minimum to a lower one;

- this process is driven by instanton.

As a first step, we study the stability of a vacuum in the context of dRGT Massive Gravity Theory with constant graviton mass

2. Setup of model

$$S = S_{MG} + S_m,$$

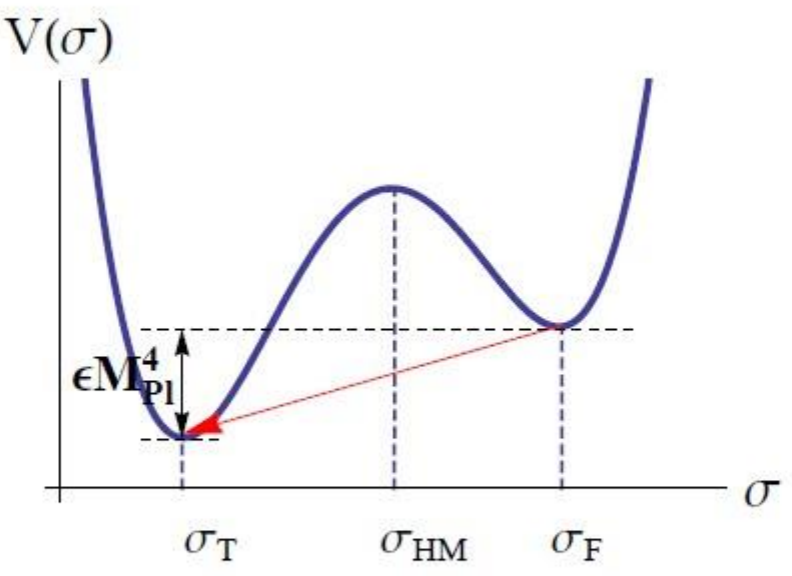
$$S_m \equiv -\int d^4x \, \sqrt{-g} \left[\frac{1}{2}(\partial\sigma)^2 + V(\sigma) \right],$$

- potential $V(\sigma)$

local minima: σ_F

global minima: σ_T

local max: σ_{HM}



- tunneling probability per unit time per unit volume

$$\Gamma/V = Ce^{-B},$$

$$B = S_E[g_{\mu\nu,B}, \phi_B] - S_E[g_{\mu\nu,F}, \phi_F],$$

↑ bounce solution ↑ 'false vacuum'

Lowest action ↓

usually, bounce solutions are explored by assuming an O(4) symmetry

➢ spacetime metric: Euclidean

$$g_{\mu\nu}dx^\mu dx^\nu = N(\xi)^2 d\xi^2 + a(\xi)^2 \Omega_{ij} dx^i dx^j,$$

$$\Omega_{ij} \equiv \delta_{ij} + \frac{K\delta_{il}\delta_{jm}x^l x^m}{1 - K\delta_{lm}x^l x^m}, \quad K > 0$$

Note: the fiducial metric may not respect the symmetry

- fiducial metric: deSitter

$$G_{ab}(\phi)d\phi^a d\phi^b \equiv -(d\phi^0)^2 + b(\phi^0)^2 \Omega_{ij} d\phi^i d\phi^j,$$

$$b(\phi^0) \equiv F^{-1}\sqrt{K}\cosh(F\phi^0).$$

↑ fiducial Hubble parameter

⟶ the O(4)-symmetric solutions are obtained by setting

$$\phi^0 = f(\xi), \quad \phi^i = x^i.$$

Inserting these ansatz into the action, we obtain the constraint equation by varying with respect with f

$$(i\dot{a} + Nb_{,f})\left[\left(3 - \frac{2b}{a}\right) + \alpha_3\left(1 - \frac{b}{a}\right)\left(3 - \frac{b}{a}\right) + \alpha_4\left(1 - \frac{b}{a}\right)^2\right] = 0,$$

$$\dot{a} \equiv \frac{da}{d\xi}$$

$$b_{,f} \equiv \frac{db}{df} = \sqrt{K}\sinh(Ff)$$

$\longrightarrow$ Branch I $\quad Nb_{,f} = -i\dot{a},$ $\quad$ Not considered below

Branch II $\quad \left(3 - \frac{2b}{a}\right) + \alpha_3\left(1 - \frac{b}{a}\right)\left(3 - \frac{b}{a}\right) + \alpha_4\left(1 - \frac{b}{a}\right)^2 = 0.$

$$\longrightarrow \quad b = X_{\pm}a, \qquad X_{\pm} \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}.$$

Friedmann equation & EOM for tunneling field

$$\begin{cases} \dfrac{3}{a^2}\left(\dfrac{da}{d\tau}\right)^2 - \dfrac{3K}{a^2} = \dfrac{1}{2}\left(\dfrac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm, \\ \dfrac{d^2\sigma}{d\tau^2} + \dfrac{3}{a}\left(\dfrac{da}{d\tau}\right)\dfrac{d\sigma}{d\tau} - V_{,\sigma}(\sigma) = 0 \end{cases}$$

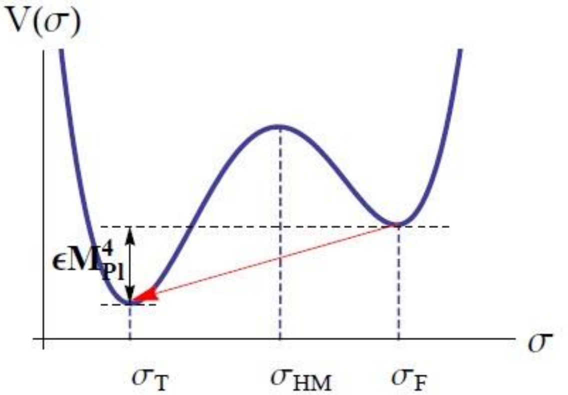
where $\mathrm{d}\tau \equiv N \mathrm{d}\xi,$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right],$$

3. Coleman-de Luccia(CDL) solutions

- CDL solutions can be found when $\sigma(0) = \sigma_{\rm T}, \quad \sigma(\tau_f) = \sigma_{\rm F}$

$$a(\tau) \begin{cases} = a_{\rm T}(\tau) \equiv H_{\rm T}^{-1}\sqrt{K}\cos\left(H_{\rm T}\tau\right) , & \tau < \tau_0 \\ = a_{\rm F}(\tau) \equiv H_{\rm F}^{-1}\sqrt{K}\cos\left(H_{\rm F}\tau + \theta_{\rm F}\right) , & \tau > \tau_0 \end{cases}$$



$$b(\tau) = X_\pm a(\tau) \implies -\left(f'(\tau)\right)^2 = \begin{cases} X_\pm^2 \dfrac{K - (a_{\rm T}H_{\rm T})^2}{K - (a_{\rm T}FX_\pm)^2} , & \tau < \tau_0 \\ X_\pm^2 \dfrac{K - (a_{\rm T}H_{\rm F})^2}{K - (a_{\rm F}FX_\pm)^2} , & \tau > \tau_0 \end{cases}$$

- difference from GR in action is the mass term

$$S^{\rm mass} \equiv -m_g^2 \int d^4x_E \sqrt{\Omega}\left(\mathcal{L}_{2E} + \alpha_3\mathcal{L}_{3E} + \alpha_4\mathcal{L}_{4E}\right)$$

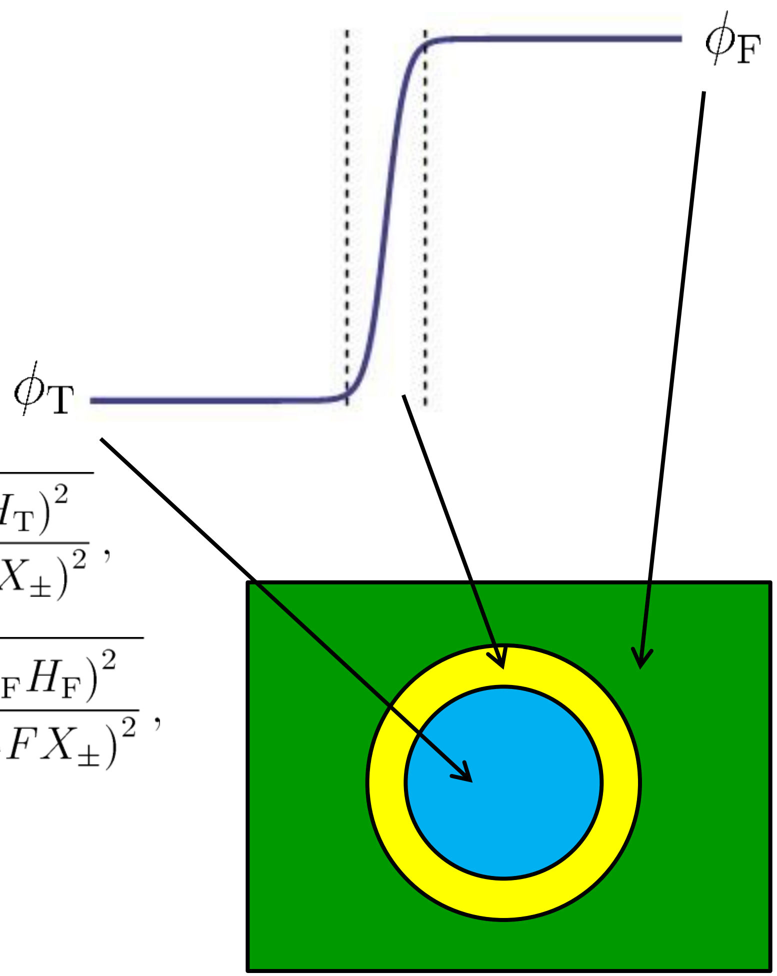
$$= 2\pi^2 K^{-\frac{3}{2}} m_g^2 Y_\pm \int {\rm d}\tau \, a^3(\tau)\sqrt{-(f')^2} ,$$

$$Y_\pm \equiv 3(1 - X_\pm) + 3\alpha_3(1 - X_\pm)^2 + \alpha_4(1 - X_\pm)^3 ,$$

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\rm inside} + B_{\rm outside} + B_{\rm wall}\ ,$$

$$\begin{cases} B_{\rm inside} \equiv S_{\rm inside} - S_{\rm F}\big|_{\tau<\tau_0}\ , \\ B_{\rm outside} \equiv S_{\rm outside} - S_{\rm F}\big|_{\tau>\tau_0}\ , \\ B_{\rm wall} \equiv S_{\rm wall} - S_{\rm F}\big|_{\tau=\tau_0}\ , \end{cases}$$



$$S_{\rm inside} = m_g^2 Y_\pm X_\pm \int d^3x\sqrt{\Omega}\int_{-\pi/(2H_{\rm T})}^{\tau_0(1-\delta)} d\tau\ a_{\rm T}^3 \sqrt{\frac{K-(a_{\rm T}H_{\rm T})^2}{K-(a_{\rm T}FX_\pm)^2}}\ ,$$

$$S_{\rm outside} = m_g^2 Y_\pm X_\pm \int d^3x\sqrt{\Omega}\int_{\tau_0(1+\delta)}^{\pi/(2H_{\rm F})} d\tau\ a_{\rm F}^3 \sqrt{\frac{K-(a_{\rm F}H_{\rm F})^2}{K-(a_{\rm F}FX_\pm)^2}}\ ,$$

$$S_{\rm wall} = m_g^2 Y_\pm \int d^3x\sqrt{\Omega}\int_{\tau_0(1-\delta)}^{\tau_0(1+\delta)} d\tau\ a^3(\tau)\sqrt{-(f')^2}$$

where $\delta \ll 1$

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\rm inside} + B_{\rm outside} + B_{\rm wall} \,,$$

$$\left\{ \begin{array}{r} B_{\rm inside} \equiv S_{\rm inside} - S_{\rm F}\big|_{\tau<\tau_0} \,, \\ B_{\rm outside} \equiv S_{\rm outside} - S_{\rm F}\big|_{\tau>\tau_0} \,, \\ B_{\rm wall} \equiv S_{\rm wall} - S_{\rm F}\big|_{\tau=\tau_0} \,, \end{array} \right.$$

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\rm inside} + B_{\rm outside} + B_{\rm wall}\,,$$

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$$\frac{3}{a^2}\left(\frac{da}{d\tau}\right)^2 - \frac{3K}{a^2} = \frac{1}{2}\left(\frac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm,$$

↓

$$a' = \sqrt{K + \frac{a^2}{3}\left[\frac{\sigma'^2}{2} - V(\sigma) - \Lambda_\pm\right]}$$

↓

$$\int_0^{\tau_0(1-\delta)} {\rm d}\tau = \int_0^{a_0}\left(\frac{da}{d\tau}\right)^{-1} {\rm d}a$$

↓

$$B_{\rm inside} = 2\pi^2 K^{-\frac{3}{2}} m_g^2 Y_\pm X_\pm \int_0^{a_0} a^3 {\rm d}a \left\{\frac{1}{\sqrt{K - a^2\Lambda_{\pm,{\rm T}}/3}}\sqrt{\frac{K-(aH_{\rm T})^2}{K-(aFX_\pm)^2}} - \frac{1}{\sqrt{K - a^2\Lambda_{\pm,{\rm F}}/3}}\sqrt{\frac{K-(aH_{\rm F})^2}{K-(aFX_\pm)^2}}\right\}$$

$$= \mathcal{O}(\epsilon)$$

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\rm inside} + B_{\rm outside} + B_{\rm wall} \,,$$

$$\begin{cases} B_{\rm inside} \equiv S_{\rm inside} - S_{\rm F}|_{\tau<\tau_0} \,, \\ B_{\rm outside} \equiv S_{\rm outside} - S_{\rm F}|_{\tau>\tau_0} \,, \\ B_{\rm wall} \equiv S_{\rm wall} - S_{\rm F}|_{\tau=\tau_0} \,, \end{cases}$$

$$\frac{3}{a^2}\left(\frac{da}{d\tau}\right)^2 - \frac{3K}{a^2} = \frac{1}{2}\left(\frac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm,$$

↓

$$a' = \sqrt{K + \frac{a^2}{3}\left[\frac{\sigma'^2}{2} - V(\sigma) - \Lambda_\pm\right]}$$

↓

$$\int_0^{\tau_0(1-\delta)} \mathrm{d}\tau = \int_0^{a_0} \left(\frac{da}{d\tau}\right)^{-1} \mathrm{d}a$$

↓

$$B_{\rm inside} = 2\pi^2 K^{-\frac{3}{2}} m_g^2 Y_\pm X_\pm \int_0^{a_0} a^3 \mathrm{d}a \left\{ \frac{1}{\sqrt{K - a^2\Lambda_{\pm,\rm T}/3}} \sqrt{\frac{K-(aH_{\rm T})^2}{K-(aFX_\pm)^2}} - \frac{1}{\sqrt{K - a^2\Lambda_{\pm,\rm F}/3}} \sqrt{\frac{K-(aH_{\rm F})^2}{K-(aFX_\pm)^2}} \right\}$$

$$= \mathcal{O}(\epsilon)$$

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\rm inside} + B_{\rm outside} + B_{\rm wall} \,,$$

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$$\frac{d^2\sigma}{d\tau^2} + \frac{3}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} - V_{,\sigma}(\sigma) = 0$$

$$\downarrow \quad \frac{1}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} \ll 1$$

$$\sigma' \simeq \sqrt{2\big[V(\sigma) - V(\sigma_{\rm T})\big]}$$

$$\downarrow \quad {\rm d}\tau = \left(\frac{{\rm d}\sigma}{{\rm d}\tau}\right)^{-1}{\rm d}\sigma$$

$$B_{\rm wall} \simeq 2\pi^2 K^{-\frac{3}{2}} a_0^3 m_g^2 Y_\pm \int_{\sigma_{\rm T}}^{\sigma_{\rm F}} \frac{{\rm d}\sigma}{\sqrt{2\big[V(\sigma) - V(\sigma_{\rm T})\big]}} \left[\sqrt{-(f')^2}\Big|_{\tau<\tau_0} - \sqrt{-(f')^2}\Big|_{\tau>\tau_0}\right]$$

$$= \mathcal{O}(\epsilon)$$

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\rm inside} + B_{\rm outside} + B_{\rm wall}\,,$$

$$\begin{cases} B_{\rm inside} \equiv S_{\rm inside} - S_{\rm F}\big|_{\tau<\tau_0}\,, \\ B_{\rm outside} \equiv S_{\rm outside} - S_{\rm F}\big|_{\tau>\tau_0}\,, \\ B_{\rm wall} \equiv S_{\rm wall} - S_{\rm F}\big|_{\tau=\tau_0}\,, \end{cases}$$

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$$\frac{1}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} \ll 1$$

$$\sigma' \simeq \sqrt{2\left[V(\sigma) - V(\sigma_{\rm T})\right]}$$

$$\mathrm{d}\tau = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\right)^{-1}\mathrm{d}\sigma$$

No difference from GR ?

$$B_{\rm wall} \simeq 2\pi^2 K^{-\frac{3}{2}} a_0^3 m_g^2 Y_\pm \int_{\sigma_{\rm T}}^{\sigma_{\rm F}} \frac{\mathrm{d}\sigma}{\sqrt{2\left[V(\sigma) - V(\sigma_{\rm T})\right]}} \left[\sqrt{-(f')^2}\Big|_{\tau<\tau_0} - \sqrt{-(f')^2}\Big|_{\tau>\tau_0}\right]$$

$$= \mathcal{O}(\epsilon)$$

- CDL as perturbations around Hawking-Moss (HM) solutions

Expand the potential $V(\sigma)$ around $\sigma = \sigma_{\rm HM}$ as follows:

$$V(\sigma) = V(\sigma_{\rm HM}) - \frac{M^2}{2}(\sigma_{\rm HM} - \sigma)^2 + \frac{m}{3}(\sigma_{\rm HM} - \sigma)^3 + \frac{\nu}{4}(\sigma_{\rm HM} - \sigma)^4 + \cdots ,$$

near the HM limit where $M^2 \equiv 4H_{\rm HM}^2(1+\chi^2)$ with $\chi^2 \ll 1$, the regular solutions are perturbatively found to be

$$a(\tau) = \tilde{H}_{\rm HM}^{-1} \cos\left(\tilde{H}_{\rm HM}\tau\right)\left[1 + \frac{\varepsilon_M^2 H_{\rm HM}^2}{8}\cos^2\left(\tilde{H}_{\rm HM}\tau\right)\right] + \mathcal{O}(\varepsilon_M^3)$$

$$\sigma(\tau) = \sigma_{\rm HM} + \varepsilon_M H_{\rm HM}\sin\left(\tilde{H}_{\rm HM}\tau\right) + \frac{\varepsilon_M^2 m}{12}\left[1 - 2\sin^2\left(\tilde{H}_{\rm HM}\tau\right)\right]$$

$$-\varepsilon_M^3 H_{\rm HM}\sin\left(\tilde{H}_{\rm HM}\tau\right)\left[\frac{3H_{\rm HM}^2 - 4\mu}{56}\cos^2\left(\tilde{H}_{\rm HM}\tau\right) - \frac{m^2}{36H_{\rm HM}^2}\sin^2\left(\tilde{H}_{\rm HM}\tau\right)\right] + \mathcal{O}(\varepsilon_M^4)$$

$\tilde{H}_{\rm HM} \equiv H_{\rm HM}(1 + H_{\rm HM}^2\varepsilon_M^2/24)$

$\mu \equiv \nu + m^2/18H_{\rm HM}^2$

$\varepsilon_M^2 \equiv 84\chi^2/(16H_{\rm HM}^2 + 9\mu)$

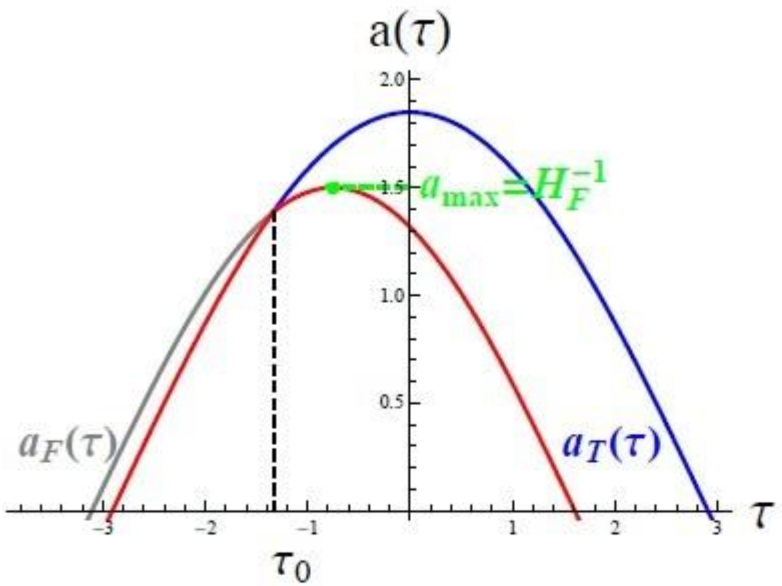
$$Y_{\pm} \equiv 3(1-X_{\pm}) + 3\alpha_3(1-X_{\pm})^2 + \alpha_4(1-X_{\pm})^3,$$

$$\delta^{(2)}S = \frac{\pi^2 m_g^2 X_{\pm} Y_{\pm} H_{\rm HM}^2 \varepsilon_M^2}{2\tilde{H}_{\rm HM}^4 \sqrt{1-\tilde{\alpha}^2}}$$

Hence, if $Y_{\pm} > 0$, HM dominates over CDL, vise versa.

In GR, perturbations in action vanish until $\mathcal{O}(\varepsilon_M^4)$, and CDL always dominate over HM.

Reconsideration of thin-wall result



$$b(\tau) \equiv F^{-1}\sqrt{K}\cosh\left(Ff(\tau)\right) = X_\pm a(\tau) \quad \longrightarrow \quad -(f')^2 = \frac{X_\pm^2 (a')^2}{K-(FX_\pm a)^2}$$

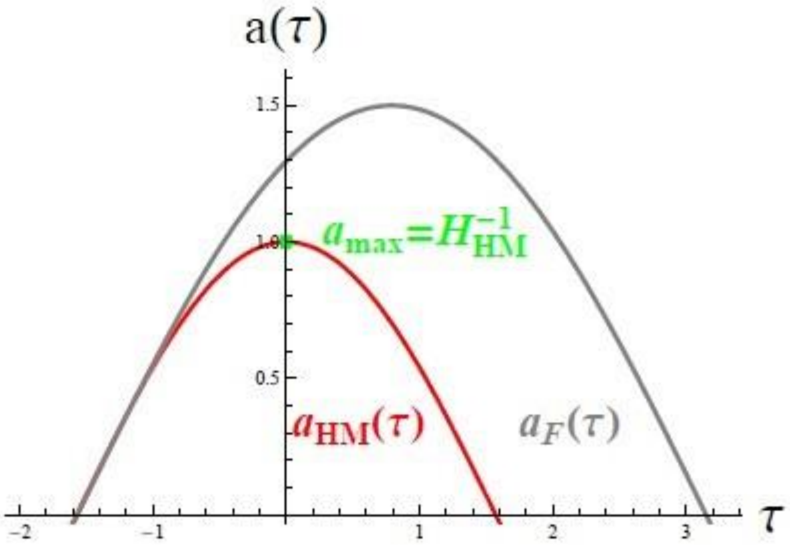
$$S^{\rm mass} = 4\pi^2 K^{-\frac{3}{2}} m_g^2 X_\pm Y_\pm \int_0^{a_{\rm max}} \frac{a^3 {\rm d}a}{\sqrt{K-(FX_\pm a)^2}}$$

$$= -\frac{4\pi^2 K^{-\frac{3}{2}} m_g^2 X_\pm Y_\pm}{3(FX_\pm)^4}\left[\sqrt{K-(FX_\pm a)^2}\left(2K+(FX_\pm a)^2\right)\right]_0^{a_{\rm max}}$$

$$B^{\mathrm{mass}}_{\mathrm{thin-wall}} \equiv S^{\mathrm{mass}} - S^{\mathrm{mass}}_{\mathrm{F}} \propto \left[\sqrt{K-(FX_{\pm}a)^2}\left(2K+(FX_{\pm}a)^2\right)\right]^{a_{\max}}_{a_{\mathrm{F,max}}} = 0\,,$$

This explains the reason why no contribution in thin-wall limit. However, in HM case, $a_{\max} = a_{\mathrm{HM,max}} \equiv H^{-1}_{\mathrm{HM}}$

$$B^{\mathrm{mass}}_{\mathrm{HM}} = -\frac{4\pi^2 K^{-\frac{3}{2}} m_g^2 X_{\pm} Y_{\pm}}{3(FX_{\pm})^4}\left[\sqrt{K-(FX_{\pm}a)^2}\left(2K+(FX_{\pm}a)^2\right)\right]^{H^{-1}_{\mathrm{HM}}}_{H^{-1}_{\mathrm{F}}} \neq 0$$



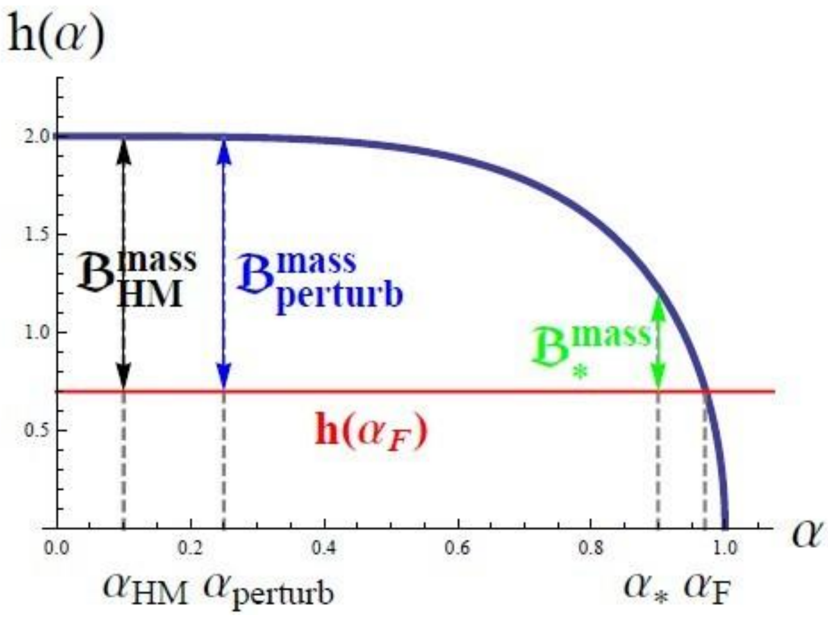
Defining

$$\mathfrak{B}^{\text{mass}} \equiv -\frac{3(FX_\pm)^4 B^{\text{mass}}}{4\pi^2 m_g^2 X_\pm Y_\pm} = \left[\sqrt{1-(FX_\pm a)^2}\left(2+(FX_\pm a)^2\right)\right]_{H_{\text{F}}^{-1}}^{a_{\max}},$$

$$\equiv h(\alpha_{max}) - h(\alpha_F) > 0$$

$$\Delta\Gamma \equiv \frac{\Gamma_{\text{MG}}}{\Gamma_{\text{GR}}} \simeq \exp\left(\frac{4\pi^2 m_g^2 Y_\pm \mathfrak{B}^{\text{mass}}}{3F^4 X_\pm^3}\right).$$

- HM solution gives largest correction term where a_{max} is smallest;
- when a_{max} increases, correction shrinks gradually;
- at thin-wall limit, the behavior of CDL solution is the same as GR.



CDL V.S. HM

Under the thin-wall approximation, one can compare the probability of CDL process to HM process as follows

$$\ln\left(\frac{P_{\rm CDL}}{P_{\rm HM}}\right) \approx 4\pi^2\left[\frac{16M_{\rm Pl}^6}{\Sigma^2} - \frac{m_g^2 M_{\rm Pl}^2 X_\pm Y_\pm}{3}\left(\frac{A(\alpha_{\rm F})}{H_{\rm F}^4} - \frac{A(\alpha_{\rm HM})}{H_{\rm HM}^4}\right)\right],$$

$$\Sigma \equiv \int_{\sigma_{\rm T}}^{\sigma_{\rm F}} d\sigma \sqrt{2[V(\sigma) - V(\sigma_{\rm T})]}$$

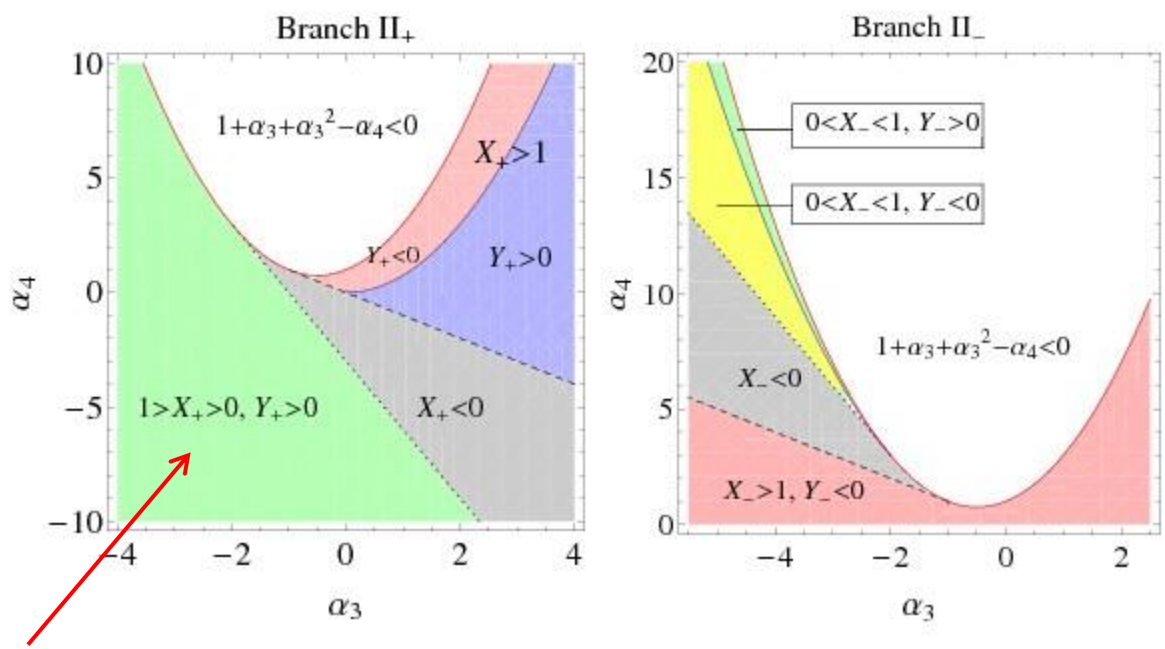
$$A(\alpha) \equiv \frac{2 - \sqrt{1-\alpha^2}(2+\alpha^2)}{\alpha^4}$$

In GR, $m_g = 0$, CDL process dominates over HM one.

However, provided that parameters and their combinations are of order unity, if $m_g > \mathcal{O}\left(M_{\rm Pl}^2 H_{\rm F}^2 \Sigma^{-1}\right) \sim \mathcal{O}\left(a_0^{-1}\right)$, HM process dominates over CDL.

Parameter $X_\pm$ is constrained by the mass of tensor mode for self-accelerating branch: A. E. Gumrukcuoglu, C. Lin and S. Mukohyama. JCAP 03, 006(2012);

$$M_{\rm GW}^2 = \begin{cases} m_g^2 X_+(1-X_+)\sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}\,, & \text{for } X_+ \\ m_g^2 X_-(X_- - 1)\sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}\,, & \text{for } X_- \end{cases} \quad \Longrightarrow \quad \begin{matrix} 1 > X_+ > 0 \\ X_- > 1 \end{matrix}$$



possible region for HM domination

Summary and future work

- We constructed a model in which the tunneling field minimally couples to the dRGT massive gravity;
- corrections to CDL tunneling change monotonically when one goes beyond thin-wall approximation until HM case;
- under the thin-wall approximation, the HM process may dominate over CDL one, it is interesting to investigate its implications;
- it would be a further work to generalize our analysis to extended massive gravity theories and bigravity theory.