Review on EiBI Inflation: Discussion on Tensor Perturbation

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Outline

1. Introduction to EiBI Gravity
2. Chaotic Inflation in GR
3. Scalar Field in EiBI Gravity
4. MPS (Maximal Pressure Solution)
5. Inflation in EiBI gravity
6. Conclusions
7. **Discussion on Tensor Perturbation**

Eddington-inspired Born-Infeld Gravity

Formalism

Einstein Gravity

Field: $g_{\alpha\beta}$

David Hilbert

$$S_{\rm EH} = \frac{1}{2}\int d^4x \sqrt{|g|}\,(R - 2\Lambda)$$

$$8\pi G = 1$$

$$\begin{aligned}\delta R &= R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu} \\ &= R_{\mu\nu}\delta g^{\mu\nu} + \nabla_\sigma \left(g^{\mu\nu}\delta\Gamma^\sigma_{\nu\mu} - g^{\mu\sigma}\delta\Gamma^\rho_{\rho\mu}\right)\end{aligned}$$

Palatini

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\sigma}\left(\underline{g_{\alpha\sigma,\beta}} + \underline{g_{\beta\sigma,\alpha}} + \underline{g_{\alpha\beta,\sigma}}\right)$$

GR

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu\Gamma^\rho_{\nu\sigma} - \partial_\nu\Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}.$$

Eddington Gravity (1924)

Field: $\Gamma^{\alpha}_{\beta\gamma}$

$$\boxed{S_{\rm Edd} = 2\kappa \int d^4x \sqrt{|R|}}$$

Arthur Eddington

- ✓ **Varying S, Integrating by parts, Eliminating a vanishing trace, we get**

$$\boxed{\nabla_{\alpha}(2\kappa\sqrt{|R|}R^{\mu\nu}) = 0,}$$

where ∇ is the covariant derivative defined in terms of $\Gamma^{\alpha}_{\beta\gamma}$

- ✓ define a new rank-2 tensor $q_{\mu\nu}$ such that $\nabla_{\alpha}(\sqrt{|q|}q^{\mu\nu}) = 0$
- ✓ theory then become

$$\boxed{2\kappa\sqrt{|R|}R^{\mu\nu} = \sqrt{|q|}q^{\mu\nu}}$$

- ✓ which can be rewritten as the Einstein field equations

if we equate $q_{\alpha\beta}$ with $g_{\alpha\beta}$ and κ with Λ^{-1}

Therefore, Eddington's action

:- viable and alternative starting point to GR

:- $S_{\rm EH} \propto \Lambda$ and $S_{\rm Edd} \propto \dfrac{1}{\Lambda}$

:- dual to GR

However, incomplete : NOT including MATTER

Later attempts to couple matter with Γ

:- start with Palatini gravitational action
coupled to matter $\boxed{I[g, \Gamma, \Psi]}$ -- no derivatives in g

:- EOM for g → $\boxed{g_{\mu\nu} = g_{\mu\nu}(\Gamma, \Psi)}$ → back into $\boxed{I[g, \Gamma, \Psi]}$

→ can eliminate g

:- $\boxed{I'[\Gamma, \Psi]}$: complicated,
but Dynamics is fully equivalent
to the original metric theory

Eddington-inspired Born-Infeld Gravity

(MERIT 1) One parameter (κ) theory

$$S_{\text{EiBI}} = \frac{1}{\kappa}\int d^4x\Big[\sqrt{-|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)|} - \lambda\sqrt{-|g_{\mu\nu}|}\,\Big] + S_M(g,\Phi)$$

(Vollick 2004, Banados-Ferreira 2010)

:- $\boxed{g_{\mu\nu} \text{ and } \Gamma^{\mu}_{\alpha\beta}}$: independent

:- Matter is in usual way (Not in sqrt)

:- For large κ -limit → Eddington limit

$$\boxed{\sqrt{|g_{\mu\nu}+\kappa R_{\mu\nu}|} \simeq \sqrt{|g|}\left[1+\frac{\kappa}{2}R+\frac{1}{8}\kappa^2(R^2-2R_{\mu\nu}R^{\mu\nu})+\mathcal{O}(\kappa^3)\right]}$$

:- For small κ-limit → Einstein limit

Field Equations

$$S_{\rm EiBI} = \frac{1}{\kappa}\int d^4x\Big[\sqrt{-|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)|} - \lambda\sqrt{-|g_{\mu\nu}|}\,\Big] + S_M(g,\Phi)$$

✓ Variation of S w.r.t. $g_{\mu\nu}$ and Γ $\Rightarrow$ 2 EOM's

EOM1: $$\frac{\sqrt{-|q|}}{\sqrt{-|g|}}\, q^{\mu\nu} = \lambda g^{\mu\nu} - \kappa T^{\mu\nu}$$ **: Relation b/t g and q via T**

EOM2: $$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}$$ **:- Auxiliary Metric**
:- Dynamical Equation

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} q^{\mu\sigma}(q_{\sigma\alpha,\beta} + q_{\sigma\beta,\alpha} - q_{\alpha\beta,\sigma})$$ **: Connection is defined by q**

$$T^{\mu\nu} = \frac{2}{\sqrt{|g|}}\frac{\delta L_M}{\delta g_{\mu\nu}} \Rightarrow \nabla^{g}_{\mu} T^{\mu\nu} = 0$$ **: EOM for matter**

: Energy-Momentum Conservation

→ Matter plays in the background metric

According to the Palatini formalism, one should consider the equations of motion by varying the action (1.1) with respect to (w.r.t) the fields $g_{\mu\nu}$ and $\Gamma^{\rho}_{\mu\nu}$ individually. Variation of the action w.r.t. $g_{\mu\nu}$ leads to the equation of motion,

$$\frac{\sqrt{-|g+\kappa R|}}{\sqrt{-|g|}}[(g+\kappa R)^{-1}]^{\mu\nu} - \lambda g^{\mu\nu} = -\kappa T^{\mu\nu}, \tag{1.2}$$

where $[(g+\kappa R)^{-1}]^{\mu\nu}$ denotes the matrix inverse. The energy-momentum tensor $T^{\mu\nu}$ is given by the usual sense,

$$T^{\mu\nu} = \frac{2}{\sqrt{-|g|}}\frac{\delta L_M}{\delta g_{\mu\nu}}. \tag{1.3}$$

For the variation of the action w.r.t. Γ, one introduces an auxiliary metric $q_{\mu\nu}$ defined by

$$q_{\mu\nu} \equiv g_{\mu\nu} + \kappa R_{\mu\nu}. \tag{1.4}$$

Then the variation of the action (1.1) w.r.t. the connection $\Gamma^{\mu}_{\rho\sigma}$ gives

$$\nabla^{\Gamma}_{\mu} q^{\rho\sigma} = 0, \tag{1.5}$$

where $q^{\rho\sigma} \equiv (q^{-1})^{\rho\sigma}$ is the matrix inverse of $q_{\rho\sigma}$, and ∇^{Γ} denotes the covariant derivative defined by the connection Γ. This equation is the metric compatibility which yields

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2}q^{\mu\sigma}(q_{\alpha\sigma,\beta} + q_{\beta\sigma,\alpha} + q_{\alpha\beta,\sigma}). \tag{1.6}$$

Therefore, Eq. (1.4) can be regarded as the equation of motion since the Ricci tensor is evaluated in terms of $q_{\rho\sigma}$ through the relation (1.6). Using Eq. (1.4), the first equation of motion (1.2) can also be simplified,

$$\sqrt{-|q|}$$

Some Immediacies (Banados-Ferreira 2010)

Equivalence to GR in Vacuum

✓

$$\boxed{\begin{aligned} &\sqrt{-q}(q^{-1})^{\mu\nu} = \lambda\sqrt{-g}g^{\mu\nu} - \cancel{\kappa T^{\mu\nu}} \\ \Rightarrow\quad &\sqrt{-q}(q^{-1})^{\mu\nu} = \lambda\sqrt{-g}g^{\mu\nu} \quad (vacuum) \end{aligned}}$$

: EOM1

✓ Putting $g_{\mu\nu} = \alpha q_{\mu\nu}$ then $g^{\mu\nu} = \frac{1}{\alpha}q^{\mu\nu}$.

✓ Substituting this we obtain

$$\frac{1}{\alpha^2}\sqrt{-g}\ \alpha\ g^{\mu\nu} = \lambda\sqrt{-g}g^{\mu\nu}$$

$$\Rightarrow \quad \therefore \alpha = \frac{1}{\lambda}.$$

$$\boxed{\Lambda = \frac{\lambda - 1}{\kappa}}$$

✓ Then,

EOM2: $\boxed{q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}}$ $\Longrightarrow$ $\boxed{R_{\mu\nu} = \frac{\lambda - 1}{\kappa}g_{\mu\nu} \equiv \Lambda g_{\mu\nu}}$

(MERIT 2) EiBI in vacuum or with only CC is the same with EH

Black Hole Solutions

Schwartzchild-de Sitter BH

SAME with Einstein Solution

Charged BH : non-vacuum

$$\boxed{ds^2 = -\psi(r)^2 f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2.}$$

$$\left\{\begin{aligned} f(r) &= \frac{r\sqrt{r^4 + \kappa q^2}}{r^4 - \kappa q^2} \int dr \left[\frac{(r^2 - q^2)(r^4 - \kappa q^2)}{r^4 \sqrt{r^4 + \kappa q^2}} - 2M \right], \\ \psi(r) &= \frac{r^2}{\sqrt{2r^4 + 2\kappa q^2}}, \end{aligned}\right.$$

$$E(r) = \frac{q}{\sqrt{r^4 + \kappa q^2}},$$ **: electric field**

Poisson Equation

Nonrelativistic Limit w/ the lowest-order correction

Metric : $ds^2 = -(1+2\Phi)dt^2 + (1-2\Psi)d\vec{x}\cdot d\vec{x},$

with $\Phi = \Psi$ **and** $T^{\mu\nu} = \rho u^\mu u^\nu$

$$\boxed{\nabla^2\Phi = -\frac{1}{2}\rho - \frac{1}{4}\kappa\nabla^2\rho.}$$ **: Poisson Equation**

Implies repulsive nature of EiBI gravity

Nonsingular Universe driven by Perfect Fluid

Banados & Ferreira for w=1/3 (2010)
IC, Kim and Moon for all w (2012)

Metric & Auxiliary Metric

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\, d\mathbf{x}^2,$$
$$q_{\mu\nu}dx^\mu dx^\nu = -X^2(t)\, dt^2 + Y^2(t)\, d\mathbf{x}^2.$$

$\boxed{\text{Let } a = e^\Omega}$

Matter-Stress Tensor

: same with GR

$$T^{\mu\nu} = (p + \rho)u^\mu u^\nu + p g^{\mu\nu}$$

$$\nabla^g_\mu T^{\mu\nu} = 0 \quad \Longrightarrow \quad \dot{\rho} + 3\dot{\Omega}(\rho + p) = 0.$$

$$p = w\rho \quad \Longrightarrow \quad \rho = \rho_0 e^{-3(1+w)\Omega}.$$

EOM 1

$$\frac{\sqrt{-|q|}}{\sqrt{-|g|}}\, q^{\mu\nu} = \lambda g^{\mu\nu} - \kappa T^{\mu\nu}$$

we get

$$X = \frac{(\lambda - \kappa p)^{3/4}}{(\lambda + \kappa\rho)^{1/4}}, \qquad \text{and} \qquad Y = [(\lambda - \kappa p)(\lambda + \kappa\rho)]^{1/4} e^{\Omega}.$$

: relation b/w q and g via matter field

EOM 2 → Friedmann Equations

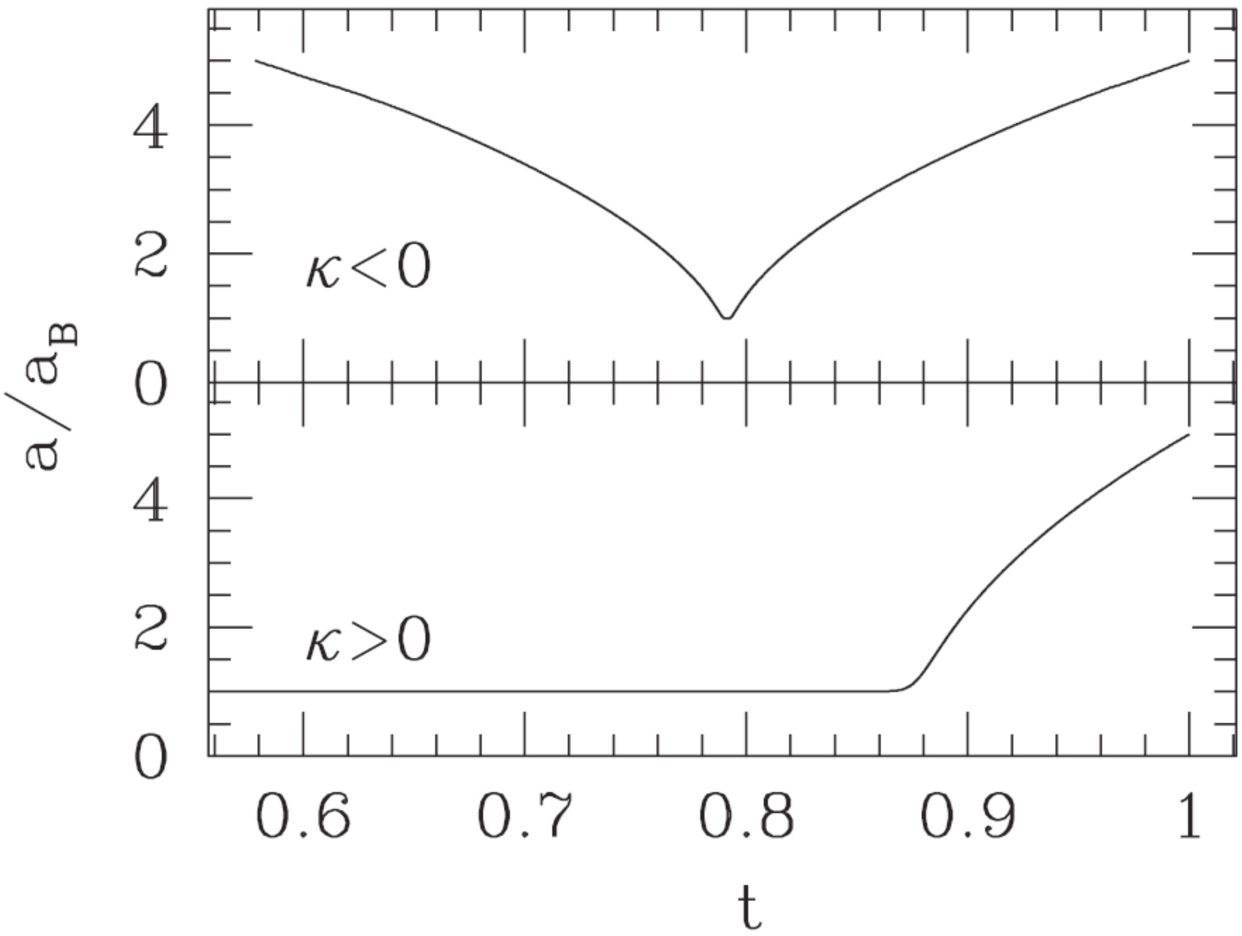
$$q_{\mu\nu} - g_{\mu\nu} = \kappa R_{\mu\nu}$$

Volume Part :

$$H^2 \equiv \dot{\Omega}^2 = \frac{(\lambda - w\kappa\rho)^2}{6\kappa}$$
$$\times \frac{(\lambda + \kappa\rho)^2 + 2(\lambda - w\kappa\rho)^{3/2}(\lambda + \kappa\rho)^{3/2} - 3(\lambda - w\kappa\rho)(\lambda + \kappa\rho)}{[(3/4)\kappa w(1 + w)(\lambda + \kappa\rho)\rho - (3/4)\kappa(1 + w)(\lambda - w\kappa\rho)\rho + (\lambda - w\kappa\rho)(\lambda + \kappa\rho)]^2}$$

1) w>0

Banados & Ferreira for w=1/3 (2010)
IC, Kim and Moon for all w (2012)



Bouncing Universe

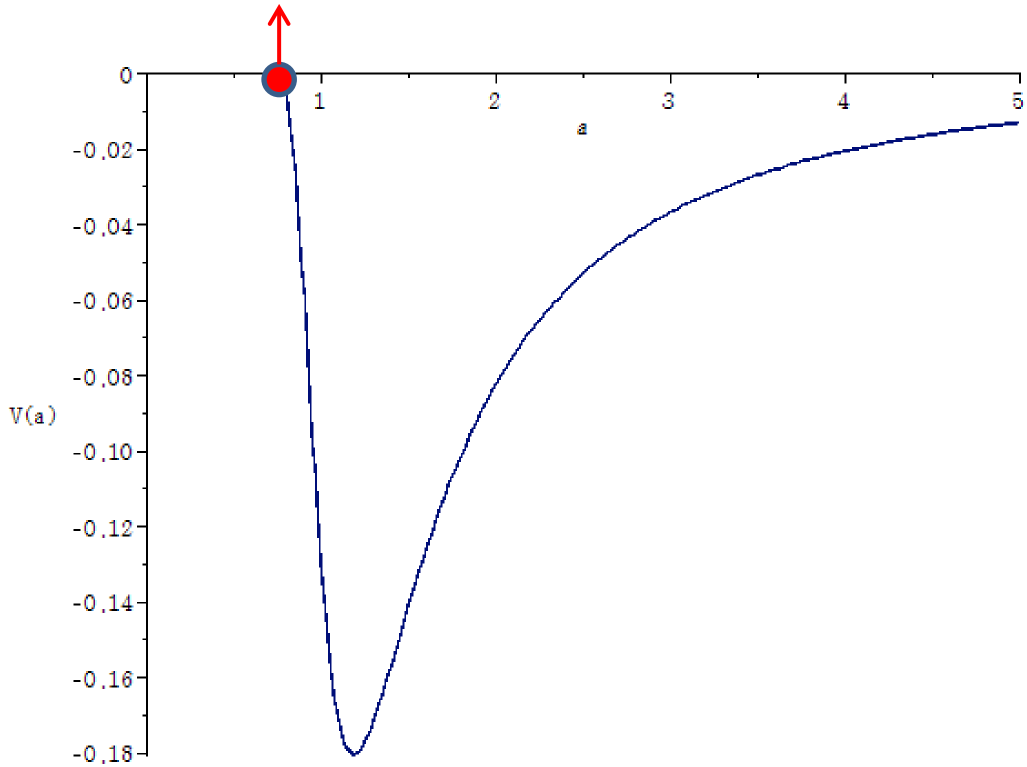
Non-Singular Universe

$$\boxed{\dot{a}^2 + V(a) = 0}$$

$$X = \frac{(\lambda - \kappa p)^{3/4}}{(\lambda + \kappa\rho)^{1/4}}, \qquad \text{and} \qquad Y = [(\lambda - \kappa p)(\lambda + \kappa\rho)]^{1/4} e^{\Omega}.$$

a_B : Maximal Pressure/Density Limit

a_B



Early Times

At a= a_B **, H=0 :** $\lambda - w\kappa\rho = 0, \quad \rho = \rho_B = \lambda / w\kappa,$

Expand near a= a_B **:** $\rho = \rho_B - \varepsilon, \quad a = a_B + \epsilon,$

Then we have

$$\boxed{H^2 \approx \frac{8\kappa w^2(1+w)^2\lambda^2}{27(1+w)^4\lambda^4}\varepsilon^2 = H_0^2\left(\frac{a}{a_B} - 1\right)^2}$$

$$\boxed{H_0^2 = \frac{8}{3\kappa}}$$

$$\boxed{a(t) \approx a_B + Ae^{H_0 t}.}$$

:- t → -infty : origin of Universe
:- finite size
:- Non-Singular Universe

Chaotic Inflation in GR

Action & Metric

$$S_M = \int d^4x \sqrt{-|g_{\mu\nu}|} \left[-\frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) \right], \qquad V(\phi) = \frac{m^2}{2}\phi^2, \qquad \boxed{8\pi G = 1}$$

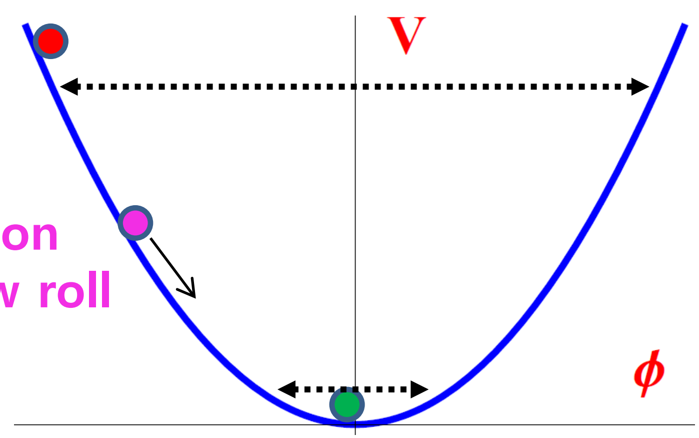
$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) d\mathbf{x}^2.$$

Field Equations & Slow-Roll Conditions

$$H^2 = \left(\frac{\dot a}{a}\right) = \frac{1}{3}\rho = \frac{1}{3}\left(\cancel{\frac{1}{2}\dot\phi^2} + V\right)$$: 1st slow-roll condition

$$\cancel{\ddot\phi} + 3H\dot\phi + V'(\phi) = 0$$: 2nd slow-roll condition

Chaotic
:- large fluctuation



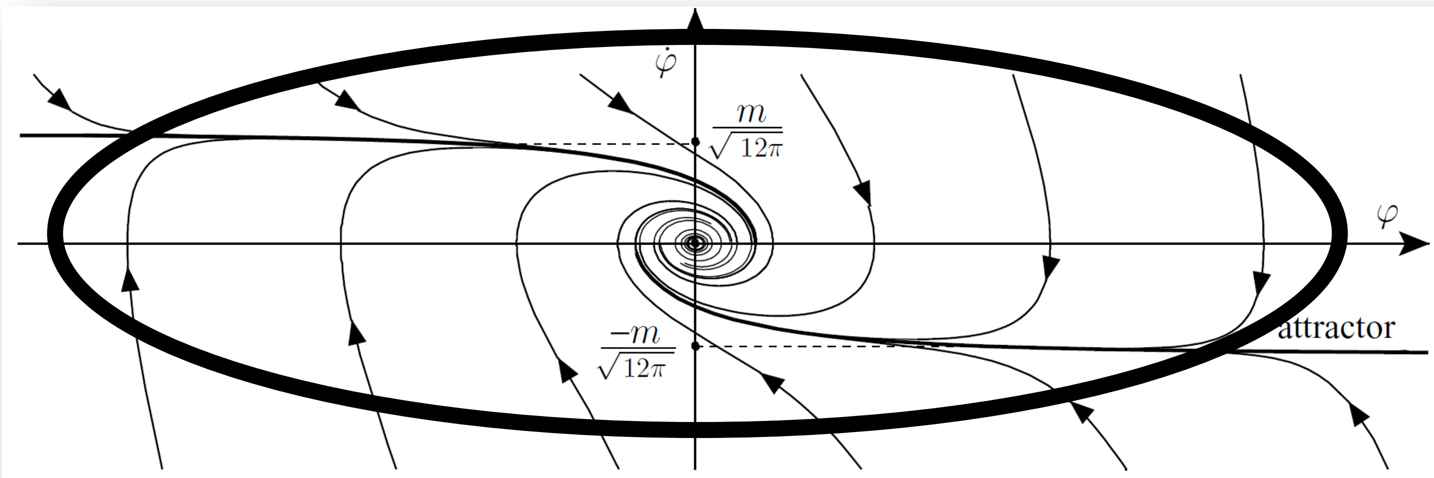
Inflation
:- slow roll

Inflaton decay :- reheating

Attractor Solution for Slow-Roll Inflation

$$\boxed{\phi(t) \approx \phi_i + \sqrt{2/3}mt, \qquad a(t) \approx a_i\, e^{\frac{1}{4}[\phi_i^2 - \phi^2(t)]},}$$

$$N \sim 70\ e\text{-foldings} \quad \begin{cases} |\phi_i| \gtrsim 10 \text{ is required} \\ m \sim 10^{-5} \text{ from observational data} \end{cases}$$



Quantum Gravity Regime

$$\boxed{\rho = K + V > M_P^2} \qquad K = \dot{\phi}^2/2 \text{ and } V = m^2\phi^2/2.$$

The curvature scale is determined mainly by H

$$\boxed{R = 6\left[\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{\ddot{a}}{a}\right)\right] = 12H^2 + 6\dot{H}} \qquad H^2 = \frac{1}{3}\rho = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right)$$

When H > M_p , Gravity Part requires quantum treatment

:- In GR, large curvature is inevitable

:- In EiBI gravity,

→ curvature scale is NOT directly related with energy scale ρ
→ maybe Quantum Gravity is avoidable
in describing the high-energy state of the scalar field

Scalar Field in EiBI Gravity

$$\boxed{S_{\rm EiBI} = \frac{1}{\kappa}\int d^4x\Big[\sqrt{-|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)|} - \lambda\sqrt{-|g_{\mu\nu}|}\,\Big] + S_M(g,\Phi)}$$

✓ Variation of S w.r.t. $g_{\mu\nu}$ and Γ $\Rightarrow$ 2 EOM's

EOM1: $$\boxed{\frac{\sqrt{-|q|}}{\sqrt{-|g|}}\, q^{\mu\nu} = \lambda g^{\mu\nu} - \kappa T^{\mu\nu}}$$ **: Relation b/t g and q via T**

EOM2: $$\boxed{q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}}$$ **:- Auxiliary Metric**
:- Dynamical Equation

$$\boxed{\Gamma^{\mu}_{\alpha\beta} = \tfrac{1}{2} q^{\mu\sigma}(q_{\sigma\alpha,\beta} + q_{\sigma\beta,\alpha} - q_{\alpha\beta,\sigma})}$$ **: Connection is defined by q**

$$\boxed{T^{\mu\nu} = \frac{2}{\sqrt{|g|}}\frac{\delta L_M}{\delta g_{\mu\nu}}} \Rightarrow \boxed{\nabla^g_\mu T^{\mu\nu} = 0}$$ **: EOM for matter**

: Energy-Momentum Conservation

→ Matter plays in the background metric

Field Equations

$$S_M = \int d^4x\sqrt{-|g|}\left[-\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi)\right]$$

$$V(\phi) = \frac{m^2}{2}\phi^2$$

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\, d\mathbf{x}^2,$$

$$q_{\mu\nu}dx^\mu dx^\nu = -X^2(t)\, dt^2 + Y^2(t)\, d\mathbf{x}^2.$$

$$X = (\lambda - \kappa p)^{3/4}(\lambda + \kappa\rho)^{-1/4}, \qquad Y = [(\lambda + \kappa\rho)(\lambda - \kappa p)]^{1/4}\, a,$$

Puts an upper limit on pressure

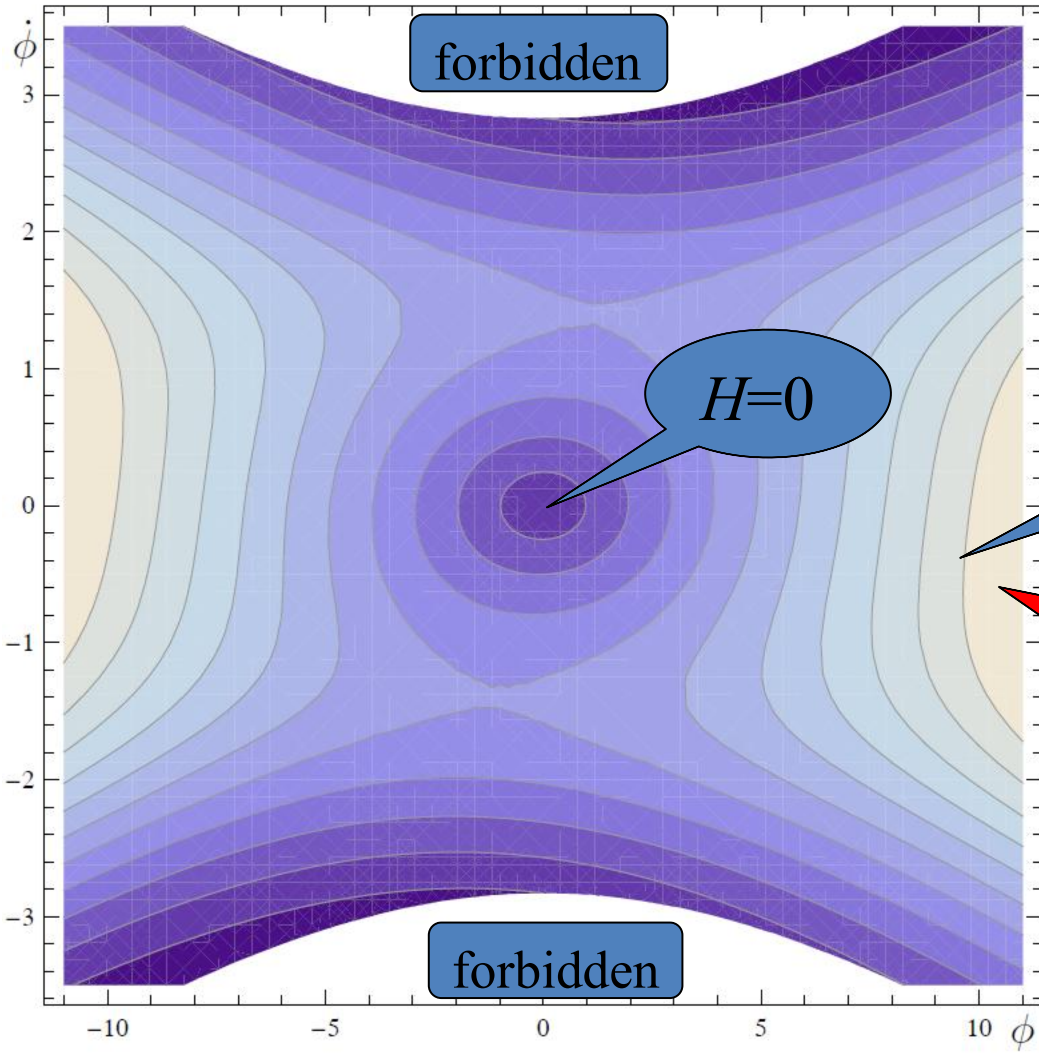
$$\rho = \dot{\phi}^2/2 + V \text{ and } p = \dot{\phi}^2/2 - V$$

$$H \equiv \frac{\dot{a}}{a} = \frac{1}{(\lambda\!\!\!^{-} + V)^2 + \dot{\phi}^4/2}\left\{-\frac{1}{2}\left(\lambda\!\!\!^{-} + V + \frac{\dot{\phi}^2}{2}\right)V'(\phi)\dot{\phi} \pm \frac{1}{\sqrt{3}}\left(\lambda\!\!\!^{-} + V - \frac{\dot{\phi}^2}{2}\right)\times \right.$$
$$\left.\left[\left(\lambda\!\!\!^{-} + V + \frac{\dot{\phi}^2}{2}\right)^{3/2}\left(\lambda\!\!\!^{-} + V - \frac{\dot{\phi}^2}{2}\right)^{3/2} - \frac{1}{\kappa}\left(\lambda\!\!\!^{-} + V + \frac{\dot{\phi}^2}{2}\right)\left(\lambda\!\!\!^{-} + V - \dot{\phi}^2\right)\right]^{1/2}\right\}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0.$$

$$\lambda\!\!\!^{-} \equiv \lambda/\kappa$$

H in Phase Space $(\phi, \dot{\phi})$ for $m = 1/4,\ \kappa = 1/4,$ and $\lambda = 1.$



Contour gradient :

$\Delta H = 0.1.$

Quantum Gravity Regime

Slow-Roll Evolution :

Same both for GR and EiBI

1st Slow-Roll Condition

$$\dot{\phi}^2 \ll V(\phi)$$

$$H \approx \sqrt{[V(\phi)+\Lambda]/3} = (m/\sqrt{6})|\phi|$$: **same as in GR**

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0}$$

integrated

$$\boxed{\mp\dot{\phi} + \sqrt{2/3}m\log\left(\pm\dot{\phi} + \sqrt{2/3}m\right) \approx \sqrt{3/8}m(\phi^2 - \phi_0^2),}$$

$$\boxed{\phi_0 \sim \phi_i}$$

2nd Slow-Roll Condition

$$\boxed{\cancel{\ddot{\phi}} + 3H\dot{\phi} + m^2\phi = 0}$$

$$\boxed{\cancel{\mp\dot{\phi}} + \sqrt{2/3}m\log\left(\pm\dot{\phi} + \sqrt{2/3}m\right) \approx \sqrt{3/8}m(\phi^2 - \phi_0^2),}$$

$$\boxed{|\dot{\phi}| \approx \sqrt{2/3}m}$$

: **usual chaotic inflation**

→ **attractor**

Backward in Time

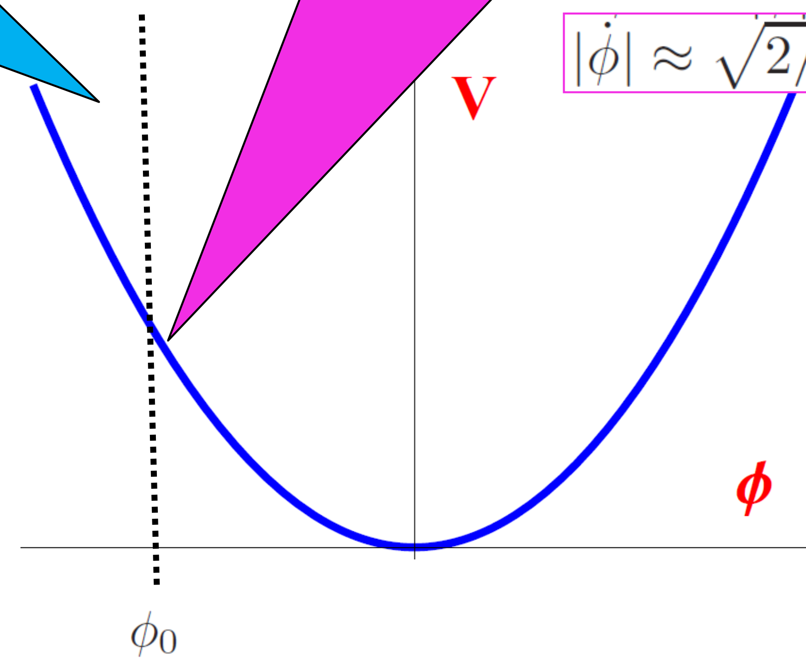
Only 1^{st} Slow-Roll Cond's Satisfied
→ $|\dot{\phi}|$ jumps rapidly

$$\dot{\phi} \propto \phi^2$$

Arrives Planck Scale soon in GR

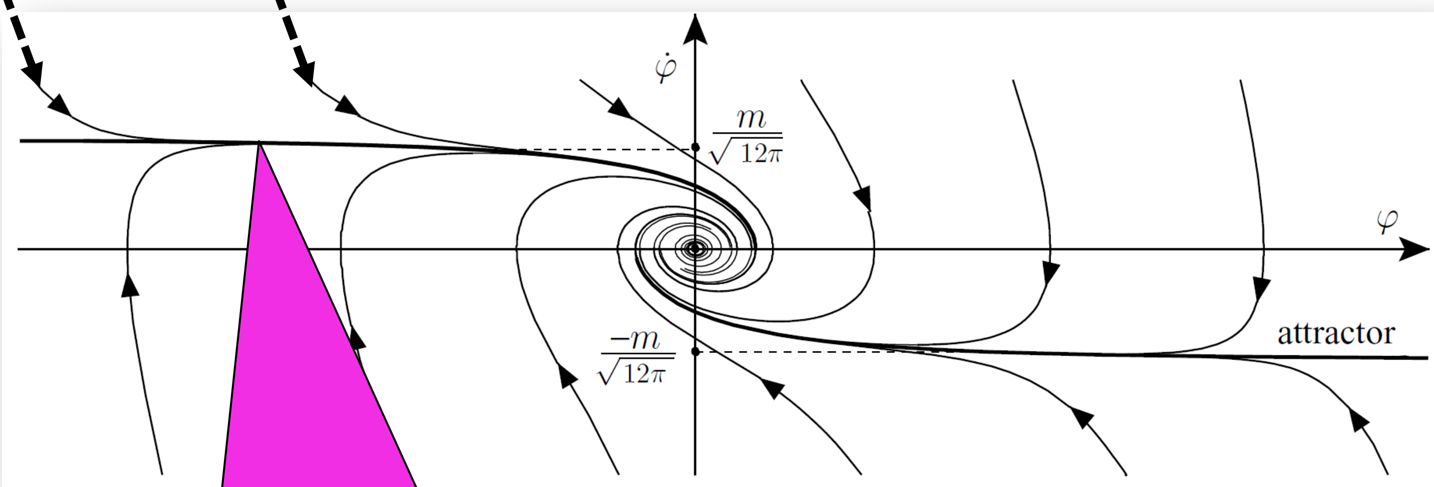
1^{st} & 2^{nd} Slow-Roll Cond's Satisfied
→ Chaotic Inflation

$$|\dot{\phi}| \approx \sqrt{2/3}m$$



Forward in Time

Only 1st Slow-Roll Cond.Satisfied
→ $|\dot{\phi}|$ drops rapidly



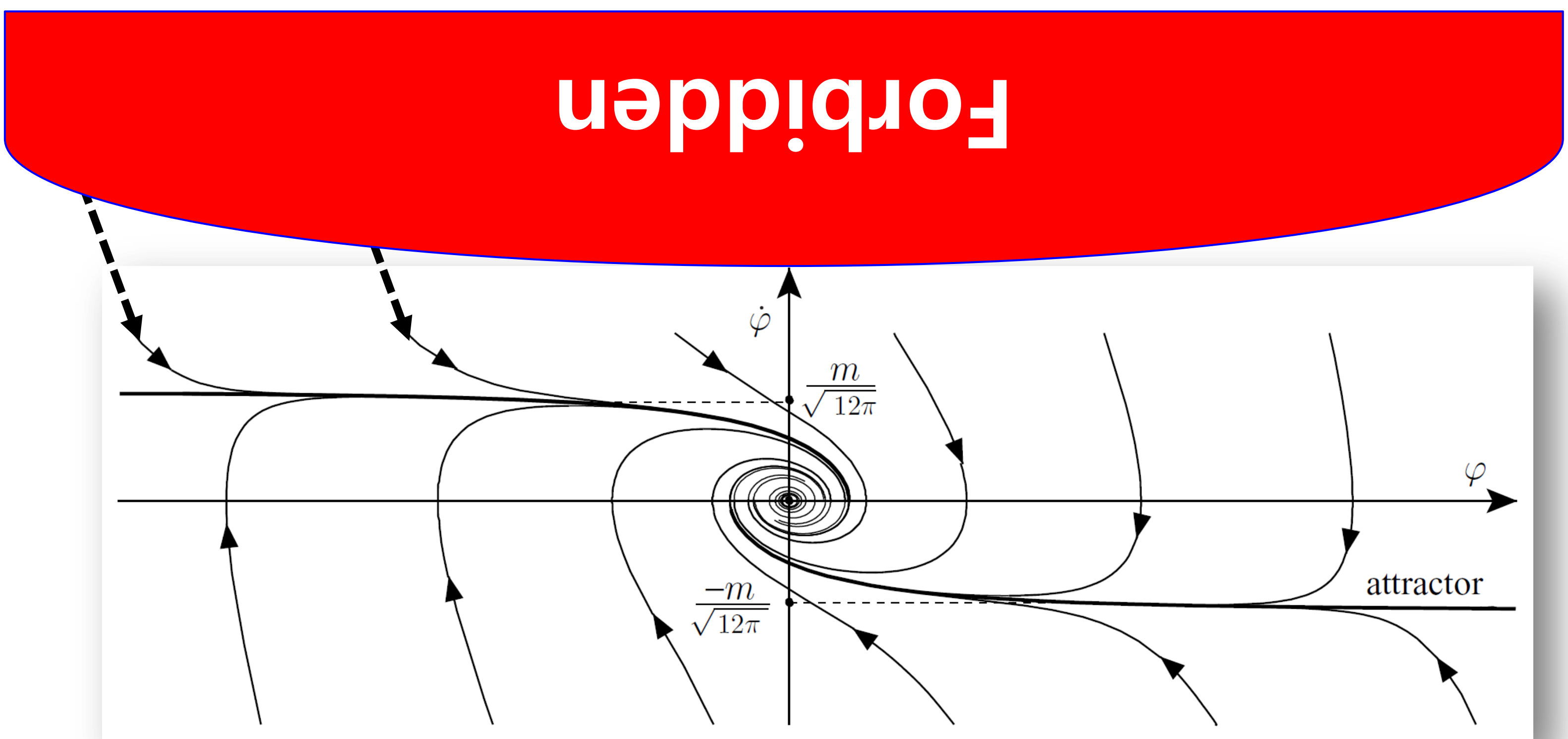
1st & 2nd Slow-Roll Cond.'s Satisfied
→ Chaotic Inflation

In EiBI gravity, there exists **an upper limit** in $\dot{\phi}^2$

→ **Maximal Pressure Condition** (MPC)

$$\boxed{\frac{1}{2}\dot{\phi}^2 - V(\phi) - \bar{\lambda} = 0,}$$

pressure



Maximal Pressure Solution (MPS)

Maximal Pressure Condition

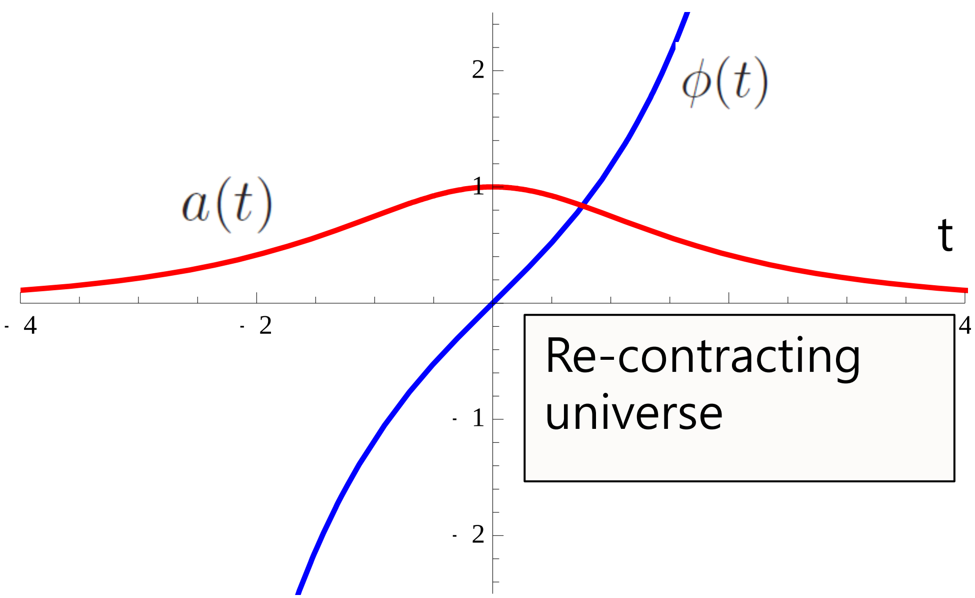
$$\frac{1}{2}\dot{\phi}^2 - V(\phi) - \bar{\lambda} = 0,$$

$$\dot{\phi} = \sqrt{2\left[V(\phi) + \bar{\lambda}\right]} = \sqrt{2\left(\frac{m^2}{2}\phi^2 + \bar{\lambda}\right)}$$

Maximal Pressure Solution

$$\phi(t) = \frac{\sqrt{2\bar{\lambda}}}{m}\sinh[m(t-t_0)],$$

$$a(t) = \frac{a_0}{(2\bar{\lambda})^{1/3}}\cosh^{-2/3}[m(t-t_0)],$$



At early stage (t <<)

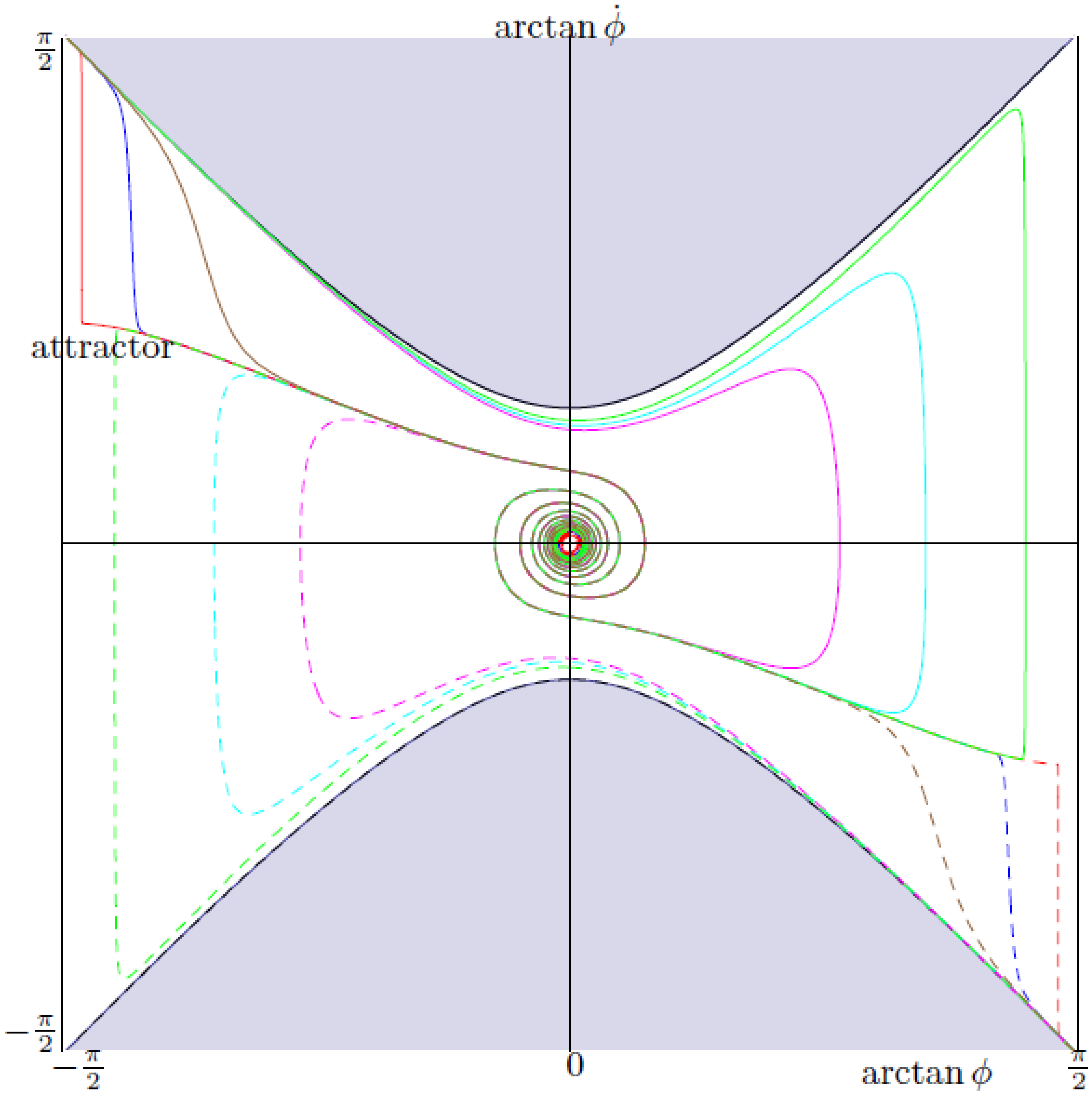
$$\boxed{\phi(t) \approx -\sqrt{\frac{\lambda\!\!\!^-}{2m^2}}\, e^{m(t-t_0)}, \qquad a(t) \approx a_0 \left(\frac{2}{\lambda\!\!\!^-}\right)^{\frac{1}{3}} e^{\frac{2}{3}m(t-t_0)}.}$$

$$\Downarrow$$

$$\boxed{H = H_{\mathrm{MPS}} \approx \frac{2}{3}m}$$

Finite Curvature !!!

:- even when the energy density is high
:- determined solely by the mass scale



$\arctan \dot{\phi}$
$\frac{\pi}{2}$
attractor
$-\frac{\pi}{2}$
$-\frac{\pi}{2}$
0
$\arctan \phi$
$\frac{\pi}{2}$

Stability of MPS: global perturbation

MPS's

$$\boxed{\dot{\phi} = U(\phi) \qquad H = -\frac{2}{3}U'(\phi).} \qquad U(\phi) \equiv \sqrt{2[V(\phi)+\lambda\!\!\!^{-}]}$$

Linear Perturbations

$$\boxed{\dot{\phi} = U(\phi)\left[1+\epsilon\psi(t)\right], \qquad H = -\frac{2}{3}U'(\phi)\left[1+\epsilon h(t)\right],}$$

$$\dot{\psi} - 2U'h = 0, \qquad h = \left(-\frac{2}{3} + \sqrt{\frac{2}{3\kappa}\frac{1}{U'}}\right)\psi.$$

Results

Exponentially increase

$$U\psi = \psi_0(2\lambda\!\!\!^{-})^{-1/6}\cosh^{-1/3}[m(t-t_0)]e^{t/t_c} \propto e^{\left(\frac{m}{3}+\sqrt{\frac{8}{3\kappa}}\right)t} \qquad (\text{as } t \to -\infty),$$

$$U'h = \psi_0(2\lambda\!\!\!^{-})^{-2/3}\left\{\sqrt{\frac{2}{3\kappa}} - \frac{2m}{3}\tanh[m(t-t_0)]\right\}\cosh^{-4/3}[m(t-t_0)]e^{t/t_c} \propto e^{\left(\frac{4m}{3}+\sqrt{\frac{8}{3\kappa}}\right)t} \qquad (\text{as } t \to -\infty)$$

EiBI Inflation

Both perturbations grow in time → MPS is unstable !!!!.

Universe starts from MPS

→ departs from MPS due to instability

→ $|\dot{\phi}|$ decreases

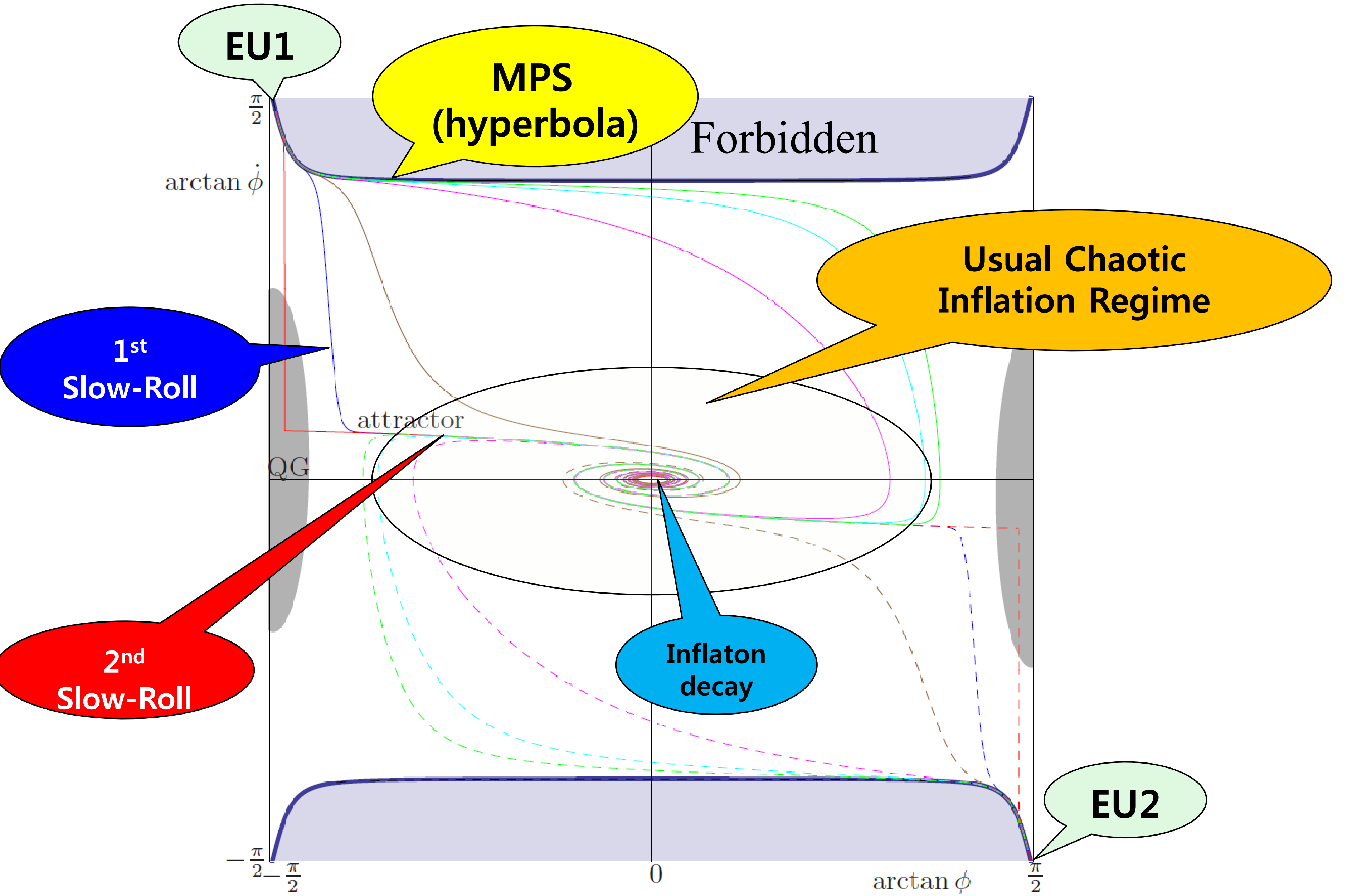
→ Universe enters 1st Slow-Roll Regime

→ $|\dot{\phi}|$ rapidly drops

→ Universe enters 2nd Slow-Roll Regime

→ Settles down to Inflationary Attractor

Numerical Evolution



Quantum Gravity

In GR, the pre-inflationary period is in Quantum Gravity Regime

:- due to high energy density

In EiBI gravity, Quantum Gravity Regime is considerably suppressed

:- kinetic energy is bounded by MPS

:- curvature around MPS ~ mass scale

On the Attractor, curvature is $H^2 \sim m^2 \phi_0^2$

:- can be small if $|\phi_0| < m^{-1}$

Therefore, MPS provides a natural NON-singular initial state for inflation

→ Non-Quantum Precursor of Inflation

Only for $V > M_P^2$ and $V \gg K$, curvature scale becomes large

:- can be avoided by varieties of evolution paths

Conclusions

A scalar field with potential $V(\phi) = \frac{m^2}{2}\phi^2$ in EiBI gravity provides
- Non-Singular
- Non-Quantum Gravitational
- so, Natural
- pre-Inflationary Stage

Followed by
- Ordinary Inflation

Scalar Perturbation Theory needs be investigated
- Some work done by Ke Yang et. al. (2013) for BF solution
- Need be investigated for EiBI Inflation

Tensor perturbation has been investigated for BF solution
by Escamilla-Rivera, Banados, Ferreira (2012)
- Need be investigated for EiBI Inflation

Discussions on Tensor Perturbation

Formulation of Tensor Perturbation in EiBI

$$\boxed{\begin{aligned} g_{\mu\nu}dx^\mu dx^\nu &= -a^2 d\eta^2 + a^2\left(\delta_{ij} + h_{ij}\right) dx^i dx^j , \\ q_{\mu\nu}dx^\mu dx^\nu &= -X^2 d\eta^2 + Y^2\left(\delta_{ij} + \gamma_{ij}\right) dx^i dx^j , \\ &= Y^2\left[-d\tau^2 + \left(\delta_{ij} + \gamma_{ij}\right) dx^i dx^j\right] , \end{aligned}}$$

$$\boxed{\partial_i h^{ij} = \partial_i \gamma^{ij} = 0 \text{ and}} \ \boxed{h = \gamma = 0.}$$

From EOM 1, one gets $h_{ij} = \gamma_{ij}$.

$$\boxed{h_{ij}(\eta, \vec{x}) = \sum_{\lambda=+,-} \int \frac{d^3k}{(2\pi)^{3/2}} \, h_\lambda(\eta, \vec{k}) \, \epsilon^\lambda_{ij}(\vec{k}) \, e^{i\vec{k}\cdot\vec{x}},}$$

From EOM 2,

$$\boxed{\frac{\kappa Y^2}{2X^2} h''_\lambda + \frac{\kappa Y^2}{2X^2}\left(3\frac{Y'}{Y} - \frac{X'}{X}\right) h'_\lambda + \frac{\kappa k^2}{2} h_\lambda = 0.}$$

$$h_\lambda'' \approx 0 \qquad \Rightarrow \qquad h_\lambda = A\eta + B.$$

Grows in Radiation-Dominated Universe [Escamilla-Rivera, Banados, Ferreira]

$$h_{ij} \propto A\delta\eta = A/a(t_e) \times \delta t$$

Tensor Perturbation of MPS grows in the same way
:- may cause a trouble

Time scale for growth of tensor perturbation : $\delta t_{\rm T} \sim \frac{1}{\epsilon} \sim \frac{1}{m}$

Time scale for growth of MPS perturbation : $\delta t_{\rm MPS} \sim \frac{1}{m/3 + \sqrt{8/3\kappa}}$

However, SAFE if $\delta t_{\rm T} > \delta t_{\rm MPS} \Rightarrow m \ll 1/\sqrt{\kappa}$

Universe leaves MPS before T.P. grows sufficiently

During Inflationary period, tensor perturbation is oscillatory