The CMB and theories of the primordial universe

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Oscillatory features in the curvature power spectrum after a sudden turn of the inflationary trajectory

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X. Gao, D. Langlois and S.M.

1205.5275 [hep-th], 1306.5680 [hep-th]

S.M. and R. Saito, in preparation

Single field Inflation

- Inflation has been very successful so far

  - Solving horizon problem, flatness problem

  - Diluting unwanted relics like magnetic monopole

  - Providing origin of the structure in the Universe

- Predictions by simple slow-roll inflation scenario

  - Curvature perturbations (adiabatic)
  - Nearly exponential expansion (scale invariance)

  - Almost negligible self-interactions (gaussian)

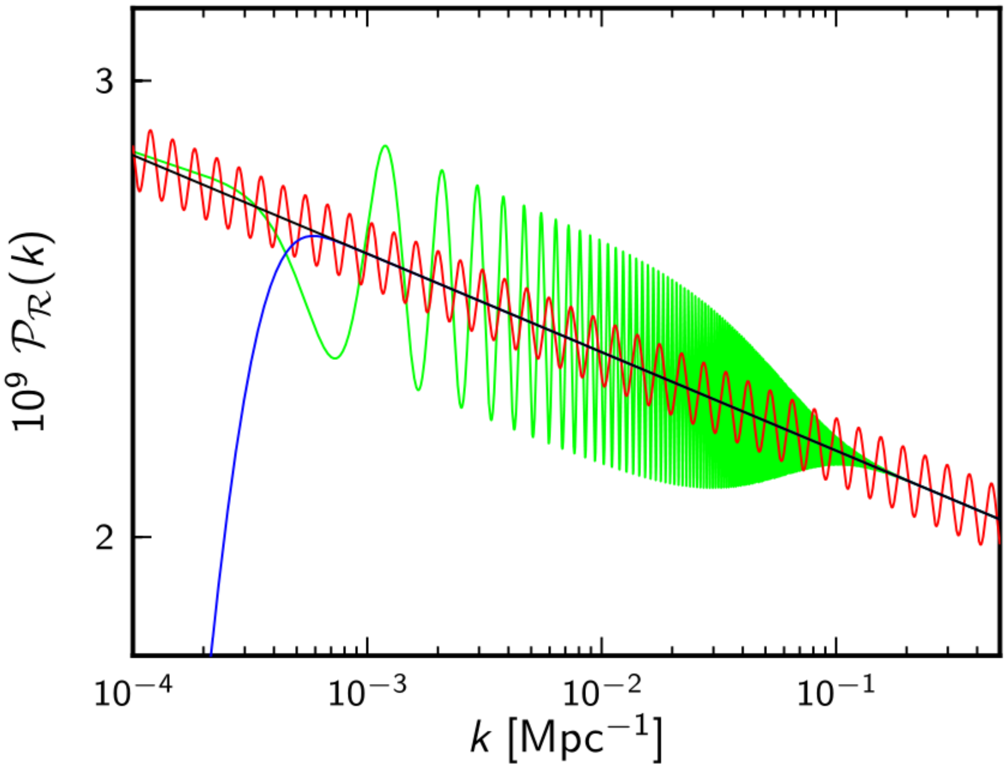
  - Perturbations are initially in Bunch-Davies vacuum state

These are consistent with current observations

Beyond the single field model

- Observational hint

  -Oscillatory features in the CMB power spectrum

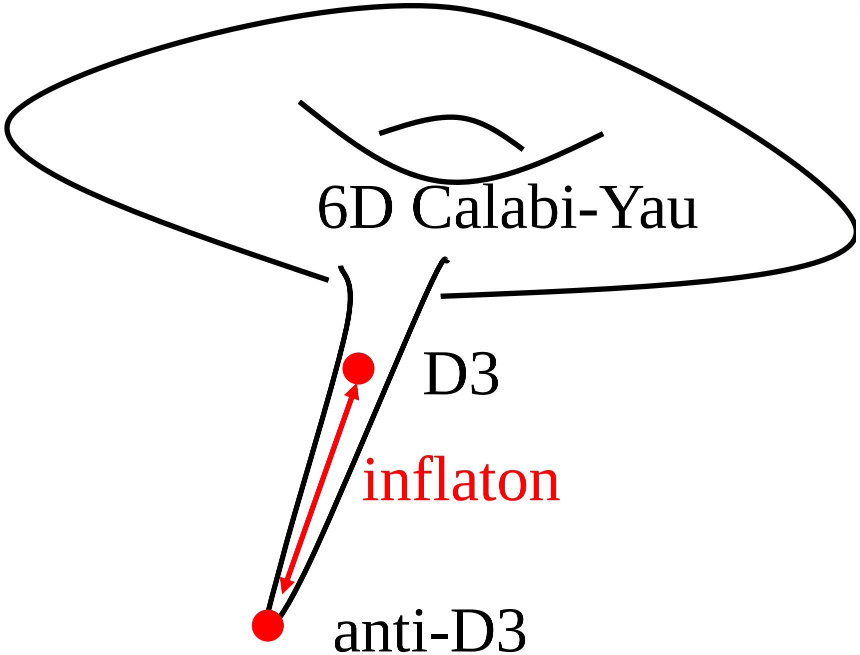


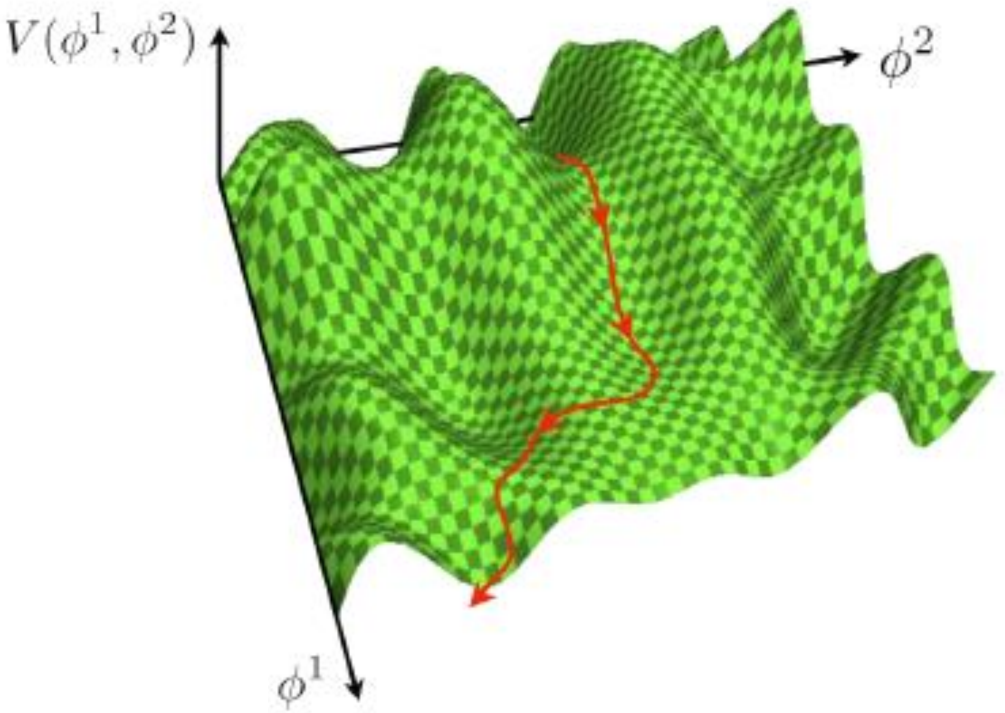
Planck `13

Introducing the oscillations periodic in t gives better fit!

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_0(k)\ \left\{1 + \alpha_{\mathrm{w}}\ \sin\left[\omega \ln\left(\frac{k}{k_*}\right) + \varphi\right]\right\}$$

Inflation in string theory





Since there are many directions in the internal space,
It is unlikely that only one degree of freedom is dynamical !

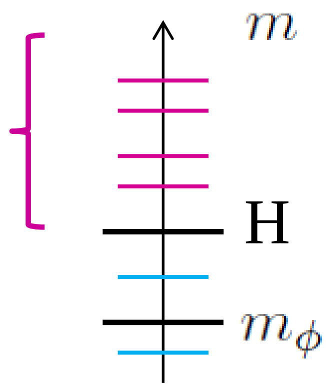
Light field and heavy field

- Conventional wisdom

If m > H, we can integrate them out

only light fields (m < H) play roles as inflatons

Integrate out



- Recent progress

We need to be careful about integrating out heavy modes, especially when inflaton trajectory is curved

Chen & Wang `09, Tolley & Wyman `09, Achucarro, Gong, Hardeman, Palma, Patil `10, Shiu & Xu `11, Pi & Sasaki `12, Saito, Nakashima, Takamizu, Yokoyama `12, …..

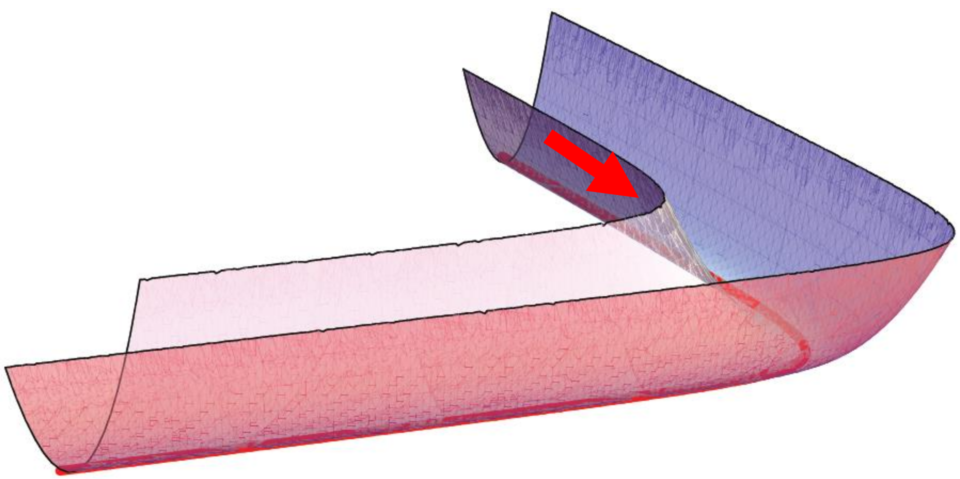
Model and background evolution

Modeling a turning trajectory

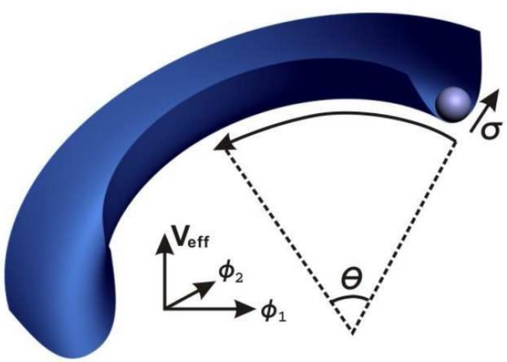
We will concentrate on a single turning process, by requiring (the minimal deviation from the standard scenario):

1) The turning process occurs in a finite time interval

2) The potential trough is asymptotically straight before and after the turn



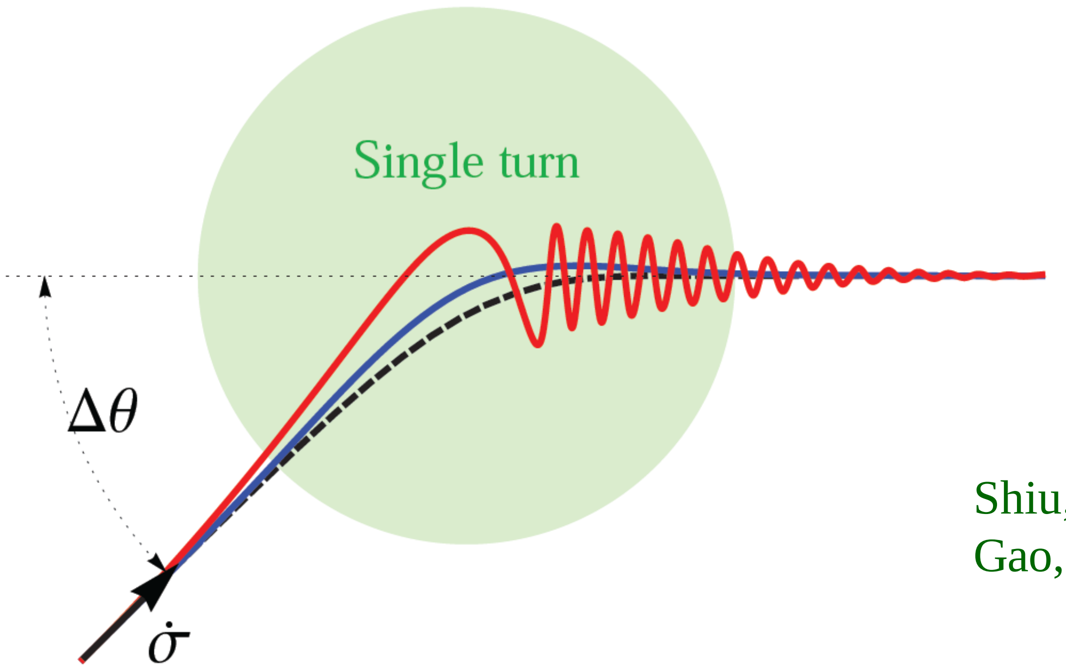
Different from ``constant turn”



Chen, Wang `09, `12

Gong, Pi, Sasaki `13

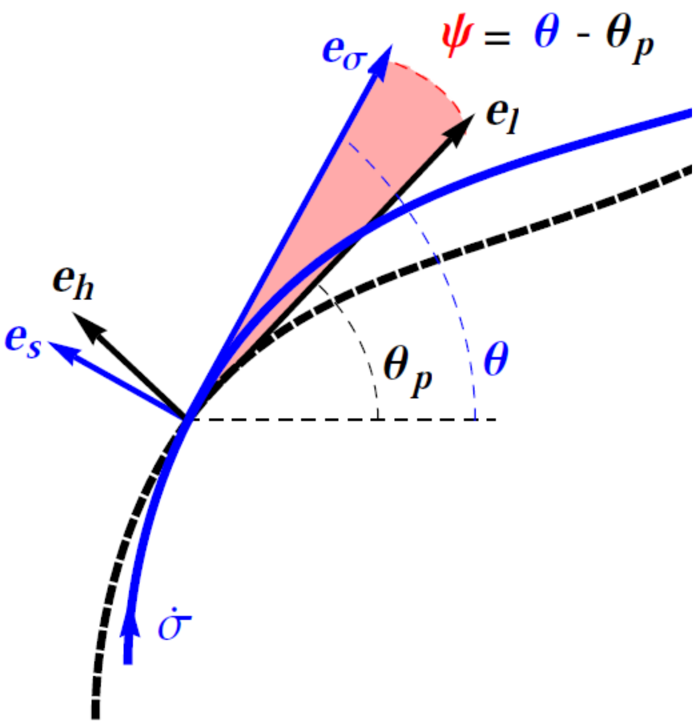
Schematic picture of the single turn



Shiu, Hu `11, Chen, `11, `12
Gao, Langlois, SM, `12, `13

- During the turn, the trajectory deviates from the bottom of the valley
- For soft turn, it smoothly relaxes to the minimum of the potential
- For sharp turn, it relaxes via oscillations around the minimum

Turning trajectory: a two-field example



The background trajectory is specified by

- Velocity

$$\ddot\sigma + 3H\dot\sigma + V_{,\sigma} = 0$$

- Direction

$$\boxed{\ddot\psi + 3H\dot\psi + m_h^2\psi \simeq -\ddot\theta_p - 3H\dot\theta_p}$$

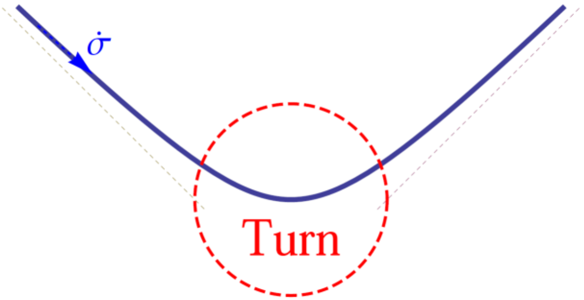
$$V_{,mn} = \mathrm{diag}\{m_l^2, m_h^2\}\,, \quad m_h \gg H \gg m_l$$

- In general, the direction of the trajectory ψ tends to change sourced by the turning light direction θ_p

- ψ behaves as a damped oscillator with frequency given by m_h

A Gaussian toy model

- Gaussian ansatz



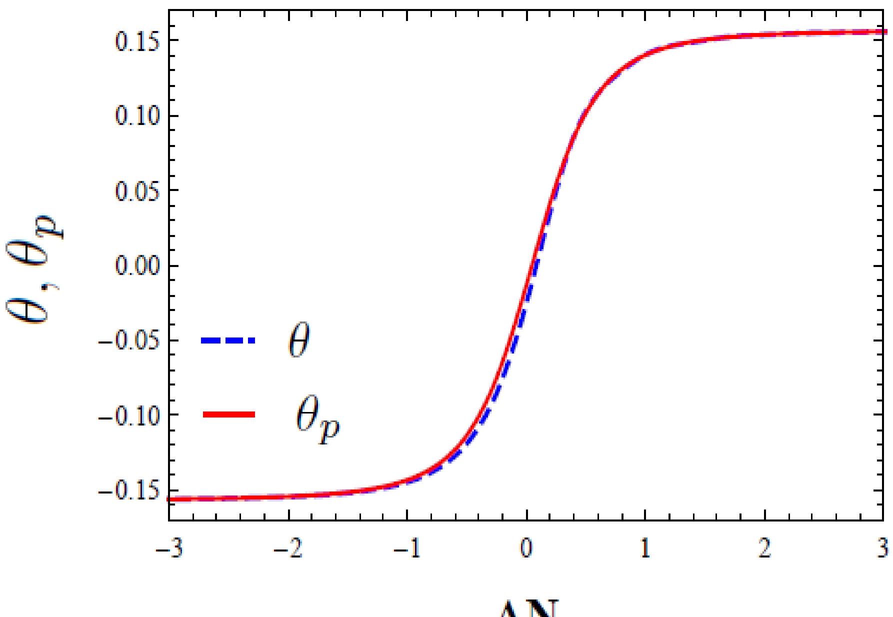
$$\dot{\theta}_p(t) = \Delta\theta \frac{\mu}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2 t^2}$$

"Energy scale" of the turn: ***μ* = 1/Δt >> *H***

The qualitative behaviors of the background trajectory depends on the ratio: $\boxed{\mu/m_h}$

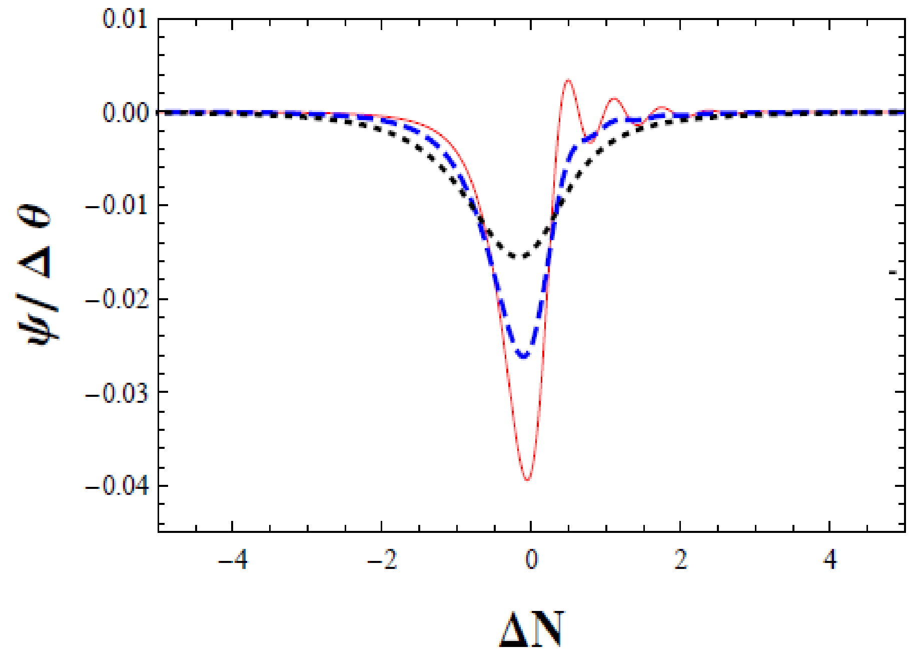
Background dynamics (soft turn $\mu/m_h < 1$)

μ/mh = 0.1 mh/H = 10



(0 corresponds to turning time)

μ/mh = 0.1, 0.15, 0.2 mh/H = 10



(0 corresponds to turning time)

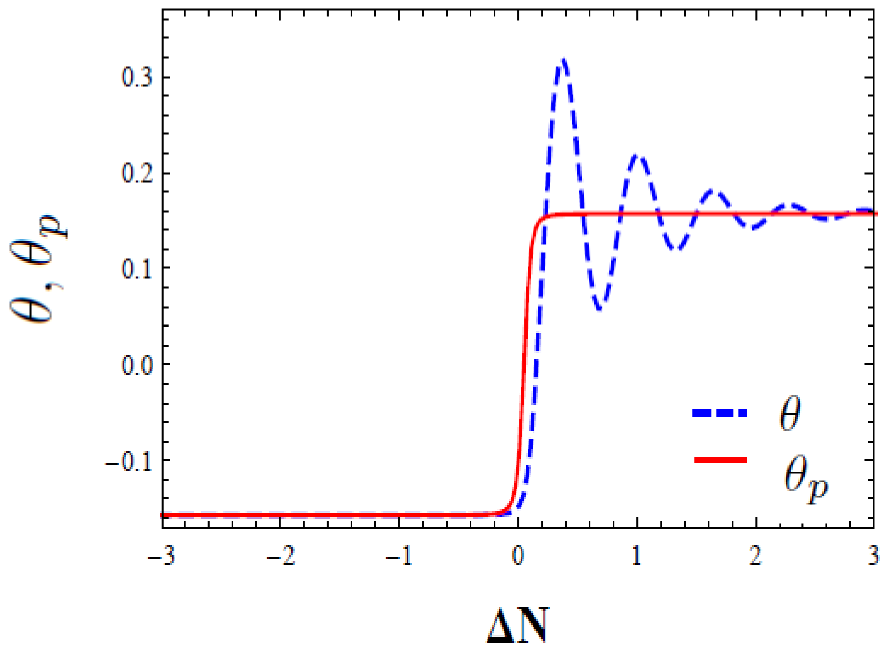
$$\boxed{\psi(t) \approx \frac{\Delta\theta}{\sqrt{2\pi}} \left(\frac{\mu}{m_h}\right)^2 e^{-\frac{1}{2}\mu^2 t^2} \left(\mu t - 3\frac{H}{\mu}\right)}$$

No oscillating response

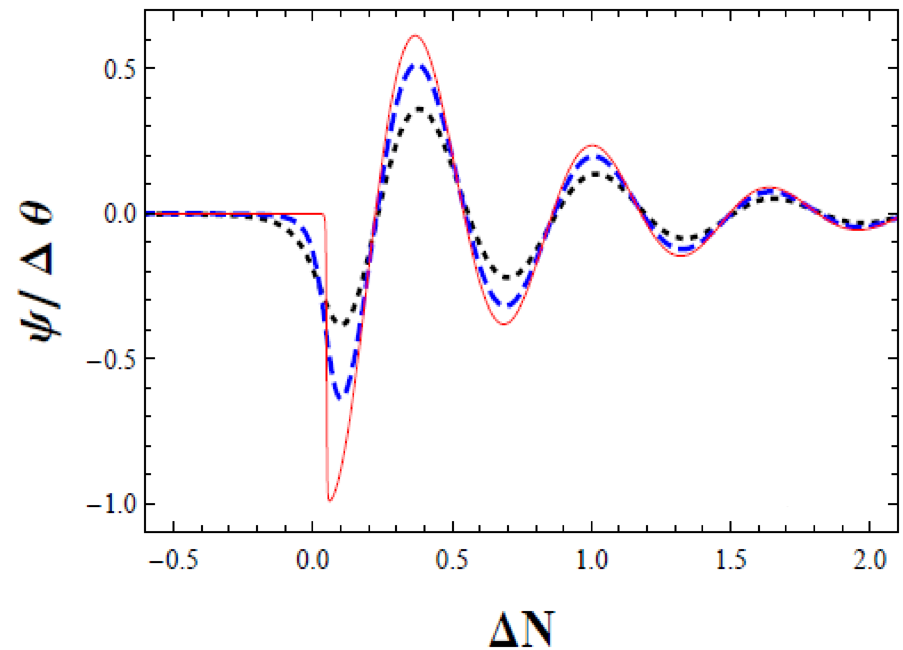
Adiabatic/entropy basis and light/heavy basis are almost same

Background dynamics (sharp turn $\mu/m_h > 1$)

μ/mh = 2 mh/H = 10



μ/mh = 1, 2, 5 mh/H = 10



$$\boxed{\psi(t) \approx -\frac{\Delta\theta}{2} e^{-\frac{m_h^2}{2\mu^2}} \operatorname{erfc}\left(-\frac{\mu t}{\sqrt{2}}\right) e^{-\frac{3}{2}Ht} \cos\left(m_h t - \mathrm{phase}\right)}$$

Soon after the turn, the trajectory starts damped oscillation around the light direction.

Oscillatory background during a sharp turn

When the turn is sharp, the oscillating trajectory will induce oscillatory parts in background quantities

- Deviation from the smooth value:

$$a = \bar{a} + \Delta a, \quad H = \bar{H} + \Delta H, \quad \epsilon = \bar{\epsilon} + \Delta\epsilon$$

- An equation of motion for $\Delta\epsilon$

$$\frac{d^2\Delta\epsilon}{dt^2} + 3\bar{H}\frac{d\Delta\epsilon}{dt} - 12\bar{\epsilon}\bar{H}^2\Delta\epsilon \;=\; 2\bar{\epsilon}\left[\left(\dot{\theta}_p + \dot{\psi}\right)^2 - \hat{m}_h^2\sin^2\psi\right]$$

Infinitely sharp turn limit (μ/mh is infinity)

$$\dot{\theta}_p = \Delta\theta\frac{\mu}{\sqrt{2\pi}}e^{-\frac{1}{2}\mu^2t^2} \longrightarrow \dot{\theta}_p = \Delta\theta\delta\left(t\right)$$

$$\psi\left(t\right) \approx -\Theta\left(t\right)\Delta\theta e^{-\frac{3}{2}\bar{H}t}\cos\left(\hat{m}_h t\right)$$

$$\boxed{\Delta\epsilon \approx \frac{\Theta\left(t\right)}{2}\bar{\epsilon}(\Delta\theta)^2e^{-3\bar{H}t}\cos\left(2\hat{m}_h t\right) + \text{non-osci}}$$

Perturbations

Adiabatic / entropic v.s. light / heavy basis

There are two convenient decompositions :

Adiabatic / entropic decomposition $\longrightarrow$ Kinematic features

Gordon, Wands, Bassett, Maartens `00
Groot Nibbelink, van Tent `01

Light / heavy decomposition $\longrightarrow$ Potential features

Gao, Langlois, SM `12, `13

- Adiabatic/entropic decomposition has special advantage, since the adiabatic mode is directly related to the curvature perturbation

- Light/heavy modes are directly related with the shape of the inflationary potential, which is (sometimes) more robust and simpler

- The final spectra for the curvature perturbation

$u_\sigma = \cos\psi u_l + \sin\psi u_h$ after the turn $\psi \to 0, \quad u_\sigma \simeq u_l$

Deviations from the single-field slow-roll

▪ Formal structure of the system:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}(\theta_m, a) \\ &= \mathcal{L}(\theta_m, \bar{a} + \Delta a) \\ &= \mathcal{L}_0(0, \bar{a}) + \mathcal{L}_{\mathrm{I}}^{(\mathrm{turn})}(\theta_m, \bar{a}) + \mathcal{L}_{\mathrm{I}}^{(\mathrm{resonance})}(0, \Delta a)\end{aligned}$$

▪ ``Free” part (SFSL limit):

$$\mathcal{L}_0^{\mathrm{l,h}} = \frac{1}{2}\left[u_{l,h}'^2 - (\partial u_{l,h})^2 - \left(\bar{a}^2 m_{l,h}^2 - \bar{a}^2 \bar{H}^2 (2-\bar{\epsilon})\right) u_{l,h}^2\right]$$

$\bar{H}$ **and** $\bar{\epsilon}$ **are evaluated by** $\bar{a}$ $\qquad u_m = a\delta\phi_m \qquad m = l, h$

▪ ``Interaction” part:

Effects 1: bending light direction (potential trough)

$$\mathcal{L}_{\mathrm{I}}^{(\mathrm{turn})} = \frac{1}{2}\theta_m'^2 u_l^2 + \frac{1}{2}\theta_m'^2 u_h^2 + 2\theta_m' u_l u_h' + \theta_m'' u_l u_h$$

Effects 2: oscillating background (resonance)

$$\mathcal{L}_{\mathrm{I}}^{(\mathrm{resonance})} = -\frac{1}{2}\left[(\Delta a)^2 m_l^2 - \Delta\left(a^2 H^2 (2-\epsilon)\right)\right] u_l^2$$

Effects (I):
Contributions from the turning light direction

Evolution equations for perturbations

$$u_m = a\delta\phi_m \qquad m = l, h \qquad\qquad {}' \equiv \frac{d}{d\eta}$$

$$\left[\begin{array}{l} u_l'' + \left(k^2 + \bar{a}^2 m_l^2 - {\theta_m}'^2 - \dfrac{\bar{a}''}{a}\right) u_l = {\theta_m}'' u_h + 2{\theta_m}' u_h', \\ u_h'' + \left(k^2 + \bar{a}^2 m_h^2 - {\theta_m}'^2 - \dfrac{\bar{a}''}{a}\right) u_h = -{\theta_m}'' u_l - 2{\theta_m}' u_l'. \end{array}\right.$$

$\bar{a}$: smooth part of the scale factor

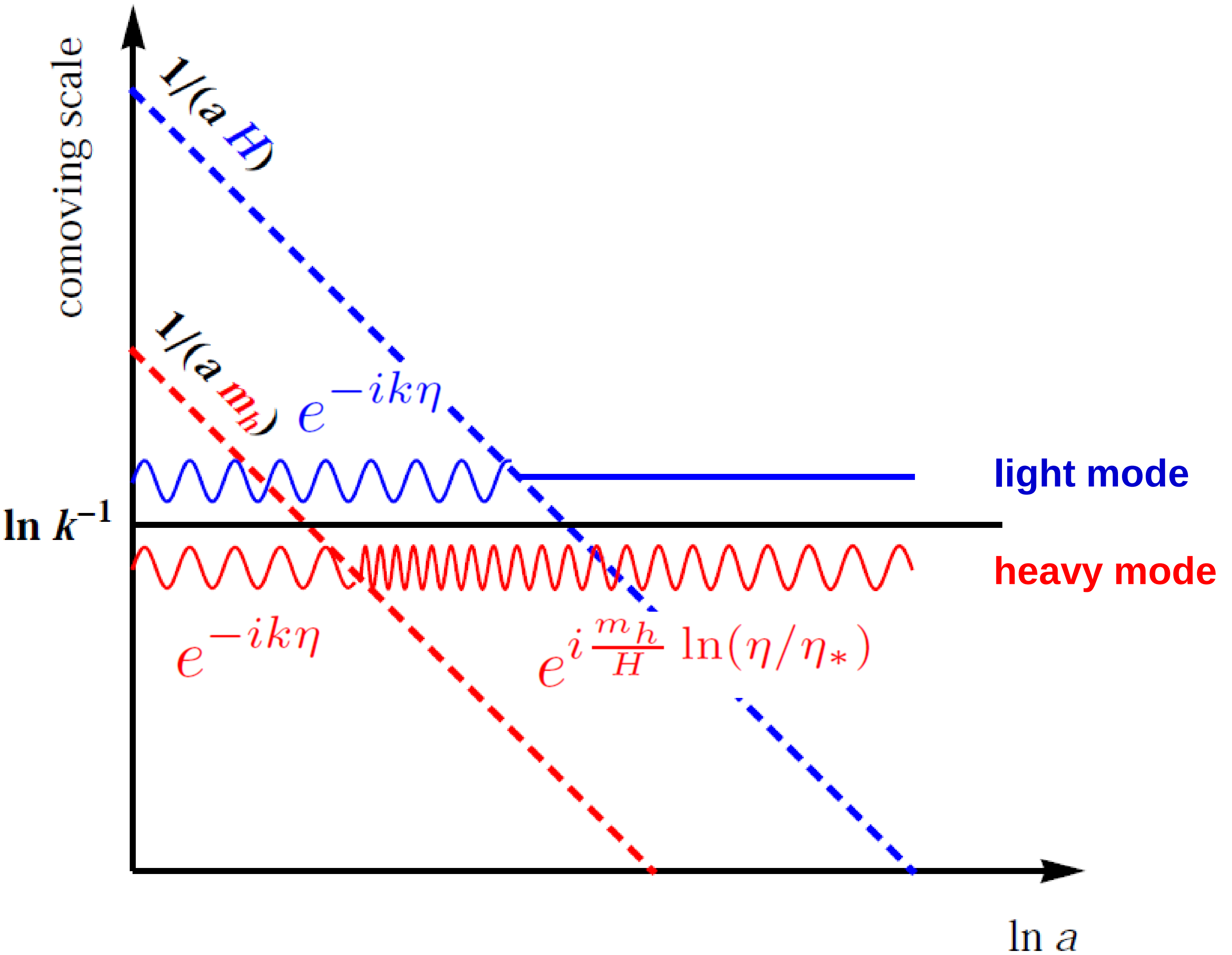
θ_m' : the turning rate of the light direction approximated by the turning rate of the potential trough

$$\dot{\theta}_m(t) \approx \dot{\theta}_p(t) = \Delta\theta \frac{\mu}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2 (t-t_*)^2}$$

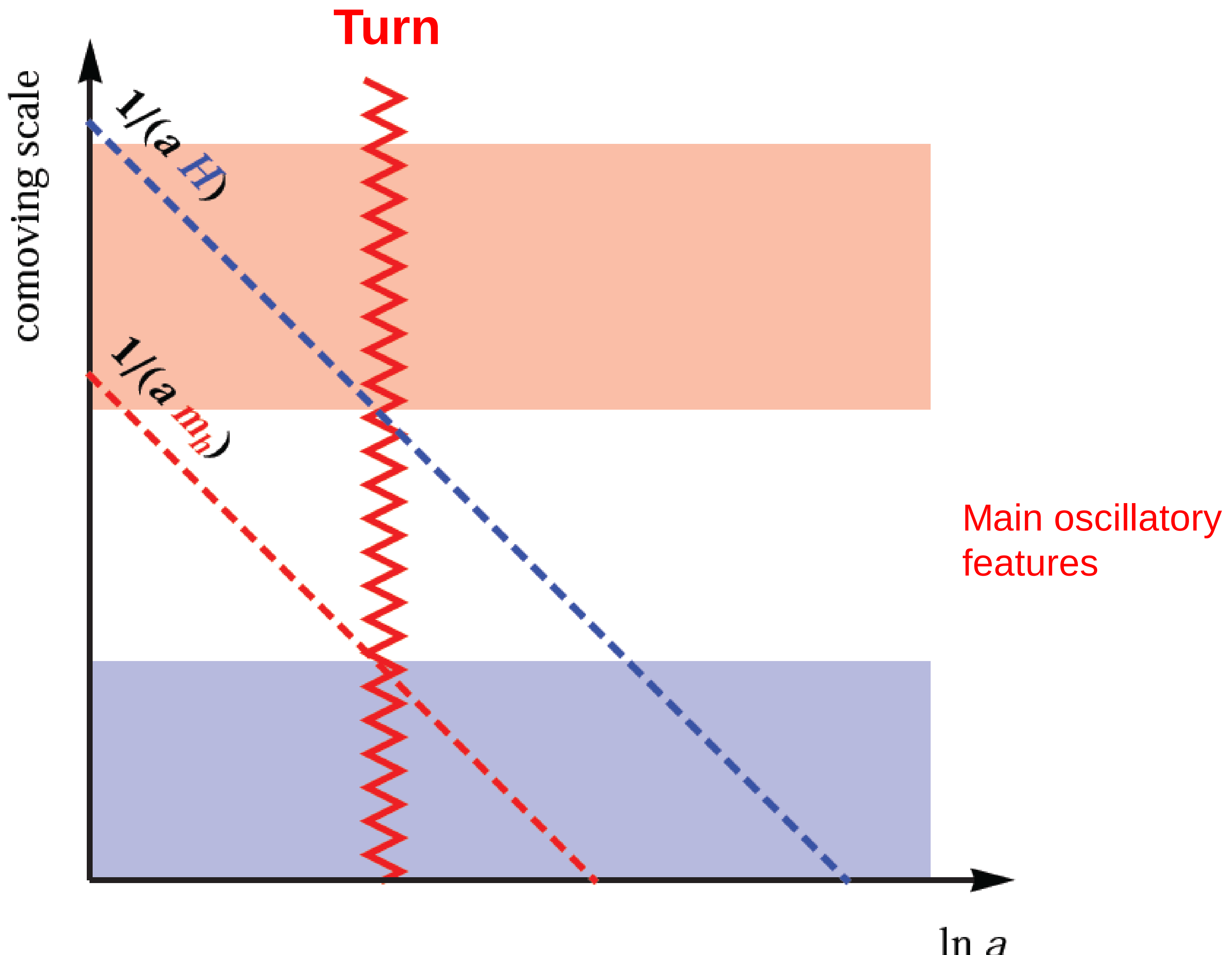
⇓

We will solve both the full two-field system and effective single light-field theory

Characteristic scales

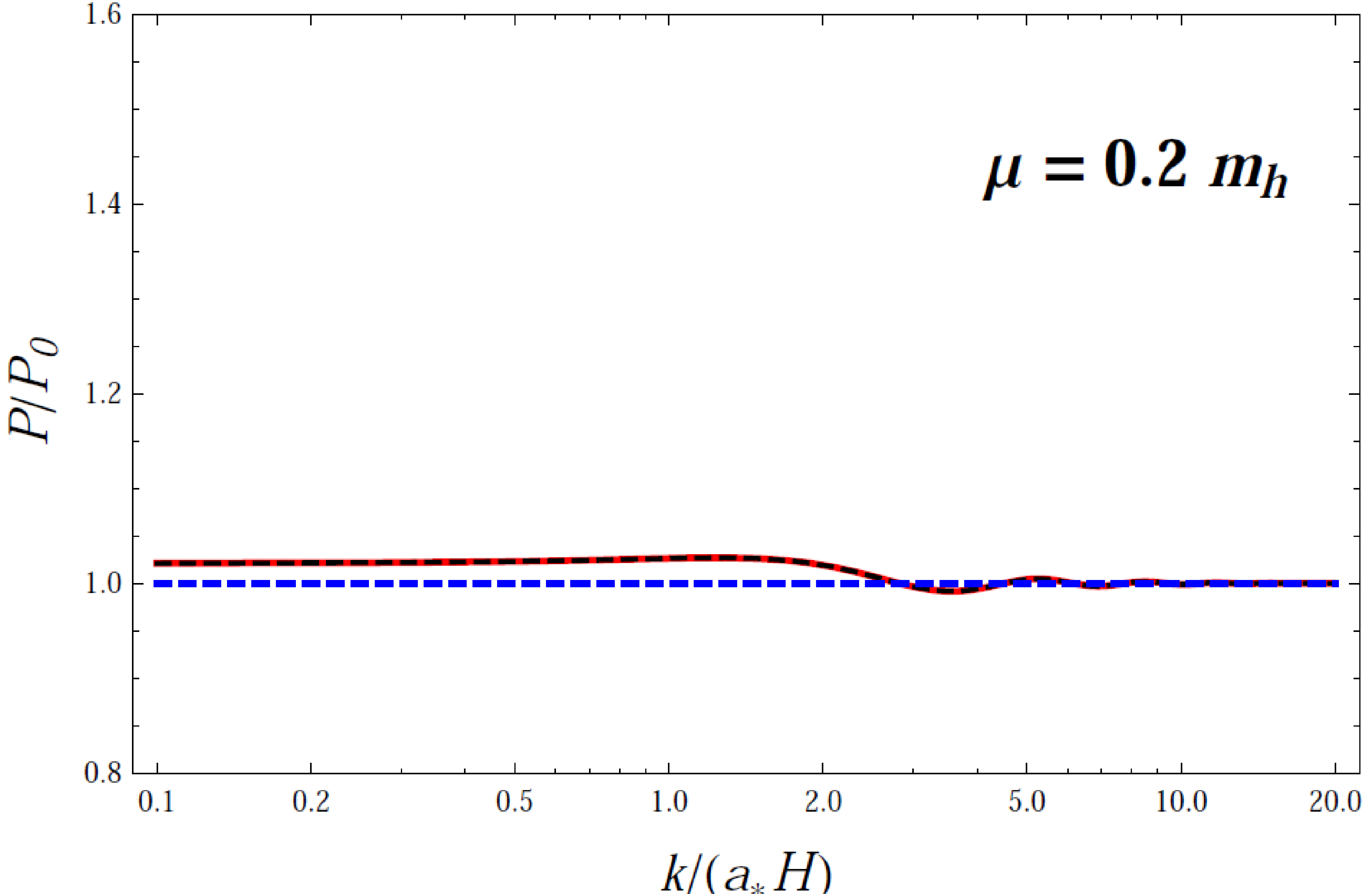


Characteristic scales



Numerics: fixed m_h

$m_h = 60H\,,\ \ \Delta\theta = \pi/30$



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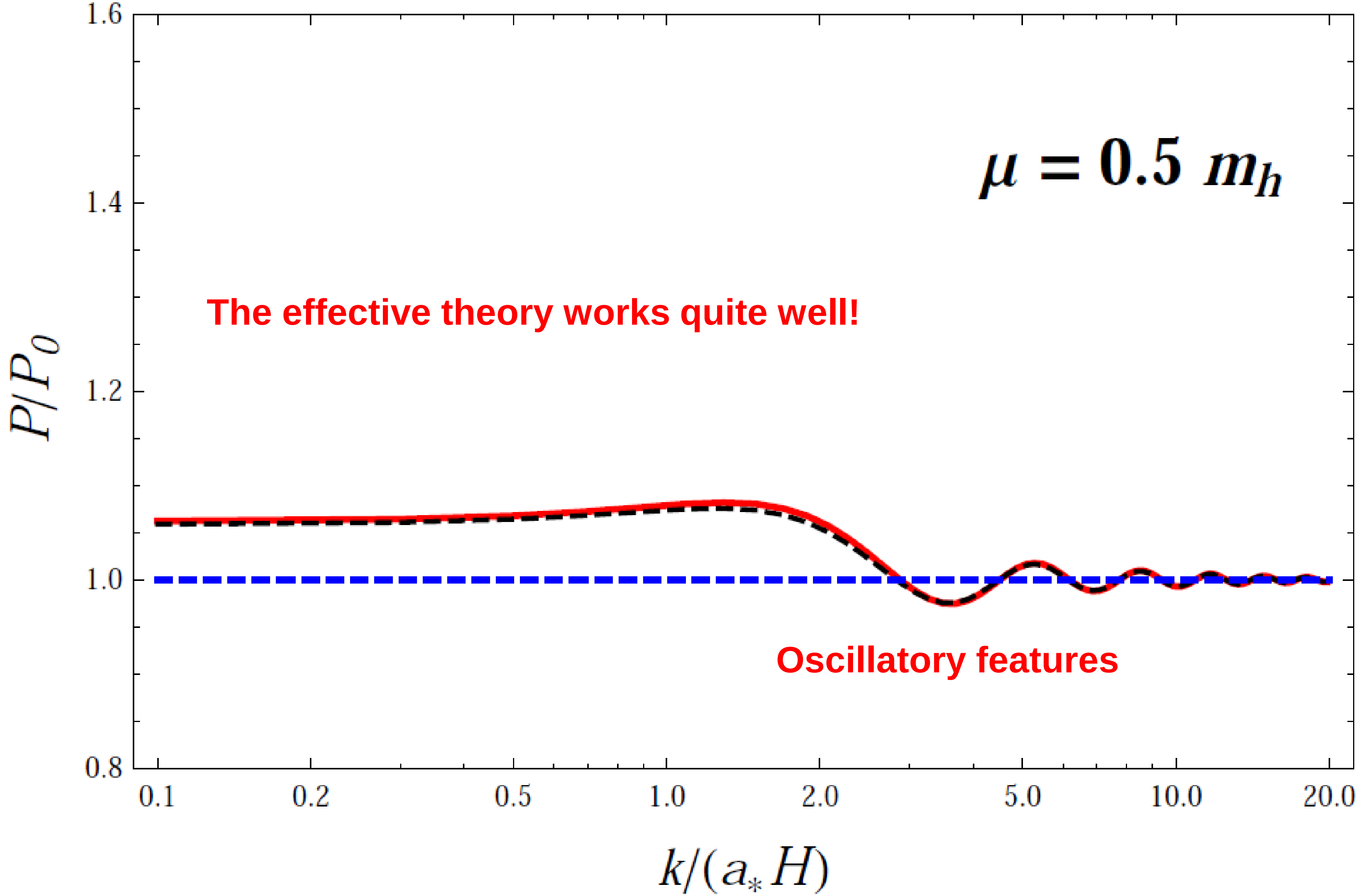

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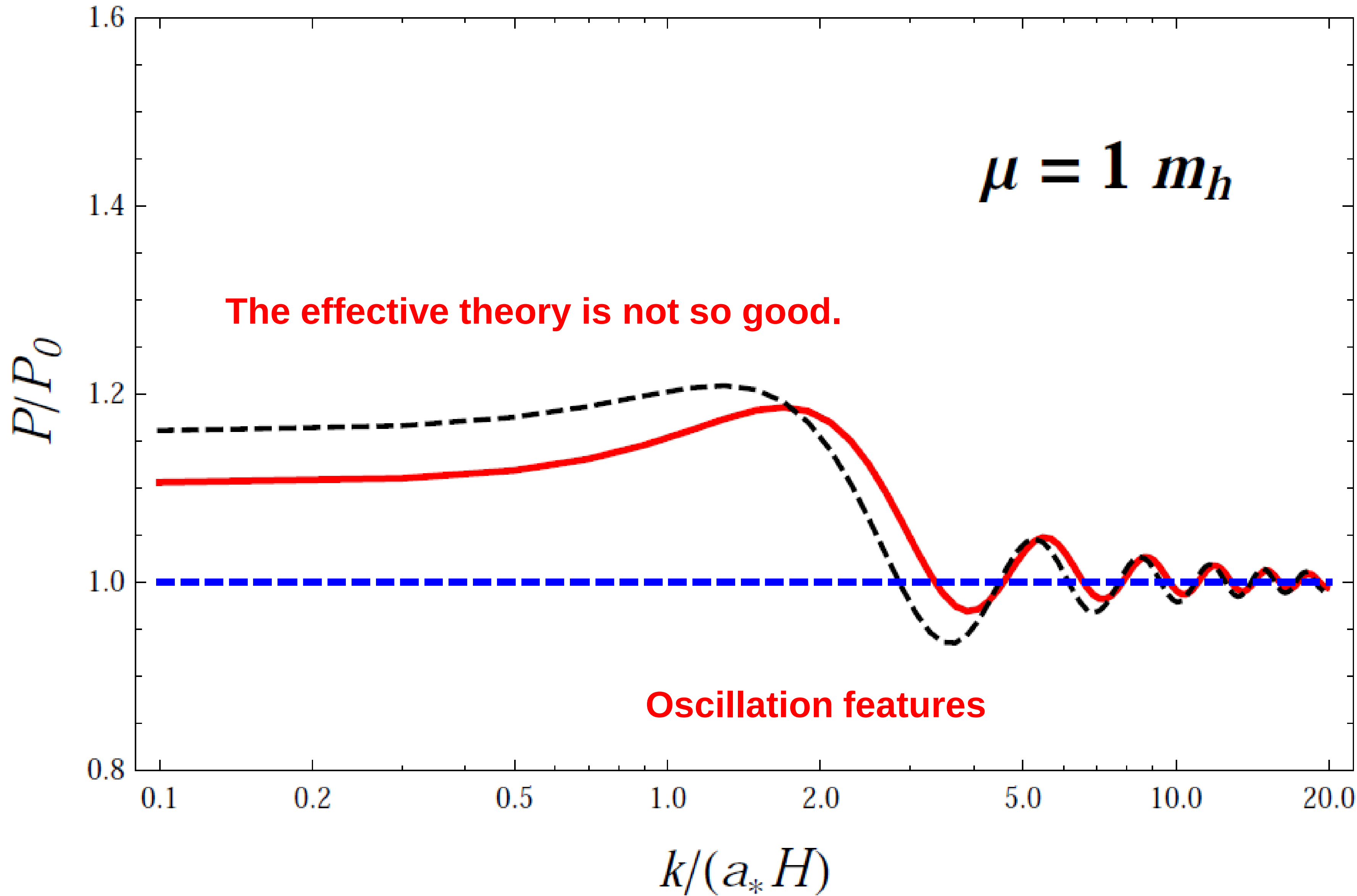

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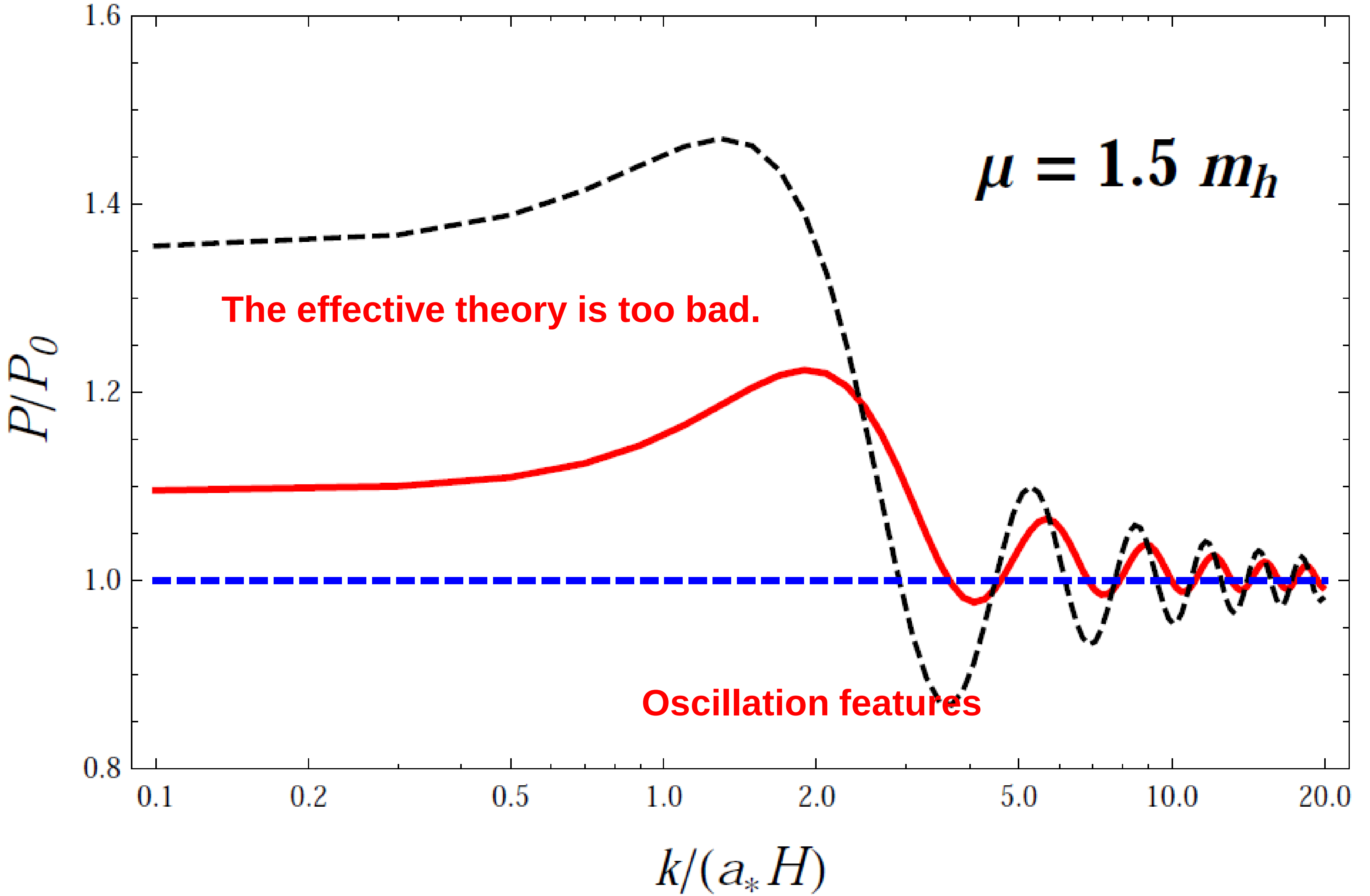


Effects (II):
Resonance from the oscillatory background

Resonance

- Lagrangian:

$$\mathcal{L}_{\rm I}^{(\rm resonance)} = -\frac{1}{2}\left[(\Delta a)^2 m_l^2 - \Delta\left(a^2 H^2 (2-\epsilon)\right)\right] u_l^2$$
$$\simeq \frac{1}{2}\Delta\left(a^2 H^2 (2-\epsilon)\right) u_l^2$$
$$\simeq -\frac{1}{2}\bar{a}^2 \bar{H}^2 (\Delta\epsilon)_{\rm osci} u_l^2$$

with

$$\Delta\epsilon \approx \frac{\Theta(t)}{2}\bar{\epsilon}(\Delta\theta)^2 e^{-3\bar{H}t}\cos\left(2\hat{m}_h t\right) + \text{non-osci}$$

(in the infinitely sharp turn limit)

Cf. Chen, `11, `12

An oscillation periodic in cosmic time t will induce resonance effect which is periodic in (ln k) in the power spectrum

neglecting isocurvature perturbation

Resonance result

- Resultant spectrum

$$\left(\frac{\Delta P}{P}\right)_{\rm res} \approx \Theta\left(\frac{k}{a_* m_h}-1\right)\frac{\sqrt{\pi}}{4}\bar{\epsilon}\left(\Delta\theta\right)^2\left(\frac{\bar{H}}{m_h}\right)^{\frac{3}{2}} \times\left(\frac{a_* m_h}{k}\right)^3\cos\left[2\frac{m_h}{\bar{H}}\ln\left(\frac{k}{a_* m_h}\right)+2\frac{m_h}{\bar{H}}-\frac{\pi}{4}\right].$$

- The oscillation is periodic in ln k
- These features appear only on very small scales $k > a_* m_h \gg a_* \bar{H}$
- The amplitude is very small $\bar{\epsilon}\left(\Delta\theta\right)^2\left(\frac{\bar{H}}{m_h}\right)^{\frac{3}{2}} \ll 1$

Smaller by factor $(H/m_h)^2$ than Chen obtained ! !

→ The resonance feature is subdominant with respect to the one caused by the turning trajectory

Summary so far

- We have studied the influence of heavy modes in inflation
- Potential basis consists of eigenvectors of Hessian matrix
- Sudden turn and mass hierarchy for inflationary trajectory

  - Sharpness of the turn is governed by the ratio μ/m_h

  - For a soft turn, effective theory is valid

  - For a sharp turn, effective theory breaks down

    Feature from the turn

$$\boxed{\frac{\Delta\mathcal{P}}{\mathcal{P}_0} \approx (\Delta\theta)^2 \, \frac{\nu}{x_*^3} e^{-\nu^2/\tilde{\mu}^2} \left(x_* \cos x_* - \sin x_*\right)^2 \qquad x_* \equiv \frac{k}{a_* H}, \; \nu = \frac{m_h}{H}, \; \tilde{\mu} = \frac{\mu}{H}}$$

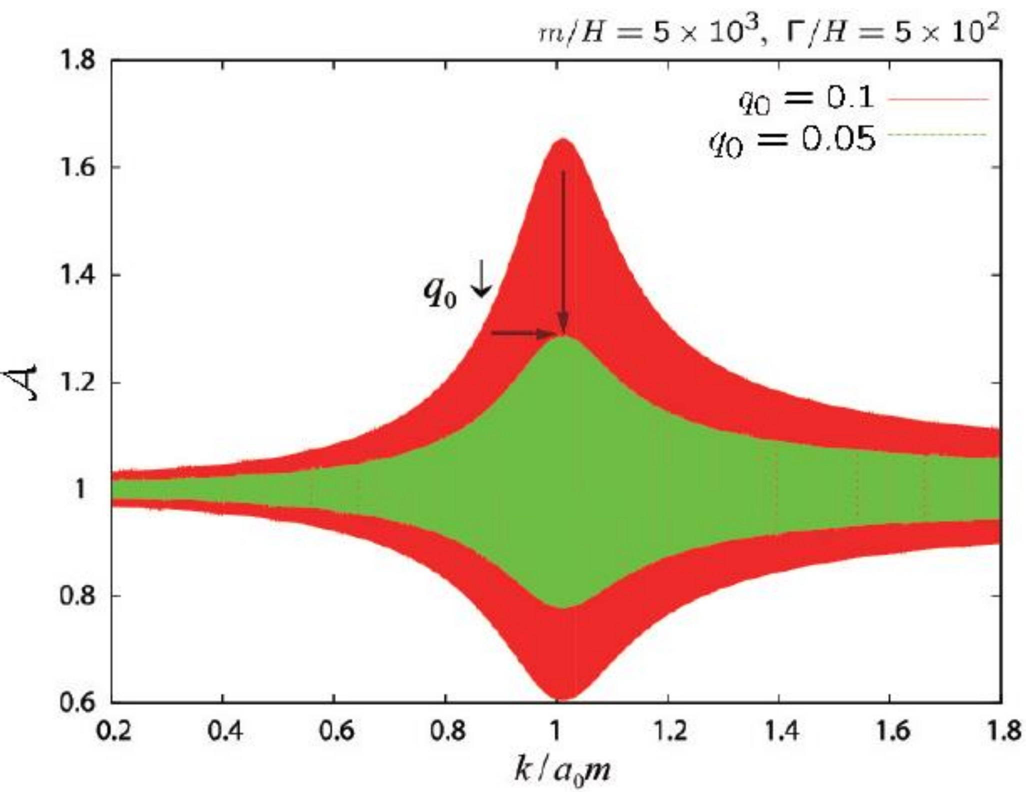
    Feature from the background oscillation is negligible

Cf. Noumi, Yamaguchi `13

Discussions

- Efficient resonance is obtained without spoiling the slow-roll when a heavy field couples with the inflaton derivatively

Saito, Nakashima, Takamizu, Yokoyama `12, Saito, Takamizu `13



$$K_d \equiv \frac{\lambda_{d1}}{4\Lambda_d^4}\underline{(\partial\chi)^2}\,\underline{(\partial\phi)^2} + \frac{\lambda_{d2}}{4\Lambda_d^4}(\partial\chi\cdot\partial\phi)^2$$

Heavy field   Inflaton

$$q_0 : (\lambda_{d1} + 6\lambda_{d2})\frac{m^2\hat{\chi}_0^2}{8\Lambda_d^4}$$

From 1206.2164 [astro.ph.CO]

Sudden turn in multi-field DBI inflation

- Multi-field DBI inflation

Langlois, Renaux-Petel, Steer, Tanaka `08, Arroja, SM, Koyama `08

-Action

$$\boxed{P(X^{IJ},\phi^K) = -\frac{1}{f(\phi^I)}\left(\sqrt{\mathcal{D}}-1\right) - V(\phi^I)}$$

with $\mathcal{D} = \det(\delta^\mu_\nu + f\,\underline{G_{IJ}\partial^\mu\phi^I\partial_\nu\phi^J})$ $\quad X^{IJ} \equiv -\frac{1}{2}\partial_\mu\phi^I\partial^\mu\phi^J$

Derivative coupling

-Background equation

$$\mathcal{D}_t\dot{\phi}^I + (3-s)H\dot{\phi}^I + \underline{c_s}G^{IJ}\left(V_{,J} - \frac{(1-c_s)^2}{2c_s}\frac{f_{,J}}{f^2}\right) = 0 \qquad s \equiv \frac{\dot{c}_s}{Hc_s}$$

-Mass matrix for the perturbations

$$\tilde{\mathcal{M}}_{IJ} = \underline{\mathcal{D}_I\mathcal{D}_J V} - \frac{(1-c_s)^2}{2c_s}\frac{\mathcal{D}_I\mathcal{D}_J f}{f^2} - \frac{(1-c_s)^3(1+3c_s)}{4c_s^3}\frac{\mathcal{D}_I f\,\mathcal{D}_J f}{f^3} + 2\dot{H}\mathcal{R}_{IKLJ}e^K e^L + \frac{(1-c_s^2)^2}{2c_s^4 f^2 H}f_{,(I}\dot{\phi}_{J)}$$
$$+ \frac{\dot{H}}{2H^2c_s^4}\left(1-c_s^2\right)\dot{\phi}_I\dot{\phi}_J - \frac{1}{a^3}\mathcal{D}_t\left[\frac{a^3}{2Hc_s^4}\left(1+c_s^2\right)\dot{\phi}_I\dot{\phi}_J\right]$$

Preliminary result

- Effective heavy mass is

→ Condition for the sharp turn is $\mu > \sqrt{c_s} m_h$

- q-parameter for the resonance is roughly $\sim (1 - c_s^2)\Delta\theta^2$

→ Resonant feature is observable for $c_s \ll 1$

Assuming the background with constant c_s and f

- Need to check

- Background solution in concrete models
- Validity of the potential basis
- Applicability of the effective single light field theory
- Structure of the resonance for the cases $c_s \ll 1$