Measuring lensing effect on temperature and polarizations

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One-day Workshop August 30, 2013

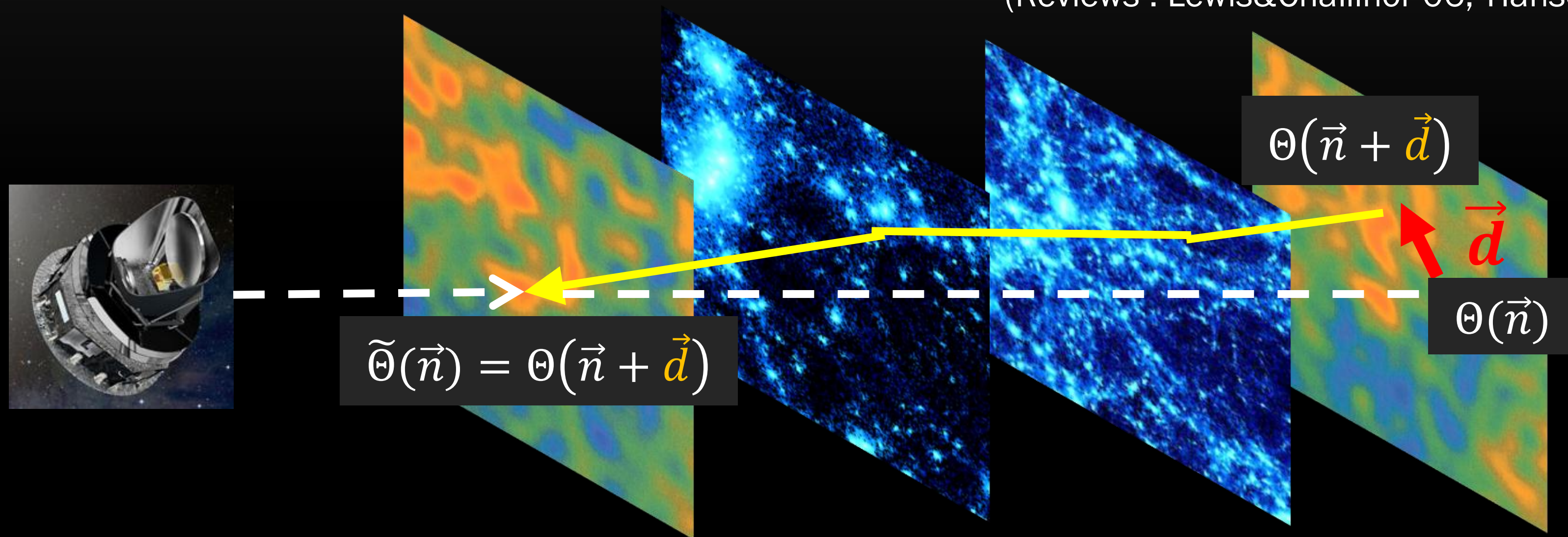
CONTENTS

1. BRIEF INTRODUCTION

CMB LENSING

- CMB Lensing = distortion of spatial pattern of CMB anisotropies

(Reviews : Lewis&Challinor'06, Hanson+'10)



Observer $(\chi = 0)$ — Comoving distance → Last scattering surface $(\chi = \chi_s)$

- Deflection angle

$$\vec{d}(\vec{n}) = \nabla \boldsymbol{\phi}(\vec{\boldsymbol{n}})$$

Lensing potential

$$\phi(\vec{n}) = -2\int_0^{\chi_s} d\chi \frac{\chi_s - \chi}{\chi\chi_s} \Psi(\eta_0 - \chi, \chi\vec{n})$$

Gravitation potential from LSS

Estimate lensing potential from lensed CMB maps, and extract cosmological information

Cosmological Application 1: Dark energy/ Massive Neutrinos

Dark energy, massive neutrinos

(see, e.g., Hu'01, Lesgourgues&Pastor'06)

Density perturbations

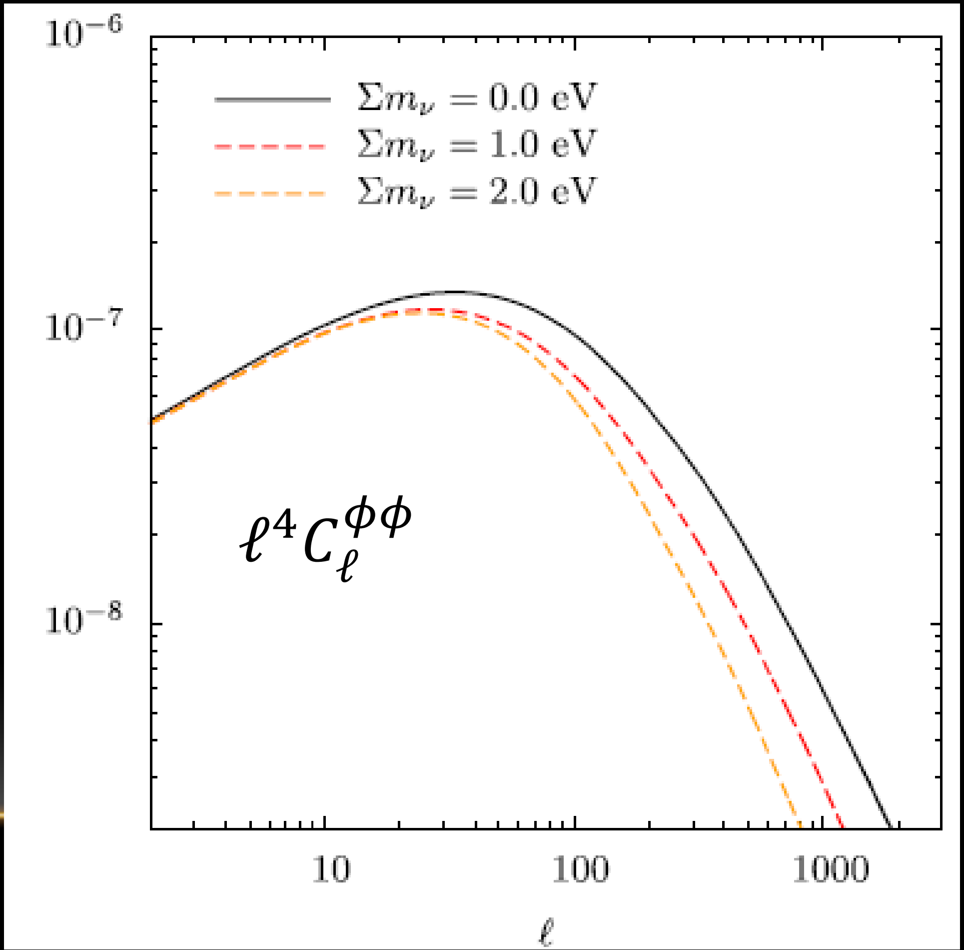
Gravitational potential

$\phi(\vec{n})$

gradient mode

$$\phi_{\ell m} = \int d^2\vec{n}\, Y_{\ell,m}(\vec{n})\phi(\vec{n})$$

$$C_\ell^{\phi\phi} = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} |\phi_{\ell m}|^2$$



COSMOLOGICAL APPLICATION 2: CURL MODE

- Even/Odd parity decomposition

(e.g., Cooray+'05; TN+'12; Book+'12; Yamauchi+'12; Yamauchi+'13 ; TN+'13)

✓ Deflection angle $$d_a = \underbrace{\partial_a \phi}_{\text{gradient}} + \epsilon_a{}^b \underbrace{\partial_b \omega}_{\text{curl}}$$

- Sources

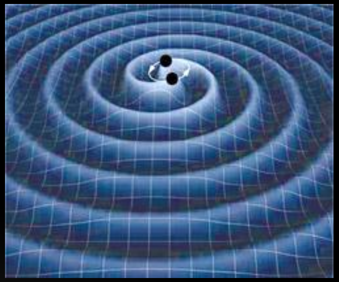
Gradient ϕ ← Scalar

Curl ϖ ← Vector, Tensor

density perturbations (linear)

Cosmic string

GWs



from NASA

Magnetic fields

from ESO

Also important for a test of systematics

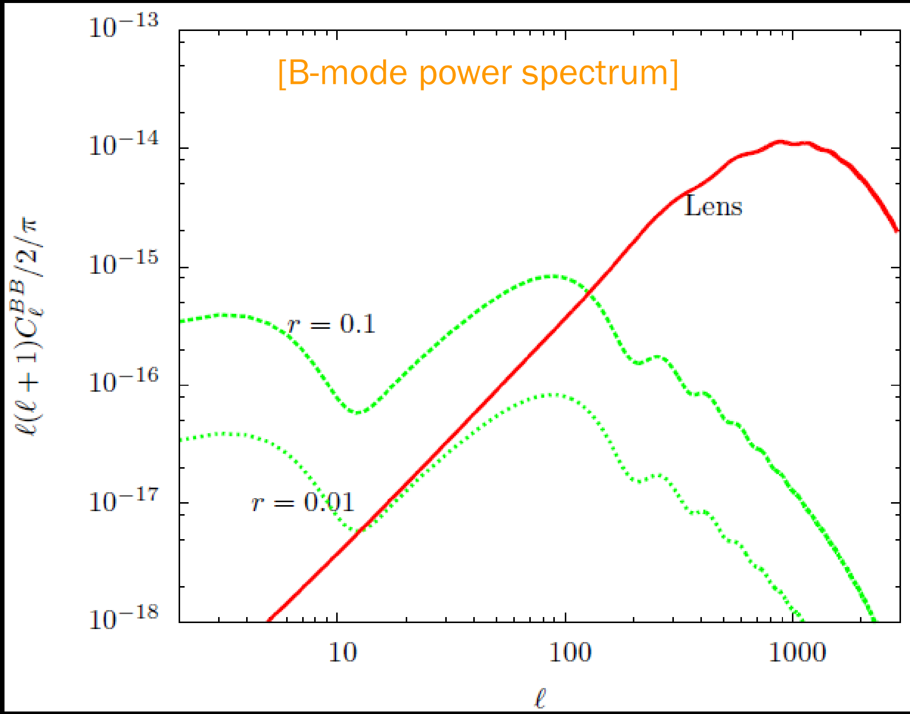
Other motivations to measure CMB lensing

✓ CMB Lensing generates B-mode and secondary non-Gaussianity

noise for primordial GWs detection

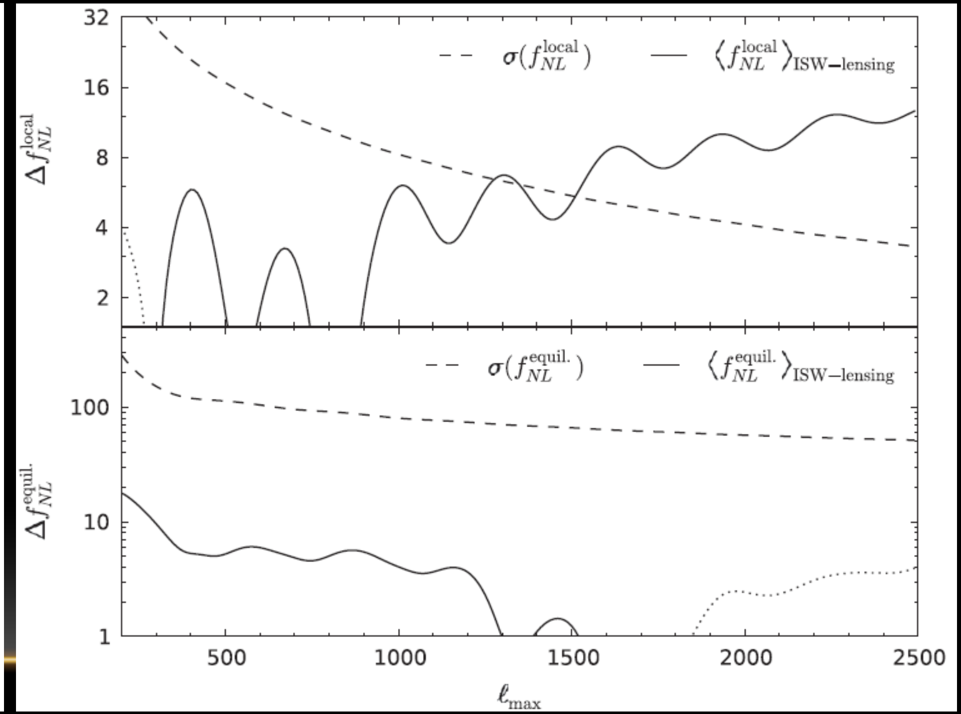
noise for primordial non-Gaussianity

Primordial GWs



e.g., Knox+'02, Kesden+'02, Smith+'09

Primordial non-Gaussianity



Hanson+'09

2. HOW TO ESTIMATE LENSING EFFECT

ESTIMATING LENSING FIELDS THROUGH …

- Angular power spectrum
  - ✓ useful to see whether the observed CMB anisotropies are lensed or not

- Lensing reconstruction = estimate lensing potentials

  (Das+'11; van Engelen+'12; PLANCK'13)
  - ✓ useful for cross-correlation studies with, e.g., cosmic shear, galaxy clustering, etc

- Minkowski Functionals (e.g., Schmalzingr+'00) may be another possible method to measure lensing effect

LENSING RECONSTRUCTION

- Basic Idea (Review: Hanson+'10)
  - ✓ Anisotropy induced by lensing creates mode coupling between different Fourier modes

$$\tilde{\Theta}_{\vec{\ell}} = \Theta_{\vec{\ell}} - \int \mathrm{d}^2\vec{L}\,\left[\,\vec{L}'\,\underline{\phi_{\vec{L}'}} + (\star\,\vec{L}')\,\underline{\varpi_{\vec{L}'}}\right]\cdot\vec{L}\,\Theta_{\vec{L}} \qquad (L' = L - \ell)$$

- Estimator for $x\ (=\phi,\varpi\ )$ (e.g., Hu&Okamoto'02; Hirata&Seljak'03a,b; Namikawa+'12)

(Filtered) observed data

$$\hat{x}_L^{(\Theta\Theta)} = A_L^{xx,(\Theta\Theta)} \int d^2\ell\ \ g_{L,\ell}^{x,(\Theta\Theta)}\ \boxed{\bar{\Theta}_\ell\bar{\Theta}_{\vec{L}-\vec{\ell}}} \qquad \bar{\Theta}_L = \frac{\tilde{\Theta}_L}{C_L^{\Theta\Theta}}$$

optimal weighting

Determined by "unbiased" and "optimal" (minimize non-lensing contributions) conditions

Lensing fields are estimated through mode-coupling (off-diagonal covariance) of CMB anisotropies

LENSING RECONSTRUCTION

- Parity decomposition of polarization

$$E_{\vec{\ell}} \pm \mathrm{i}\, B_{\vec{\ell}} = -\int \mathrm{d}^2 n\, \mathrm{e}^{-\mathrm{i}n\ell} (Q \pm iU) e^{\mp 2i\varphi_\ell}$$

Stokes Q and U parameters

✓ Similar to temperature case, anisotropy induced by lensing creates mode coupling between different Fourier modes

$$\tilde{E}_{\vec{\ell}} = E_{\vec{\ell}} - \int d^2L \big[\, \vec{L}'\, \underline{\phi_{\vec{L}'}} + (\star\, \vec{L}')\, \underline{\varpi_{\vec{L}'}} \big] \cdot \vec{L}\, (E_{\vec{L}} \cos 2\varphi_{L,\ell} - B_L \sin 2\varphi_{L,\ell})$$

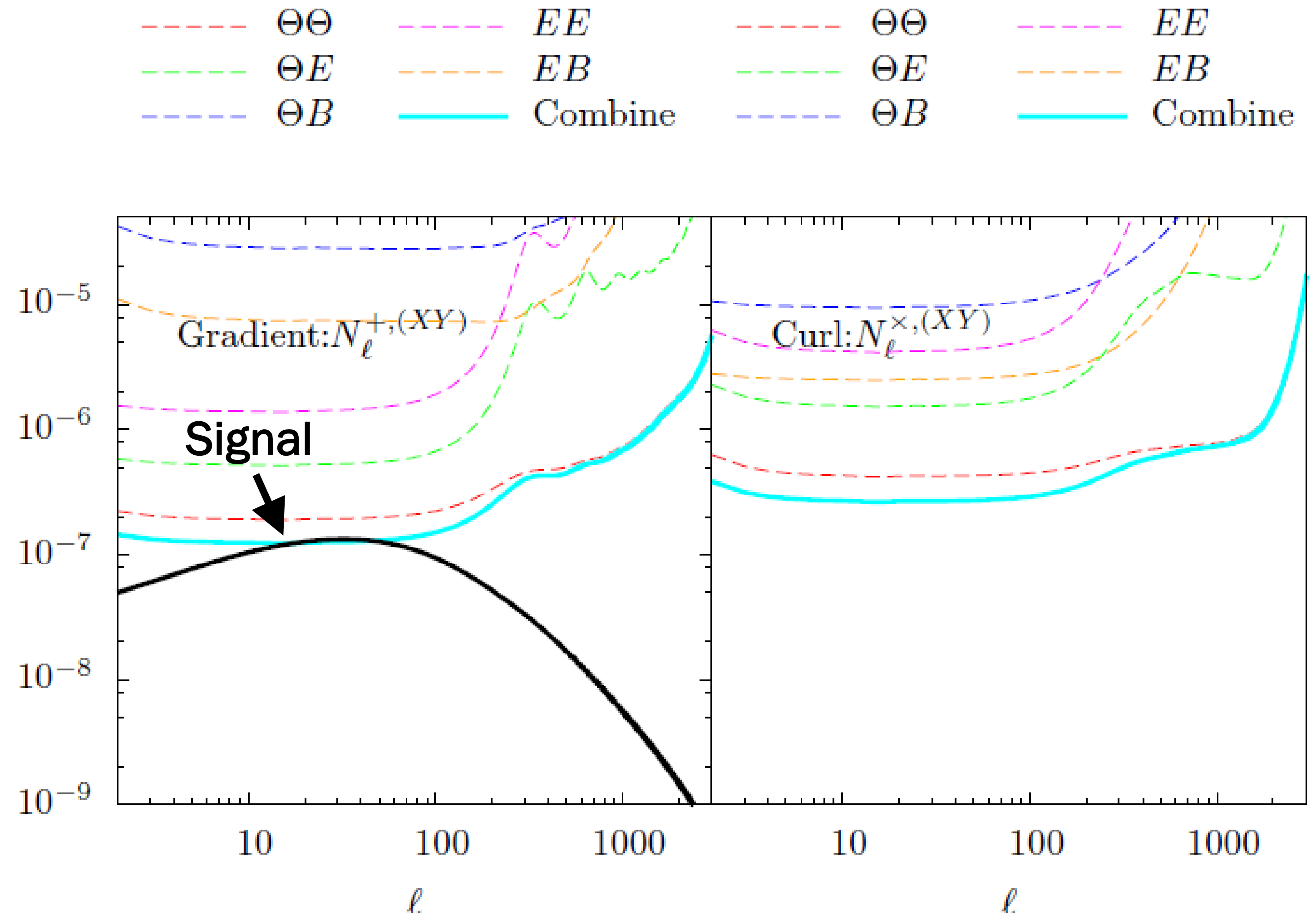
$$\tilde{B}_{\vec{\ell}} = B_{\vec{\ell}} - \int d^2L \big[\, \vec{L}'\, \phi_{\vec{L}'} + (\star\, \vec{L}')\, \varpi_{\vec{L}'} \big] \cdot \vec{L}\, (B_{\vec{L}} \cos 2\varphi_{L,\ell} + E_L \sin 2\varphi_{L,\ell})$$

- Generalizing quadratic estimator

$$\hat{x}_L^{(XY)} = A_L^{xx(XY)} \int d^2\ell \;\; g_{L,\ell}^{x,(XY)} \;\; \bar{X}_\ell \bar{Y}_{\vec{L}-\vec{\ell}} \qquad (X, Y = \Theta, E, B)$$

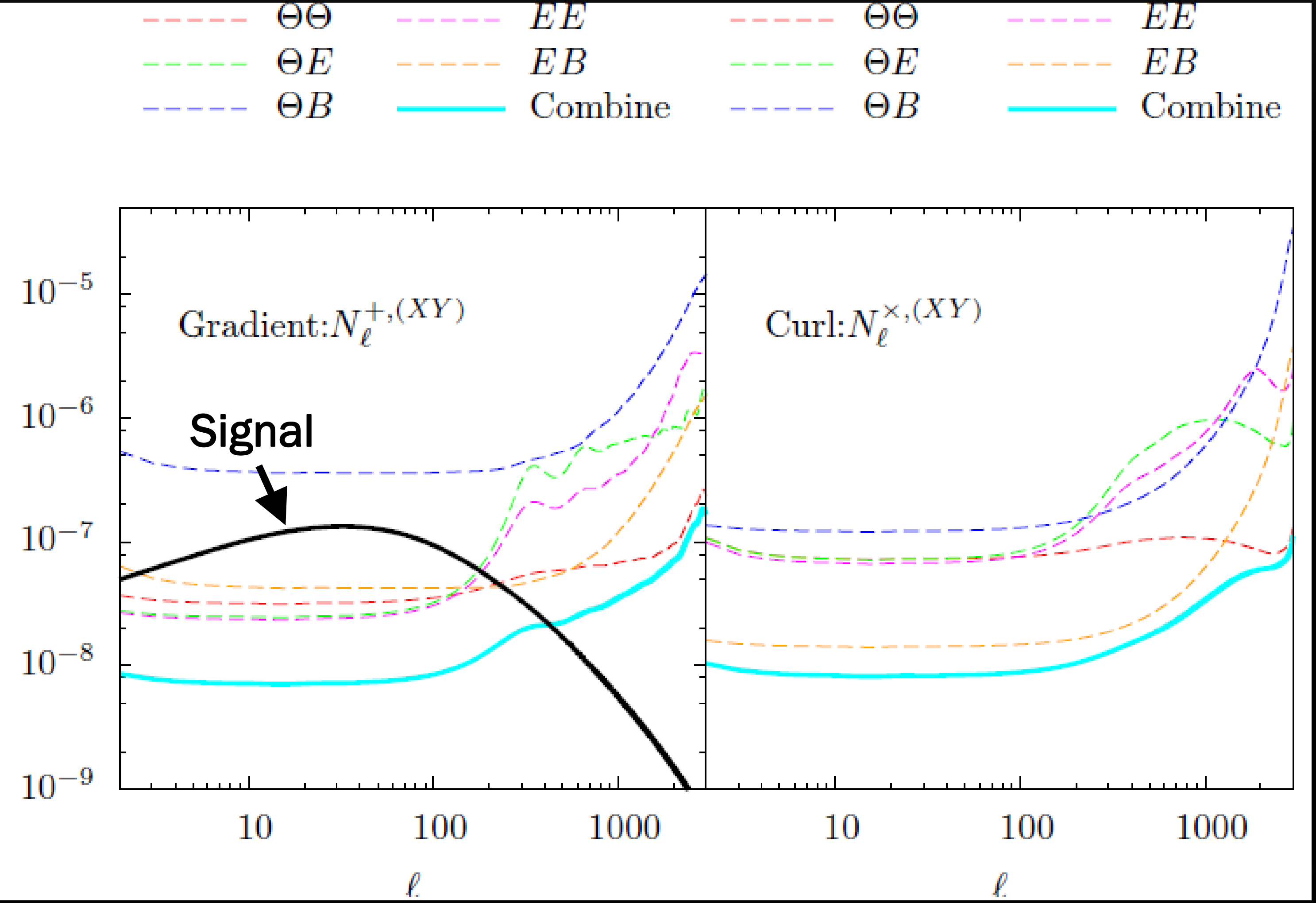
LENSING RECONSTRUCTION

- Signal and noise (Planck)



LENSING RECONSTRUCTION

- Signal and noise (ground based experiment like SPTpol, PolarBear, ACTPol)



✓ Near future, polarizations are quite useful to reconstruct lensing fields.

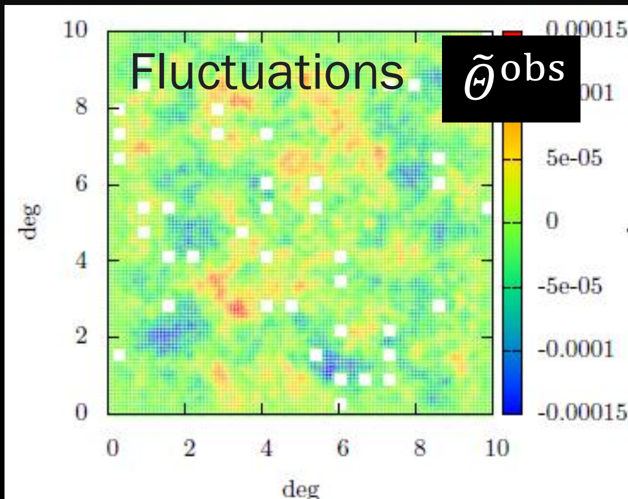
3. BIAS-HARDENED ESTIMATOR FOR LENSING RECONSTRUCTION FROM CMB MAPS

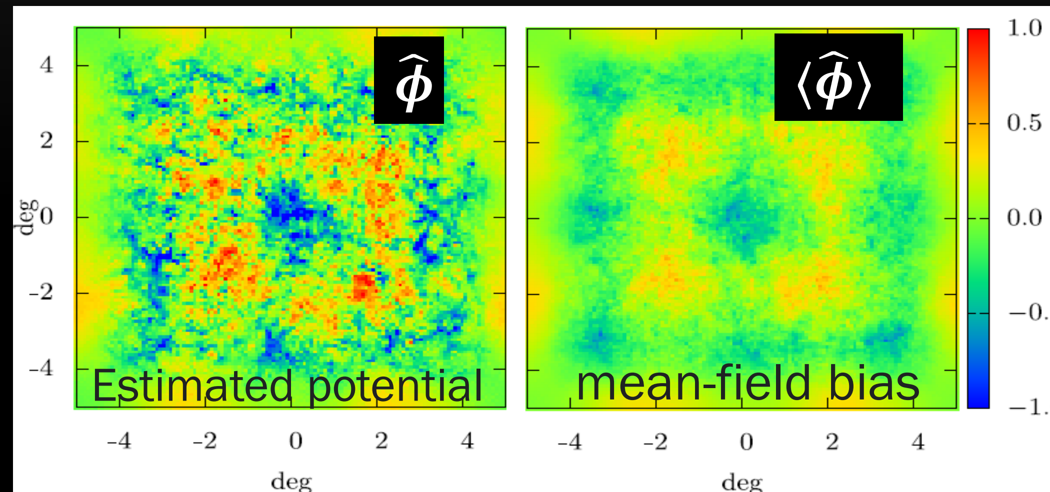
Based on *TN*, Hanson & Takahashi (2013)

TN, Hanson & Takahashi in prep.

MEAN-FIELD BIAS (MASK)

- Survey boundary, points source masks





- Mean-field bias

$$\tilde{\Theta}^{\mathrm{obs}}(\vec{n}) = (1 - M(\vec{n}))\tilde{\Theta}(\vec{n})$$

$$M(\vec{n}) = \begin{cases} 0 & \text{(observed region)} \\ 1 & \text{(otherwise)} \end{cases}$$

Window function

$$\rightarrow \quad \tilde{\Theta}^{\mathrm{obs}}_{\vec{\ell}} = \tilde{\Theta}_{\vec{\ell}} - \int d^2L \; M_{\vec{\ell}-\vec{L}}\tilde{\Theta}_{\vec{L}}$$

$$\rightarrow \quad \langle \hat{x}^{(\Theta\Theta)}_{\vec{L}} \rangle = \underline{R^{xM,(\Theta\Theta)}_L M_L}$$

mean-field bias

$$\left(R^{xM,(\Theta\Theta)}_L \equiv A^{xx,(\Theta\Theta)}_L \int \mathrm{d}^2L \, g^{x,(\Theta\Theta)}_{\ell,L} f^{M,(\Theta\Theta)}_{\ell,L} \right)$$

$$f^{M,(\Theta\Theta)}_{\ell,L} = -C^{\Theta\Theta}_L - C^{\Theta\Theta}_{|\ell-L|}$$

✓ The situation is similar for polarizations (Q,U)

MEAN-FIELD BIAS (UNRESOLVED PS)

- Unresolved point sources/inhomogeneous noise

Data model must be

$$X^{obs}(\vec{n}) = X(\vec{n}) + n^X(\vec{n}) \qquad (X = \Theta, E, B)$$

Assumptions:

$$\langle n^X(\vec{n}) n^Y(\vec{n}')\rangle = S^{XY}(\vec{n})\delta(\vec{n} - \vec{n}')$$

$$\langle \tilde{X} n^Y \rangle = 0$$

- Mean-field bias

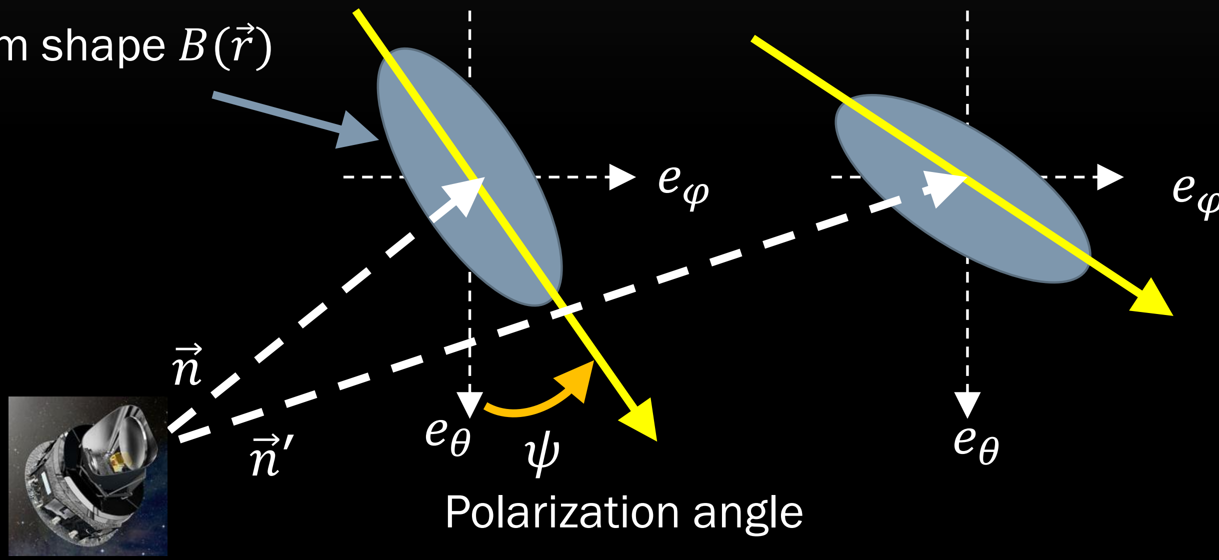
$$\langle \hat{x}_{\vec{L}}^{(XY)} \rangle = R_L^{xS,(XY)} S_L^{XY} \qquad \left(R_L^{xS,(XY)} \equiv A_L^{xx,(XY)} \int dL\, g_{\ell,L}^x f_{\ell,L}^{S,(XY)} \right)$$

$$f_{\ell,L}^{S,(XY)} = 1$$

MEAN-FIELD BIAS (BEAM)

- Polarization angle systematics associated with beam

(e.g., Souradeep+'01; Ng'05; Shimon+'08)



$$\tilde{X}^{obs}(\vec{n};\psi) = \int \mathrm{d}^2\vec{r}\, B(\vec{r};\psi)\, \tilde{X}(\vec{n}-\vec{r})$$

If ψ depends on sky position, the observed anisotropies have off-diagonal covariance

- Mean-field bias

For two-beam experiment, $$\langle \hat{x}_{\vec{\ell}} \rangle = \sum_{p=0,\pm} \sum_{n} R_\ell^{x,(n,p)} \psi_\ell^{(n,p)}$$

EXPRESSION FOR MEAN-FIELD BIAS

- Mean-field bias $\alpha = \Theta\Theta, \Theta E, \ldots$

$$\langle \hat{x}_L^{(\alpha)} \rangle = \sum_{y=\phi,\varpi,M,\ldots} R_L^{xy,(\alpha)}\, y_L^{(\alpha)}$$

$$\left(R_\ell^{xy,(\alpha)} \equiv A_\ell^{xx,(\alpha)} \int dL\, g_{\ell,L}^{x,(\alpha)}\, f_{\ell,L}^{y,(\alpha)} \right)$$

	Masking
$\Theta\Theta$	$-\widetilde{C}_L^{\Theta\Theta} - \widetilde{C}_{L'}^{\Theta\Theta}$
ΘE	$-\widetilde{C}_L^{\Theta E} \cos 2\varphi_{\boldsymbol{L},\boldsymbol{L}'} - \widetilde{C}_{L'}^{\Theta E}$
EE	$(\widetilde{C}_L^{\rm EE} + \widetilde{C}_{L'}^{\rm EE}) \cos\varphi_{\boldsymbol{L},\boldsymbol{L}'}$
ΘB	$-\widetilde{C}_L^{\Theta E} \sin 2\varphi_{\boldsymbol{L},\boldsymbol{L}'}$
EB	$(\widetilde{C}_L^{\rm EE} + \widetilde{C}_{L'}^{\rm BB}) \sin\varphi_{\boldsymbol{L},\boldsymbol{L}'}$
BB	$(\widetilde{C}_L^{\rm BB} + \widetilde{C}_{L'}^{\rm BB}) \cos\varphi_{\boldsymbol{L},\boldsymbol{L}'}$

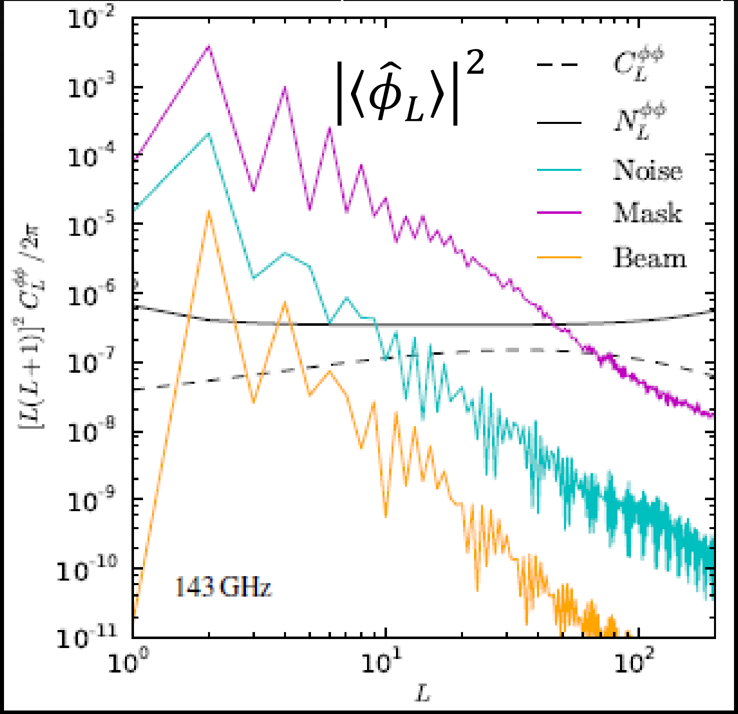
	Masking (with pure E and B modes)
$\Theta\Theta$	0
ΘE	$(L/L')^2 \widetilde{C}_L^{\Theta E} + \widetilde{C}_{L'}^{\Theta E}$
EE	$(L/L')^2 \widetilde{C}_L^{\rm EE} + (L \leftrightarrow L')$
ΘB	0
EB	0
BB	$(L/L')^2 \widetilde{C}_L^{\rm BB} + (L \leftrightarrow L')$

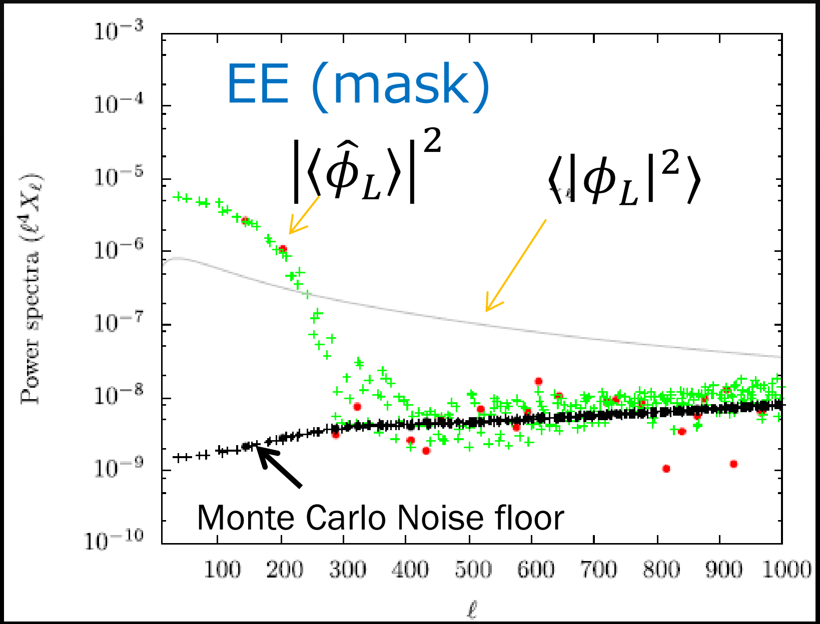
	Beam, $p = 0$
$\Theta\Theta$	$[\mathcal{B}_{\boldsymbol{L}}^{(\Theta,\Theta),(n,0)} \widetilde{C}_L^{\Theta\Theta} + \mathcal{B}_{\boldsymbol{L}}^{(\Theta,E),(n,0)} \widetilde{C}_L^{\Theta E}] + (\boldsymbol{L} \leftrightarrow \boldsymbol{L}')$
ΘE	$\mathcal{B}_{\boldsymbol{L}'}^{(\Theta,\Theta),(n,0)} \widetilde{C}_{L'}^{\Theta E} + \mathcal{B}_{\boldsymbol{L}'}^{(\Theta,E),(n,0)} \widetilde{C}_{L'}^{\rm EE}$
EE	0
ΘB	$\mathcal{B}_{\boldsymbol{L}'}^{(\Theta,B),(n,0)} \widetilde{C}_{L'}^{\rm BB}$
EB	0
BB	0
	Beam, $p = \pm$
$\Theta\Theta$	0
ΘE	$\mathrm{e}^{\mp 2\mathrm{i}\varphi_{\boldsymbol{L}'}} [\mathcal{B}_{\boldsymbol{L}}^{(E,\Theta),(n,\pm)} \widetilde{C}_L^{\Theta\Theta} + \mathcal{B}_{\boldsymbol{L}}^{(E,E),(n,\pm)} \widetilde{C}_L^{\Theta E}]$
EE	$\mathrm{e}^{\mp 2\mathrm{i}\varphi_{\boldsymbol{L}'}} [\mathcal{B}_{\boldsymbol{L}}^{(E,\Theta),(n,\pm)} \widetilde{C}_L^{\Theta E} + \mathcal{B}_{\boldsymbol{L}}^{(E,E),(n,\pm)} \widetilde{C}_L^{\rm EE}] + (\boldsymbol{L} \leftrightarrow \boldsymbol{L}')$
ΘB	$\mathrm{e}^{\mp 2\mathrm{i}\varphi_{\boldsymbol{L}'}} [\mathcal{B}_{\boldsymbol{L}}^{(B,\Theta),(n,\pm)} \widetilde{C}_L^{\Theta\Theta} + \mathcal{B}_{\boldsymbol{L}}^{(B,E),(n,\pm)} \widetilde{C}_L^{\Theta E}]$
EB	$\mathrm{e}^{\mp 2\mathrm{i}\varphi_{\boldsymbol{L}'}} [\mathcal{B}_{\boldsymbol{L}}^{(B,\Theta),(n,\pm)} \widetilde{C}_L^{\Theta E} + \mathcal{B}_{\boldsymbol{L}}^{(B,E),(n,\pm)} \widetilde{C}_L^{\rm EE}] + \mathrm{e}^{\mp 2\mathrm{i}\varphi_{\boldsymbol{L}}} \mathcal{B}_{\boldsymbol{L}'}^{(E,B),(n,\pm)} \widetilde{C}_{L'}^{\rm BB}$
BB	$\mathrm{e}^{\mp 2\mathrm{i}\varphi_{\boldsymbol{L}'}} \mathcal{B}_{\boldsymbol{L},n}^{(B,B),(n,\pm)} \widetilde{C}_L^{\rm BB} + (\boldsymbol{L} \leftrightarrow \boldsymbol{L}')$

SIGNIFICANCE OF MEAN-FIELD BIAS

✓ Mean field bias

(Planck collaboration'13)





✓ In conventional method, we compute $R_\ell^{\phi M}$ with Cl's to estimate $\langle\hat{\phi}_{\vec{\ell}}\rangle$, and then subtract as $\hat{\phi}_L - \langle\hat{\phi}_L\rangle$

✓ This method rely entirely on the knowledge of $R_\ell^{\phi M}$, but $\hat{\phi}$ would be biased due to uncertainties of e.g., Cl's,

We need alternative method for cross-check

BIAS-HARDENED ESTIMATOR

- ✓ We formulate an estimator as follows

  1. Simiar to $\hat{x}$, we formulate estimator for $a\ (= M, S, \Psi^{(n,p)})$
  2. $\hat{a}$ also has mean-field bias, so we combine $\hat{a}$ and $\hat{\phi}$ to construct an estimator which has no mean-field bias:

$$\hat{x}_L^{(\alpha)} = \sum_{y=\phi,\varpi,M,\ldots} \{R_L^{-1}\}^{x,y,(\alpha)}\, \hat{y}_L^{(\alpha)} \qquad \left(\{R_L\}^{x,y,(\alpha)} = A_\ell^{xx,(\alpha)} \int d^2\ell\, g_{L,\ell}^{x,(\alpha)} f_{L,\ell}^{x,(\alpha)}\right)$$

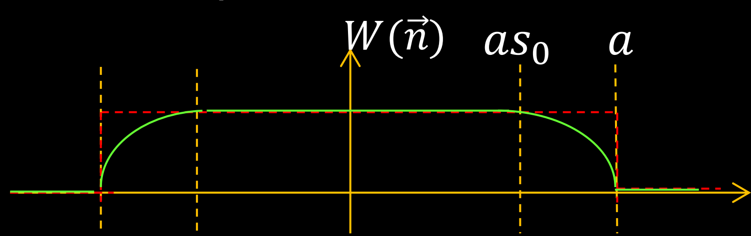
- ✓ Comparing with the conventional approach, uncertainty in R_L^{xy} propagates to estimator in a different way, so the above estimator would utilize for cross check (more robust but a bit noisy than the conventional approach)

- ✓ Higher order terms of a is ignored

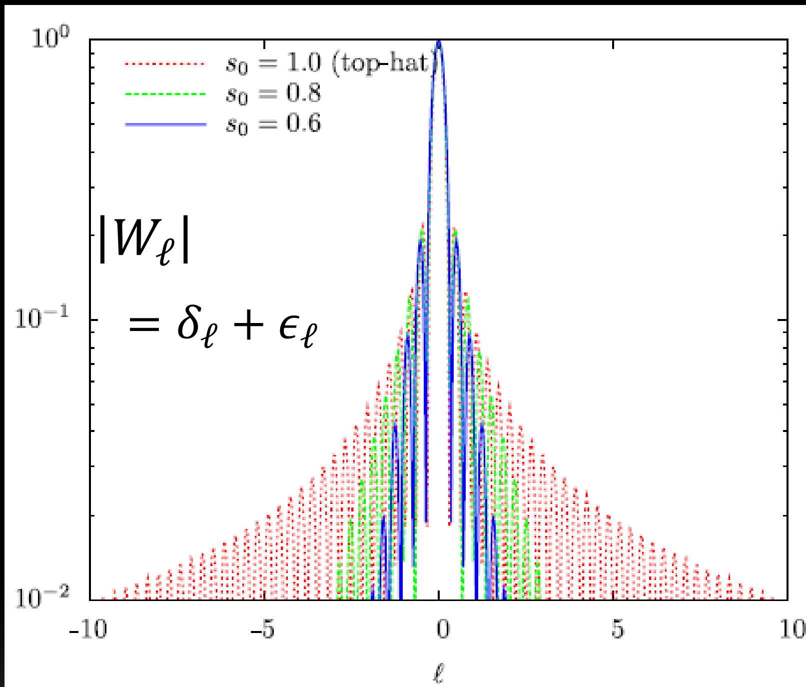
- ✓ It would be possible to estimate origin of unknown systematics (e.g., patchy reionization, motion of the earth, unresolved point sources, etc)

NUMERICAL TEST

- **Purpose**
  - ✓ For mask, the assumption $\epsilon_\ell \ll 1$ is not always satisfied
  - ✓ With "filtering" (suppress ϵ_ℓ), we test how well the bias-hardened estimator works
- **Filtering for survey boundary**
  1. Apodization: window function is modified so that the Fourier counterpart becomes δ-like function



$$X^{obs}_{\vec{\ell}} = \int d^2L \, W_{\vec{\ell}-\vec{L}} \tilde{X}_{\vec{L}} \sim \tilde{X}_{\vec{\ell}}$$

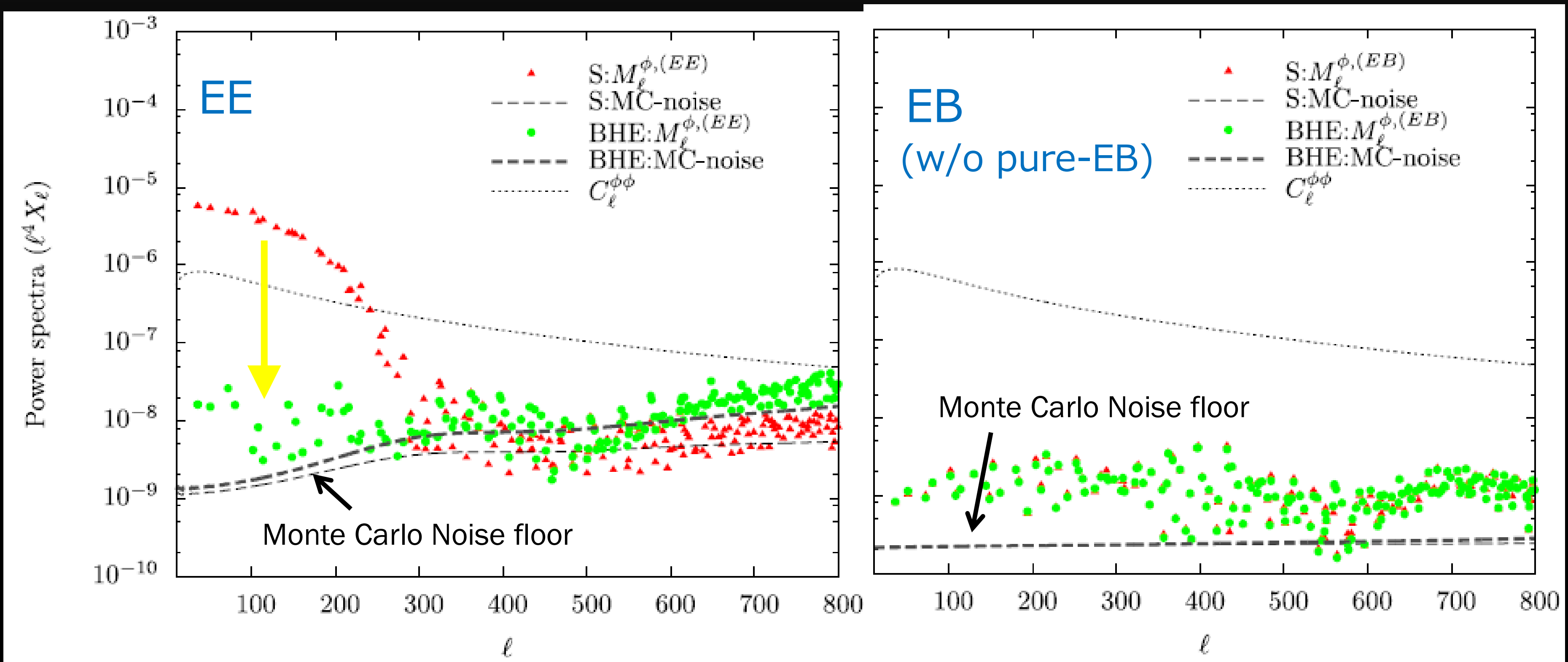


  2. Pure-EB estimator (e.g., Smith+'06)

- **Simulated lensed map made by Takahashi-san**
  - ✓ $5 \times 5 \ \mathrm{deg}^2$
  - ✓ 1024^2 grids
  - ✓ 100 realizations

NUMERICAL RESULTS

- Mean-field bias from masking [Gradient mode]

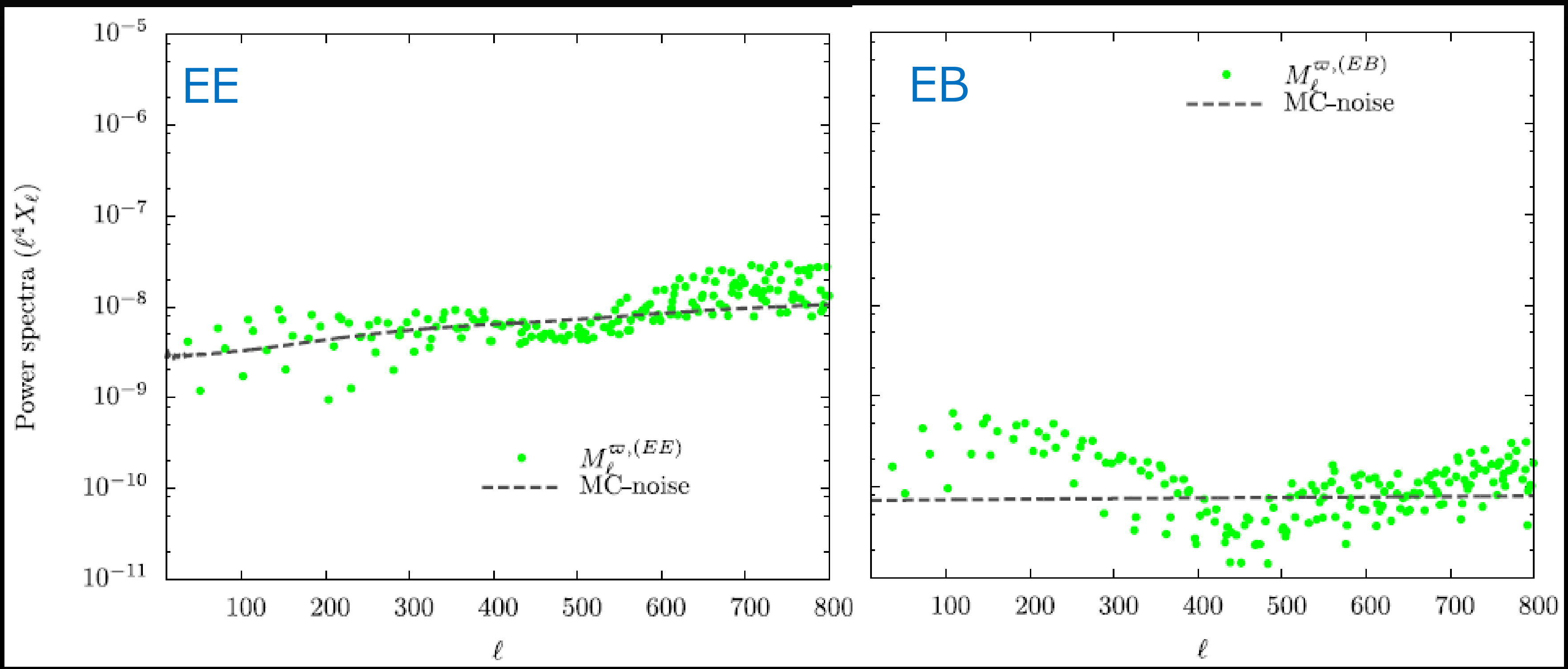


EE: Bias-hardened estimator suppresses mean-field bias down to MC noise level

EB: Even without pure-EB estimator, mean-field bias from masking is negligible compared to the signal

NUMERICAL RESULTS

- Mean-field bias from masking [Curl mode]

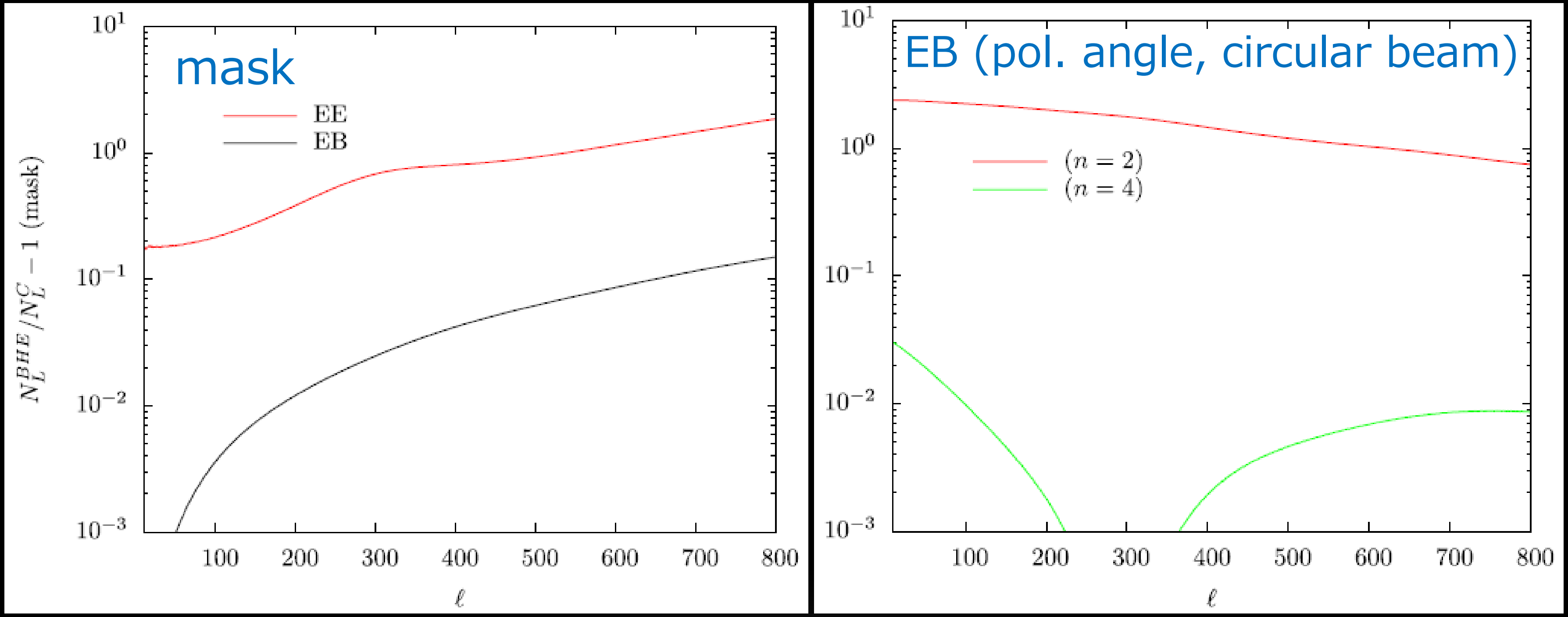


Mean-field bias from masking is negligible

LOSS OF SIGNAL-TO-NOISE

- ✓ One concern for using bias-hardened estimator is the loss of signal-to-noise.

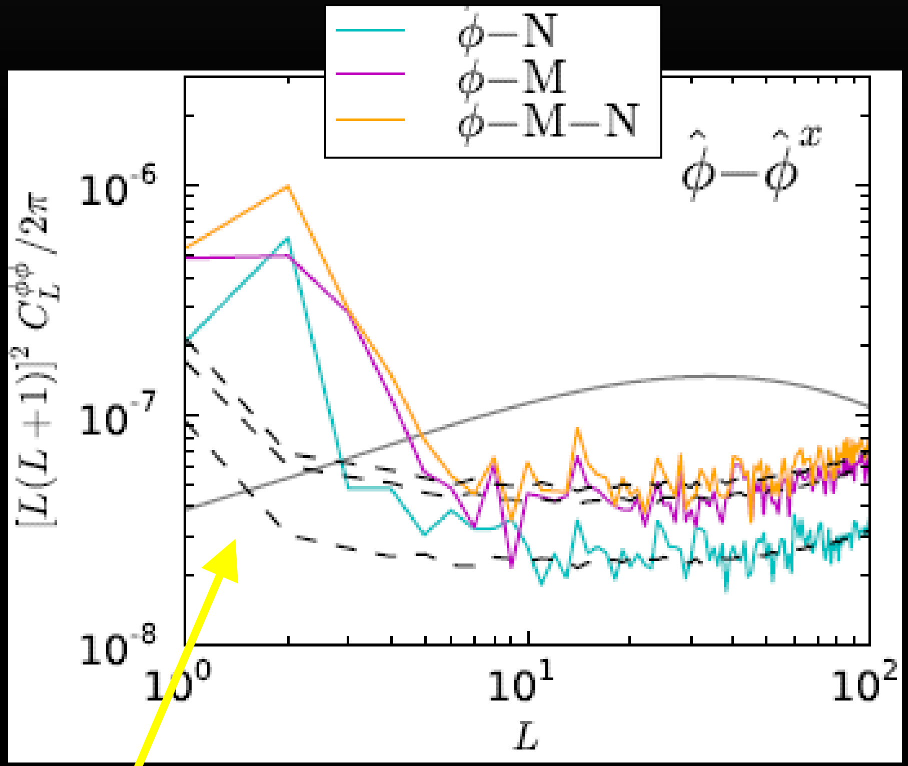
[Fractional difference of noise level between BHE and conventional]



- ✓ The loss of S/N is not so significant (but depends on scale)

APPLICATION TO PLANCK DATA (TEMPERATURE)

- Difference of the results between bias-hardened estimator and conventional method



(Planck collaboration'13)

Unexpected discrepancy

(so they conservatively use L>40 for parameter estimation)

LENSING POWER SPECTRUM ESTIMATE

- ✓ For cosmology we are interested in C_ℓ^{xx} rather than x

- ✓ From $\hat{x}$ to C_ℓ^{xx} (e.g., Kesden+'03; Hanson+'11)

$$\langle |\hat{x}_L^{XY}|^2 \rangle = \int d\vec{\ell}_1 \int d\vec{\ell}_2 \, F^{x,(XY)}_{L,\ell_1} F^{x,(XY)}_{L,\ell_2} \langle \bar{X}^*_{\ell_1} \bar{Y}^*_{\vec{L}-\vec{\ell}_1} \bar{X}_{\ell_2} \bar{Y}_{\vec{L}-\vec{\ell}_2} \rangle$$

$$= M_\ell^x + N_\ell^{x,(0)} + \underbrace{\boxed{C_\ell^{xx}} + N_\ell^{x,(1)}}_{O(C_\ell^{xx})} + \underbrace{N_\ell^{x,(2)}}_{O([C_\ell^{xx}]^2)} + \cdots$$

due to non-lensing anisotropy (residual mean-field bias)

from intrinsic scatter of CMB anisotropies (Gaussian bias)

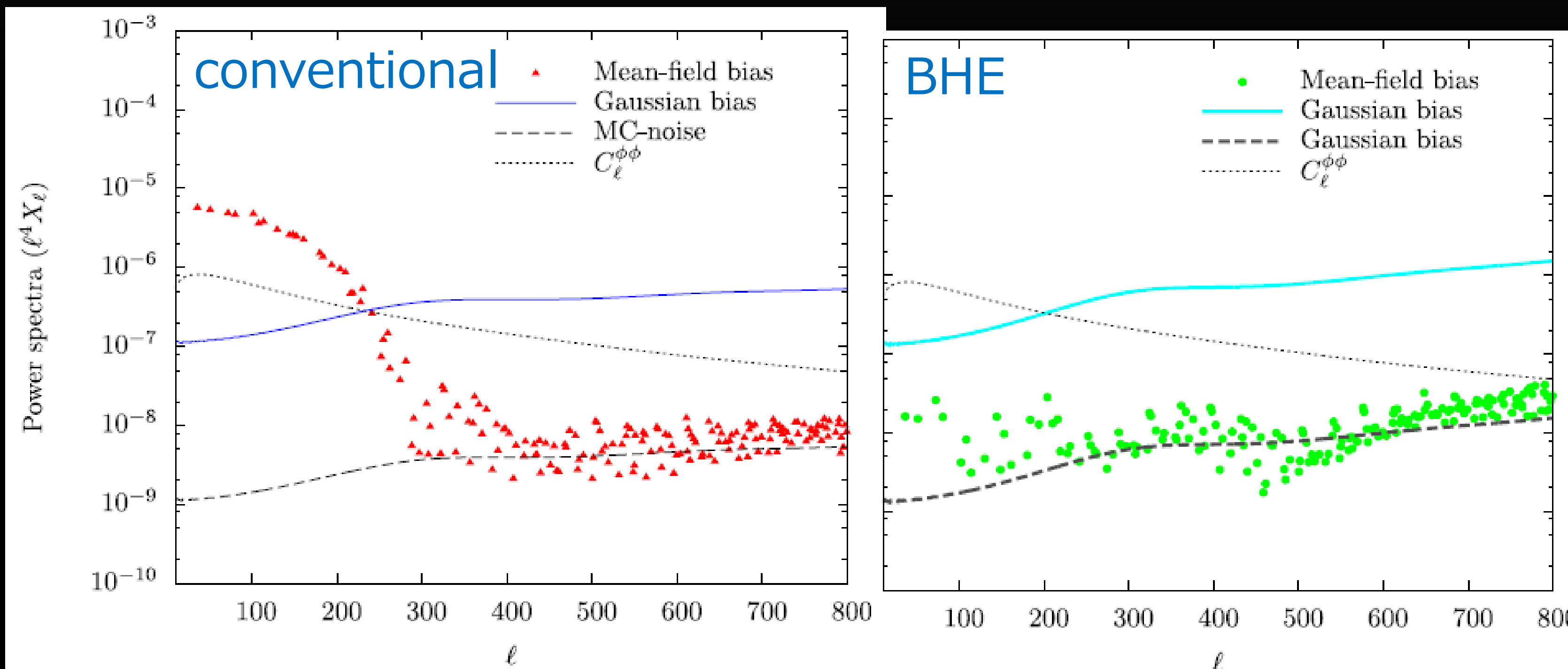
Generated by lensing

In estimating power spectrum, we have to know many bias terms accurately

SIGNIFICANCE OF BIAS TERMS ON CL ESTIMATE

- Mean-field bias and Gaussian bias in the power spectrum estimate

[Gradient mode, EE-estimator]



✓ Bias-hardened estimator suppress mean-field bias enough to ignore in the power spectrum estimates

✓ Gaussian bias, however, is significant and should be accurately corrected

ESTIMATOR FOR GAUSSIAN BIAS

- Gaussian bias estimate

✓ Conventional (e.g., Hu'01)

$$\hat{N}_{\ell}^{x,(0)} = 2\int d\vec{\ell}_1 \int d\vec{\ell}_2\, F^{x}_{L,\ell_1} F^{x}_{L,\ell_2} \bar{C}_{\ell_1,\ell-\ell_2} \bar{C}_{\ell-\ell_1,\ell_2} \qquad (\bar{C}_{L_1,L_2} \equiv \langle \bar{\mathrm{X}}_{L_1} \bar{\mathrm{Y}}^{*}_{L_2} \rangle)$$

✓ Our approach

- Naturally derived as an optimal trispectrum estimator with maximum likelihood approach

$$\hat{N}_{\ell}^{x,(0)} = 2\int d\vec{\ell}_1 \int d\vec{\ell}_2\, F^{x}_{L,\ell_1} F^{x}_{L,\ell_2} [2\bar{C}_{\ell_1,\ell-\ell_2} \bar{\mathrm{X}}_{\ell-\ell_1} \bar{\mathrm{Y}}^{*}_{\ell_2} - \bar{C}_{\ell_1,\ell-\ell_2} \bar{C}_{\ell-\ell_1,\ell_2}]$$

- More accurate than previous method (e.g., $\bar{C}_{L_1,L_2}$)

(if $\bar{C}_{L_1,L_2} \rightarrow \bar{C}_{L_1,L_2} + \delta\bar{C}_{L_1,L_2}$, the bias propagates as 2nd order of $\delta\bar{C}_{L_1,L_2}$)

Summary

- We present estimators to mitigate

  1) mean field bias (from masking, point sources, beam, etc)

  2) Gaussian bias

- Using numerical test, we found that the mean-field bias from masking is suppressed by combining “bias-hardened estimator” and some filtering approach

- Noise level would be degraded at most by factor of 2-3