Non-linear curvature perturbation in multi-field inflation models with non-minimal coupling

The CMB and theories of the primordial universe
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Models with non-minimal coupling

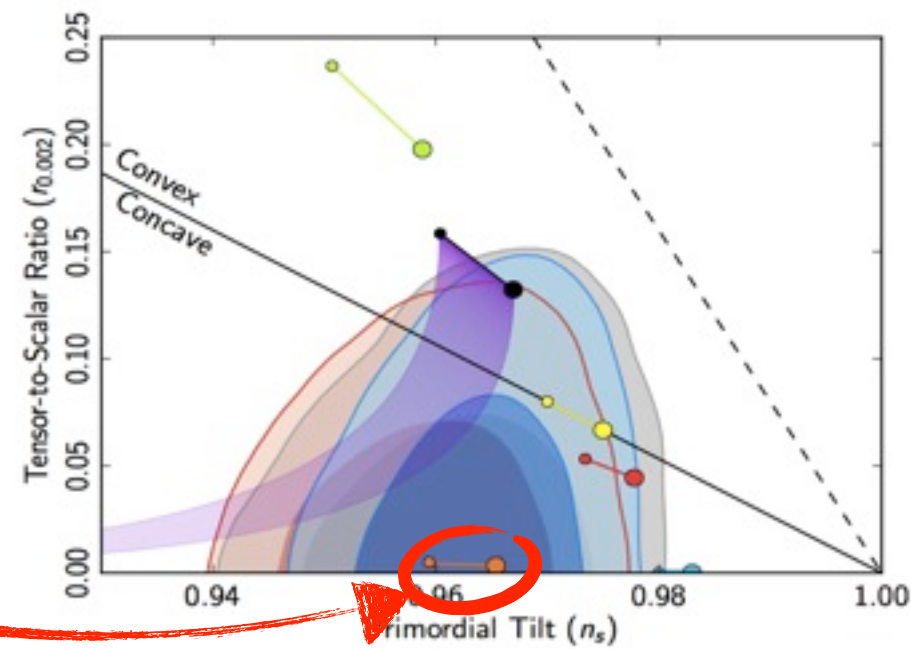
- In the context of unifying theories, modified gravity and renormalisation, **non-minimal coupling** of scalar fields to the Ricci Scalar **is common**.

e.g.

$$(\Phi = df / dR)$$

$$S_{f(R)} = \int d^4x\sqrt{-g}\,f(R) \;\Rightarrow\; \int d^4x\sqrt{-g}\left(\Phi R + \ldots\right)$$

$$S_{Higgs} = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(M_{pl}{}^2 + \xi h^2\right)R + \ldots\right]$$



- Models with non-minimal coupling are **favoured by** ***Planck***
  Kaiser & Sfakianakis '13, Kallosh & Linde '13

- In general we **expect multiple fields** in context of unifying theories

$\Rightarrow$ Let us consider the general class of models with action of the form

$$\boxed{S = \int d^4x\sqrt{-g}\left\{f(\phi)R - \frac{1}{2}h_{ab}g^{\mu\nu}\,\partial_\mu\phi^a\,\partial_\nu\phi^b - V(\phi) + L_{matter}\right\}}$$

The Jordan and Einstein frames

- By making a conformal transformation $g_{\mu\nu} = \dfrac{\tilde{g}_{\mu\nu}}{2f}$ $\quad S_{ab} = \dfrac{1}{2f}\left(h_{ab} + \dfrac{3f_a f_b}{f} \right)$

$$\Rightarrow \boxed{S = \int d^4x\sqrt{-\tilde{g}}\left\{ \frac{\tilde{R}}{2} - \frac{1}{2} S_{ab}\tilde{g}^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - \tilde{V}(\phi) + \tilde{L}_{matter} \right\}} \qquad \tilde{V} = \frac{V}{4f^2}$$

- Original frame called **Jordan frame - matter minimally coupled**
- New frame called **Einstein frame** - **canonical Einstein Hilbert gravity** but **mass-, length- and time-scales become spacetime dependent!**
- Calculations seem easier in the Einstein frame - **how are the Jordan and Einstein frame quantities related?**
- In particular, we are interested in $\zeta \leftrightarrow \dfrac{\Delta T}{T}$

$\Rightarrow$

- How are ζ and $\tilde{\zeta}$ related?
- What is the effect of non-minimal coupling on CMB spectrum?

Linear order: single field case Makino & Sasaki '91

- Introduce **canonically normalised field** $\tilde{\phi}$ satisfying $\dfrac{d\tilde{\phi}}{d\phi} = \sqrt{S_{\phi\phi}}$
- Decompose **metric in Jordan frame** as

$$ds^2 = a(\eta)^2 \left\{ -(1+2AY)d\eta^2 - 2BY_i d\eta dx^i + \left[(1+2\mathcal{R})\,\delta_{ij} + 2H_T \frac{1}{k^2} Y_{,ij} \right] dx^i dx^j \right\}$$

- Make similar decomposition in Einstein frame, but with tildes everywhere, then **require** $d\tilde{s}^2 = 2 f ds^2$

$$\Rightarrow \quad \boxed{\widetilde{\mathcal{H}} = \mathcal{H} + \frac{f'}{2f} \qquad \widetilde{\mathcal{R}} = \mathcal{R} + \frac{\delta f}{2f}}$$

- Substitute these relations into definition of $\tilde{\zeta}$

$$\boxed{\tilde{\zeta} \equiv \widetilde{\mathcal{R}} - \frac{\widetilde{\mathcal{H}}}{\tilde{\phi}'}\delta\tilde{\phi} = \mathcal{R} - \frac{\mathcal{H}}{\phi'}\delta\phi = \zeta}$$

- i.e. **the comoving curvature perturbation is frame independent!**

Linear order: multi-field case

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- Cannot canonically normalise all fields

$$\delta q \propto \delta T^0{}_i$$

- Using the fact that $\zeta \simeq \mathcal{R} + \frac{H}{\rho + p}\delta q$ in the JF and similarly in EF

$$\Rightarrow \quad \boxed{\zeta - \tilde{\zeta} \simeq \mathcal{A}_{ab}\mathcal{K}^{ab} + \mathcal{B}_{ab}\dot{\mathcal{K}}^{ab}}$$

- Where $\mathcal{K}^{ab} = \delta\phi^a\dot{\phi}^b - \delta\phi^b\dot{\phi}^a$ are isocurvature perturbations and $\mathcal{A}_{ab}$ and $\mathcal{B}_{ab}$ just depend on background quantites.

$\Rightarrow$

- Difference between ζ and $\tilde{\zeta}$ a direct consequence of isocurvature modes
- In the absence of isocurvature modes, $\mathcal{K}^{ab} = 0$, the two curvature perturbations do coincide.
- Also find $\dot{\tilde{\zeta}} = 0 \not\Leftrightarrow \dot{\zeta} = 0$, i.e. evolutions can be very different

Beyond linear order

- Following Gong et al. '11 (1107.1840), define metric and conformal transformation as

$$g_{ij} = a^2 e^{2\mathcal{R}} \gamma_{ij}, \quad \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \tilde{g}_{ij} = \tilde{a}^2 e^{2\tilde{\mathcal{R}}} \tilde{\gamma}_{ij}, \quad \Omega = \Omega_0 e^{\Delta\Omega}$$

$$\Longrightarrow \quad \boxed{\widetilde{\mathcal{R}} = \mathcal{R} + \Delta\Omega}$$

$\Longrightarrow$ Curvature perturbations **same in gauge where $\Delta\Omega = 0$**

- Single field case: $\Omega = \Omega(\phi) \Rightarrow \Delta\Omega = 0 \Rightarrow \delta\phi = 0$

$\Delta\Omega = 0$ **coincides with comoving gauges** in both frames

$$\Longrightarrow \quad \boxed{\tilde{\zeta} = \zeta}$$

- Multi-field case: $\Delta\Omega = 0$ **does not necessarily coincide with the comoving gauges.**

$$\Longrightarrow \quad \boxed{\tilde{\zeta} \neq \zeta}$$

δN formalism in Jordan and Einstein frames

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- Have established $\tilde{\zeta} \neq \zeta$ in multi-field case
- We **already know how to calculate $\tilde{\zeta}$ and its spectral properties**

$\Longrightarrow$ can we relate $\tilde{\zeta}$ back to ζ beyond linear order using the δN?

- δN formalism states that $\zeta = \delta N = N(t_*, t_\diamond; x) - N_0(t_*, t_\diamond)$

No. of e-foldings between initial flat slice and final constant energy slice

Fiducial background no. of e-foldings

- δN expansions in the Jordan and Einstein frames are:

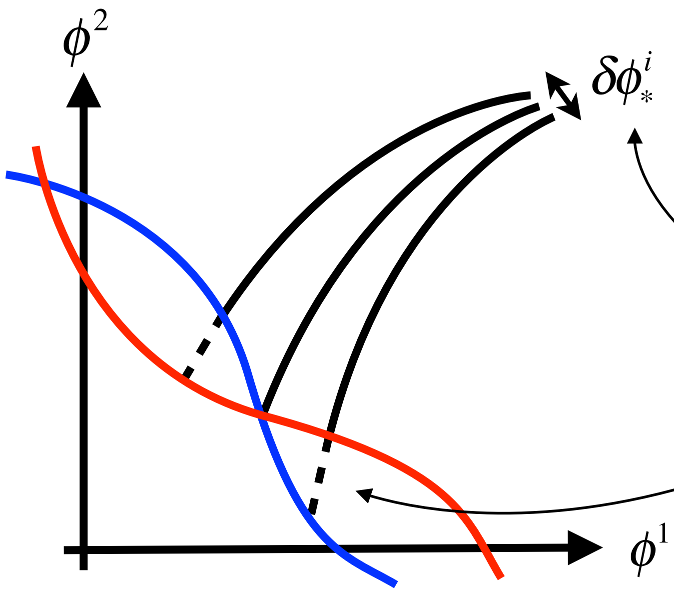
$$\zeta = \delta N = N_a \delta\phi^a_{\mathcal{R}} + \frac{1}{2} N_{ab} \delta\phi^a_{\mathcal{R}} \delta\phi^b_{\mathcal{R}} + \ldots$$

$$\tilde{\zeta} = \delta \tilde{N} = \tilde{N}_a \delta\phi^a_{\tilde{\mathcal{R}}} + \frac{1}{2} \tilde{N}_{ab} \delta\phi^a_{\tilde{\mathcal{R}}} \delta\phi^b_{\tilde{\mathcal{R}}} + \ldots$$

- **Two differences:**
  - Dependence on initial conditions of N and $\tilde{N}$
  - Definition of initial flat hypersurface field perturbations

δN formalism in Jordan and Einstein frames

- Diagrammatically:



$\omega = const.$ defines final **constant energy** surface in **Jordan frame**

$\tilde{\omega} = const.$ defines final **constant energy** surface in **Einstein frame**

$\tilde{N}_a \neq N_a$

$\delta\phi^a_{\mathcal{R}} \neq \delta\phi^a_{\tilde{\mathcal{R}}}$

- First turn to the relation $\tilde{N}_a \leftrightarrow N_a$

$$\left.\begin{aligned} N &= \int_{\mathcal{R}=0}^{\omega=const.} \mathcal{H} d\eta \\ \tilde{N} &= \int_{\tilde{\mathcal{R}}=0}^{\tilde{\omega}=const.} \tilde{\mathcal{H}} d\eta \\ &= \int_{\tilde{\mathcal{R}}=0}^{\tilde{\omega}=const.} \mathcal{H} d\eta + \frac{1}{2}\ln\left(\frac{f_{\tilde{\omega}=const}}{f_{\tilde{\mathcal{R}}=0}}\right) \end{aligned}\right\} \Longrightarrow \boxed{\tilde{N}_a = N_a^{\tilde{\omega}} - \frac{f_a}{2f}\bigg|_* + \frac{f_b}{2f}\bigg|_\diamond \frac{\partial \phi^b_{\tilde{\omega}}}{\partial \phi^a_*}}$$

$$N_a^{\tilde{\omega}} = \frac{\partial}{\phi^a_*}\int_*^{\tilde{\omega}=const.} \mathcal{H} d\eta$$

δN formalism in Jordan and Einstein frames

- Next turning to the relation $\delta\phi^a_{\mathcal{R}} \leftrightarrow \delta\phi^a_{\tilde{\mathcal{R}}}$
- Using the definition of **Sasaki-Mukhanov variables** to first order:

$$\left.\begin{aligned} \delta\phi^a_{\mathcal{R}} &= \delta\phi^a - \frac{\phi'^a}{\mathcal{H}}\mathcal{R} \\ \tilde{\mathcal{R}} &= \mathcal{R} + \frac{\delta f}{2f}. \end{aligned}\right\} \Longrightarrow \delta\phi^a_{\mathcal{R}} = \left(\delta^a_b + \frac{\phi'^a f_b}{2f\mathcal{H}}\right)\delta\phi^b_{\tilde{\mathcal{R}}}$$

- Combining with $\tilde{N}_a = N^{\tilde{\omega}}_a - \left.\frac{f_a}{2f}\right|_* + \left.\frac{f_b}{2f}\right|_\diamond \frac{\partial\phi^b_{\tilde{\omega}}}{\partial\phi^a_*}$
- Terms circled in red cancel and we find:

$$\boxed{\begin{aligned} \zeta - \tilde{\zeta} = {}& (N_a - N^{\tilde{\omega}}_a)\delta\phi^a_{\tilde{\mathcal{R}}} + \frac{1}{2}(N_{ab} - N^{\tilde{\omega}}_{ab})\delta\phi^a_{\tilde{\mathcal{R}}(1)}\delta\phi^b_{\tilde{\mathcal{R}}(1)} \\ & - \left.\frac{f_b}{2f}\right|_\diamond \frac{\partial\phi^b_{\tilde{\omega}}}{\partial\phi^a_*}\delta\phi^a_{\tilde{\mathcal{R}}} - \left(\left.\frac{f_{cd}}{2f}\right|_\diamond \frac{\partial\phi^c_{\tilde{\omega}}}{\partial\phi^a_*}\frac{\partial\phi^d_{\tilde{\omega}}}{\partial\phi^b_*} - \left.\frac{f_c f_d}{2f^2}\right|_\diamond \frac{\partial\phi^c_{\tilde{\omega}}}{\partial\phi^a_*}\frac{\partial\phi^d_{\tilde{\omega}}}{\partial\phi^b_*} + \left.\frac{f_c}{2f}\right|_\diamond \frac{\partial^2\phi^c_{\tilde{\omega}}}{\partial\phi^a_*\partial\phi^b_*}\right)\delta\phi^a_{\tilde{\mathcal{R}}(1)}\delta\phi^b_{\tilde{\mathcal{R}}(1)}. \end{aligned}}$$

- In an **adiabatic limit:**

$$\left.\begin{aligned} &\omega = const. \Leftrightarrow \tilde{\omega} = const. \\ &\partial\phi^a_{\tilde{\omega}} / \partial\phi^b_* = 0 \end{aligned}\right\} \Longrightarrow \boxed{\tilde{\zeta} = \zeta}$$

Correlation functions

- Given the expansion for $\tilde{\zeta}$

$$\tilde{\zeta} = \delta\tilde{N} = \tilde{N}_a \delta\phi^a_{\tilde{\mathcal{R}}} + \frac{1}{2}\tilde{N}_{ab}\delta\phi^a_{\tilde{\mathcal{R}}}\delta\phi^b_{\tilde{\mathcal{R}}} + \dots$$

  we can determine its correlation functions if we know those of $\delta\phi^a_{\tilde{\mathcal{R}}}$

- The fields are minimally coupled in the E.F. and we know $\langle \delta\phi^a_{\tilde{\mathcal{R}}}\delta\phi^b_{\tilde{\mathcal{R}}}\rangle$ and $\langle \delta\phi^a_{\tilde{\mathcal{R}}}\delta\phi^b_{\tilde{\mathcal{R}}}\delta\phi^c_{\tilde{\mathcal{R}}}\rangle$ (See e.g. Elliston *et al* '12)

$$\langle \delta\phi^a_{\tilde{\mathcal{R}}}(\boldsymbol{k}_1)\delta\phi^b_{\tilde{\mathcal{R}}}(\boldsymbol{k}_2)\rangle = (2\pi)^3\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2)\frac{\tilde{H}^2}{2k^3}S^{ab}$$

  so we obtain the standard results

Einstein frame

$$\mathcal{P}_{\tilde{\zeta}}(k) = \tilde{N}_a\tilde{N}_b S^{ab}\left(\frac{\tilde{H}}{2\pi}\right)^2 \qquad \tilde{f}_{NL} = \frac{\tilde{N}_a\tilde{N}_b\nabla_c\nabla_d\tilde{N}S^{ac}S^{bd}}{\left[\tilde{N}_e\tilde{N}_f S^{ef}\right]^2}$$

Correlation functions

- In the Jordan frame we have the expansion

$$\zeta = \delta N = N_a \delta\phi^a_{\mathcal{R}} + \frac{1}{2} N_{ab} \delta\phi^a_{\mathcal{R}} \delta\phi^b_{\mathcal{R}} + \ldots$$

using the relation $\delta\phi^a_{\mathcal{R}} \leftrightarrow \delta\phi^a_{\tilde{\mathcal{R}}}$ and knowing $\langle \delta\phi^a_{\tilde{\mathcal{R}}} \delta\phi^b_{\tilde{\mathcal{R}}} \rangle$ and $\langle \delta\phi^a_{\tilde{\mathcal{R}}} \delta\phi^b_{\tilde{\mathcal{R}}} \delta\phi^c_{\tilde{\mathcal{R}}} \rangle$ we can determine correlation functions of ζ

Jordan frame

$$\Rightarrow \quad \mathcal{P}_\zeta(k) = \mathcal{N}_a \mathcal{N}_b S^{ab} \left(\frac{\tilde{H}}{2\pi} \right)^2 \qquad f_{NL} = \frac{\mathcal{N}_a \mathcal{N}_b \mathcal{N}_{cd} S^{ac} S^{bd}}{[\mathcal{N}_e \mathcal{N}_f S^{ef}]^2}$$

$$\mathcal{N}_a = N_a - \frac{f_a}{2f} \qquad \text{and} \qquad \mathcal{N}_{ab} = \nabla_a \nabla_b N - \frac{\nabla_a \nabla_b f}{2f} + \frac{f_a f_b}{2f}$$

- Exactly the same form as E.F. but with but $\tilde{N}_a \leftrightarrow \mathcal{N}_a$ etc

- We have

$$\mathcal{N}_a - \tilde{N}_a = N_a - N_a^{\tilde{w}} - \frac{f_b}{2f} \bigg|_{\diamond} \frac{\partial \phi^b_{\tilde{w}}}{\partial \phi^a_*}$$

$\Rightarrow$ **Equivalence in adiabatic limit follows from $\mathcal{N}_a = \tilde{N}_a$**

Analytically soluble examples

- Using $dN = Hdt$ and $W = V/f^2$ the slow-roll eom take the form

$$\frac{d\phi^a}{dN} = -2fh^{ab}\frac{W_b}{W}.$$

$\Longrightarrow$ **analytically soluble if**

a function of only a'th field

$$2fh^{ab}\frac{W_b}{W} = \frac{g^{(a)}(\phi^a)}{F(\boldsymbol{\phi})} \quad \Longrightarrow \quad \frac{1}{g^{(a)}(\phi^a)}\frac{d\phi^a}{dN} = -\frac{1}{F(\boldsymbol{\phi})}$$

a function of all the fields

- Taking $h_{ab} = \delta_{ab} \Rightarrow$ require **W is either sum or product separable**
- Introduce the new coordinates $q^a = qn^a$ satisfying

$$\ln q^a = \int \frac{d\phi^a}{g^{(a)}(\phi^a)} \qquad \text{and} \qquad \sum_a (n^a)^2 = 1.$$

- Eom now written as

$$\frac{d\ln q}{dN} = -\frac{1}{F(q,n^a)} \qquad \text{and} \qquad \frac{dn^a}{dN} = 0, \qquad \Longrightarrow \qquad \boxed{N = \int_\diamond^* F\, d\ln q}$$

Non-minimally coupled "spectator" example

- Take a two field example ϕ = inflaton, minimally coupled
  W product separable: χ = "spectator", non-minimally coupled but non-dynamical, i.e. $\chi' = 0$

- χ **does contribute to the curvature perturbation** due to its non-minimal coupling

- At linear order we find

$$\boxed{\zeta - \tilde{\zeta} \simeq \frac{f_\chi}{2f\epsilon_\diamond}\delta\chi_{\tilde{\mathcal{R}}}}$$

difference due to "spectator" field

(slow-roll parameter at final time)

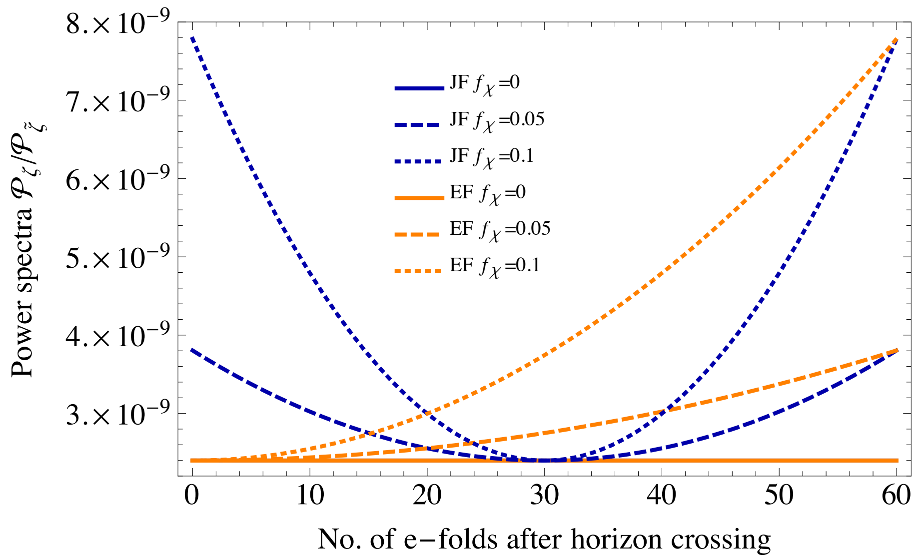
- **Difference a result of the difference in definition of the final constant energy surface:**

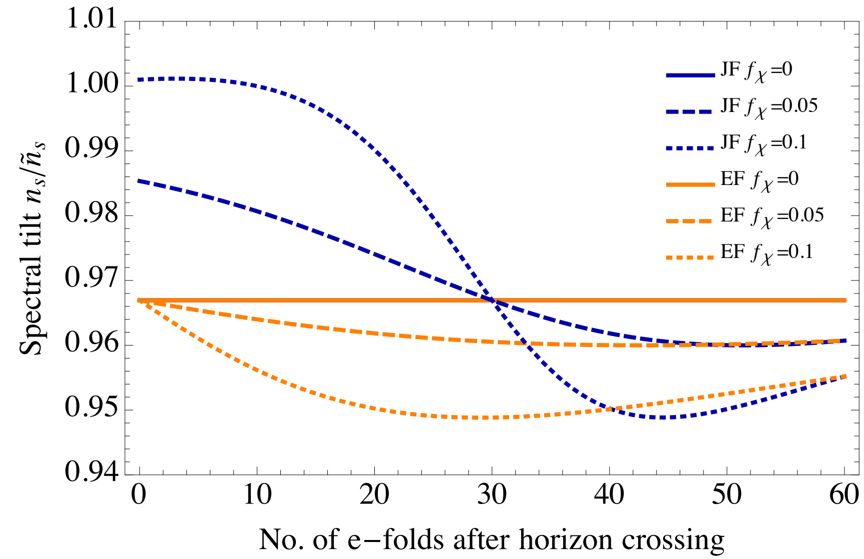
$$\text{JF:} \quad \rho = 3H^2 = \frac{fW}{2} \qquad\qquad \text{EF:} \quad \tilde{\rho} = 3\tilde{H}^2 = \frac{W}{4}$$

- For $|\zeta| \sim |\tilde{\zeta}|$ we consider $f_\chi/\sqrt{f} \sim \mathcal{O}(\sqrt{\epsilon})$

Non-minimally coupled "spectator" example

- Consider the explicit example $V = V^{(\phi)}V^{(\chi)}$ and $V^{(\phi)} = m^2\phi^{2p}$
- For $p=1,\quad 2f = V^{(\chi)} = 1,\quad W_\chi = W_{\chi\chi} = f_{\chi\chi} = 0,\quad m^2 = 1.94\times10^{-11}$ plot power spectra and tilts in JF and EF as functions of N:





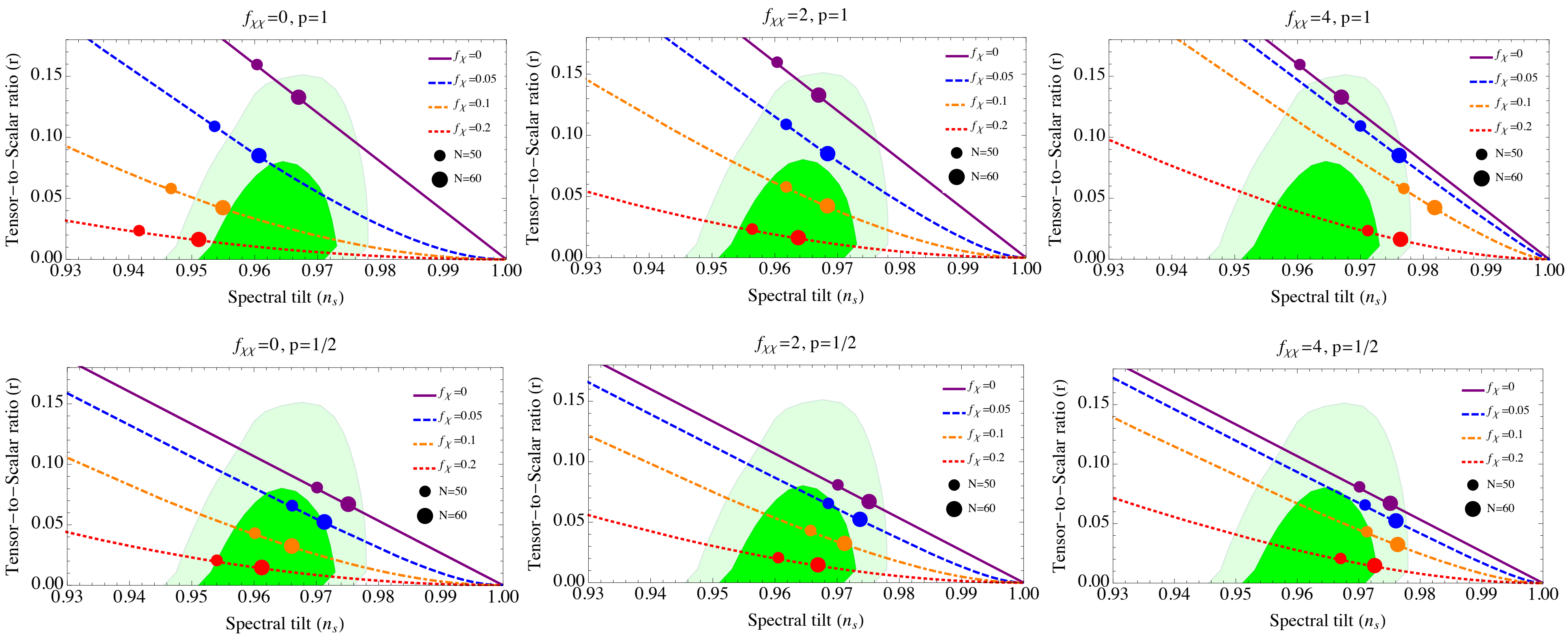
- **Evolution** in the two frames is **very different**
- The **difference** between frames goes as $1/\epsilon_\diamond$

$$\boxed{\zeta - \tilde{\zeta} \simeq \frac{f_\chi}{2f\epsilon_\diamond}\delta\chi_{\tilde{\mathcal{R}}}}$$

$\Rightarrow$
- **is initially large** as $\epsilon_\diamond \ll 1$
- **finally the frames agree to leading order** as $\epsilon_\diamond \sim 1$

Non-minimally coupled "spectator" example

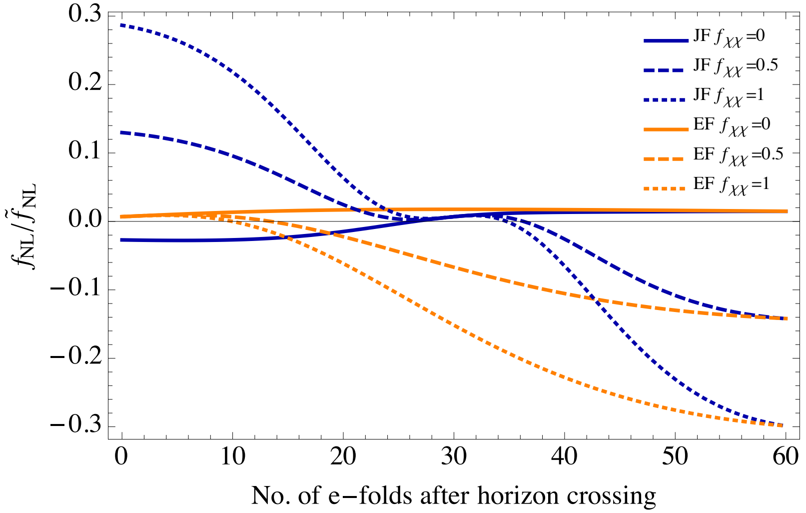
- For p=1, p=1/2 and a range of f_χ and $f_{\chi\chi}$ we plot end-of-inflation predictions for n_s and r.

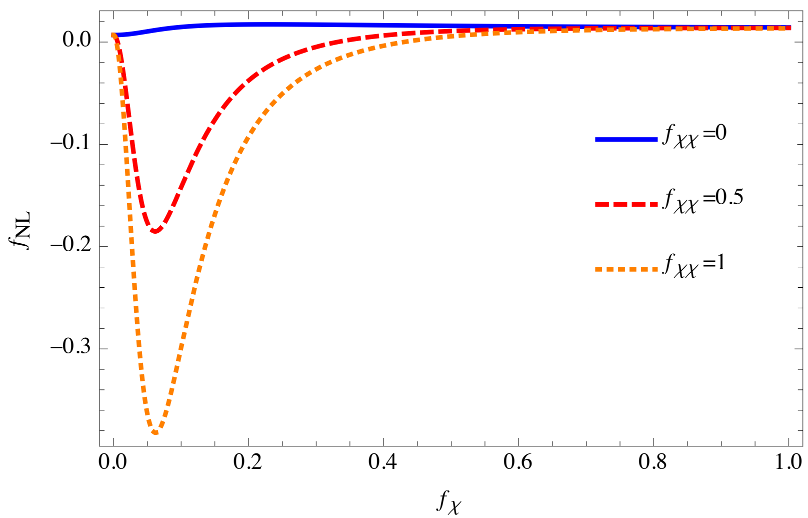


- Contribution from $\delta\chi$ **suppresses tensor-to-scalar ratio** and gives rise to **redder tilt as f_χ is increased.**
- **Non-zero $f_{\chi\chi}$ gives rise to a more blue-tilted spectrum**.
- **Predictions** can be brought **within 68% CL of Planck**

Non-minimally coupled "spectator" example

- Going beyond linear order we find $f_{NL}, \tilde{f}_{NL}, f_{NL} - \tilde{f}_{NL} \sim \mathcal{O}(1) \times f_{\chi\chi}$
- In our analytic calculation of f_{NL} we require the curvature of the Einstein frame field-space to be negligible, which in turn requires $f_{\chi\chi} \ll 1$





Upper plot: The evolution of f_{NL} and $\tilde{f}_{NL}$ for $f_{\chi} = 0.1$ and a range of $f_{\chi\chi}$ at the limit of validity of the analytic results.

Lower plot: Dependence of final f_{NL} on f_{χ} for same range of $f_{\chi\chi}$.

- **Evolution** in two frames **very different**
- Both **f_{NL} and $\tilde{f}_{NL}$ are very small**
- **Strong dependence of f_{NL} on f_{χ}**

Non-minimally coupled multi-brid example

M. Sasaki. '08 for original model

- Non-minimally coupled extension of the multi-brid inflation model with

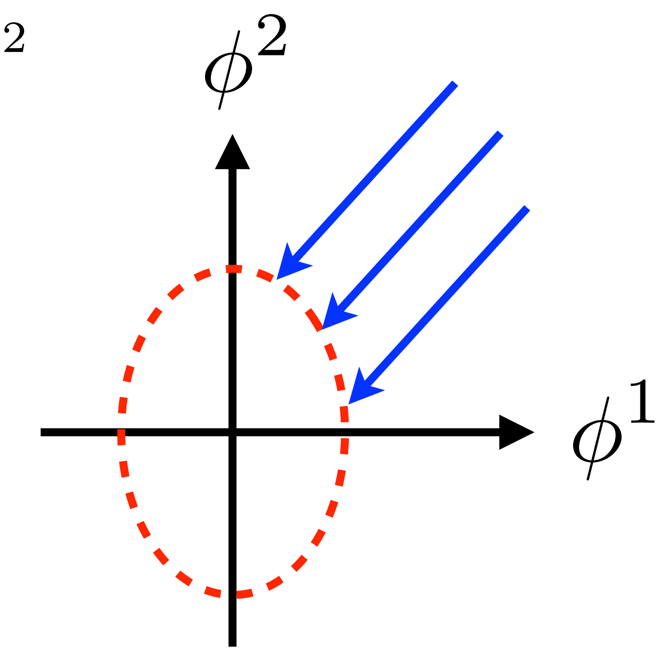
$$V = V_0 \exp\left[\sum_a m_a \phi^a\right] \qquad f = \sqrt{f_0}\exp\left[\sum_a \frac{z_a}{2}\phi^a\right]$$

- End of inflation determined by **tachyonic instability** of χ field:

$$V_0 = \frac{1}{2}\sum_a w_a^2 (\phi^a)^2 \chi^2 + \frac{\lambda}{4}\left(\chi^2 - \frac{\sigma^2}{\lambda}\right)^2 ,$$

- Becomes unstable for $\sum_a w_a^2 (\phi^a)^2 < \sigma^2$
- **End of inflation** condition **frame independent**
- In two field case:

$$\phi^1_\diamond = \sigma \cos\gamma / w_1 \qquad \phi^2_\diamond = \sigma \sin\gamma / w_2 .$$



Non-minimally coupled multi-brid example

- Find

$$\boxed{\begin{aligned}\delta N^\sigma = &\frac{1}{2f_\diamond}\frac{w_2\sin\gamma\delta\phi_*^2 + w_1\cos\gamma\delta\phi_*^1}{M_2w_2\sin\gamma + M_1w_1\cos\gamma} + \frac{1}{2f_\diamond}\frac{(w_1w_2)^2}{2\sigma}\frac{(M_1\delta\phi_*^2 - M_2\delta\phi_*^1)^2}{(M_2w_2\sin\gamma + M_1w_1\cos\gamma)^3}\\ &-\frac{N_*}{2}(z_1\delta\phi_*^1 + z_2\delta\phi_*^2) + \frac{N_*}{8}(z_1\delta\phi_*^1 + z_2\delta\phi^2)^2 - \frac{\delta\mathcal{S}}{8f_\diamond(M_2w_1\cos\gamma + M_2w_2\sin\gamma)^2},\end{aligned}}$$

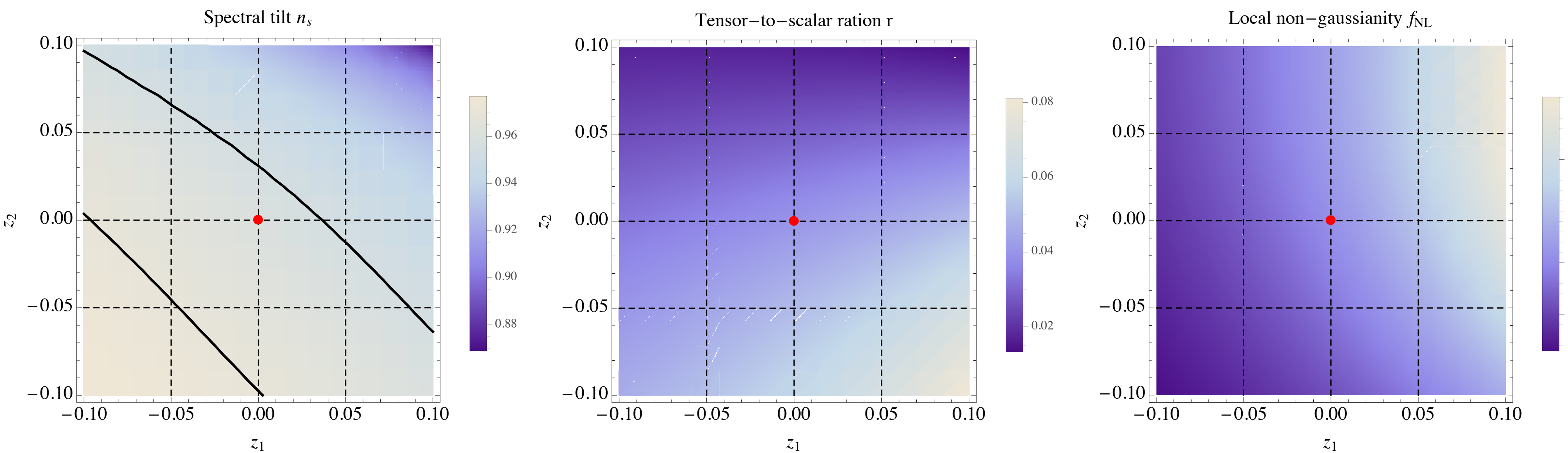
$$(M_1 = m_1 - z_1 \quad \text{and} \quad M_2 = m_2 - z_2)$$

- Terms on first line are also present in minimally coupled case.
- First two terms on 2nd line are **new terms due to non-minimal coupling** and **are significant**
- $\delta\mathcal{S}$ contains **additional second-order contributions**, but they are **negligible**

Non-minimally coupled multi-brid example

- To compare with original model, set

$$M_1^2=(m_1-z_1)^2=0.005,\quad M_2^2=(m_2-z_2)^2=0.035,$$
$$\gamma\to 0,\quad \omega_1=\omega_2=0.1,\quad 2\sqrt{f_0}=1\ \text{ and }\ N_*=60$$



- For $z_1=z_2=0$ have $(n_s,r,f_{NL})=(0.96,0.04,4.1)$ (red dots)
- **r and f_{NL} within 68% CL from Planck for all z_1 and z_2**, but do vary
- **Observational constraint $n_s=0.9697\pm0.0073$ can be used to constrain z_1 and z_2.** (see black contours)

Aside on observational equivalence

N. Deruelle & M. Sasaki 1007.3563
T. Chiba & M. Yamaguchi '13 1308.1142

- Fact that $\zeta \neq \tilde{\zeta}$ is an indication that they are not directly observable

- Conformally related frames are **observationally indistinguishable**
- The **physical interpretation may differ** from frame to frame

e.g. Start with Einstein gravity and FLRW metric:

$$ds^2 = a^2(\eta)\left(-d\eta^2 + \delta_{ij}dx^i dx^j\right) \quad \Longrightarrow$$ Observe **redshift effect** due to expansion

Make conformal transformation $\Omega = 1/a(\eta)$

$$d\tilde{s}^2 = -d\eta^2 + \delta_{ij}dx^i dx^j \quad \Longrightarrow$$ No expansion! What happened to redshift?

But, **electron masses now time dependent** $\tilde{m}(\eta) = \dfrac{m}{\Omega} = a(\eta)m$

$\Longrightarrow$ Atomic transition energies vary with time $\propto \tilde{m}(\eta)$ $\Longrightarrow$ **Observationally indistinguishable from redshift effect**

Conclusions

- Using the δN formalism we have determined the non-linear relation between ζ and $\tilde{\zeta}$. Found that
  - **definition of the initial flat surface does not effect $\zeta - \tilde{\zeta}$**
  - **definition of the final constant energy surface is important**

- Using the relation $\delta\phi^a_{\tilde{\mathcal{R}}} \leftrightarrow \delta\phi^a_{\mathcal{R}}$ **we could also determine the correlation functions of ζ as well as $\tilde{\zeta}$**

- Saw that ζ **and** $\tilde{\zeta}$ and their spectral properties **can evolve differently**

- In the non-minimally coupled "spectator" field model we found that the additional **contribution of the spectator field to the curvature perturbation tended to reduce the tensor-to-scalar ratio and allow for a tuneable tilt**, allowing us to bring predictions into agreement with recent observations.

- In the non-minimally coupled extension of the multi-brid inflation model we found that **observational constraints on the tilt could be used to constrain the form of non-minimal coupling**.