No-boundary proposal toward good inflation models

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Quantum Gravity/cosmology

Probability of observables that depend on models

Guideline to phenomenological models

Phenomenology of String/Particle/cosmology

Modified Gravity

Theoretical requirements?

observational expectations?

cosmological observations

Contents

1. What is the no-boundary proposal?
2. How to use the no-boundary wave function?
3. Good inflation models

Based on

- Hwang, Sahlmann and DY, arXiv:1107.4653
- Hwang, Lee, Sahlmann and DY, arXiv:1203.0112
- Hwang, Kim, Lee, Sahlmann and DY, arXiv:1207.0359
- Hwang, Lee, Stewart, DY and Zoe, arXiv:1208.6563
- Sasaki, DY and Zhang, arXiv:1307.5948
- Saito, Sasaki, DY and Zhang, in preparation
- Hwang, Park and DY, in preparation

What is the no-boundary proposal?

Brief introduction

Problem of singularity

The singularity theorem:

Our universe should begin from the initial singularity. How to resolve?

Maybe, by using the Schrodinger equation for fields:

so-called, the Wheeler-DeWitt equation.

$$\left(G_{ijkl}\frac{\delta}{\delta\gamma_{ij}}\frac{\delta}{\delta\gamma_{kl}}+\gamma^{1/2}\;{}^{(3)}R\right)\Psi[{}^{(3)}\mathcal{G}]=0$$

3-metric (and fields) ∈superspace

(quantized)
Hamiltonian constraint

wave function of universe

No-boundary proposal

What is the boundary condition of WDW eqn?

Perhaps, the ground state?

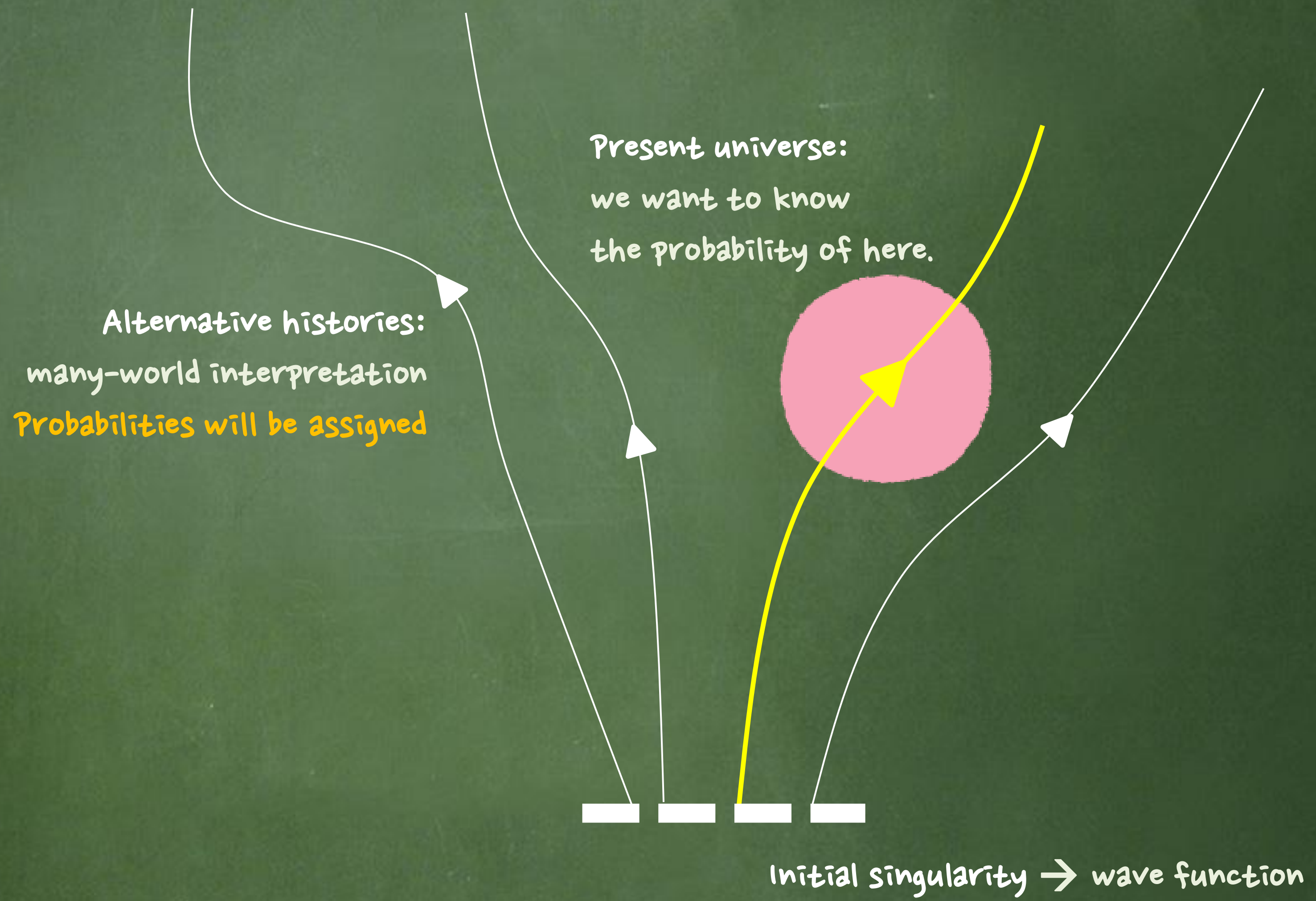
Hartle-Hawking wave function

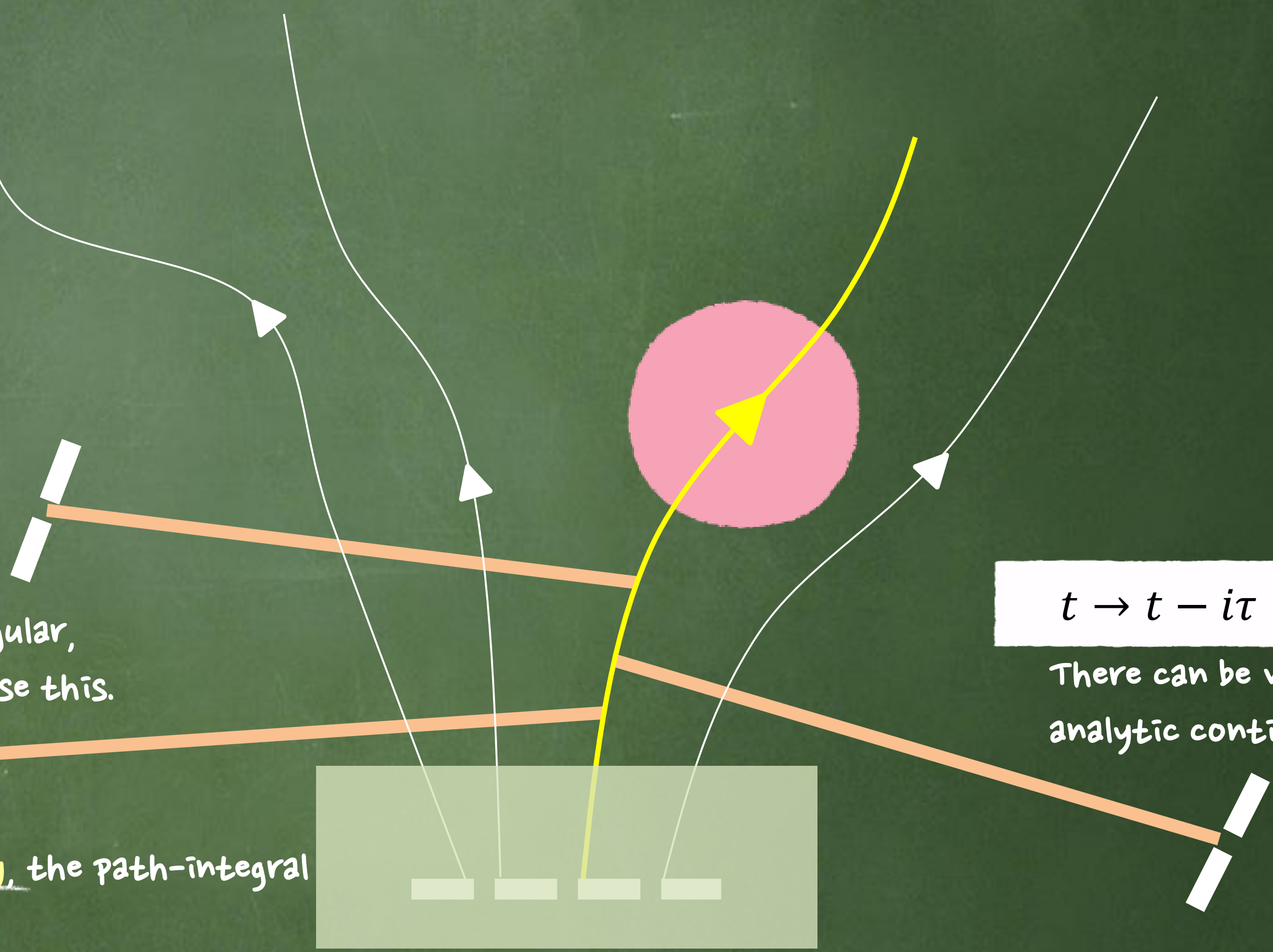
Euclidean action

$$\Psi_0[h_{ij}] = N \int \delta g \exp(-I_E[g])$$

path integral

over regular compact manifold


Present universe:
we want to know
the probability of here.
Alternative histories:
many-world interpretation
Probabilities will be assigned
Initial singularity → wave function

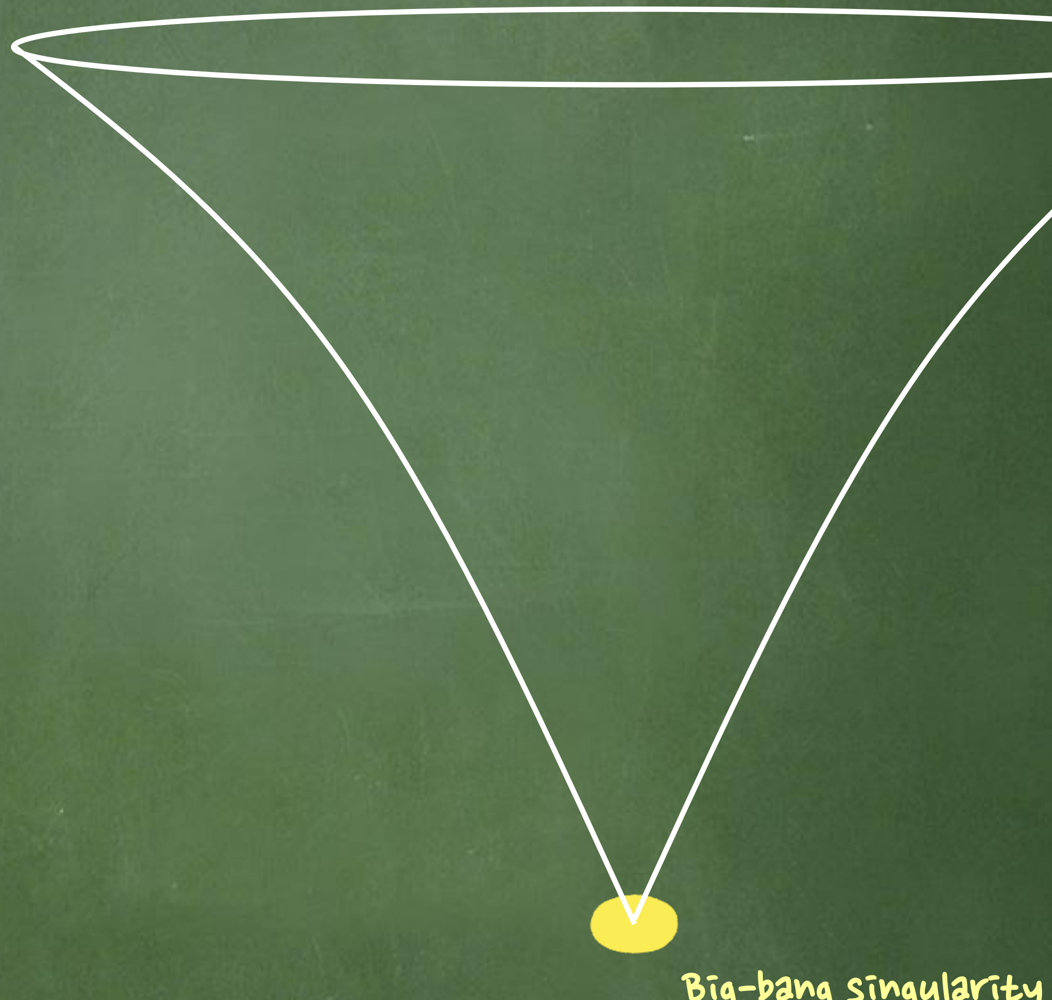


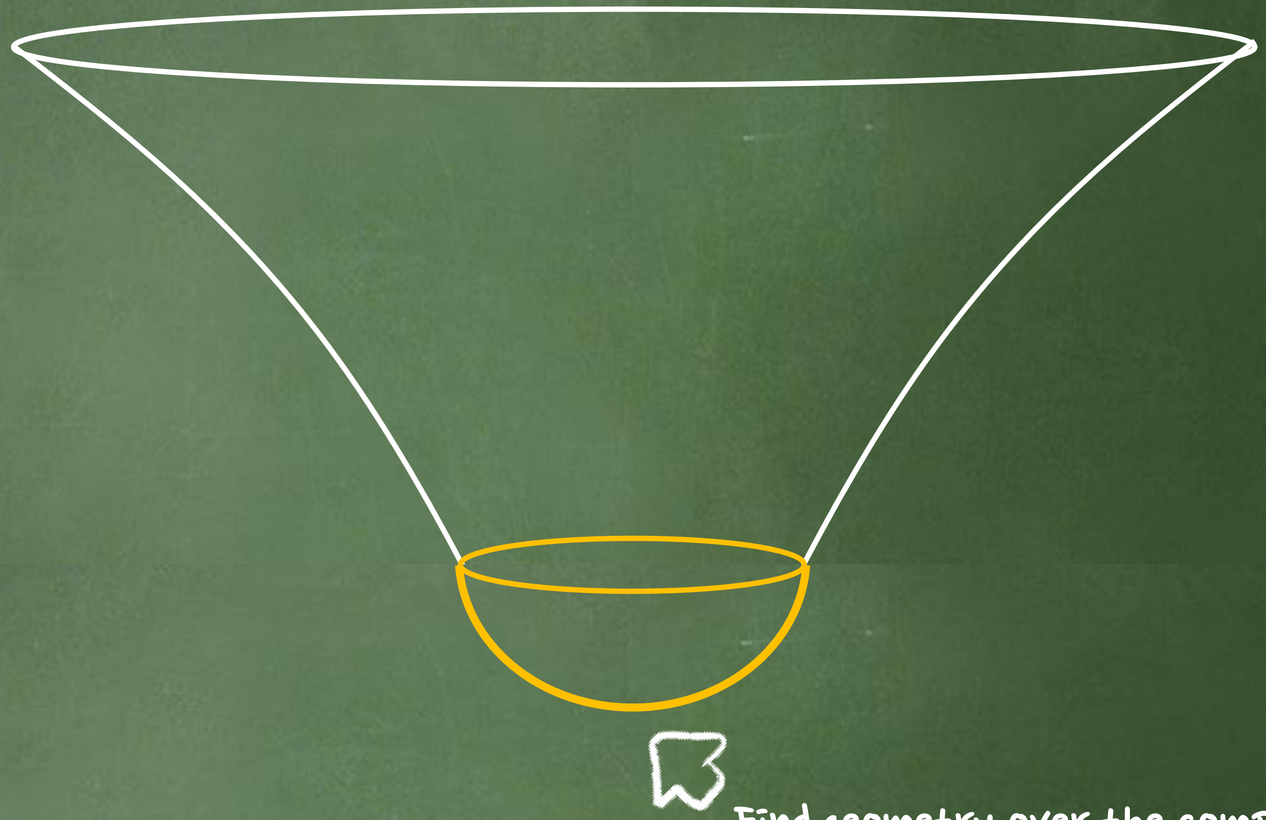
If this path is regular,
then we will choose this.

$$t \to t - i\tau$$

There can be various analytic continuations.

Due to analyticity, the path-integral still makes sense,
even though we analytically continue to Euclidean time.


Big-bang singularity



Find geometry over the complex time,
until the geometry to be regular.

How to use the no-boundary wave function?

use of fuzzy instantons

Fuzzy instantons

In general, all functions (metric and fields) should be complex functions (Halliwell and Hartle, 1990).

An on-shell solution of Euclidean complexified fields are called by $\boxed{\text{fuzzy instanton}}$ (Hartle, Hawking and Hertog, 2007).

How to calculate path integral?

Approximation 1: Mini-superspace

$$ds_{\mathrm{E}}^2 = N^2(\eta)d\eta^2 + \rho^2(\eta)(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2))$$

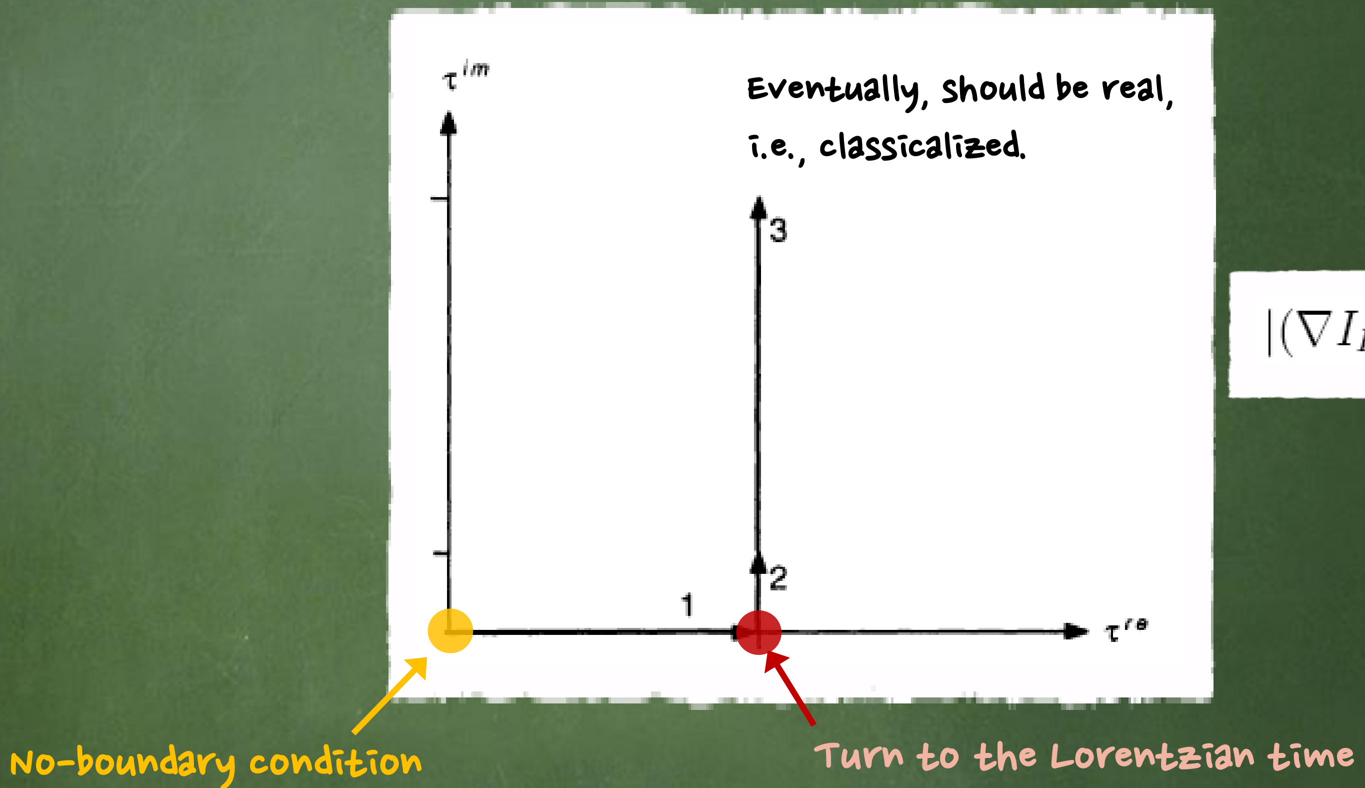
Approximation 2: Steepest-descent (sum-over fuzzy instantons)

$$\Psi_{\mathrm{HH}}(q) \approx \sum_{\mathrm{ext}} P(q_{\mathrm{ext}}) e^{-S_{\mathrm{E}}[q_{\mathrm{ext}}]/\hbar}$$

Additional constraint to fuzzy instantons: classicality

$$|(\nabla I_R)^2| \ll |(\nabla S)^2|$$

Time contour



$$|(\nabla I_R)^2| \ll |(\nabla S)^2|$$

Example (Einstein gravity)

$$S_{\mathrm{E}} = -\int d^4x \sqrt{+g}\left(\frac{1}{16\pi}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right)$$

$$\begin{aligned}\ddot{\phi} &= -3\frac{\dot{\rho}}{\rho}\dot{\phi} \pm V', \\ \ddot{\rho} &= -\frac{8\pi}{3}\rho\left(\dot{\phi}^2 \pm V\right)\end{aligned}$$

$$\begin{aligned}\rho(0)^{\mathfrak{Re}} = \rho(0)^{\mathfrak{Im}} &= 0, \\ \dot{\rho}(0)^{\mathfrak{Re}} &= 1, \\ \dot{\rho}(0)^{\mathfrak{Im}} &= 0, \\ \dot{\phi}(0)^{\mathfrak{Re}} = \dot{\phi}(0)^{\mathfrak{Im}} &= 0.\end{aligned}$$

Initial conditions for no-boundary

$$\phi(0) = \phi_0 e^{i\theta}$$

The remained initial conditions.

For given initial field amplitude,
by tuning the phase angle and the turning point,
we find a classical fuzzy instanton!

$$\underline{\rho}(t=0) = \rho(\eta = X), \quad \underline{\dot{\rho}}(t=0) = i\dot{\rho}(\eta = X),$$
$$\underline{\phi}(t=0) = \phi(\eta = X), \quad \underline{\dot{\phi}}(t=0) = i\dot{\phi}(\eta = X).$$

Junction conditions at the turning point

Summarize

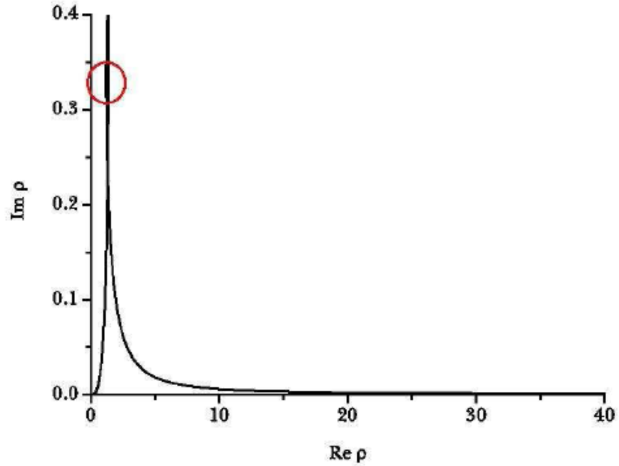
Step 1: We impose the mini-superspace metric.

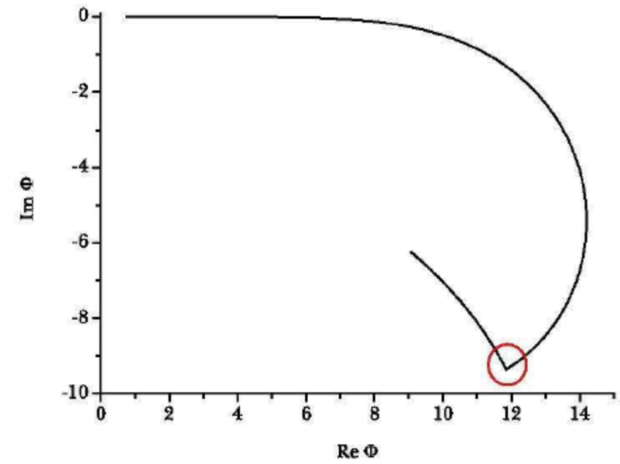
Step 2: We only consider on-shell solutions (fuzzy instantons), that begins from the no-boundary condition.

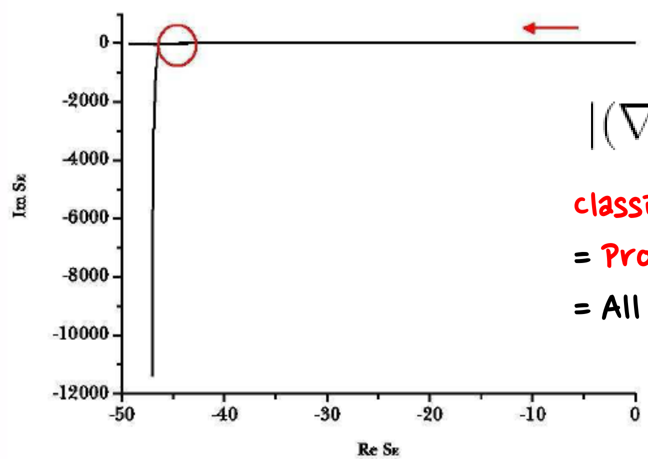
Step 3: Between on-shell fuzzy instantons, we only restrict that satisfies classicality.

- on-shell: dof = 8+1
- on-shell + no-boundary: dof = 8+1 − 6
- on-shell + no-boundary + classicality: dof = 8+1 − 6 − 2 = 1

One typical example of fuzzy instanton







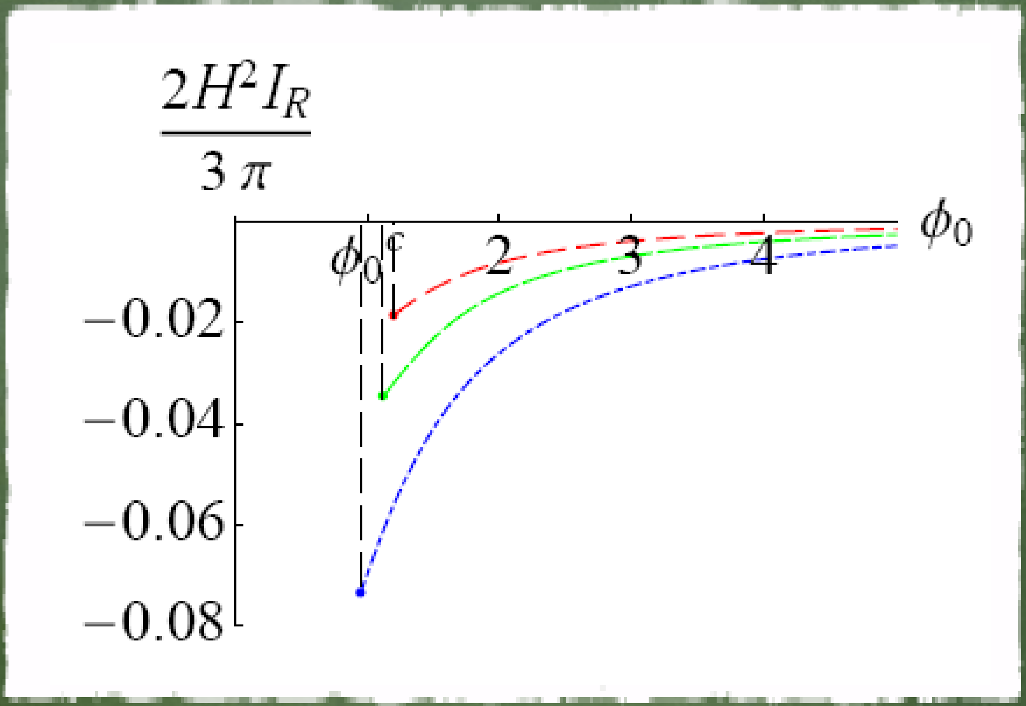
$$|(\nabla I_R)^2| \ll |(\nabla S)^2|$$

classicality is satisfied for a long Lorentzian time.

= Probability is a constant over the time.

= All fields are reallized.

Example (Einstein gravity, quadratic potential)



(Hartle, Hawking and Hertog, 2007)

Good inflation models

Preference of large e-foldings

Does this prefer inflation?

Traditionally, Euclidean probability does not prefer inflation.

Possible answers:

1. No inflation (Ekpyrotic, big bounce, string gas cosmology, etc.)
2. Not ground state (Vilenkin's tunneling proposal)
3. Not wave function (Susskind, landscape + multiverse + anthropic)
4. Additional weighting (Hartle-Hawking-Hertog)

Is there any better explanation, apart from these unsatisfactory opinions?

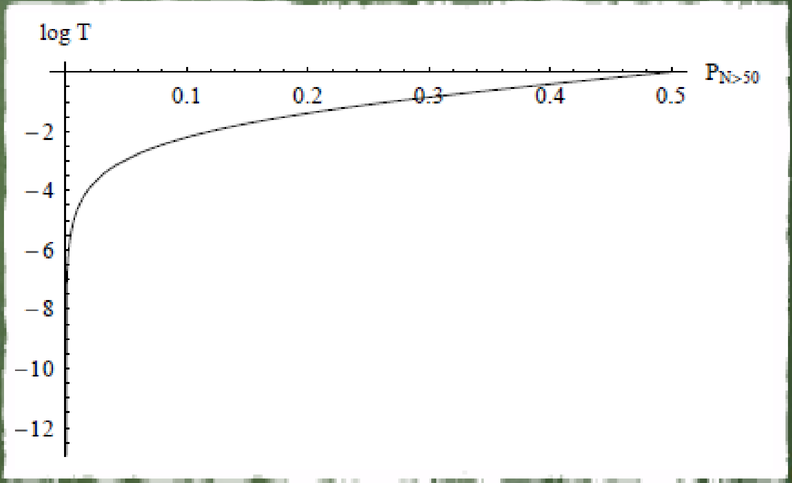
criterion of a good model (Hwang, Park and DY, in preparation)

Typicality for a given hypothesis

$$\mathcal{T} \equiv \frac{P[\mathcal{N} \geq 50]}{P[\mathcal{N} < 50]}$$

For a given probability cutoff, there is corresponding typicality bound.

If the typicality is smaller than the bound, then we reject 棄却 the hypothesis.



criterion of a good model (Hwang, Park and DY, in preparation)

Reduced action that does not depend on energy scale of inflation but depend on the shape of potential

$$\Delta \equiv \left| \tilde{S}_{\mathrm{E}}[\phi_{\mathrm{M}}] - \tilde{S}_{\mathrm{E}}[\phi_{\mathrm{top}}] \right|$$

Ratio of field space that allows sufficient and insufficient e-folding.

$$\Xi \equiv \log \frac{\phi_{50} - \phi_{\mathrm{top}}}{\phi_{\mathrm{M}} - \phi_{50}}$$

Then, the typicality is presented by the competition of three factors: potential shape, energy scale, and the field space of large e-folds

$$-\frac{2}{V_0}\Delta + \Xi \gtrsim \log \mathcal{T}_{\mathrm{cut}}$$

Three ways to prefer inflation

1. The first inflation began at large energy scale.

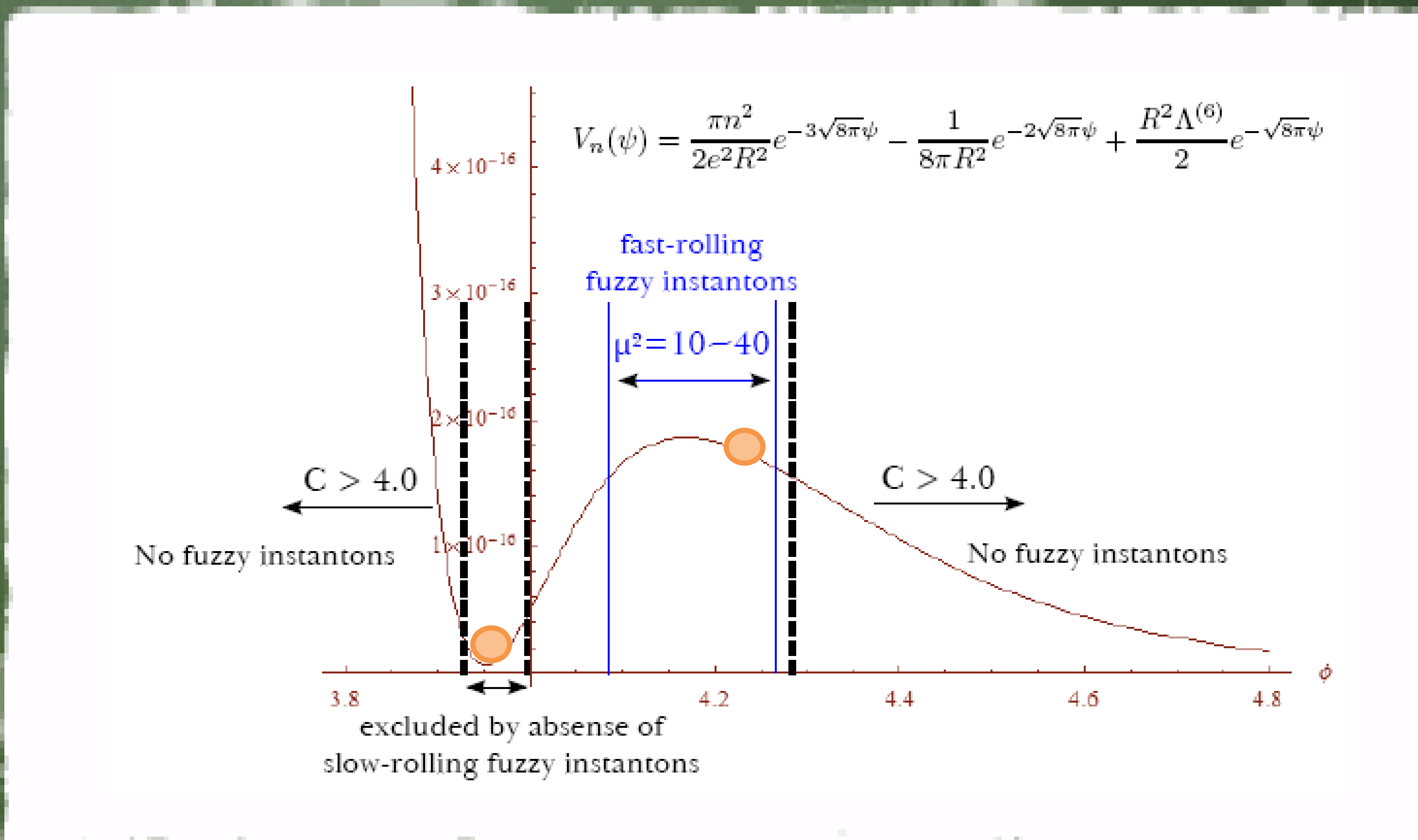
$$-\frac{2}{V_0}\Delta + \Xi \gtrsim \log \mathcal{T}_{\rm cut}$$

2. Potential shape is finely tuned.

3. There is something new effects.

CASE1: First inflation was near-Planck scale?

(Hwang, Sahlmann and DY, 2011; Hwang, Lee, Sahlmann and DY, 2012)



1/2 stable vs. 1/2 unstable

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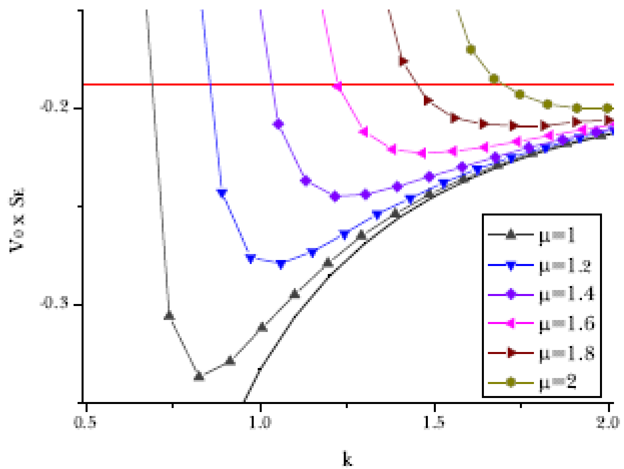

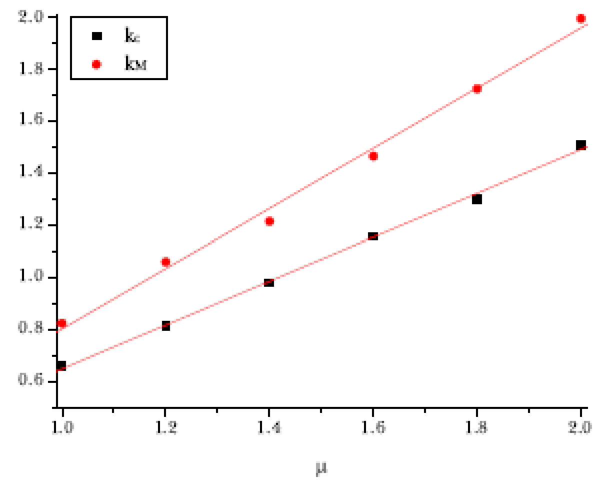

$V_0 \simeq 1$ limit

$$\langle \mathcal{N} \rangle \simeq 17.59 \times \mu^{-1.93} \times k_{\min}[\mu] + 17.59 \times 1.44 \times \mu^{-1.93},$$

$$\Delta \mathcal{N} \simeq 25.33 \times \mu^{-1.93}.$$

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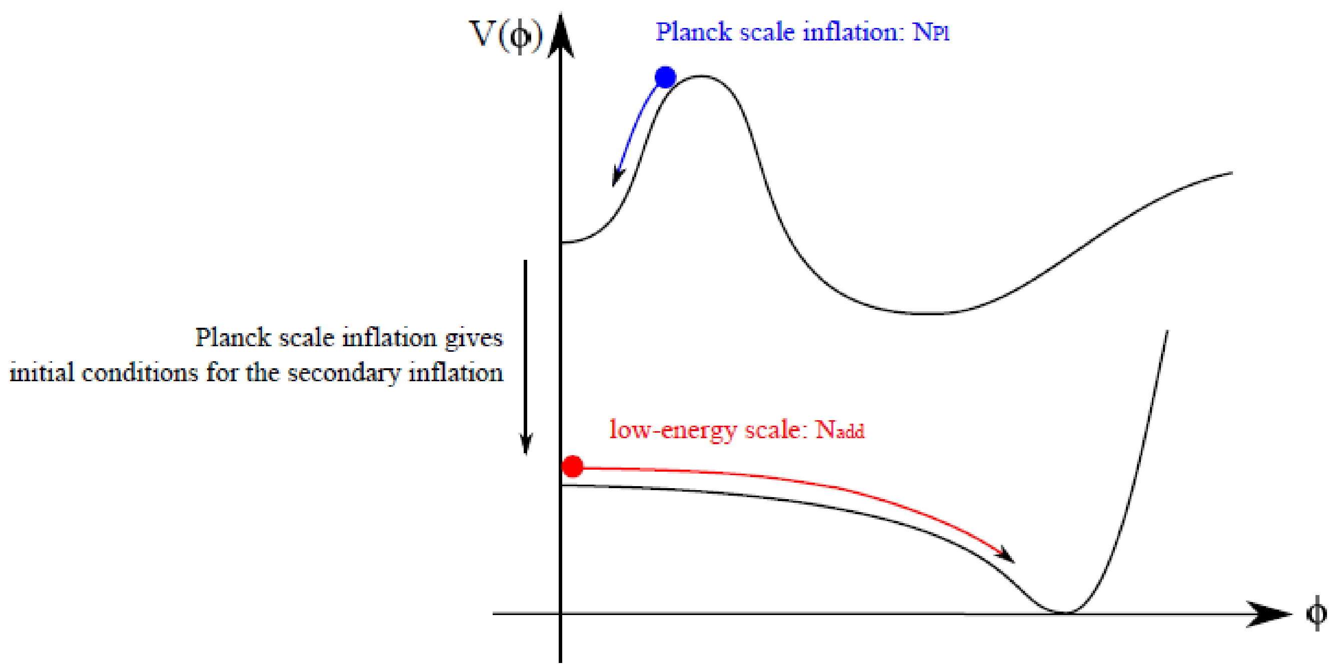
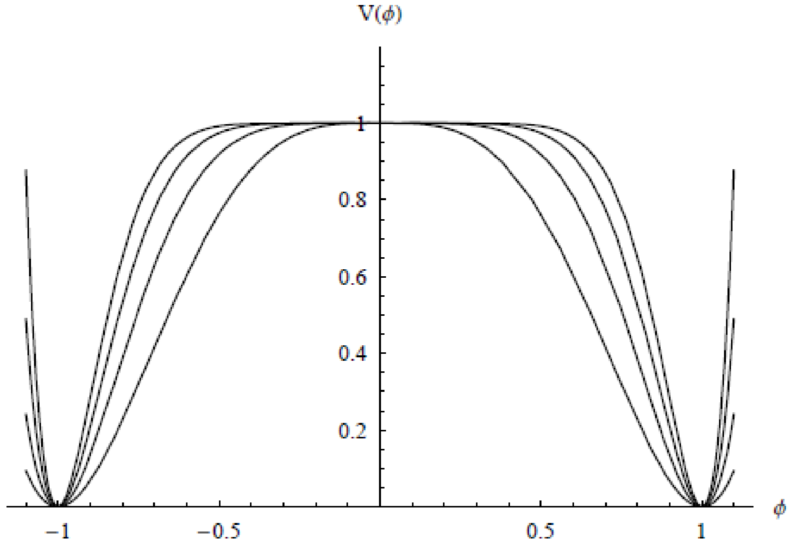
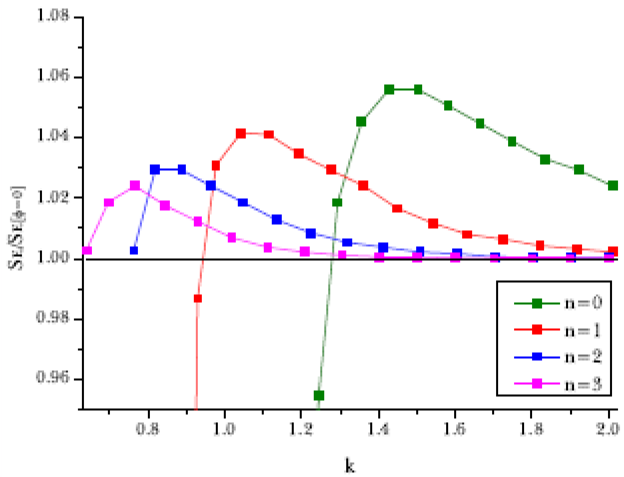


CASE2: Fine-tuning of the potential (Hwang, Park and DY, in preparation)





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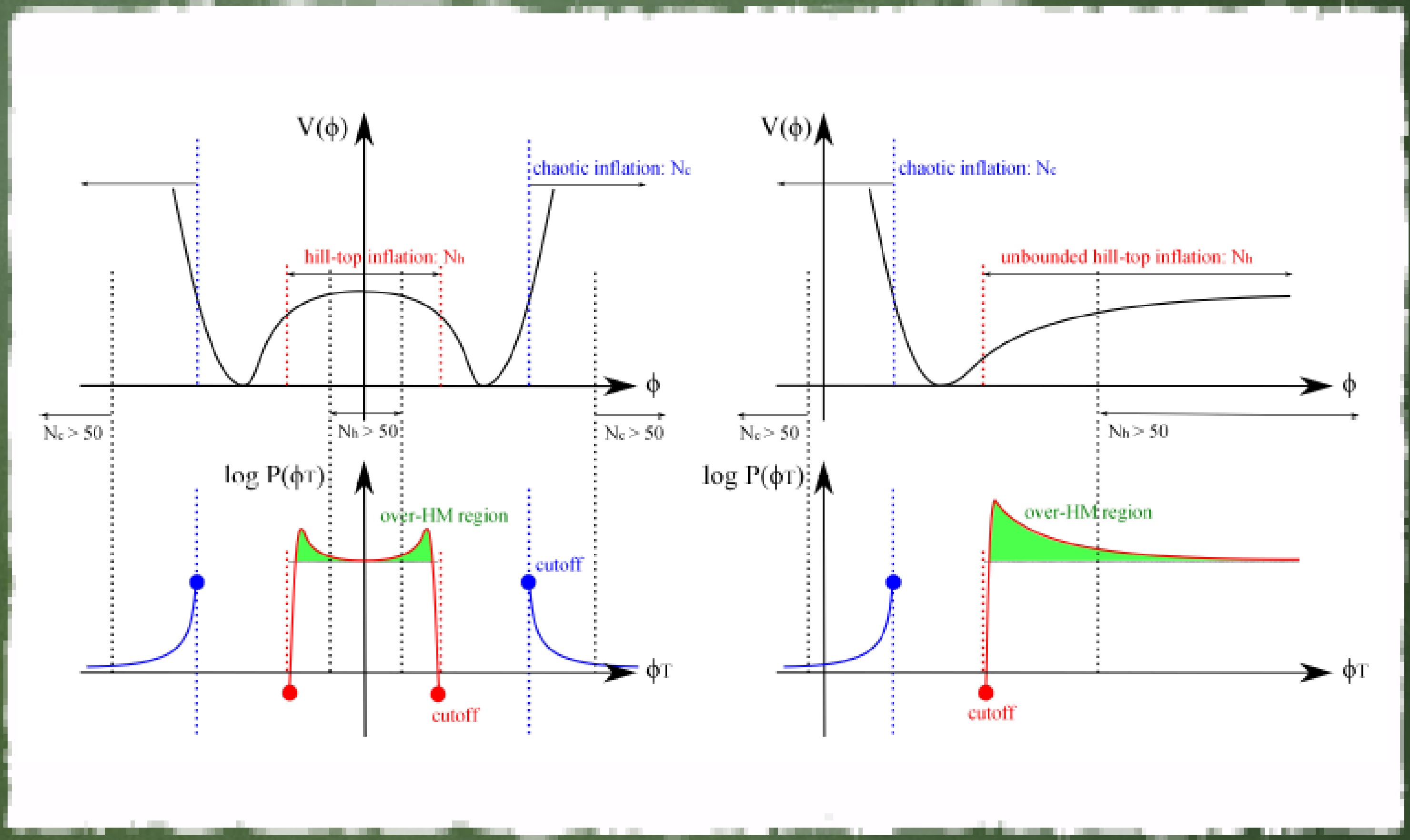
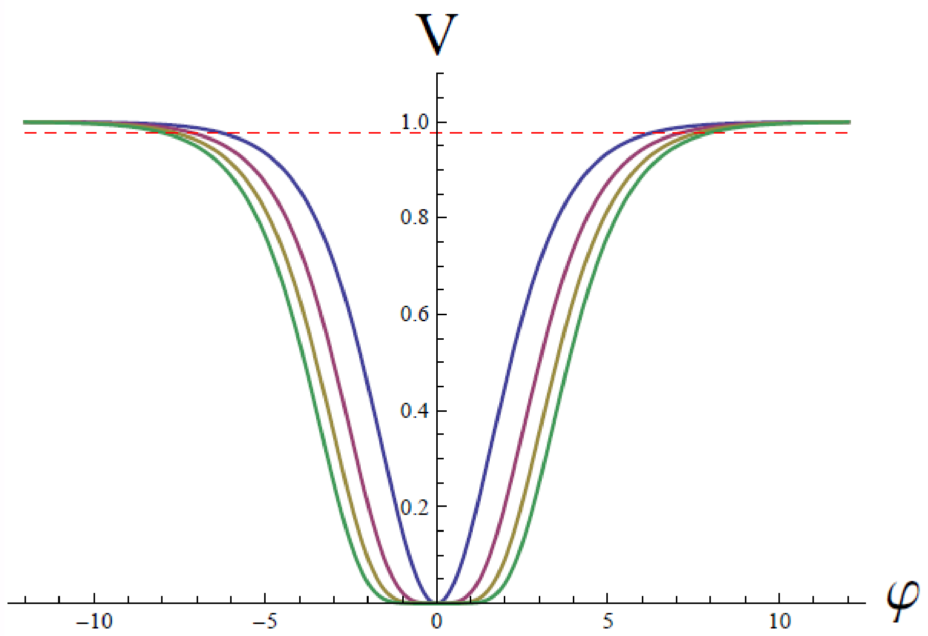


CASE2: Starobinski-like model (Kallosh and Linde, 2013)

$$\mathcal{L}_{\text{total}} = \sqrt{-g_J}\left[\frac{R(g_J)}{2}\left(1-\frac{\phi^2}{6}\right) - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4}\left(\phi^2-6\right)^2\right]$$



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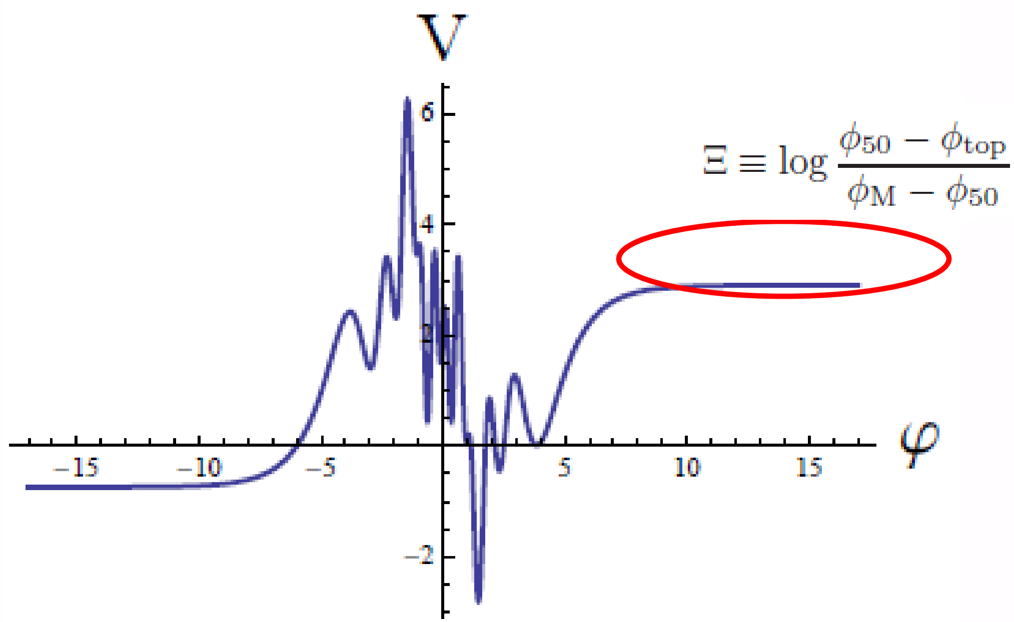
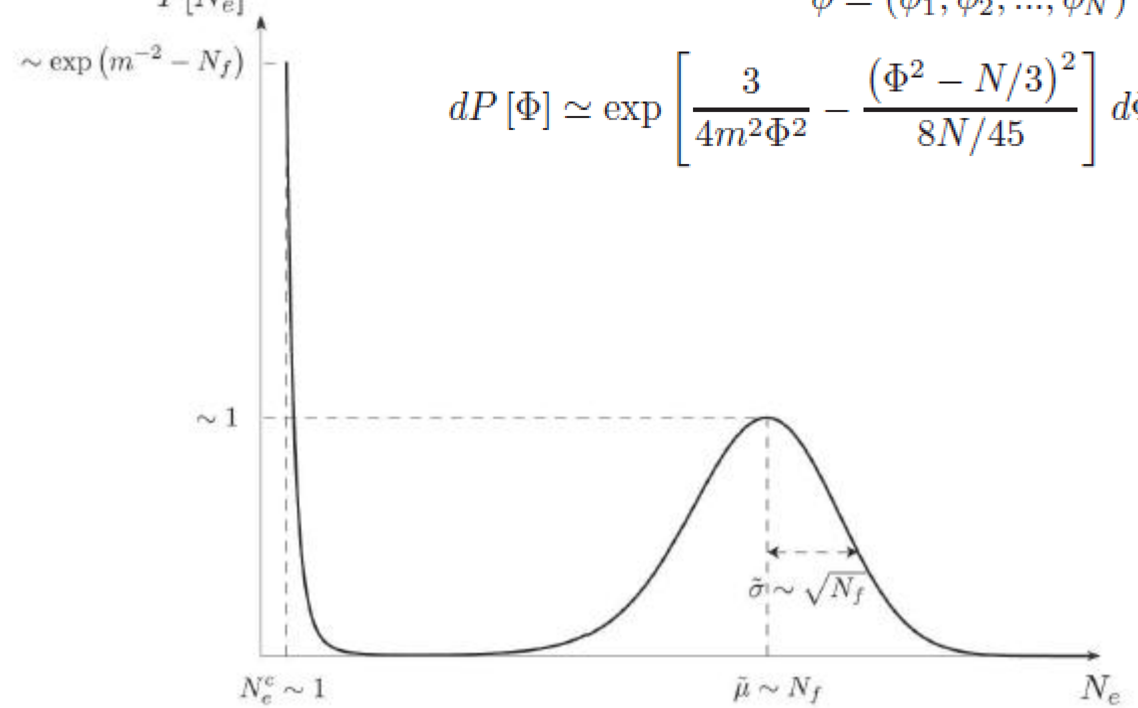


CASE3: New ingredient from multi-field inflation

(Hwang, Kim, Lee, Sahlmann and DY, 2012)

$$\vec{\phi} = (\phi_1, \phi_2, ..., \phi_N)$$

$$dP\,[\Phi] \simeq \exp\left[\frac{3}{4m^2\Phi^2} - \frac{\left(\Phi^2 - N/3\right)^2}{8N/45}\right] d\Phi$$

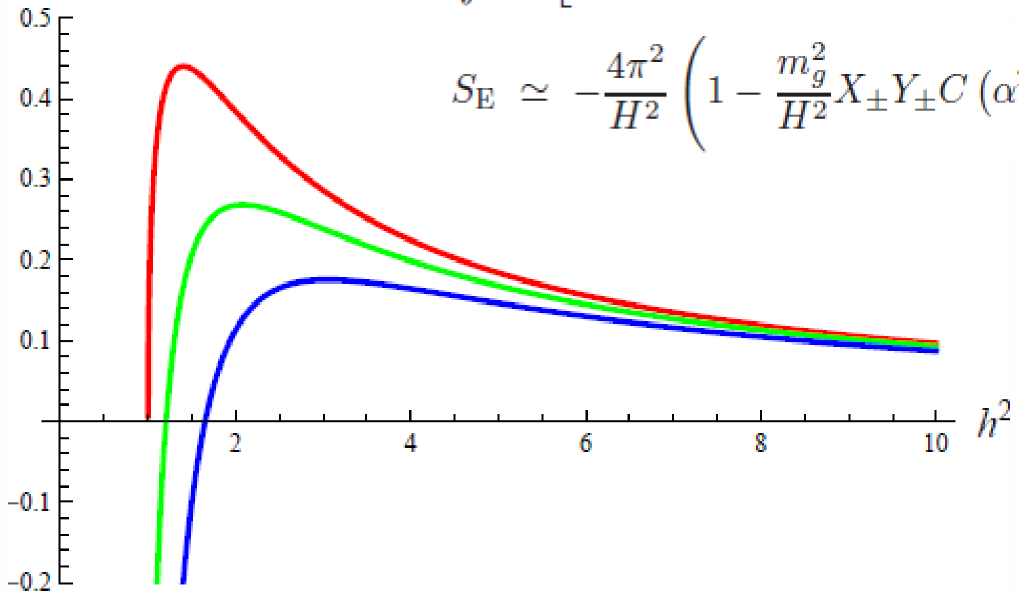


CASE3: New ingredient from massive gravity

(Sasaki, DY and Zhang, 2013)

$$S_{\mathrm{E}} = 2\pi^2 \int d\tau \left[2a^3 V_{\mathrm{eff}} - 6a - m_g^2 a^3 Y_{\pm} \sqrt{-\dot{f}^2} \right]$$

$$S_{\mathrm{E}} \simeq -\frac{4\pi^2}{H^2} \left(1 - \frac{m_g^2}{H^2} X_{\pm} Y_{\pm} C\left(\alpha^2\right) \right)$$



conclusion

- Now is the time to select inflation models.
- No-boundary wave function is useful to judge a good inflation hypothesis.
- There are three ways to satisfy a good inflation model.
  1. If inflation began at the high energy scale, then it can prefer large e-foldings, as well as moduli/dilaton stabilization.
  2. Starobinski-like model can be helpful to explain sufficiently large e-foldings, although we need justification.
  3. New ingredients can be introduced by multi-field dynamics or modified gravity (e.g., massive gravity).