Hawking-Moss and Coleman-de Luccia instantons in dRGT Massive Gravity

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Based on:

YZ, Ryo Saito and Misao Sasaki, JCAP 02(2013)029 [1210.6224]
Misao Sasaki, Dong-han Yeom and YZ, 1307.5948
Ryo Saito, Misao Sasaki, Dong-han Yeom and YZ, in preparison

Outline

1. A Review on Massive Gravity
2. Setup of model
3. Hawking-Moss solutions
4. Coleman-de Luccia solutions
5. Conclusion and Future Prospects

1. A review on Massive Gravity

"Can a graviton have mass ?"

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h| \ll 1$$

To the lowest order in h, one finds the Lagrangian:

$$L = L_{\mathrm{EH}}(h) + \frac{m_g^2}{2}\left(h_{\mu\nu}h^{\mu\nu} + \alpha h^2\right) \ ,$$

decompose $h_{\mu\nu} = h^{\perp}_{\mu\nu} + \partial_{(\mu}A^{\perp}_{\nu)} + \partial_\mu\partial_\nu\chi,$

where $\partial^\mu h^{\perp}_{\mu\nu} = \partial^\mu A^{\perp}_\mu = 0,$

$$L \supset -\frac{m_g^2}{2}\left[(\partial_\mu\partial_\nu\chi)^2 + \alpha(\Box\chi)^2\right] \ ,$$

So to avoid higher-order derivatives, we impose

$$\alpha = -1 \quad \Longrightarrow \quad \text{Fierz-Pauli 1939}$$

$$\boxed{L = L_{\mathrm{EH}}(h) + \frac{m_g^2}{2}\left(h_{\mu\nu}h^{\mu\nu} - h^2\right) \ ,}$$

- The unique massive gravity theory in linear level without ghost in Minkowski background;
- Diffeomorphism invariance is broken due to mass term.

- Boulware-Deser ghost (Boulware & Deser '72)

  If consider non-Minkowski background (e.g. FLRW), there appears a sixth mode which is a ghost

General Relativity (GR): $$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}R\,,$$

In 3+1 dim, for symmetric tensor $g_{\mu\nu}$, the propagating degrees of freedom (dof) can be counted as:

$$6 \quad -4 \qquad = 2$$

Lagrangian multiplier

Helicity ± 2

Such situation changes in the Massive Gravity Theory.

In Massive Gravity (MG), the mass of graviton is non-vanishing, which breaks the gauge invariance

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}[R(g) - m^2 V(g)]$$

$$\supset -\frac{m^2}{16\pi G}\int d^4x \sqrt{\gamma} N V(\gamma, N, N^i)$$

Generally speaking, the dof is

$$6 - 0 = 6$$

No Lagrangian multiplier…

Helicity $\pm 2, \ \pm 1, \ 0,$

In Ma shing,
which

Gene

Recently, a non-linear construction of massive gravity theory (dRGT) is proposed, where the BD ghost is removed by specially designed non-linear terms, so that the lapse function N becomes a Lagrangian Multiplier, which removes the ghost degree of freedom.

< A simple example >

physical $$ds^2 = -N^2 dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)\ ,$$

reference $$ds_f^2 = -dt^2 + dx^2\ ,$$

set $$N^i = 0\,,$$

$$g^{-1}f = \left(\begin{array}{cc} -1/N^2 & 0 \\ 0 & \gamma^{ik} \end{array}\right)\left(\begin{array}{cc} -1 & 0 \\ 0 & \delta_{kj} \end{array}\right) = \left(\begin{array}{cc} 1/N^2 & 0 \\ 0 & \gamma^{ik}\delta_{kj} \end{array}\right)$$

define $$\boxed{K^\mu_\nu \equiv \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu_{\ \nu}},$$

$$\Rightarrow \quad K^\mu_\nu = \left(\begin{array}{cc} 1 - 1/N & 0 \\ 0 & \delta^i_j - \sqrt{\gamma^{ik}\delta_{kj}} \end{array}\right)$$

$$\boxed{L = L_{\rm EH} - m_g^2 M_{\rm pl}^2 \sqrt{-g}\,{\rm det}'\left(\delta^\mu_\nu + \beta K^\mu_\nu\right)},$$

$$L = L_{\rm EH} - m_g^2 M_{\rm pl}^2 \sqrt{-g}\,\underline{\det{}'\left(\delta^\mu_\nu + \beta K^\mu_\nu\right)}\ ,$$

$$N\sqrt{\gamma}$$

$$\left[1+\beta\left(1-\frac{1}{N}\right)\right]\det\left((1+\beta)\delta^i_j - \beta\sqrt{\gamma^{ik}\delta_{kj}}\right)$$

$$L = L_{\rm EH} - m_g^2 M_{\rm pl}^2 \sqrt{\gamma}[N(1+\beta)-\beta]\det\left((1+\beta)\delta^i_j - \beta\sqrt{\gamma^{ik}\delta_{kj}}\right)$$

Mass term is linear in lapse function $\Longrightarrow$ Langrangian multiplier

$$\frac{\partial L}{\partial N} = H - m_g^2 M_{\rm pl}^2 \sqrt{\gamma}(1+\beta)\det\left((1+\beta)\delta^i_j - \beta\sqrt{\gamma^{ik}\delta_{kj}}\right) = 0$$

Recover the Hamitonian constraint

For non-vanishing shift function case, the situation becomes more complicated, but we can still recover the Hamiltonian constraint by redefining a new shift function:

$$N^i = n^i + Nm^i(\gamma_{ij},\ n^i)\ ,$$

So that the corresponding mass term again is linear in lapse function:

$$N\sqrt{g^{-1}f} = A(\gamma_{ij}, n^i) + NB(\gamma_{ij},\ n^i)\ .$$

Non-linear Massive Gravity (dRGT)

C. de Rham, G. Gabadadze, Phys. Rev. D 82, 044020 (2010);
C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett 106, 231101 (2011);
S. F. Hassan and R. A. Rosen, JHEP 1107, 009 (2011)

$$S_{MG} = \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right],$$

where $[\mathcal{K}] = tr\left(K_\mu^\nu\right)$

$$\mathcal{L}_2 = \frac{1}{2} \left([\mathcal{K}]^2 - [\mathcal{K}^2] \right),$$

$$\mathcal{L}_3 = \frac{1}{6} \left([\mathcal{K}]^3 - 3 [\mathcal{K}] [\mathcal{K}^2] + 2 [\mathcal{K}^3] \right),$$

$$\mathcal{L}_4 = \frac{1}{24} \left([\mathcal{K}]^4 - 6 [\mathcal{K}]^2 [\mathcal{K}^2] + 3 [\mathcal{K}^2]^2 + 8 [\mathcal{K}] [\mathcal{K}^3] - 6 [\mathcal{K}^4] \right),$$

$$\mathcal{K}_\nu^\mu \equiv \delta_\nu^\mu - \sqrt{g^{\mu\sigma} G_{ab}(\phi) \partial_\nu \phi^a \partial_\sigma \phi^b}.$$

fiducial metric

Stuckelberg field

Self-accelerating solution is found in context of non-linear massive gravity, where two branches exist with effective cosmological constant consists of a contribution from mass of graviton. A. E. Gumrukcuoglu et. al. JCAP 106, 231101(2011);

$$\Lambda_\pm = -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right)\pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right],$$

There seems to be some hope to explain the current acceleration, but…

1. Very small m_g^2 from observation;

2. Non-linear instability problem when $m_g^2 = const$.

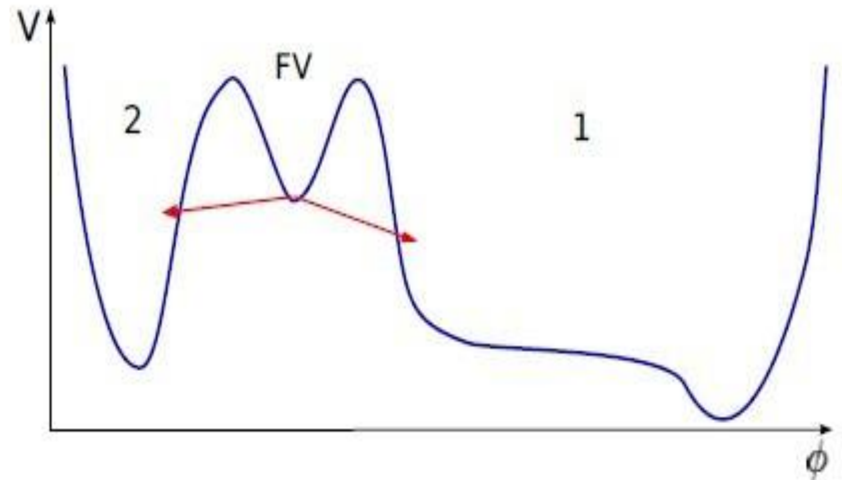
A. De Felice et. al. Phys. Rev. Lett. 109, 171101 (2012)

It is interesting and necessary to consider the mass-varying case in dRGT massive gravity theory, where the mass of graviton decays from some large value to the current value: e.g. $m_g = m_g(\sigma)$

One hopeful scenario is the dependence on a tunneling field.

• the field can (and will) tunnel from a metastable minimum to a lower one;

• this process is driven by instanton.



S. Coleman and F. de Luccia, Phys.Rev. D21, 3305, (1980)

As a first step, we study the stability of a vacuum in the context of non-linear Massive Gravity with constant graviton mass

2. Setup of model

$$S = S_{MG} + S_m,$$

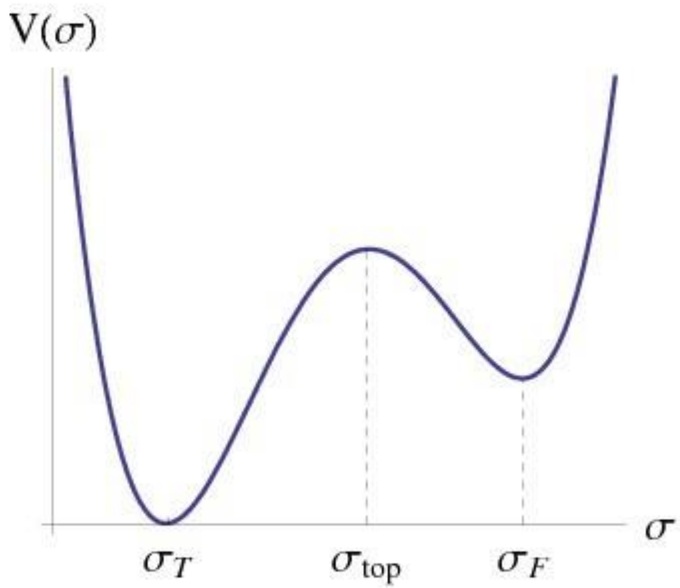
$$S_m \equiv -\int d^4x \, \sqrt{-g} \left[\frac{1}{2}(\partial\sigma)^2 + V(\sigma) \right],$$

- potential $V(\sigma)$

  local minima: σ_F

  global minima: σ_T

  local max: $\sigma_{\rm top}$



- tunneling probability per unit time per unit volume

$$\Gamma/V = Ce^{-B},$$

$$B = S_E[g_{\mu\nu,B}, \phi_B] - S_E[g_{\mu\nu,F}, \phi_F],$$

bounce solution

'false vacuum'

Lowest action

usually, bounce solutions are explored by assuming an O(4) symmetry

➢ spacetime metric: Euclidean

$$g_{\mu\nu}dx^\mu dx^\nu = N(\xi)^2 d\xi^2 + a(\xi)^2 \Omega_{ij} dx^i dx^j,$$

$$\Omega_{ij} \equiv \delta_{ij} + \frac{K\delta_{il}\delta_{jm}x^l x^m}{1 - K\delta_{lm}x^l x^m}, \quad K > 0$$

Note: the fiducial metric may not respect the symmetry

- fiducial metric: deSitter

$$G_{ab}(\phi)d\phi^a d\phi^b \equiv -(d\phi^0)^2 + b(\phi^0)^2 \Omega_{ij} d\phi^i d\phi^j,$$

$$b(\phi^0) \equiv F^{-1}\sqrt{K}\cosh(F\phi^0).$$

fiducial Hubble parameter

$\longrightarrow$ the O(4)-symmetric solutions are obtained by setting

$$\phi^0 = f(\xi), \quad \phi^i = x^i.$$

Inserting these ansatz into the action, we obtain the constraint equation by varying with respect with f

$$(i\dot{a} + Nb_{,f})\left[\left(3 - \frac{2b}{a}\right) + \alpha_3\left(1 - \frac{b}{a}\right)\left(3 - \frac{b}{a}\right) + \alpha_4\left(1 - \frac{b}{a}\right)^2\right] = 0,$$

$$\dot{a} \equiv \frac{da}{d\xi} \qquad b_{,f} \equiv \frac{db}{df} = \sqrt{K}\sinh(Ff)$$

$$\rightarrow \begin{cases} \text{Branch I} & Nb_{,f} = -i\dot{a}, \qquad \text{Not considered below} \\ \text{Branch II} & \left(3 - \frac{2b}{a}\right) + \alpha_3\left(1 - \frac{b}{a}\right)\left(3 - \frac{b}{a}\right) + \alpha_4\left(1 - \frac{b}{a}\right)^2 = 0. \end{cases}$$

$$\rightarrow \boxed{b = X_\pm a, \qquad X_\pm \equiv \frac{1 + 2\,\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}.}$$

Friedmann equation & EOM for tunneling field

$$\begin{cases} \dfrac{3}{a^2}\left(\dfrac{da}{d\tau}\right)^2 - \dfrac{3K}{a^2} = \dfrac{1}{2}\left(\dfrac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm, \\[2ex] \dfrac{d^2\sigma}{d\tau^2} + \dfrac{3}{a}\left(\dfrac{da}{d\tau}\right)\dfrac{d\sigma}{d\tau} - V_{,\sigma}(\sigma) = 0 \end{cases}$$

where $\mathrm{d}\tau \equiv N \mathrm{d}\xi,$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right],$$

3. Hawking-Moss(HM) solutions

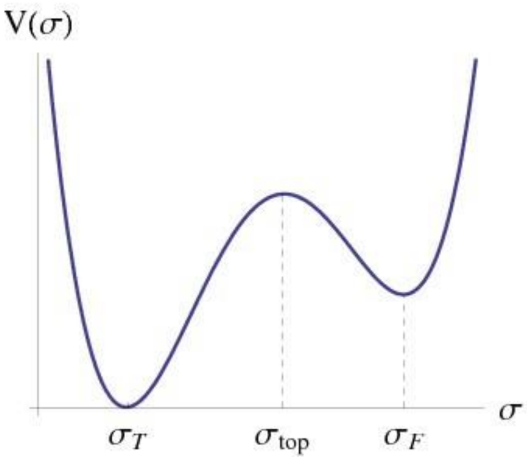
- HM solutions can be found at the local maximum of the potential

$\sigma = \sigma_{\rm top}$

$$a_{\rm HM}(\tau) = H_{\rm HM}^{-1}\sqrt{K}\cos\left(H_{\rm HM}\tau\right) ,$$

$\boxed{\mathrm{d}\tau \equiv N\mathrm{d}\xi}$

$\boxed{H_{\rm HM} \equiv \sqrt{\frac{\Lambda_{\pm} + V(\sigma_{top})}{3}}}$



- inserting this result into the Euclidean action and evaluate by integrating in the range $H_{\rm HM}\tau = -\pi/2 \longrightarrow \pi/2$, we finally express the HM action

$$\boxed{Y_\pm \equiv 3(1-X_\pm) + 3\alpha_3(1-X_\pm)^2 + \alpha_4(1-X_\pm)^3},$$

$$S_E[a_{\rm HM}, \sigma_{\rm top}] = \int {\rm d}^3x\sqrt{\Omega}\int_{-\pi/2H_{\rm HM}}^{\pi/2H_{\rm HM}} {\rm d}\tau\ a_{HM}^3 \left(2\Lambda_{\pm,{\rm eff}} - \frac{6K}{a_{\rm HM}^2} + m_g^2 Y_\pm \sqrt{-\left(\frac{df_{\rm HM}}{d\tau}\right)^2}\right)$$

$$\boxed{\Lambda_{\pm,{\rm eff}} \equiv \Lambda_\pm + V(\sigma_{\rm top})}$$

$$b_{\rm HM} = F^{-1}\sqrt{K}\cosh(F f_{\rm HM}) = X_\pm a_{\rm HM} \Longrightarrow \boxed{\left(\frac{df_{\rm HM}}{d\tau}\right)^2 = \frac{X_\pm^2 \sin^2(H_{\rm HM}\tau)}{\alpha_{\rm HM}^2 \cos^2(H_{\rm HM}\tau) - 1}}$$

Different from GR!

$$V(\sigma_F) > 3(X_\pm F)^2 \Longleftarrow \alpha_{\rm HM} \equiv X_\pm \frac{F}{H_{\rm HM}} \in [0,1]$$

$$\alpha_F = X_\pm F/H_F < \alpha_{HM}$$

- Note: for the Minkowski fiducial metric, $b_{\rm HM} = \sqrt{K} f_E$, by setting $f_E = -if$

$$\left(\frac{df_{E,{\rm HM}}}{d\tau}\right)^2 = -X_\pm \sin(H_{\rm HM}\tau)$$

so we recover the Minkowski one by setting $\alpha_{\rm HM} = 0$.

$$\boxed{Y_\pm \equiv 3(1-X_\pm) + 3\alpha_3(1-X_\pm)^2 + \alpha_4(1-X_\pm)^3},$$

$$S_E[a_{\rm HM}, \sigma_{\rm top}] = -\frac{8\pi^2}{H_{\rm HM}^2}\left[1 - \frac{Y_\pm X_\pm}{6\alpha^4}\left(\frac{m_g}{H_{\rm HM}}\right)^2\left(2-\sqrt{1-\alpha^2}(2+\alpha^2)\right)\right]$$

standard HM solution

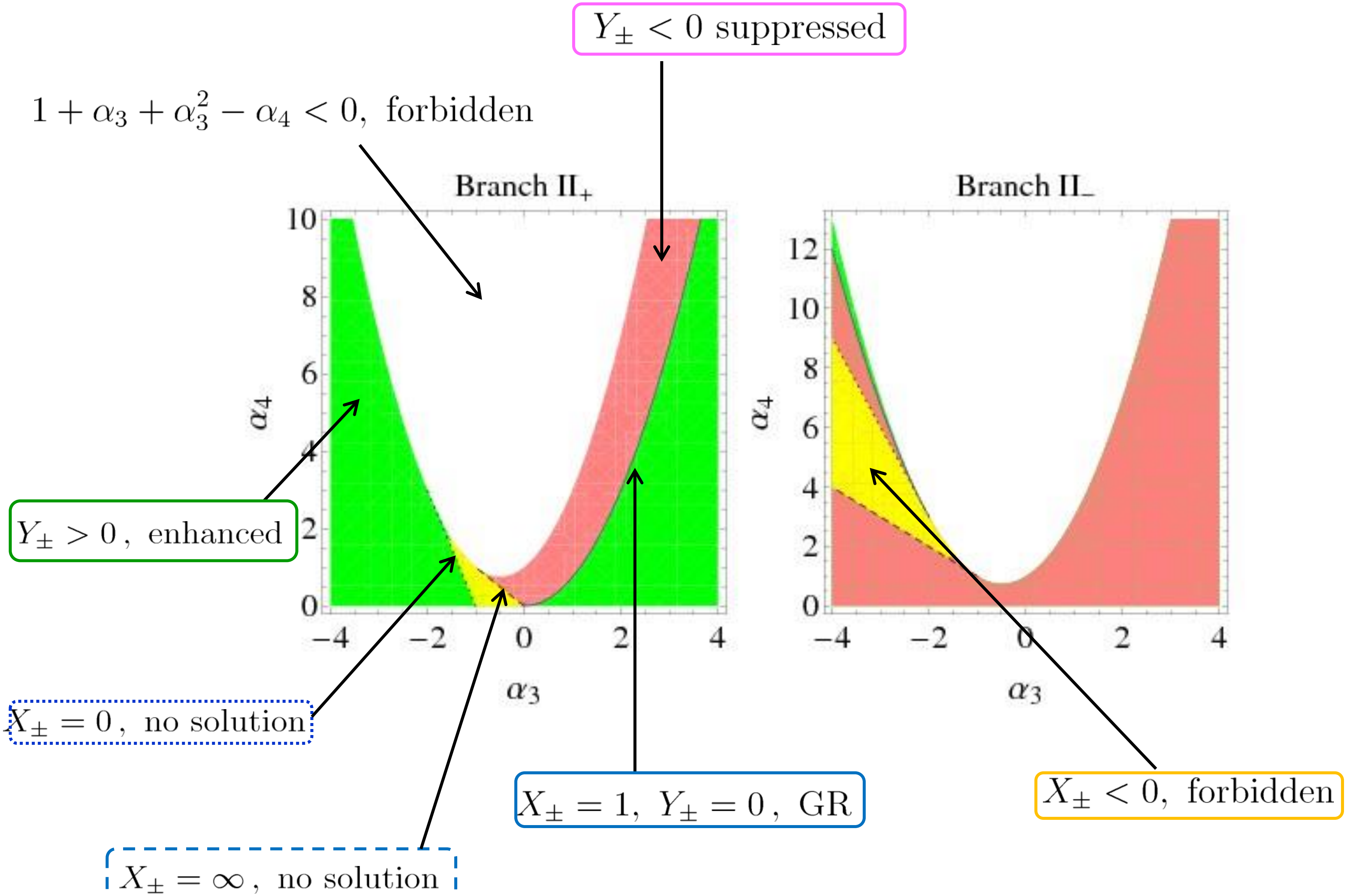
Correction due to the mass of graviton

$$\alpha \equiv X_\pm \frac{F}{H_{\rm HM}} \leq 1$$

Comparing with GR case, recalling the tunneling probability $\Gamma \propto e^{-B}$, we obtains:

$$\Delta B \equiv B^{(\rm MG)} - B^{(\rm GR)} = AY_\pm\,, \quad A<0$$

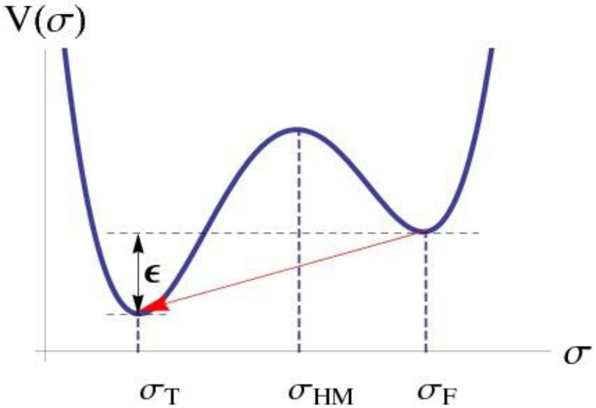
HM tunneling rate is enhanced for $Y_\pm > 0$,
suppressed for $Y_\pm < 0$.


$Y_\pm < 0$ suppressed
$1 + \alpha_3 + \alpha_3^2 - \alpha_4 < 0$, forbidden
Branch II$_+$
Branch II$_-$
α_4
α_3
$Y_\pm > 0$, enhanced
$X_\pm = 0$, no solution
$X_\pm = 1$, $Y_\pm = 0$, GR
$X_\pm < 0$, forbidden
$X_\pm = \infty$, no solution

4. Coleman-de Luccia(CDL) solutions

- CDL solutions can be found when $\sigma(0) = \sigma_{\mathrm{T}}, \quad \sigma(\tau_f) = \sigma_{\mathrm{F}}$

$$a(\tau) \begin{cases} = a_{\mathrm{T}}(\tau) \equiv H_{\mathrm{T}}^{-1}\sqrt{K}\cos\left(H_{\mathrm{T}}\tau\right) , & \tau < \tau_0 \\ = a_{\mathrm{F}}(\tau) \equiv H_{\mathrm{F}}^{-1}\sqrt{K}\cos\left(H_{\mathrm{F}}\tau + \theta_{\mathrm{F}}\right) , & \tau > \tau_0 \end{cases}$$



$$b(\tau) = X_{\pm} a(\tau) \implies -\left(f'(\tau)\right)^2 = \begin{cases} X_{\pm}^2 \dfrac{K - (a_{\mathrm{T}} H_{\mathrm{T}})^2}{K - (a_{\mathrm{T}} F X_{\pm})^2} , & \tau < \tau_0 \\ X_{\pm}^2 \dfrac{K - (a_{\mathrm{T}} H_{\mathrm{F}})^2}{K - (a_{\mathrm{F}} F X_{\pm})^2} , & \tau > \tau_0 \end{cases}$$

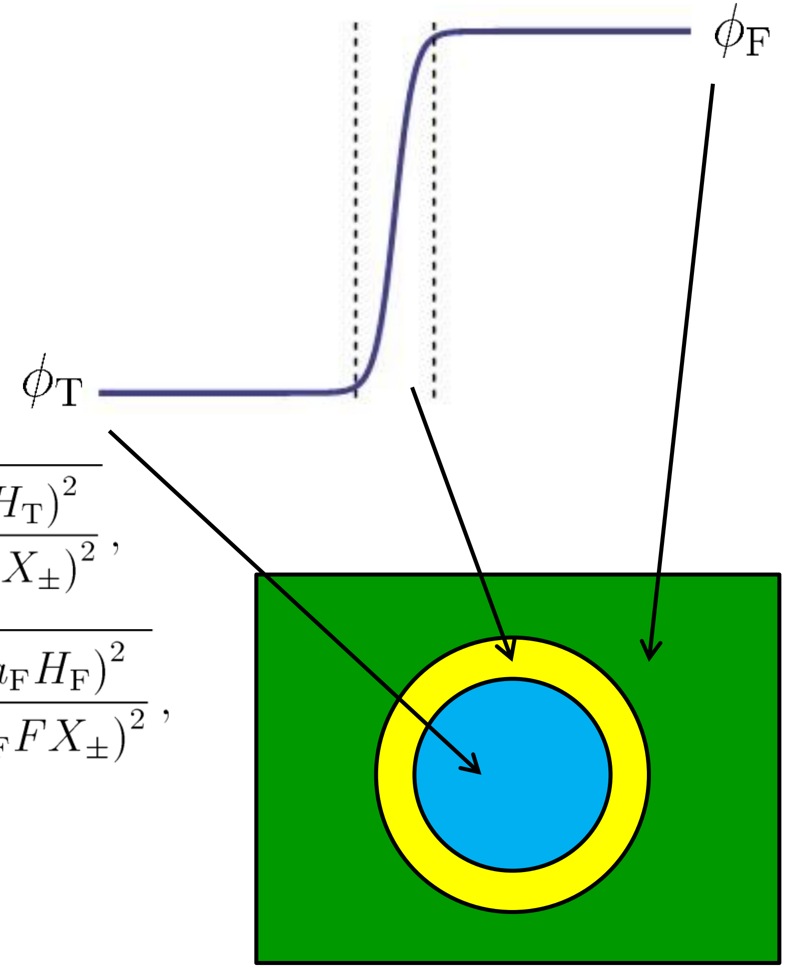
- difference from GR in action is the mass term

$$\boxed{\begin{aligned} S^{\mathrm{mass}} &\equiv -m_g^2 \int d^4 x_E \sqrt{\Omega}\left(\mathcal{L}_{2E} + \alpha_3 \mathcal{L}_{3E} + \alpha_4 \mathcal{L}_{4E}\right) \\ &= 2\pi^2 K^{-\frac{3}{2}} m_g^2 Y_{\pm} \int \mathrm{d}\tau \; a^3(\tau)\sqrt{-(f')^2} , \end{aligned}}$$

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\rm inside} + B_{\rm outside} + B_{\rm wall} \,,$$

$$\begin{cases} B_{\rm inside} \equiv S_{\rm inside} - S_{\rm F}\big|_{\tau<\tau_0} \,, \\ B_{\rm outside} \equiv S_{\rm outside} - S_{\rm F}\big|_{\tau>\tau_0} \,, \\ B_{\rm wall} \equiv S_{\rm wall} - S_{\rm F}\big|_{\tau=\tau_0} \,, \end{cases}$$



$$S_{\rm inside} = m_g^2 Y_\pm X_\pm \int {\rm d}^3x \sqrt{\Omega} \int_{-\pi/(2H_{\rm T})}^{\tau_0(1-\delta)} {\rm d}\tau \; a_{\rm T}^3 \sqrt{\frac{K-(a_{\rm T}H_{\rm T})^2}{K-(a_{\rm T}FX_\pm)^2}} \,,$$

$$S_{\rm outside} = m_g^2 Y_\pm X_\pm \int {\rm d}^3x \sqrt{\Omega} \int_{\tau_0(1+\delta)}^{\pi/(2H_{\rm F})} {\rm d}\tau \; a_{\rm F}^3 \sqrt{\frac{K-(a_{\rm F}H_{\rm F})^2}{K-(a_{\rm F}FX_\pm)^2}} \,,$$

$$S_{\rm wall} = m_g^2 Y_\pm \int {\rm d}^3x \sqrt{\Omega} \int_{\tau_0(1-\delta)}^{\tau_0(1+\delta)} {\rm d}\tau \; a^3(\tau) \sqrt{-(f')^2}$$

where $\delta \ll 1$

- thin-wall approximation: Coleman & de Luccia, 1980

$$B = B_{\text{inside}} + B_{\text{outside}} + B_{\text{wall}} \,,$$

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$$\frac{3}{a^2}\left(\frac{da}{d\tau}\right)^2 - \frac{3K}{a^2} = \frac{1}{2}\left(\frac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm,$$

$$\downarrow$$

$$a' = \sqrt{K + \frac{a^2}{3}\left[\frac{\sigma'^2}{2} - V(\sigma) - \Lambda_\pm\right]}$$

$$\downarrow$$

$$\int_0^{\tau_0(1-\delta)} \mathrm{d}\tau = \int_0^{a_0} \left(\frac{da}{d\tau}\right)^{-1} \mathrm{d}a$$

$$\downarrow$$

$$B_{\text{inside}} = 2\pi^2 K^{-\frac{3}{2}} m_g^2 Y_\pm X_\pm \int_0^{a_0} a^3 \mathrm{d}a \left\{ \frac{1}{\sqrt{K - a^2\Lambda_{\pm,\mathrm{T}}/3}} \sqrt{\frac{K - (aH_{\mathrm{T}})^2}{K - (aFX_\pm)^2}} - \frac{1}{\sqrt{K - a^2\Lambda_{\pm,\mathrm{F}}/3}} \sqrt{\frac{K - (aH_{\mathrm{F}})^2}{K - (aFX_\pm)^2}} \right\}$$

$$= \mathcal{O}(\epsilon)$$

- thin-wall approximation: Coleman & de Luccia, 1980

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$$\frac{3}{a^2}\left(\frac{da}{d\tau}\right)^2 - \frac{3K}{a^2} = \frac{1}{2}\left(\frac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm,$$

$$\Big\downarrow$$

$$a' = \sqrt{K + \frac{a^2}{3}\left[\frac{\sigma'^2}{2} - V(\sigma) - \Lambda_\pm\right]}$$

$$\Big\downarrow$$

$$\int_0^{\tau_0(1-\delta)} \mathrm{d}\tau = \int_0^{a_0} \left(\frac{da}{d\tau}\right)^{-1} \mathrm{d}a$$

$$\Big\downarrow$$

$$B_{\rm inside} = 2\pi^2 K^{-\frac{3}{2}} m_g^2 Y_\pm X_\pm \int_0^{a_0} a^3 \mathrm{d}a \left\{ \frac{1}{\sqrt{K - a^2\Lambda_{\pm,\rm T}/3}} \sqrt{\frac{K - (aH_{\rm T})^2}{K - (aFX_\pm)^2}} - \frac{1}{\sqrt{K - a^2\Lambda_{\pm,\rm F}/3}} \sqrt{\frac{K - (aH_{\rm F})^2}{K - (aFX_\pm)^2}} \right\}$$

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- thin-wall approximation: Coleman & de Luccia, 1980

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$$\frac{d^2\sigma}{d\tau^2} + \frac{3}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} - V_{,\sigma}(\sigma) = 0$$

$$\Big\downarrow \quad \frac{1}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} \ll 1$$

$$\sigma' \simeq \sqrt{2\big[V(\sigma) - V(\sigma_{\text{T}})\big]}$$

$$\Big\downarrow \quad \mathrm{d}\tau = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\right)^{-1}\mathrm{d}\sigma$$

$$B_{\text{wall}} \simeq 2\pi^2 K^{-\frac{3}{2}} a_0^3 m_g^2 Y_\pm \int_{\sigma_{\text{T}}}^{\sigma_{\text{F}}} \frac{\mathrm{d}\sigma}{\sqrt{2\big[V(\sigma) - V(\sigma_{\text{T}})\big]}} \left[\sqrt{-(f')^2}\Big|_{\tau<\tau_0} - \sqrt{-(f')^2}\Big|_{\tau>\tau_0}\right]$$

$$= \mathcal{O}(\epsilon)$$

- thin-wall approximation: Coleman & de Luccia, 1980

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$$\frac{d^2\sigma}{d\tau^2} + \frac{3}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} - V_{,\sigma}(\sigma) = 0$$

$$\Big\downarrow \quad \frac{1}{a}\left(\frac{da}{d\tau}\right)\frac{d\sigma}{d\tau} \ll 1$$

$$\sigma' \simeq \sqrt{2\left[V(\sigma) - V(\sigma_{\rm T})\right]}$$

$$\Big\downarrow \quad {\rm d}\tau = \left(\frac{{\rm d}\sigma}{{\rm d}\tau}\right)^{-1} {\rm d}\sigma$$

No difference from GR ?

$$B_{\rm wall} \simeq 2\pi^2 K^{-\frac{3}{2}} a_0^3 m_g^2 Y_\pm \int_{\sigma_{\rm T}}^{\sigma_{\rm F}} \frac{{\rm d}\sigma}{\sqrt{2\left[V(\sigma) - V(\sigma_{\rm T})\right]}} \left[\sqrt{-(f')^2}\bigg|_{\tau<\tau_0} - \sqrt{-(f')^2}\bigg|_{\tau>\tau_0}\right]$$

$$= \mathcal{O}(\epsilon)$$

- CDL as perturbations around HM

Expand the potential $V(\sigma)$ around $\sigma = \sigma_{\rm HM}$ as follows:

$$V(\sigma) = V(\sigma_{\rm HM}) - \frac{M^2}{2}(\sigma_{\rm HM} - \sigma)^2 + \frac{m}{3}(\sigma_{\rm HM} - \sigma)^3 + \frac{\nu}{4}(\sigma_{\rm HM} - \sigma)^4 + \cdots,$$

near the HM limit where $M^2 \equiv 4H_{\rm HM}^2(1+\chi^2)$ with $\chi^2 \ll 1$, the regular solutions are perturbatively found to be

$$a(\tau) = \tilde{H}_{\rm HM}^{-1}\cos\left(\tilde{H}_{\rm HM}\tau\right)\left[1 + \frac{\varepsilon^2 H_{\rm HM}^2}{8}\cos^2\left(\tilde{H}_{\rm HM}\tau\right)\right] + \mathcal{O}(\varepsilon^3)$$

$$\sigma(\tau) = \sigma_{\rm HM} + \varepsilon H_{\rm HM}\sin\left(\tilde{H}_{\rm HM}\tau\right) + \frac{\varepsilon^2 m}{12}\left[1 - 2\sin^2\left(\tilde{H}_{\rm HM}\tau\right)\right]$$

$$-\varepsilon^3 H_{\rm HM}\sin\left(\tilde{H}_{\rm HM}\tau\right)\left[\frac{3H_{\rm HM}^2 - 4\mu}{56}\cos^2\left(\tilde{H}_{\rm HM}\tau\right) - \frac{m^2}{36H_{\rm HM}^2}\sin^2\left(\tilde{H}_{\rm HM}\tau\right)\right] + \mathcal{O}(\varepsilon^4)$$

$\tilde{H}_{\rm HM} \equiv H_{\rm HM}(1 + H_{\rm HM}^2\varepsilon^2/24)$

$\mu \equiv \nu + m^2/18H_{\rm HM}^2$

$\varepsilon^2 \equiv 84\chi^2/(16H_{\rm HM}^2 + 9\mu)$

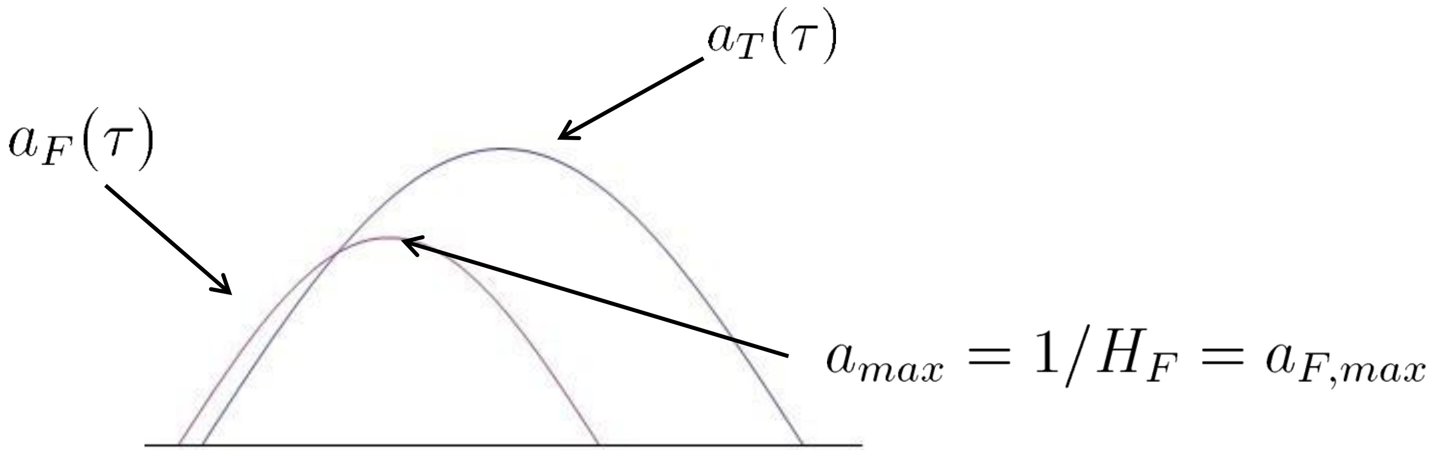
$$\delta^{(2)}S = \frac{\pi^2 m_g^2 X_\pm Y_\pm H_{\mathrm{HM}}^2 \varepsilon^2}{2\tilde{H}_{\mathrm{HM}}^4\sqrt{1-\tilde{\alpha}^2}}$$

Hence, if $Y_\pm > 0$, HM dominates over CDL, vise versa.

$\updownarrow$

In GR, perturbations in action vanish until $\mathcal{O}(\varepsilon^4)$, and CDL always dominate over HM.

Reconsideration of thin-wall result



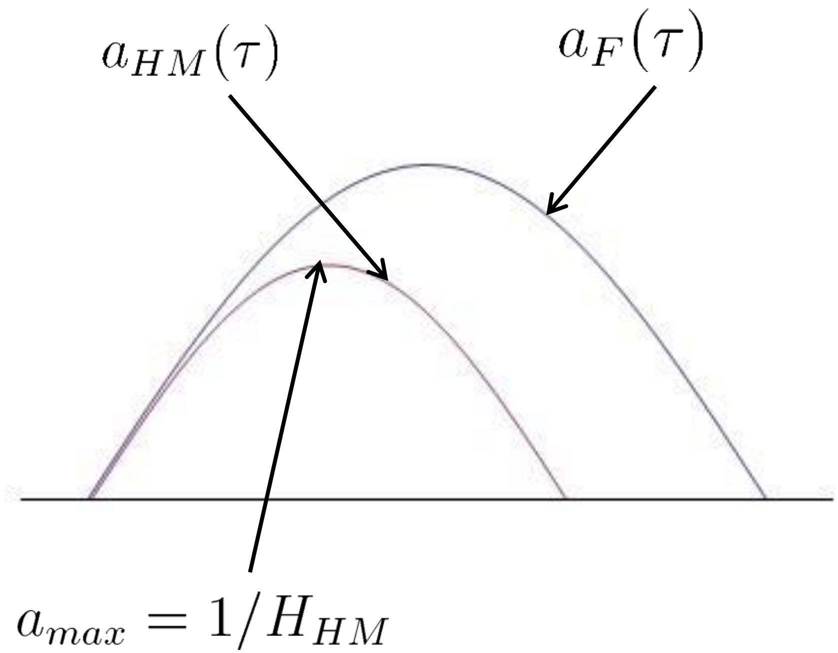
$$b(\tau) \equiv F^{-1}\sqrt{K}\cosh\left(Ff(\tau)\right) = X_{\pm}a(\tau) \quad \longrightarrow \quad -(f')^2 = \frac{X_{\pm}^2(a')^2}{K-(FX_{\pm}a)^2}$$

$$\boxed{\begin{aligned} S^{\mathrm{mass}} &= 4\pi^2 K^{-\frac{3}{2}} m_g^2 X_{\pm} Y_{\pm} \int_0^{a_{\max}} \frac{a^3 \mathrm{d}a}{\sqrt{K-(FX_{\pm}a)^2}} \\ &= -\frac{4\pi^2 K^{-\frac{3}{2}} m_g^2 X_{\pm} Y_{\pm}}{3(FX_{\pm})^4} \left[\sqrt{K-(FX_{\pm}a)^2}\left(2K+(FX_{\pm}a)^2\right)\right]_0^{a_{\max}} \end{aligned}}$$

$$B^{\mathrm{mass}}_{\mathrm{thin-wall}} \equiv S^{\mathrm{mass}} - S^{\mathrm{mass}}_{\mathrm{F}} \propto \left[\sqrt{K-(FX_{\pm}a)^2}\left(2K+(FX_{\pm}a)^2\right)\right]^{a_{\mathrm{max}}}_{a_{\mathrm{F,max}}} = 0\,,$$

This explains the reason why no contribution in thin-wall limit. However, in HM case, $a_{\mathrm{max}} = a_{\mathrm{HM,max}} \equiv H^{-1}_{\mathrm{HM}}$

$$B^{\mathrm{mass}}_{\mathrm{HM}} = -\frac{4\pi^2 K^{-\frac{3}{2}} m_g^2 X_{\pm} Y_{\pm}}{3(FX_{\pm})^4}\left[\sqrt{K-(FX_{\pm}a)^2}\left(2K+(FX_{\pm}a)^2\right)\right]^{H^{-1}_{\mathrm{HM}}}_{H^{-1}_{\mathrm{F}}} \neq 0$$



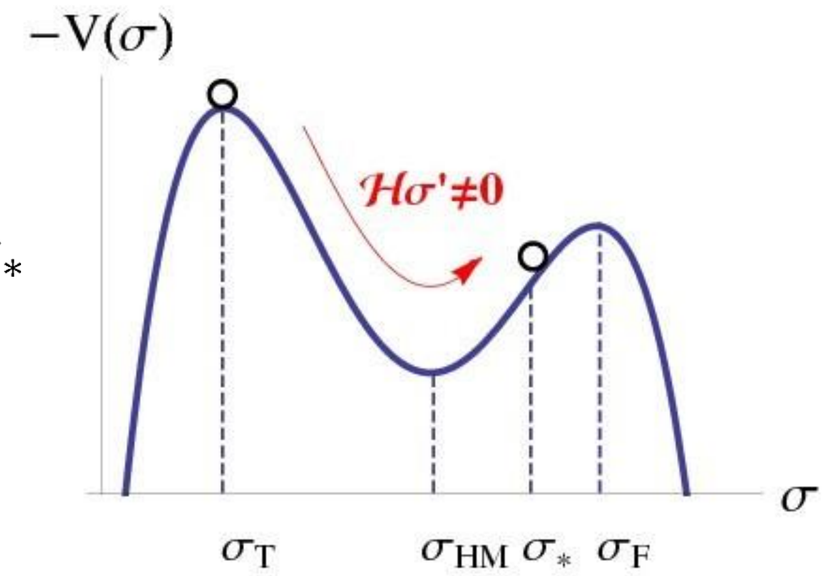
Now we consider deviations from thin-wall limit:

non-vanishing friction term

↓

cannot reach σ_F, instead stop at σ_* where $V(\sigma_*) > V(\sigma_{\rm F})$. By using

$$\frac{3}{a^2}\left(a'^2 - K\right) = \frac{\sigma'^2}{2} - V(\sigma) - \Lambda_\pm$$



and $\mathrm{d}a_{*,\max}/\mathrm{d}\tau = 0$, provided that $\sigma'^2/2 \ll V(\sigma_*), \quad K = 1$

$$a_{\max} = a_{*,\max} \simeq H_*^{-1} \equiv \sqrt{\frac{3}{V(\sigma_*) + \Lambda_\pm}} < a_{\mathrm{F},\max} \equiv H_{\mathrm{F}}^{-1}$$

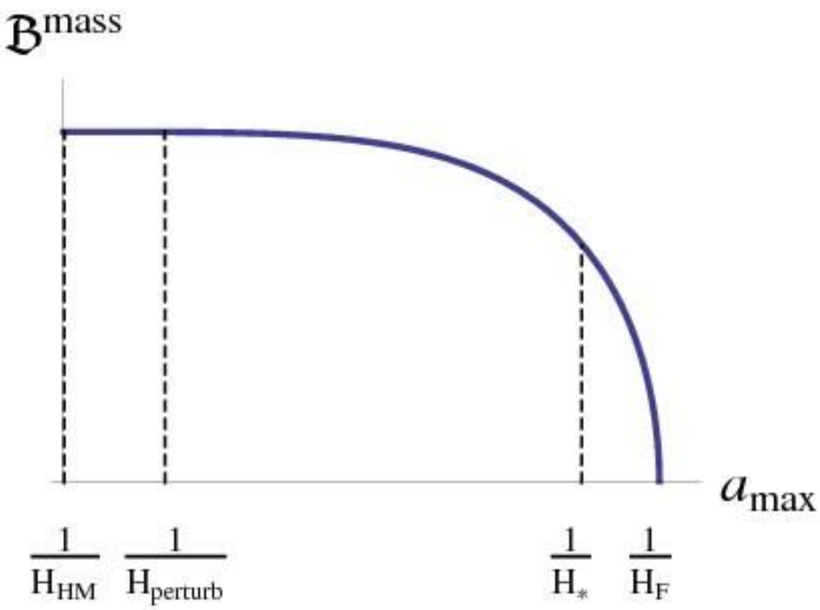
$$\longrightarrow \quad B_*^{\rm mass} = -\frac{4\pi^2 m_g^2 X_\pm Y_\pm}{3(FX_\pm)^4}\left[\sqrt{1-(FX_\pm a)^2}\left(2+(FX_\pm a)^2\right)\right]_{H_{\rm F}^{-1}}^{H_*^{-1}} \neq 0$$

Deviations from thin-wall limit leads to corrections to CDL tunneling rate!

Defining

$$\mathfrak{B}^{\rm mass} \equiv -\frac{3(FX_\pm)^4 B^{\rm mass}}{4\pi^2 m_g^2 X_\pm Y_\pm} = \left[\sqrt{1-(FX_\pm a)^2}\,\left(2+(FX_\pm a)^2\right)\right]_{H_{\rm F}^{-1}}^{a_{\rm max}},$$

- HM solution gives largest correction term where a_{max} is smallest;
- when a_{max} increases, correction shrinks gradually;
- at thin-wall limit, the behavior of CDL solution is the same as GR.



Summary and future work

- We constructed a model in which the tunneling field minimally couples to the non-linear massive gravity;
- corrections to HM solution from mass term is found, which implies suppression or enhancement of tunneling rate, depending on the choices of parameters;
- there appears constraint on the height of potential for false vacuum;
- corrections to CDL tunneling changes monotonically with respect to the thickness of the wall;
- it would be a further work to investigate the case where the tunneling field couples to the non-linear massive gravity non-minimally, e.g. $m_g = m_g(\sigma)$