Branes and Stringy Geometry

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based on work with Fabio Riccioni

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In this talk I will review some recent advances in the classification of branes and show how an interesting pattern arises: the so-called 'wrapping rules'

I will try to relate these results to stringy geometry

Outline

An Update on Branes

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Brane Classification

Branes and 'Stringy Geometry'

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Summary and Open Issues

The IIA/IIB superstring

From the world-volume point of view the IIA/IIB superstring is described by a worldvolume action for the embedding super-coordinates $\{X^\mu, \theta^\alpha\}(\sigma, \tau)$ which describe the coupling to the background fields

after gauge-fixing the action is worldvolume supersymmetric

bosonic part string action

$$\mathcal{L}_{\mathrm{F1}}(D=10) \;=\; \underbrace{\mathrm{T}_{\mathrm{F1}}\sqrt{-g}}_{\text{NG term}} + \underbrace{B_2}_{\text{WZ term}}$$

worldvolume multiplet = 8 + 8 scalar multiplet with 16 supercharges

From Strings to Branes

Branes are an essential part of (non-perturbative) string theory

The NS-NS 2-form B_2 suggests a half-supersymmetric string

Similarly, the 3-form C_3 of 11D sugra couples to a half-susy M2-brane

Its dual 6-form C_6 couples to the half-supersymmetric M5-brane

sugra potential $\leftrightarrow$ *half-supersymmetric brane*

Does it always work as simple as that?

Strings and T-duality

In $D < 10$ we have a singlet NS-NS 2-form B_2 as well as 1-forms $B_{1,A}$ $(A = 1, \ldots, 2d)$ that transform as a vector under the T-duality group SO(d,d) with $d = 10 - D$

To construct a gauge-invariant WZ term

$$\mathcal{L}_{\mathrm{WZ}}(D < 10) = B_2 + \eta^{AB} \mathcal{F}_{1,A} B_{1,B}$$

we need to introduce "extra scalars" $b_{0,A}$ via $\mathcal{F}_{1,A} = d\, b_{0,A} + B_{1,A}$

Counting the Bosonic Worldvolume D.O.F.

$$D = 10 \quad : \qquad (10-2) = 8\,,$$

$$D < 10 \quad : \qquad (D-2) + 2(10-D) \neq 8\,!$$

Twice too many 'extra scalars' $b_{0,A}$ $\rightarrow$ 'doubled geometry'

Hull, Reid-Edwards (2006-2008)

Self-duality conditions on the extra scalars $b_{0,A}$ give correct counting

'Wess-Zumino term requirement'

the construction of a gauge-invariant WZ term may require, besides the embedding coordinates, the introduction of a number of extra worldvolume p-form potentials

worldvolume supersymmetry requires that these worldvolume fields fit into a vector (scalar) or tensor multiplet with 16 supercharges

Does the 'WZ term requirement' always lead to the rule that

potential $\Leftrightarrow$ half-susy brane ?

Dirichlet branes

D-branes have two properties:

- their worldvolume dynamics is described by a vector multiplet
- their tension scales as $\mathrm{T} \sim 1/g_s$

The worldvolume action contains a DBI-VA action:

$$\mathcal{L} \sim e^{-\phi}\sqrt{g + \mathcal{F}_2} + \mathcal{L}_{\mathrm{WZ}} \qquad\qquad \mathcal{F}_2 = d\, b_1 - B_2$$

D-branes in D=10 dimensions

$$\mathcal{L}_{\mathrm{WZ}}(D=10) = \mathrm{T}_{\mathrm{Dp}} e^{\mathcal{F}_2} C \,, \qquad \mathcal{F}_2 = d\,b_1 - B_2$$

- IIA : p even IIB : p odd

$$C = C_{p+1} \oplus C_{p-1} \oplus C_{p-3} \oplus \ldots$$

$D = 10$: $(10-p-1) + (p-1) = 8$: vector multiplet

Counting worldvolume d.o.f. in $D < 10$

D-branes transform as spinors under T-duality

$$\mathcal{L}_{\mathrm{WZ}}(D \leq 10) = (\mathrm{T}_{\mathrm{Dp}})^{\alpha} \left[e^{\mathcal{F}_2} e^{\mathcal{F}_{1,A}\Gamma^A} C \right]_{\alpha}$$

$\alpha\,(A)$ is spinor (vector) index of SO(d,d) Riccioni + E.B. (2010)

$$D \leq 10 \ : \qquad (D-p-1) + (p-1) + 2(10-D) \neq 8\,!$$

only half of the $\mathcal{F}_{1,A}\Gamma^A$ contributes to a particular spinor component !

'Defect Branes'

'defect branes' are branes with two transverse directions

Greene, Shapere, Vafa, Yau (1990); Gibbons, Green, Perry (1996), Vafa (1996)

they include 'cosmic strings', 'seven-branes' and 'exotic branes'

- number of half-supersymmetric defect branes $<$ number of $(D-2)$-form potentials $\neq$ number of dual scalars

Example: IIB supergravity

IIB supergravity contains:

- two scalars (axion and dilaton)
- three 8-form potentials $A_8^{\alpha\beta} = A_8^{\beta\alpha}$ $(\alpha = 1, 2)$ that transform as the **3** of SL(2,$\mathbb{R}$)
- two half-supersymmetric branes: the D7-brane and its S-dual

How does this follow from the WZ requirement ?

$$\mathcal{L}_{\text{WZ}} \sim \text{Q}^{\alpha\beta}\left[A_{8,\alpha\beta} + A_{6,(\alpha}\mathcal{F}_{2,\beta)} + \cdots\right] \qquad \alpha = 1,2$$

$\mathcal{F}_{2,\alpha} = d a_{1,\alpha} - A_{2,\alpha}$: two worldvolume vectors

only two out of three branes, with charges Q^{11} and Q^{22}, contain a single worldvolume vector

the third brane, with charge Q^{12}, contains two worldvolume vectors that cannot be part of a worldvolume multiplet with 16 supercharges

Aim

- *We wish to classify the single BPS branes of (toroidally compactified) string theory using supergravity as a low-energy approximation*

- we consider the branes of maximal supergravity only

For the branes of non-maximal supergravity, see the talk by Fabio tomorrow

We first have to answer the following question:

What are the T-duality representations of the $(D-2)$-form, $(D-1)$-form and D-form potentials of $3 \leq D \leq 11$ maximal supergravity?

- $(D-2)$-form potentials couple to defect branes. They are dual to scalars
- $(D-1)$-form potentials couple to domain walls. They are dual to integration constants, sometimes called embedding tensors
- D-form potentials couple to space-filling branes. They have no curvature

New supergravity development

- The T-duality representations of the $(D-2)$-form, $(D-1)$-form and D-form potentials in $3 \leq D \leq 11$ maximal supergravity have been determined using three different techniques:

  - closure of the supersymmetry algebra

    de Roo, Hartong, Howe, Kerstan, Ortín, Riccioni + E.B. (2005-2010)

  - using the embedding tensor technique

    for a review, see de Wit, Nicolai, Samtleben (2008)

  - using the very extended Kac-Moody algebra E_{11}

    Riccioni, West (2007); Nutma + E.B. (2007)

    a similar analysis can be done for E_{10}, see e.g. Nicolai, Fischbacher (2002)

Question

given a $(p+1)$-form which components of its T-duality representation couple to a half-supersymmetric brane?

Answer

There is a simple group-theoretical characterization of which components of the T-duality representation couple to a half-supersymmetric brane

- this talk: only maximal supergravity
- Fabio's talk: half-maximal and quarter-maximal supergravity

for a derivation based on the WZ term requirement, see Riccioni + E.B (2010)

for an alternative E_{11} derivation, see Kleinschmidt (2011)

Outline

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Summary and Open Issues

It is convenient to decompose the U-duality representations of the potentials in terms of representations of the T-duality group SO(d,d) and a scaling symmetry $\mathbb{R}^+$:

$$\mathrm{E}_{11-D} \supset \mathrm{SO}(d,d) \times \mathbb{R}^+$$

The scaling weight α determines the dependence of the brane tension T on the string coupling constant g_s via

$$\mathrm{T} \sim (g_s)^{\alpha}$$

and is invariant under dimensional reduction

A Universal Pattern

α	potentials	branes
$\alpha = 0$	$B_{1,A}\,,\, B_2$	fundamental
$\alpha = -1$	$C_{2n+2,a}\,,\, C_{2n,\dot{a}}$	Dirichlet
$\alpha = -2$	$D_{D-4},\, D_{D-3,A},\, D_{D-2,A_1A_2},\, D_{D-1,A_1\cdots A_3},\, D_{D,A_1\cdots A_4}$	solitonic
$\vdots$	$\vdots$	$\vdots$

A $(a, \dot{a})$ are vector (spinor)-indices of T-duality

10D : $\alpha = 0, -1, -2, -3, -4$ universal behaviour for $-4 < \alpha \leq 0$

'The $SO(d,d)$ Light-cone Rule'

Consider a potential in an a.s. representation $[A_1 \dots A_n]$ of $SO(d,d)$

$$A = 1, \cdots, 2d$$

Choose a basis of light-like coordinates for $SO(d,d)$:

$$A = i\pm = x_i \pm t_i\,, \qquad\qquad i = 1, \dots, d$$

- only the components $i_1 \pm i_2 \pm \dots i_n \pm$ anti-symmetric in the $i's$ correspond to a half-supersymmetric brane

Examples

- $B_{1,A} = B_{1,i\pm}$: $2d$ fundamental 0-branes

- $D_{D-2,AB} = D_{D-2,i\pm j\pm}$: $2d(d-1)$ instead of

  $d(2d-1) = 2d(d-1) + d$ solitonic defect-branes

remember: $A = 1, \ldots, 2d$ but $i = 1, \ldots, d$

The branes of maximal supergravity

we classified all branes of maximal supergravity and determined their worldvolume theories

an interesting pattern arises !

Outline

An Observation

supergravity is not complete!

'Standard Geometry'

the wrapping rules of 'standard geometry'

$$\text{any brane} \quad \left\{ \begin{array}{lcl} \text{wrapped} & \rightarrow & \text{undoubled} \\ \text{unwrapped} & \rightarrow & \text{undoubled} \end{array} \right.$$

only works for D-branes !

Counting D-branes

Dp-brane	IIA/IIB	9	8	7	6	5	4	3
0	1/0	1	2	4	8	16	32	64
1	0/1	1	2	4	8	16	32	64
2	1/0	1	2	4	8	16	32	64
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
8	1/0	1						
9	0/1							

spinors $(\mathrm{Dp})_\alpha\,,\qquad \alpha = 1\cdots 2^{9-D}$

Fundamental Branes

the wrapping rules of fundamental branes are given by

$$\mathrm{T}_{\mathsf{F}} \sim 1\ : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{doubled} \\ \text{unwrapped} & \rightarrow \quad \text{undoubled} \end{cases}$$

the extra input comes from pp-waves

Two points of view:

'new objects' (pp-waves) or 'stringy geometry' (doubled geometry)

Counting Fundamental Branes

Fp-brane	IIA/IIB	9	8	7	6	5	4	3
0		2	4	6	8	10	12	14
1	1/1	1	1	1	1	1	1	1

$(\mathrm{F}0)_A$ and F1

$$A = 1, \ldots, 2(10 - D)$$

Solitonic Branes with $T \geq 3$

In 9D we have two solitonic 5-branes coming from a wrapped NS5-brane and a KK monopole

$$\text{10D KK monopole}: \quad \begin{cases} 5+1 \text{ worldvolume directions} \\ 1 \text{ isometry direction} \\ 3 \text{ transverse directions} \end{cases}$$

This leads to the following 'dual' wrapping rule:

$$\mathrm{T_S} \sim (g_s)^{-2} \ : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{undoubled} \\ \text{unwrapped} & \rightarrow \quad \text{doubled} \end{cases}$$

Counting Solitonic Branes with $T \geq 3$

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	
1					1	10		
2				1	8			
3			1	6				
4		1	4					
5	1/1	2						

S(D-5)-brane and S(D-4)-brane$_A$

Solitonic Branes with $T \leq 2$

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	84
1					1	10	60	280
2				1	8	40	160	560
3			1	6	24	80	240	
4		1	4	12	32	80		
5	1/1	2	4	8	16			

dual wrapping rule reproduces precisely the counting of all solitonic branes obtained from the WZ term requirement !

Question

what is the 10D origin of the solitonic branes with $T \leq 2$ *?*

Two points of view

- supergravity can be extended with a set of mixed-symmetry tensors (with an underlying E_{11} symmetry structure) that couple to 'generalized monopoles' which form a new class of extended objects within string theory

see, e.g., Lozano-Tellechea, Ortín (2001)

see also work by de Boer and Shigemori (2010, 2012) and talks by Giusto, Kimura and Warner

- generalized monopoles are an alternative way of looking at doubled geometry

An Example

compactifying generalized monopoles $\Leftrightarrow$ flux compactification in DFT

10D origin	mixed-symmetry	flux (a=1,2,3)
NS5 (5_2)	D_6	H_{abc} (1)
KK5 (5_2^1)	$D_{7,1}$	$f^a{}_{bc}$ (9)
5_2^2	$D_{8,2}$	$Q^{ab}{}_c$ (9)
5_2^3	$D_{9,3}$	R^{abc} (1)

The 7D solitonic domain wall 6-forms $D_{6,[ABC]}$ $(A = 1, \ldots, 6)$ transform as 20 under $SO(3,3)$. These 6-forms are dual to (constant) fluxes

see also Haßler, Lüst (2013); Kimura, Sasaki (2013); Chatzistavrakidis, Gautason, Moutsopoulos, Zagermann (2013)

The general picture

The branes with fixed $\alpha = 0, -1, -2, -3, -4$ satisfy specific wrapping rules

$$\mathrm{T} \sim (g_s)^{-3} \ : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{doubled} \\ \text{unwrapped} & \rightarrow \quad \text{doubled} \end{cases}$$

$$\mathrm{T} \sim (g_s)^{-4} \ : \quad \left\{ \ \text{wrapped} \quad \rightarrow \quad \text{doubled} \right.$$

The branes with $\alpha < -4$ do not have a higher-dimensional brane origin. So far we did not identify any wrapping rules for them

Outline

Summary

- In this talk I gave an update on the branes of maximal supergravity and showed how their classification leads to effective wrapping rules

- I discussed the relation with doubled geometry

- For branes of non-maximal supergravity, see talk by Fabio

What you should remember

Relating branes in different dimensions requires extra information encoded by either generalised monopoles or doubled geometry !

Can we give a better formulation of supergravity ?