Geometries of D1-D5-P states

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Based on

- S.G., L. Martucci, M. Petrini, R. Russo: `arXiv:1306.1745`
- S.G., R. Russo: `arXiv:1311.5536`

Introduction: the D1-D5-P system

- Paradigmatic example of BPS black hole in string theory (Strominger, Vafa)

Type II B

	$\mathbb{R}$	$\times$	$\mathbb{R}^4$	$\times$	S^1	$\times$	T^4
	t		x^i		y		z^a
$D1:$	$-$		$\circ$		$-$		$\circ$
$D5:$	$-$		$\circ$		$-$		$-$
$P:$	$-$		$\circ$		$-$		$\circ$

The “naive” D1-D5-P geometry

$$ds^2 = -\frac{2}{\sqrt{Z_1 Z_2}}\, dv \left(du + \frac{\mathcal{F}}{2}\, dv \right) + \sqrt{Z_1 Z_2}\, ds_4^2 + \sqrt{\frac{Z_1}{Z_2}}\, ds_{T^4}^2$$

$$v = \frac{t+y}{\sqrt{2}} \quad , \quad u = \frac{t-y}{\sqrt{2}} \quad , \quad ds_4^2 = dx^i dx^i$$

$$Z_i = 1 + \frac{Q_i}{r^2} \quad , \quad \mathcal{F} = -\frac{Q_p}{r^2}$$

$$Q_1 : \text{ D1 charge} \quad , \quad Q_2 : \text{ D5 charge} \quad , \quad Q_p : \text{ P charge}$$

- The geometry has an “extremal” horizon of finite area at $r = 0$.
- The geometry only depends on the charges: “no-hair theorem”.

The near-horizon decoupling limit

In the limit

$$r^2\,,\ Q_p \ll Q_1\,,\ Q_2 \quad \Rightarrow \quad Z_i \to Z_i - 1$$

one obtains a geometry that is $\mathrm{AdS}_3 \times S^3 \times T^4$ asymptotically $(r^2 \gg Q_p)$

$$ds^2 \approx -\frac{r^2}{\sqrt{Q_1 Q_2}}\, 2\, du\, dv + \frac{\sqrt{Q_1 Q_2}}{r^2}\, dr^2 + \sqrt{Q_1 Q_2}\, d\Omega_3^2 + \sqrt{\frac{Q_1}{Q_2}}\, ds^2_{T^4}$$

This limit is described by a CFT:

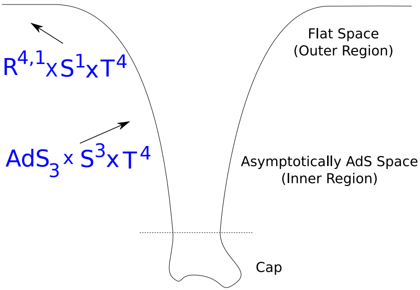
(4,4) sigma model with target space (a deformation of) $(T^4)^{n_1 n_5}/S_{n_1 n_5}$

$$\Updownarrow$$

D1-D5 CFT

Microstate geometries

- There exist solutions of supergravity that reduce to the “naive” D1-D5-P geometry for large r up to $O\left(\frac{1}{r^2}\right)$ (and hence carry the same charges as the D1-D5-P black hole) but are completely smooth and have no horizon.
- In the near-horizon limit these geometries are dual to states of the D1-D5 CFT.



Conjecture: these geometries account for (a finite fraction of) the entropy of the D1-D5-P black hole.

(Mathur, Bena, Warner,. . .)

Our goal

- To develop systematic methods to construct geometries that
  - carry D1,D5,P charges and preserve 4 supersymmetries;
  - are dual to states of the D1-D5 CFT.

  Note: for geometries carrying only two charges (egs. D1-D5) this problem has been solved in full generality.

  (Lunin, Mathur; Kanitscheider, Skenderis, Taylor)

- We will follow the steps:
  - find the general supergravity ansatz that preserves the same susy as D1-D5-P;
  - select solutions dual to known states of the D1-D5 CFT: apply symmetries of the CFT to known microstates.

General D1-D5-P ansatz I

We apply methods of Generalized Geometry in 10D (Tomasiello)

- $\exists\, \epsilon_1, \epsilon_2$ Majorana-Weyl Killing spinors;
- the spinor bilinear $K \equiv -\frac{1}{2}(\bar{\epsilon}_1 \Gamma^M \epsilon_1 + \bar{\epsilon}_2 \Gamma^M \epsilon_2)\, \partial_M$ is a Killing vector;
- we restrict to the case K is null $\Rightarrow\ K = \frac{\partial}{\partial u}$;
- other spinor bilinears

$$(i) \quad \chi \equiv -\frac{1}{2}(\bar{\epsilon}_1 \Gamma^M \epsilon_1 - \bar{\epsilon}_2 \Gamma^M \epsilon_2)\, dx^M \quad 1-\text{form}$$

$$(ii) \quad \Psi \equiv -32\, \epsilon_1 \otimes \bar{\epsilon}_2\, \Gamma_{(10)} \quad \text{polyform}$$

satisfy

$$d\chi = i_K H \quad , \quad (d - H) \wedge (e^{-\phi}\, \Psi) = i_K F + \chi \wedge F$$

(H , F NSNS and RR field strengths);

General D1-D5-P ansatz II

- the missing susy equations come from the v-component of the gravitino variation $\Rightarrow$ extra constraints;
- the previous equations plus the Bianchi identities for H and F imply all the equations of motion apart from the vv-component of Einstein equations $\Rightarrow$ $R_{vv} + \ldots = 0$;
- split the metric as $10 \to \overset{(u,v)}{2} + \overset{x^i}{4} + \overset{z^a}{4}$ and restrict (for simplicity) to geometries that are homogeneous and isotropic with respect to T^4 $\Rightarrow$ non-trivial dependence on (v, x^i);
- impose that the susy preserved is of D1-D5 type:

$$\Gamma_{D_1}\,\epsilon_2 = \epsilon_1 \quad , \quad \Gamma_{D5}\,\epsilon_2 = \epsilon_1$$

General D1-D5-P ansatz III

- the various conditions constrain the metric to be of the form

$$ds^2 = -\frac{2\alpha}{\sqrt{Z_1 Z_2}}(dv+\beta)\left[du+\omega+\frac{\mathcal{F}}{2}(dv+\beta)\right]+\sqrt{Z_1 Z_2}\,ds_4^2+\sqrt{\frac{Z_1}{Z_2}}\,ds_{T^4}^2$$

$$\text{with} \quad \alpha = \frac{Z_1 Z_2}{Z_1 Z_2 - Z_4^2}$$

$$e^{2\phi} = \ldots \quad , \quad B = \ldots \quad , \quad F = \ldots$$

- all fields of type IIB sugra can be excited;

General D1-D5-P ansatz IV

$$ds^2 = -\frac{2\alpha}{\sqrt{Z_1 Z_2}}(dv+\beta)\Big[du+\omega+\frac{\mathcal{F}}{2}(dv+\beta)\Big] + \sqrt{Z_1 Z_2}\, ds_4^2 + \sqrt{\frac{Z_1}{Z_2}}\, ds_{T^4}^2$$

- the sugra equations can be organized in such a way that the problem reduces to an almost linear one

  (Bena, SG, Shigemori, Warner)

- the non-linear part involves ds_4^2 , β :
  - ds_4^2 admits an almost hyperkäler structure

    $$J_A \wedge J_B = -2\,\delta_{AB}\,\mathrm{vol}_4 \quad , \quad J_A = -*_4 J_A \quad , \quad dJ_A = \partial_v(\beta \wedge J_A)$$

  - $D\beta = *_4 D\beta \quad \text{with} \quad D \equiv d - \beta \wedge \partial_v\,;$

- the equations for Z_1 , Z_2 , Z_4 , ω , $\mathcal{F}$ ... are linear (but inhomogeneous), if solved in the right order.

v-dependence

- The sugra equations simplify for v-independent geometries:
  - ds_4^2 is hyperkähler;
  - $d\beta = *_4 d\beta$.
- On general grounds, geometries are expected to be v-independent if they are dual to eigenstates of momentum.
- Typically classical geometries are dual to coherent states (egs. $e^{\alpha\, a_{-n}}|\psi_0\rangle$ with a_{-n} carrying momentum n)

  $\Rightarrow$ generic 3-charge microstates are v-dependent.

D1-D5 geometries (Lunin, Mathur; Kanitscheider, Skenderis, Taylor)

- D1-D5 microstates are v-independent.
- They have a flat 4D base: $ds_4^2 = dx^i\, dx^i$.
- The (T^4-isometric) geometries are encoded in a profile in $\mathbb{R}^5$ $(F^i(v'), \mathcal{F}(v'))$:

$$\beta = \frac{-A+B}{\sqrt{2}}\ ,\ A = -\frac{Q_2}{L}\int_0^L \frac{dv'\,\dot{F}^i}{|x-F|^2} dx^i\ ,\ dB = -*_4\, dA$$

$$Z_2 = 1 + \frac{Q_2}{L}\int_0^L \frac{dv'}{|x-F|^2}\ ,\ Z_1 = 1 + \frac{Q_2}{L}\int_0^L \frac{dv'\,|\dot{F}|^2}{|x-F|^2}$$

$$Z_4 = -\frac{Q_2}{L}\int_0^L \frac{dv'\,\dot{\mathcal{F}}}{|x-F|^2}\ ,\ \omega = -\frac{A+B}{\sqrt{2}}$$

- $\beta \neq 0$: KK-monopole dipole charge from binding D1 and D5.

Adding momentum to D1-D5 I

- Start from a D1-D5 geometry and take its near-horizon limit:

$$Z_1 \to Z_1 - 1 \ , \ Z_2 \to Z_2 - 1 \quad \Rightarrow \quad \text{asymptotically AdS geometry}$$

- Act with an operator of the CFT chiral algebra that preserves susy and adds momentum:
  - L_{-n} : Virasoro;
  - J^{α}_{-n} : $SU(2)_L$ R-symmetry;
  - $J^{z^a}_{-n}$: $U(1)^4$ T^4 translations.

  On the gravity side this is equivalent to performing a change of coordinates that does not vanish at the boundary of AdS.

  At the perturbative level, this method was applied by Mathur, Saxena, Srivastava; Mathur, Turton; Shigemori.

Adding momentum to D1-D5 II

- Rewrite metric and gauge fields in the form of the general ansatz and read-off $ds_4^2\,,\ \beta\,,\ Z_I\,,\ \omega\,,\ \mathcal{F}$.

  One generates a v-dependent solution of the sugra equations that is asymptotically $\mathrm{AdS}_3 \times S^3 \times T^4$.

- Extend the metric back to the asymptotically flat region, where the metric reduces to $\mathbb{R}^{4,1} \times S^1 \times T^4$.

  Note: the replacement $Z_1 \to Z_1 + 1\ ,\ Z_2 \to Z_2 + 1$, leaving the remaining metric functions unchanged, does not produce a solution of the sugra constraints.

  This is a non-trivial problem that can be solved exactly in some cases.

Case 1: v-independent ds_4^2

- Assume that ds_4^2 , β do not depend on v after the change of coordinates.
- Assume also that ds_4^2 , β have a tri-holomorphic isometry $\frac{\partial}{\partial\tau}$ $\Rightarrow$

$$ds_4^2 = V^{-1}(d\tau + A)^2 + V\, ds_3^2 \quad , \quad *_3 dA = dV$$

$$\beta = \frac{K_3}{V}\,(d\tau + A) + \xi \quad , \quad *_3 d\xi = -dK_3$$

Egs: $ds_4^2 = \text{flat}\ \mathbb{R}^4 \quad \Rightarrow \quad \tau = \phi + \psi \ , \ V = \frac{1}{r}$

- The remaining metric coefficients may depend on v and τ.

Case 1: v-independent ds_4^2 - An example

- Start from the D1-D5 geometry associated with the profile

$$F^1 + iF^2 = a\, e^{\frac{2\pi i\, v'}{L}} \;,\; F^3 = F^4 = 0 \;,\; \mathcal{F} = -b \sin\Big(\frac{2\pi\, v'}{L}\Big)$$

The geometry depends on the angle of the $(1,2)$ plane but β does not.

- Apply the operator $e^{\chi(J^+_{-1} - J^-_1)} = e^{\Sigma}\; e^{\chi(J^+_0 - J^-_0)}\; e^{-\Sigma}$

$\phi \to \phi + \frac{t}{R}\,,\, \psi \to \psi + \frac{y}{R} \leftarrow$ spectral flow   rotation in $\mathbb{R}^4$

Note: when $b = 0$ the action of this operator is trivial.

- One can see that ds_4^2 and β are left invariant, while the other metric coefficients transform non-trivially under this operator.

Case 1: v-independent ds_4^2 - The solution I

- Niehoff, Warner found a general way to solve the sugra constraints in Case 1:

  (almost) all metric coefficients can be expressed algebraically in terms of harmonic functions for the covariant Laplacian $*_4 D *_4 D$.

  Egs: $$Z_1 = L_1 + \frac{K_2 K_3}{V} \quad , \quad Z_2 = L_2 + \frac{K_1 K_3}{V} \qquad \text{with}$$

  $$\partial_\tau K_{2,1} + \partial_v L_{1,2} = 0 \quad , \quad *_4 D *_4 DK_{1,2} = *_4 D *_4 DL_{1,2} = 0$$

  To find ω one has to solve a system of partial differential equations.

Case 1: v-independent ds_4^2 - The solution II

- To construct an asymptotically flat 3-charge geometry one starts from the geometric data generated by the chiral algebra transformation, extracts the corresponding harmonic functions, and replaces

$$L_1 \to 1 + L_1 \quad , \quad L_2 \to 1 + L_2$$

  The coefficients of other harmonic functions have to be modified to preserve regularity and absence of Dirac-Misner strings.

- There is a unique asymptotically flat solution that solves all the regularity constraints and matches the near-horizon solution.
- The asymptotic charges of the solution match the values expected from the CFT (more on this in Rodolfo's talk).

Case 2: v-dependent ds_4^2

- After the change of coordinates ds_4^2 and β are v-dependent.
- An example:
  - Start from the 2-charge metric (in the near-horizon limit) with
    $$F^1 + iF^2 = a\, e^{\frac{2\pi i\, v'}{L}}\, ,\, ,\; F^3 = F^4 = \mathcal{F} = 0;$$
  - Apply the transformation $e^{\epsilon\,(J^3_{-n} - J^3_n)}$ which is equivalent to

$$(*)\;\; \psi \to \psi - \epsilon \sin\left(\frac{n\sqrt{2}v}{R}\right),\; \phi \to \phi - \epsilon \sin\left(\frac{n\sqrt{2}v}{R}\right) \Leftrightarrow x^i \to x^i - f^i(v)$$

  - The coordinate transformation generates the 3-charge near-horizon geometry; ds_4^2, β depend on v.
- The Niehoff, Warner method does not apply directly: how to construct the asymptotically flat solution?
- Note: the transformation $(*)$ cannot be applied to the asymptotically flat solution because it screws up the asymptotic structure.

Case 2: v-dependent ds_4^2 - The solution

- Strategy: apply the Niehoff, Warner method before the change of coordinates (when ds_4^2 and β are v-independent) to generate a new non-explicitly asymptotically flat solution that becomes explicitly asymptotically flat after the transformation $(*)$.
- The new solution needs to have the following asymptotic limit

$$Z_{1,2} \to 1 \quad , \quad \omega \to -\dot{f}_i(v)dx^i \quad , \quad \mathcal{F} \to -|\dot{f}(v)|^2$$

Note: these boundary conditions force all metric coefficients (apart from ds_4^2, β) to be v-dependent.

- Result: there is a unique solution with the correct asymptotic behavior that is regular.

Case 2: relation with "Vachaspati transform"

- The solution generating technique we employ to add momentum to D1-D5 geometries is analogous to the one used by Dabholkar et al. and Callan at al. to construct D1-P geometries:
  - start from the “naive” D1 solution;
  - turn on $\omega = -\dot{f}_i(v)dx^i \quad , \quad \mathcal{F} = -|\dot{f}(v)|^2$;
  - perform the coordinate shift $x^i \to x^i - f^i(v)$.
- One important technical difference: the D1 solution has

$$\beta = 0 \Rightarrow \frac{\partial}{\partial u} \text{ is hypersurface orthogonal}$$

$\Rightarrow$ constant ω and $\mathcal{F}$ solve the equations of motion.

- Moreover, for D1-P solutions all equations are linear and homogeneous $\Rightarrow$ one can superpose solutions

Multiply-wound profiles

- One can also considers profiles that are multiply-wound along S^1 :

$$f^i\left(v + 2\pi\, w\, \frac{R}{\sqrt{2}}\right) = f^i(v)\,, \quad w > 1$$

- This is crucial to obtain the full D1-P entropy.
- The shift $x^i \to x^i - f^i(v)$ is not defined globally on spacetime.
- An equivalent description: $f^i(v) \to f^i_\alpha(v)\,,\ \alpha = 1, \ldots, w$ with

$$f^i_\alpha\left(v + 2\pi\, \frac{R}{\sqrt{2}}\right) = f^i_{\alpha+1}(v)$$

- For D1-P: use linearity to superpose solutions for different α.
- For D1-D5-P: the problem is non-linear.
  How to construct geometries for multiply-wound profiles?

Summary

- We have determined the general class of susy solutions carrying D1,D5,P charges.
- Within this class, we have constructed geometries dual to BPS states of the D1-D5 CFT with momentum.
- The class of microstates we can construct is very particular: chiral algebra descendants of RR ground states.
- It is non-trivial that a unique, regular and asymptotically flat solution exists with the quantum numbers predicted by the CFT.

Outlook

- Main open problems:
  - can one extend the construction of microstate geometries to more generic states?
    Note: constructing geometries corresponding to multi-wound profiles would be a big step in this direction;
  - can microstate geometries account for a finite fraction of the 3-charge entropy?
- Extract more informations from the geometries we have:
  - compute holographically 1-point functions of chiral operators in the 3-charge microstates and compare with the CFT;
    (Kanitscheider, Skenderis, Taylor)
  - compute holographically the entanglement entropy in the microstate geometries.
    (Ryu, Takayanagi)

Entanglement entropy in black hole microstates

- Most computations of entanglement entropy are done in the vacuum or the thermal state $\Rightarrow \mathrm{AdS}_3 \times S^3 \times T^4$.
- T^4-isotropic microstates can be easily reduced on T^4 $\Rightarrow$ ds_6^2 6D Einstein metric.
- In the near-horizon limit ds_6^2 is asymptotically $\mathrm{AdS}_3 \times S^3$ but depends non-trivially on S^3.
- The Ryu, Takayanagi holographic recipe for computing entanglement entropy applies to asymptotically AdS_3 spaces.
- A small generalization: consider a 1D domain A and a geodesic (with respect to ds_6^2) 4-manifold $\Gamma_A = \gamma_A \times S^3$, where γ_A is a curve at $t = \mathrm{const.}$ such that $\partial\gamma_A = \partial A$; the entanglement entropy is

$$S_A = \frac{\mathrm{Area}(\Gamma_A)}{4G_6}$$

- Can one compare with the D1-D5 CFT prediction? (work in progress with R. Russo)