Duality covariant geometry for brane systems

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I. INTRODUCTION
II. MANIFESTLY T-DUAL FORMULATION
III. MANIFESTLY T-DUAL SUPERSPACE FROM TYPE II SUPERSTRING
IV. M-BRANES & D-BRANES
V. CONCLUSIONS

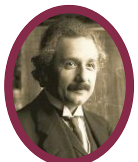
References

- Superspace with manifest T-duality from type II superstring.1403.3887[hep-th] , Kiyoshi Kamimura, Warren Siegel & M.H.
- M5 &SO(5,5). ‘13; SL(5) & M2.’12, K.Kamimura& M.H.; D-branes.’12, Tetsuji Kimura& M.H.
- Natural curvature for manifest T-duality. ‘13,M.Polacek & Siegel.
- Two viebein formalism for string inspired axionic gravity; Superspace duality in low-energy superstrings; Manifest duality in low-energy superstrings.’93 W.Siegel
- Randomizing the superstring. ‘94 W. Siegel.

I. Introduction

motivation

◆ Gravity theory beyond Einstein's gravity



"general covariance" ⇒ Einstein gravity theory

" **Manifest T-duality** " ⇒ effective gravity of string

(T-duality symmetry in massless sector of string)

◆ New string background geometry beyond Riemann

- Double field theory

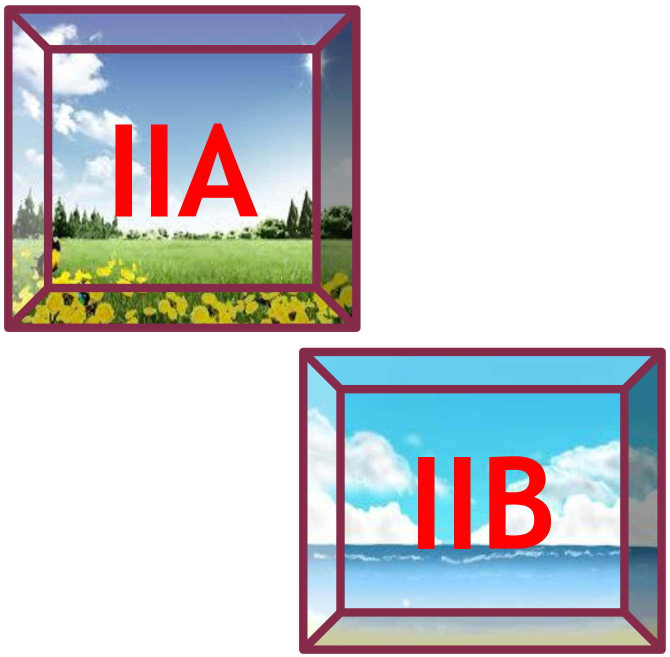
  '93 Siegel, '09 Hull&Zwiebach,'10 Hohm, Kwaw; Jeon, Lee, Park; Thompson; Tseytlin,...

- Generalized geometry

  '02 Hitchin, '04 Gualtieri, '07 Hull '08 Pacchco & Waldram, '09 Grana, Louis, Sim,'10 Berman & Perry,'11 Coimbra, Strickland-Constable ...; Koerber, Baraglia, Dabholkar ,Copland, Reid-Edwards Godazgar's, West...

T-duality in SUGRA's

- Hidden T-duality



a background

- Manifest T-duality

All backgrounds

T-duality in gravity vs string

momenta ⇔winding

Hidden in target space

Effective gravity action

$$\mathcal{L} = R + (\partial\phi)^2 + (\partial B)^2$$

$$(G+B) \to \frac{a(G+B)+b}{c(G+B)+d}$$

- Fractional transf. of components fields under O(d,d)

Manifest in worldsheet

String Hamiltonian

$$\mathcal{H}_\tau = (p, \partial_\sigma x)\mathcal{M}\begin{pmatrix} p \\ \partial_\sigma x \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} G^{mn} & G^{mp}B_{pn} \\ -B_{mp}G^{pn} & G_{mn} - B_{mp}G^{pq}B_{qn} \end{pmatrix}$$

$$\mathcal{M} = E^T E, \;\; E \to EA$$

- Linear transf. of vielbein under O(d,d)

Q. How to approach it?

★ SUGRA IN SUPERSPACE

FROM SUPERSTRING

II. Manifestly T-dual formalism

1. Affine Lie algebra
2. Vielbein
3. Torsions
4. Dimensional reductions

II-1. Affine Lie algebra

flat

'93 Siegel

- Bosonic string current: $\mathring{\triangleright}_M = (p_m, \partial_\sigma x^m)$
- Algebra:

$$[\mathring{\triangleright}_M(\sigma), \mathring{\triangleright}_N(0)] = f_{MN}{}^K \mathring{\triangleright}_K \delta(\sigma) + \eta_{MN} \partial_\sigma \delta(\sigma)$$

Jacobi identity ⇒

$$f_{[MN}{}^K f_{L]K}{}^J = 0 \ \& \ f_{MNK} \equiv f_{MN}{}^L \eta_{LK} = f_{[MNK]}$$

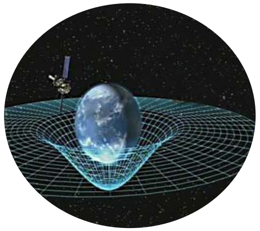
symmetric group metric

$$\eta_{MN} = \begin{array}{c} p_m \\ \partial_\sigma x^m \end{array} \begin{pmatrix} & \delta_m^n \\ \delta_n^m & \end{pmatrix}$$

totally antisymmetric

★ nondegenerate group metric required.

II-2. Vielbein



curved

- Covariant derivative: $\triangleright_A = E_A{}^M \mathring{\triangleright}_M$
- Algebra:

$$[\triangleright_A(\sigma), \triangleright_B(0)\} = T_{AB}{}^C \triangleright_C \delta(\sigma) + \underline{\eta_{AB}\partial_\sigma \delta(\sigma)}$$

Same as flat

- Orthonormal condition on $E_A{}^M$

$$\boxed{E\eta E^T = \eta \quad , \quad \eta_{MN} = \eta_{AB}}$$

totally antisym.torsion

$$T_{ABC} = (D_{[A}E_B{}^M)E_{C]M} + E_A{}^M E_B{}^N E_C{}^L f_{MNL}$$

★ Vielbein can be orthonormal O(d,d)

- Hamiltonian constraint:

$$\boxed{\mathcal{H}_\tau = \mathring{\triangleright}_M \mathcal{M}^{MN} \mathring{\triangleright}_N = \triangleright_A \hat{\delta}^{AB} \triangleright_B} \qquad \boxed{\hat{\delta} = \begin{pmatrix} \mathbf{1} & \\ & \mathbf{1} \end{pmatrix}}$$

O(d,d)/O(d)^2

#(G, B)= d^2

Bosonic string

- Vielbein: $E_A{}^M = \begin{pmatrix} e_a{}^m & e_a{}^n B_{nm} \\ 0 & e_m{}^a \end{pmatrix}$

$$E^T \hat{\delta} E = \mathcal{M} = \begin{pmatrix} G^{mn} & G^{mp} B_{pn} \\ -B_{mp} G^{pn} & G_{mn} - B_{mp} G^{pq} B_{qn} \end{pmatrix}$$

★ Vielbein is O(d,d)/O(d)^2

★ G&B are coset parameter

- Diffeo. constraint:

$$\boxed{\mathcal{H}_\sigma = \mathring{\triangleright}_M \eta^{MN} \mathring{\triangleright}_N = \triangleright_A \eta^{AB} \triangleright_B} \qquad \boxed{\eta = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}}$$

- derivatives

$$[\mathring{\triangleright}_M, \Phi(Z)] = (D_M \Phi)$$

$$[\int \mathcal{H}_\sigma, \Phi(Z)] = \partial_\sigma \Phi = \mathring{\triangleright}_M \eta^{MN} (D_N \Phi)$$

- condition on double coordinates

$$\mathcal{H}_\sigma = \mathring{\triangleright}_m \mathring{\triangleright}^m = p_m \partial_\sigma x^m = 0$$

★ σ-Diffeo. Is given by string algebra basis.

★ It gives cond. In double coordinate space

- T-duality covariant gauge symmetry

$$\delta \hat{E}_A = \left[\hat{E}_A, \hat{\lambda}\right]_T$$

zero-mode part of string algebra!

$$[E_A{}^M \mathring{\triangleright}_M, \lambda^N \mathring{\triangleright}_N] = \delta E_A{}^M \mathring{\triangleright}_M + \cancel{c\,\partial_\sigma \delta}$$

- General coordinate transf. & gauge transf.

$$\begin{cases} \delta G_{mn} = \lambda^l \partial_l G_{mn} + \partial_{(m} \lambda^l G_{l|n)} \\ \delta B_{mn} = \lambda^l \partial_l B_{mn} + \partial_{[m} \lambda^l B_{l|n]} + \partial_{[m} \lambda_{n]} \end{cases}$$

★ zero-mode string alg. Gives T-dual gauge sym.

II-3. Torsions

- Covariant derivative & curvature

$$\triangleright_A = \omega_A{}^S S + e_A{}^M P_M \qquad [\triangleright_A, \triangleright_B] = R_{AB}{}^S S + T_{AB}{}^C \triangleright_C$$

- Constrain torsion by hand

$$T_{PP}{}^P = 0 \Rightarrow \underline{\omega_{PP}{}^P = e(\partial e)e}, \boxed{\omega_{PPP} = \partial B}$$

NS-NS 3 form

- Curvature tensor

$$R_{PP}{}^S = \partial\omega + \omega\omega$$

curvature in terms of ω's

★ T-dual cov. geometry is given !

II-3. Torsions

- Covariant derivative & curvature

$$\triangleright_A = \boxed{\omega_A{}^S} S + e_A{}^M P_M \qquad [\triangleright_A, \triangleright_B] = \boxed{R_{AB}{}^S} S + T_{AB}{}^C \triangleright_C$$

$$\boxed{E_P{}^S} \qquad \boxed{T_{PP}{}^S}$$

- Constrain torsion by hand

$$T_{PP}{}^P = 0 \Rightarrow \underline{\omega_{PP}{}^P = e(\partial e)e}, \boxed{\omega_{PPP} = \partial B}$$

NS-NS 3 form

- Curvature tensor

$$R_{PP}{}^S = \partial\omega + \omega\omega$$

curvature in terms of ω's

★ T-dual cov. geometry is given !

String algebra

'13 Polacek & Siegel
"Natural curvature for
manifest T-duality"

nondegenerate Poincare

$$[P_m, P_n] = \Sigma_{mn} + \eta_{mn}\partial_\sigma\delta$$
$$[S_{mn}, \Sigma^{lk}] = \delta^k_{[m}\Sigma^l_{n]} + \eta_{S\Sigma}\partial_\sigma\delta$$

$$\overset{\circ}{\triangleright}_M = (S, P, \Sigma)\ \times 2$$

- metric

$$\eta_{MN} = \begin{array}{c} S \\ P \\ \Sigma \end{array} \begin{pmatrix} & & \mathbf{1} \\ & \mathbf{1} & \\ \mathbf{1} & & \end{pmatrix}$$

- Vielbein

$$E_A{}^M = \begin{array}{c} S \\ P \\ \Sigma \end{array} \begin{pmatrix} \mathbf{1} & & \\ \omega & e & \\ r & -\omega^T & \mathbf{1} \end{pmatrix}$$

NS-NS 3 form
& torsion

★ T-dual geometry is given by Poincare/Lorentz

★ NS-NS 3form & torsion are mixed under T

II-4. Dimensional reduction

- Constraints by symmetry generators

$$\tilde{\Sigma}_\pm = \frac{\partial}{\partial Z^{\Sigma}{}_\pm} + \cdots = 0$$

$$\tilde{P}_+ - \tilde{P}_- = 0 \;\Rightarrow\; \tilde{P}_+ + \tilde{P}_- \to \tilde{P} = \frac{\partial}{\partial x}$$

$$S_+ = S_- = 0 \;\Rightarrow\; Z^S_\pm = 0$$

$$\tilde{S} = \tilde{S}_+ + \tilde{S}_- = [x, \partial_P]$$

- Coordinates: $(\cancel{Z^S, Z^{S'}}, x, \cancel{x'}, \cancel{Z^\Sigma, Z^{\Sigma'}})$

★ Symmetry generator commutes with covariant derivative, so it does not change local structure.

'13 Polacek & Siegel

RECIPE:
manifestly T-dual gravity

1. Affine Lie algebra & double generators
2. Make covariant derivatives with vielbein
3. Constrain torsions by hands
4. Break manifest T-duality to hidden one

III. Manifestly T-dual superspace from type II superstring

1. Affine Lie algebra
2. Vielbein
3. Torsions
4. Dimensional reductions

RECIPE:

'14 Kamimura, Siegel & M.H.

manifestly T-dual **superspace**

1. Supestring algebra $\times 2$

2. Supervielbein

3. Constrain torsions **by kappa symmetry**

4. Break manifest T-duality to hidden one

RECIPE:

‘14 Kamimura, Siegel & M.H.

manifestly T-dual **superspace**

1. Supestring algebra $\times 2$

III-1. Superstring algebra

- Non-degenerate super-Poincare

susy

$$\overset{\circ}{\triangleright}_M = (S_{mn}, D_\mu, P_m, \Omega^\mu, \Sigma^{mn}) \times 2$$

Dim. 0 1/2 1 3/2 2

- Metric:

$$\eta_{MN} = \begin{array}{c c} & \begin{array}{ccccc} S & D & P & \Omega & \Sigma \end{array} \\ \begin{array}{c} S \\ D \\ P \\ \Omega \\ \Sigma \end{array} & \begin{pmatrix} & & & & 1 \\ & & & -1 & \\ & & 1 & & \\ & 1 & & & \\ 1 & & & & \end{pmatrix} \end{array}$$

★ 2 sets of P, (D,Ω),(S,Σ) are used as sust.alg.

Nondegenerate super-poincare algbera

Dim.

1 $$\{D, D\} = \not{P}$$

3/2 $$[D, P_m] = \gamma_m \Omega$$

2 $$[P_m, P_n] = \underline{\Sigma_{mn}} + \eta_{mn}\underline{\partial_\sigma \delta}$$

$$[D, \Omega] = \underline{\Sigma^{mn}}\gamma_{mn} + \eta_{D\Omega}\underline{\partial_\sigma \delta}$$

$$[S_{mn}, \Sigma^{lk}] = \delta^k_{\underline{[m}\Sigma^l_{n]}} + \eta_{S\Sigma}\underline{\partial_\sigma \delta}$$

RECIPE:

‘14 Kamimura, Siegel & M.H.

manifestly T-dual **superspace**

1. Supestring algebra $\times 2$

2. Supervielbein

III-2. Supervielbein

$$
E_A{}^M = \begin{array}{c} S \\ D \\ P \\ \Omega \\ \Sigma \end{array} \left(\begin{array}{c|ccc|c} \mathbf{1} & & & & \\ \hline \omega_\alpha & e_\alpha{}^\nu & B_\alpha{}^n & B_{\alpha\nu} & \\ \omega_a & \psi_a{}^\nu & e_a{}^n & B_{a\nu} & \\ \omega^\alpha & F^{\alpha\nu} & \psi^{\alpha n} & e^\alpha{}_\nu & \\ \hline r & -\omega^T & -\omega^T & -\omega^T & \mathbf{1} \end{array} \right)
$$

III-2. Supervielbein

Super B fields

Super connections
NS-NS field strength

Gravity
NS-NS B fields

$$
E_A{}^M = \begin{array}{c} S \\ D \\ P \\ \Omega \\ \Sigma \end{array}
\left(\begin{array}{c|ccc|c}
1 & & & & \\ \hline
\omega_\alpha & e_\alpha{}^\nu & B_\alpha{}^n & B_{\alpha\nu} & \\
\omega_a & \psi_a{}^\nu & e_a{}^n & B_{a\nu} & \\
\omega^\alpha & F^{\alpha\nu} & \psi^{\alpha n} & e^\alpha{}_\nu & \\ \hline
r & -\omega^T & -\omega^T & -\omega^T & 1
\end{array}\right)
$$

R-R field strength　　gravitino

★ NS-NS, R-R field strengths are mixed

III-2. Supervielbein

Dim.

0 -1/2 -1

$$E_A{}^M = \begin{array}{c} S \\ D \\ P \\ \Omega \\ \Sigma \end{array} \left(\begin{array}{c|ccc|c} 1 & & & & \\ \hline \omega_\alpha & e_\alpha{}^\nu & B_\alpha{}^n & B_{\alpha\nu} & \\ \omega_a & \psi_a{}^\nu & e_a{}^n & B_{a\nu} & \\ \omega^\alpha & F^{\alpha\nu} & \psi^{\alpha n} & e^\alpha{}_\nu & \\ \hline r & -\omega^T & -\omega^T & -\omega^T & 1 \end{array} \right)$$

Prepotential

2 3/2 1 1/2

★ Bμν turns out to be a prepotential of all fields

RECIPE:

'14 Kamimura, Siegel & M.H.

manifestly T-dual **superspace**

1. Supestring algebra $\times 2$

2. Supervielbein

3. Constrain torsions **by kappa symmetry**

III-3. κ-sym. constrains torsions

- Fermionic constraits: $D_\alpha = 0$

$$\boxed{\{D_\alpha, D_\beta\} = \not{P}_{\alpha\beta}}\ , \quad P^2 = 0$$

- 1st class ---kappa-symmetry generator

$$D_\alpha \not{P} = 0 \quad \Rightarrow \not{A}\not{B}\not{C}\not{D} = 0$$

- 2nd class ---must be eliminated

$$D_\alpha = 0$$

- K-symmetric Virasoro constraints

Dim.		
2	$\mathcal{A} = \frac{1}{2}P^2 + \Omega D + \frac{1}{2}\Sigma S$	Virasoro
3/2	$\mathcal{B}^\mu = D\not{P} - \not{S}\Omega$	Kappa-symmetry
1	$\mathcal{C}_{\mu\nu} = D_\mu D_\nu + (\not{S}\not{P})_{[\mu\nu]}$	(2nd class) 2
3	$\mathcal{D}_m = D\gamma_m\partial_\sigma D + \Sigma_{mn}S^{nl}P_l$	
0	$S_{mn} = S_{m'n'} = 0$	Lorentz

- Consistency of κ-sym. ⇒Torsion constraints.

$$\triangleright_\alpha = 0$$

$$[\triangleright_\alpha, \mathcal{A}] \approx 0$$

$$\Leftrightarrow \quad T_{\alpha BC'} = 0 \text{ for } B, C = (S, D_\alpha, P)$$

$$\{\triangleright_\alpha, \mathcal{B}\} \approx 0$$

$$\Leftrightarrow \quad \boxed{\{\triangleright_\alpha, \triangleright_\beta\} \approx \triangleright_P}$$

$$T_{\alpha\beta P} = (\gamma_P)_{\alpha\beta} \ , \ T_{\alpha\beta X} = 0 \text{ for } X = (S, S', D_\alpha, D_{\alpha'}, P')$$

$$T_{\alpha PY} = 0 \text{ for } Y = (S, S', D_{\alpha'}, P, P', \Omega, \Omega')$$

★ All vielbein fields are written by a prepotential $\boxed{B_{\mu\nu} = E_D{}^\Omega = E_{DD}}$

RECIPE:

'14 Kamimura, Siegel & M.H.

manifestly T-dual **superspace**

1. Supestring algebra $\times 2$

2. Supervielbein

3. Constrain torsions **by kappa symmetry**

4. Break manifest T-duality to hidden one

- Dimensional reduction

$$\tilde{\Sigma}_\pm = \tilde{\Omega}_\pm = 0$$
$$\tilde{P}_+ - \tilde{P}_- = 0 \Rightarrow \tilde{P}_+ + \tilde{P}_- \to \tilde{P}$$
$$S_\pm = 0$$
$$\tilde{S}_+ + \tilde{S}_- \to \tilde{S} = x\frac{\partial}{\partial x} + \theta\partial_\alpha + \theta'\partial_{\alpha'}$$

- Coordinates

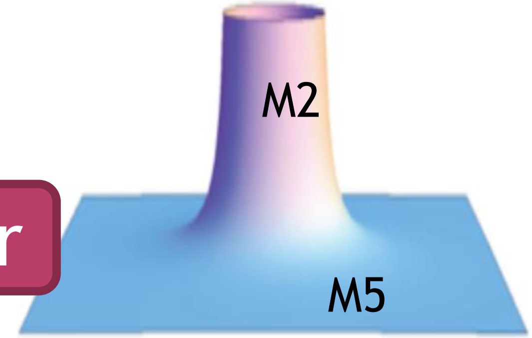
$$(\cancel{Z^S}, \cancel{Z^{S'}}, \theta, \theta', x, \cancel{x'}, \cancel{Z^\Omega}, \cancel{Z^{\Omega'}}, \cancel{Z^\Sigma}, \cancel{Z^\Sigma})$$

★ Dim. Red.gives usual coordinate space

IV. M & D-branes

1. Affine Lie algebra
2. Vielbein

M5-brane



- M5 algebra base: O(5,5)spinor

$$\begin{aligned}\mathring{\triangleright}_M &= (\mathring{\triangleright}_m,\ \mathring{\triangleright}^{[2]mn},\ \mathring{\triangleright}^{[5]}) \\ &= (p_m,\ E^{ij}\partial_i x^m \partial_j x^n,\ \partial_{[i_1} x^{[m_1} \cdots \partial_{i_5]} x^{m_5]})\end{aligned}$$

$m=1,\cdots,5$

- "metric" $\boxed{\mathcal{H}_i = p_m \partial_i x^m + \epsilon_{ij_1\cdots j_4} E^{j_1 j_2} E^{j_3 j_4}}$

$$\mathring{\triangleright}_M \rho^{MN} \mathring{\triangleright}_N = 0 \ \&\ \ \rho^{MN} = \begin{pmatrix} & a & b \\ a & b & \\ b & & \end{pmatrix}$$

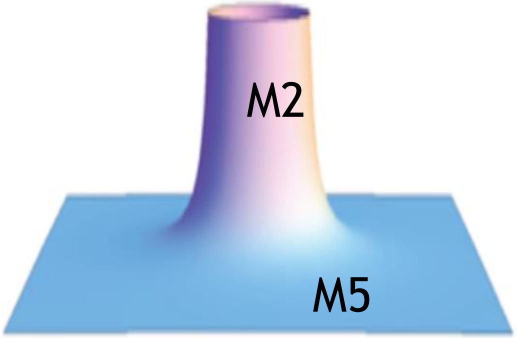
O(5,5) vector

$$\begin{aligned} a\ \ (\mathring{\triangleright}\Gamma^m \mathring{\triangleright}) &= \mathring{\triangleright}_n \mathring{\triangleright}^{[2]nm} = 0 \\ b\ \ (\mathring{\triangleright}\Gamma_m \mathring{\triangleright}) &= \mathring{\triangleright}_m \mathring{\triangleright}^{[5]} + \epsilon_{mn_1\cdots n_4} \mathring{\triangleright}^{[2]n_1 n_2} \mathring{\triangleright}^{[2]n_3 n_4} = 0 \end{aligned}$$

'13 Kamimura & M.H.

- Vielbein

$$E_A{}^M = \begin{pmatrix} e_a{}^m & C^{[3]} & C^{[6]} + C^{[3]} \wedge C^{[3]} \\ & (e_m{}^a)^2 & C^{[3]} \\ & & (e_m{}^a)^5 \end{pmatrix}$$



- Gauge symmetry

$$\delta_\xi E_a{}^M = [\xi, E_a]_{\mathrm{M5}}$$

$$\begin{cases} \delta_\xi G = \mathcal{L}_\xi G \\ \delta_\xi C^{[3]} = \mathcal{L}_\xi C^{[3]} + d\xi^{[2]} \\ \delta_\xi C^{[6]} = \mathcal{L}_\xi C^{[6]} + d\xi^{[5]} + C^{[3]} \wedge d\xi^{[2]} \end{cases}$$

General transf.

Gauge transf.

★ M5 alg. is closed with self-dual gauge field & define gauge sym.

D3-brane

'12 Kimura & M.H.

- D3 algebra base:

DBI U(1) field

$$\begin{aligned}\mathring{\triangleright}_M &= (\mathring{\triangleright}m, \mathring{\triangleright}^m | \mathring{\triangleright}^{[1]m}, \mathring{\triangleright}^{[3]mnl}) \\ &= (p_m, E^i\partial_i x^m | \epsilon^{ijk}F_{ij}\partial_k x^m, \epsilon^{ijk}\partial_i x^m \partial_j x^n \partial_k x^l)\end{aligned}$$

- "metric"

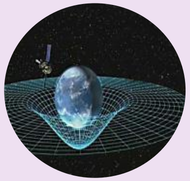
$$\boxed{\begin{aligned}\mathcal{H}_i = p_m\partial_i x^m + F_{ij}E^j \\ \Rightarrow \quad \mathring{\triangleright}_M \rho^{MN} \mathring{\triangleright}_N = 0\end{aligned}}$$

★ Dp alg. is closed with DBI gauge field & give gauge transf.

$$\rho^{MN} = \left(\begin{array}{cc|cc} & a & b & c \\ a & & c & \\ \hline b & c & & \\ c & & & \end{array}\right) \Leftrightarrow \begin{cases} a & \mathring{\triangleright}_m \mathring{\triangleright}^m = 0 \\ b & \mathring{\triangleright}_m \mathring{\triangleright}^{[1]m} = 0 \\ c & \mathring{\triangleright}_m \mathring{\triangleright}^{[3]mnl} + \mathring{\triangleright}^{[n} \mathring{\triangleright}^{[1]|l]} = 0 \end{cases}$$

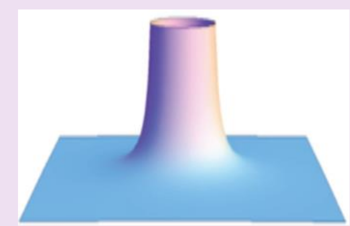
- Vielbein:

$$E_a{}^M = e_a{}^n \left(\delta_n^m, \ B_{nm} | C^{[2]}_{nm}, \ C^{[4]}_{nm_1m_2m_3} + C^{[2]}_{nm_1} C^{[2]}_{m_2m_3}\right)$$



V. Conclusions

- Duality covariant geometry is formulated from brane algebras:
  - Superspace with manifestly T-duality from superstring (nondegenerate super-Poincare)
  - Kappa-Virasoro gives torsion constraints
  - Bμν is a prepotential



- Future problems
  - R-R sector, D-branes, M branes, exotic branes
  - effective theory of F theory
  - • • •