Exceptional Field Theory

Olaf Hohm

- O.H., H. Samtleben, reformulation of DFT, arXiv:1307.0039
- U-duality covariant gravity, arXiv:1307.0509
- Exceptional field theory, arXiv:1308.1673 [PRL]
  $E_{6(6)}$, arXiv:1312.0614 , $E_{7(7)}$, arXiv:1312.4542,
  Ehlers group and $E_{8(8)}$ [to appear]

Extension of double field theory:

- O.H., Hull & Zwiebach, 1003.5027, 1006.4823; W. Siegel hep-th/9305073

Kyoto, March 2014

Plan of the talk:

- Review of double field theory
- Exceptional field theory for $\mathrm{E}_{6(6)}$: M-theory and type IIB
- Exceptional field theory for $\mathrm{E}_{7(7)}$: new fields and dual graviton
- Exceptional field theory for $\mathrm{E}_{8(8)}$ & $\mathrm{SL}(2,\mathbb{R})$ Ehlers group in $D = 4$ Einstein gravity

Review Double Field Theory

Reformulation (Extension?) of spacetime action for massless string fields:

$$S_{\text{NS}} = \int d^D x \sqrt{-g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk} + \frac{1}{4} \alpha' R^{ijkl} R_{ijkl} + \cdots \right]$$

generalized metric and doubled coordinates $X^M = (\tilde{x}_i, x^i)$,

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{kl} b_{lj} \end{pmatrix} \in O(D,D)$$

DFT Action (dilaton density $e^{-2d} = e^{-2\phi}\sqrt{-g}$):

$$S_{\text{DFT}} = \int d^{2D} X \, e^{-2d} \, \mathcal{R}(\mathcal{H}, d) \quad \xrightarrow{\tilde{\partial}^i = 0} \quad S_{\text{NS}}\big|_{\alpha'=0}$$

generalized curvature scalar

$$\mathcal{R} \equiv 4\,\mathcal{H}^{MN} \partial_M \partial_N d - \partial_M \partial_N \mathcal{H}^{MN} - 4\,\mathcal{H}^{MN} \partial_M d\, \partial_N d + 4 \partial_M \mathcal{H}^{MN}\, \partial_N d$$
$$+ \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL}\, \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL}\, \partial_K \mathcal{H}_{NL}$$

Gauge transformations and generalized Lie derivatives

In DFT gauge invariance governed by generalized Lie derivatives

$$\widehat{\mathcal{L}}_\xi \mathcal{H}_{MN} = \xi^P \partial_P \mathcal{H}_{MN} + \left(\partial_M \xi^P - \partial^P \xi_M\right) \mathcal{H}_{PN} + \left(\partial_N \xi^P - \partial^P \xi_N\right) \mathcal{H}_{MP}$$

$$\widehat{\mathcal{L}}_\xi \left(e^{-2d}\right) = \partial_M \left(\xi^M e^{-2d}\right)$$

Invariance and closure, $[\widehat{\mathcal{L}}_{\xi_1}, \widehat{\mathcal{L}}_{\xi_2}] = \widehat{\mathcal{L}}_{[\xi_1,\xi_2]_C}$, modulo strong constraint

$$\eta^{MN} \partial_M \partial_N = 2\tilde{\partial}^i \partial_i = 0 \qquad \eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

C-bracket

$$\left[\xi_1, \xi_2\right]_C^M = \xi_1^N \partial_N \xi_2^M - \frac{1}{2} \xi_{1N} \partial^M \xi_2^N - (1 \leftrightarrow 2)$$

non-trivial Jacobiator non-zero but 'trivial'

$$\left[\left[\xi_1, \xi_2\right]_C, \xi_3\right]_C^M + \text{cycl.} = \frac{1}{6} \partial^M \left(\left[\xi_1, \xi_2\right]_C^N \xi_{3N} + \text{cycl.}\right)$$

Kaluza-Klein rewriting of Double Field Theory

$D = n + d$ split $X^{\widehat{M}} = (\tilde{x}_\mu, x^\mu, Y^M)$, $M = 1, \ldots 2d$, setting $\tilde{\partial}^\mu = 0$

$$\mathcal{H}_{\widehat{M}\widehat{N}}\,,\ \ \widehat{d} \quad \rightarrow \quad \{\ g_{\mu\nu}\,,\ \ B_{\mu\nu}\,,\ \ \phi\,,\ \ \mathcal{H}_{MN}\,,\ \ A_\mu{}^M\ \}$$

Gauge structure given by D-bracket $\big[\Lambda,\ \cdot\ \big]_D \equiv \widehat{\mathcal{L}}_\Lambda$:

$$\delta_\Lambda A_\mu{}^M \equiv\ \partial_\mu \Lambda^M + \big[\Lambda, A_\mu\big]_D^M$$

Naive curvature *not gauge covariant*,

$$F_{\mu\nu}{}^M \ =\ 2\partial_{[\mu}A_{\nu]}{}^M - \big[A_\mu, A_\nu\big]_C^M \quad \Rightarrow \quad \delta_\Lambda F_{\mu\nu}{}^M \ =\ \widehat{\mathcal{L}}_\Lambda F_{\mu\nu}{}^M + \partial^M\big(\partial_{[\mu}\Lambda^N A_{\nu]N}\big)$$

$\rightarrow$ compensating 2-form potential ['tensor hierarchy', de Wlt, Samtleben]

$$\mathcal{F}_{\mu\nu}{}^M \ =\ F_{\mu\nu}{}^M - \partial^M B_{\mu\nu}$$

Covariant field strength for 2-form $\left[\, D_\mu = \partial_\mu - A_\mu{}^M \partial_M + \cdots \,\right]$

$$\mathcal{H}_{\mu\nu\rho} \;=\; 3\Big(D_{[\mu} B_{\nu\rho]} + A_{[\mu}{}^N \partial_\nu A_{\rho]N} - \frac{1}{3} A_{[\mu N} \big[A_\nu , A_{\rho]} \big]_C^N \Big)$$

satisfying Bianchi identity

$$3 D_{[\mu} \mathcal{F}_{\nu\rho]}{}^M + \partial^M \mathcal{H}_{\mu\nu\rho} \;=\; 0$$

Complete DFT action in Kaluza-Klein variables:

$$S \;=\; \int d^n x \, d^{2d} y \, e \, e^{-2\phi} \Big(\widehat{R} + 4 g^{\mu\nu} D_\mu \phi D_\nu \phi - \frac{1}{12} \mathcal{H}^{\mu\nu\rho} \mathcal{H}_{\mu\nu\rho} + \frac{1}{8} g^{\mu\nu} D_\mu \mathcal{H}^{MN} D_\nu \mathcal{H}_{MN} - \frac{1}{4} \mathcal{H}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}_{\mu\nu}{}^N - V \Big)$$

with ‘potential’

$$V(\phi, \mathcal{H}, g) \;=\; -\mathcal{R}(\phi, \mathcal{H}) - \frac{1}{4} \mathcal{H}^{MN} \partial_M g^{\mu\nu} \, \partial_N g_{\mu\nu}$$

c.f. Maharana-Schwarz [1993]; but includes ‘internal’ $Y^M = (\tilde{y}_m, y^m)$

Analogue for 11-dimensional supergravity/M-theory?

Cremmer-Julia [1979]: torus reduction of $D = 11$ SUGRA

$\rightarrow$ $\mathsf{E}_{6(6)}$ $[D = 5]$, $\mathsf{E}_{7(7)}$ $[D = 4]$, $\mathsf{E}_{8(8)}$ $[D = 3]$

Larger mathematical framework that explains/makes it manifest?

Hillmann [2009]: truncation of $D = 11$ SUGRA in $4 + 7$ split,

keeping only 'internal' field components and coordinates,

$$G_{MN} = \begin{pmatrix} e^{2\Delta}\eta_{\mu\nu} & 0 \\ 0 & g_{mn}(y) \end{pmatrix} , \qquad \text{etc.}$$

extending coordinates to fundamental $\mathbf{56} \Rightarrow \mathsf{E}_{7(7)}$ covariant action

more recently: other groups, geometry, covariant section constraints, etc.

[Berman & Perry (2010), Coimbra, Strickland-Constable & Waldram (2011), etc.]

Complete $D = 11$ SUGRA?? duality transformations in $D = 11$??

Exceptional Field Theory I: $\mathsf{E}_{6(6)}$

Coordinates (x^μ, Y^M), $\mu = 0, \ldots, 4$, M : fundamental $\mathbf{27}$ of $\mathsf{E}_{6(6)}$

Invariant tensors under $\mathbf{27}$ and dual $\overline{\mathbf{27}}$: d_{MNK}, d^{MNK}

Section constraint: [Coimbra et.al. (2011), Berman et.al. (2012)]

$$d^{MNK}\,\partial_N \otimes \partial_K \;=\; 0$$

Generalized Lie derivative

$$\mathbb{L}_\Lambda V^M \;\equiv\; \Lambda^K \partial_K V^M - 6\,\mathbb{P}^M{}_N{}^K{}_L\,\partial_K \Lambda^L\, V^N + \lambda\,\partial_P \Lambda^P\, V^M$$

with projector $\mathbb{P}^M{}_N{}^K{}_L$ onto adjoint; $\lambda(V)$ density weight.

Closure, $[\,\mathbb{L}_{\Lambda_1}, \mathbb{L}_{\Lambda_2}\,] \;=\; \mathbb{L}_{[\Lambda_1,\Lambda_2]_\mathsf{E}}$, according to ‘E-bracket’

$$\big[\Lambda_1, \Lambda_2\big]_\mathsf{E}^M \;=\; 2\Lambda^K_{[1}\partial_K\Lambda^M_{2]} - 10\,d^{MNP}d_{KLP}\,\Lambda^K_{[1}\partial_N\Lambda^L_{2]}$$

Non-trivial (but ‘trivial’) Jacobiator

$$J^M(\Lambda_1,\Lambda_2,\Lambda_3) \propto d^{MKR}\partial_K\Big(d_{RPL}\big(\,[\Lambda_1,\Lambda_2]^P_\mathsf{E}\,\Lambda^L_3 + \text{cycl.}\Big)$$

Fields transform with $\mathsf{E}_{6(6)}$ gen. Lie derivatives w.r.t. $\Lambda^M(x,Y)$

$A_\mu{}^M$ gauge field for *local* Λ^M: $\qquad \delta A_\mu{}^M \equiv \mathcal{D}_\mu \Lambda^M$

Covariant curvature

$$\mathcal{F}_{\mu\nu}{}^M \equiv 2\partial_{[\mu} A_{\nu]}{}^M - \big[A_\mu, A_\nu\big]_{\mathsf{E}}^M + 10\, d^{MNK}\partial_K B_{\mu\nu N}$$

3-form field strength $\mathcal{H}_{(3)M}$ of 2-forms by generalized Bianchi identity

$$3\,\mathcal{D}_{[\mu}\mathcal{F}_{\nu\rho]}{}^M \;=\; 10\, d^{MNK}\partial_K \mathcal{H}_{\mu\nu\rho\, N}$$

Complete action for fields $(e_\mu{}^a, \mathcal{M}_{MN}, A_\mu{}^M, B_{\mu\nu M})$, $\mathcal{M} \in \mathsf{E}_{6(6)}$:

$$S = \int d^5x\, d^{27}Y\, e\Big(\widehat{R} + \frac{1}{24}\, g^{\mu\nu}\mathcal{D}_\mu \mathcal{M}^{MN}\,\mathcal{D}_\nu \mathcal{M}_{MN} \\ - \frac{1}{4}\,\mathcal{M}_{MN}\mathcal{F}^{\mu\nu M}\mathcal{F}_{\mu\nu}{}^N + e^{-1}\mathcal{L}_{\mathsf{top}} - V(\mathcal{M}, e)\Big)$$

Topological CS-like term:

$$S_{\text{top}} = \kappa \int d^{27}Y \int_{\mathcal{M}_6} \Big(d_{MNK}\, \mathcal{F}^M \wedge \mathcal{F}^N \wedge \mathcal{F}^K - 40\, d^{MNK} \mathcal{H}_M \wedge \partial_N \mathcal{H}_K \Big)$$

'Potential' term:

$$V = \frac{1}{2}\mathcal{M}^{MN}\partial_M\mathcal{M}^{KL}\partial_L\mathcal{M}_{NK} - \frac{1}{24}\mathcal{M}^{MN}\partial_M\mathcal{M}^{KL}\partial_N\mathcal{M}_{KL}$$
$$- e^{-1}\partial_M e\, \partial_N\mathcal{M}^{MN} - \frac{1}{8}\mathcal{M}^{MN}e^{-1}\partial_M e\, e^{-1}\partial_N e - \frac{1}{32}\mathcal{M}^{MN}\partial_M g^{\mu\nu}\partial_N g_{\mu\nu}$$

Solving section constraint: reduce $\text{E}_{6(6)}$ under $\text{SL}(6) \times \text{SL}(2)$:

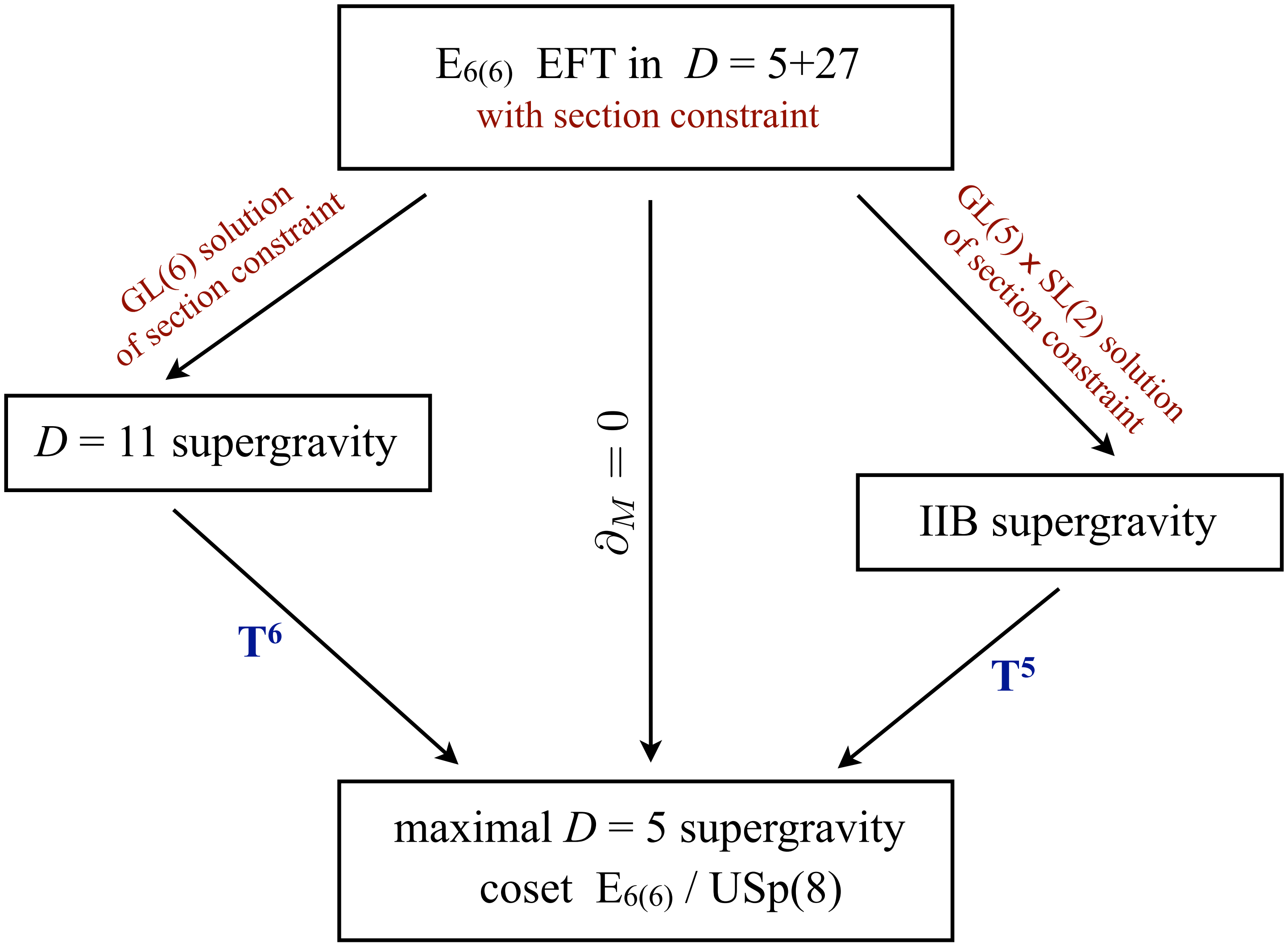
$$\mathbf{27} \;\rightarrow\; (15, 1) + (6, 2)$$

1) 6 coordinates from $SL(2)$ doublet $\Rightarrow$ $D = 11$ supergravity

2) break to $SL(5) \Rightarrow$ 5 coordinates from $15 \Rightarrow$ unbroken $SL(2)$: IIB sugra

'unification' of M-theory and type IIB, c.f. type II DFT

[Hohm, Ki Kwak, Zwiebach (2011)]



$E_{6(6)}$ EFT in $D = 5+27$
with section constraint
GL(6) solution
of section constraint
$\partial_M = 0$
GL(5) × SL(2) solution
of section constraint
$D = 11$ supergravity
IIB supergravity
T^6
T^5
maximal $D = 5$ supergravity
coset $E_{6(6)}$ / USp(8)

Exceptional Field Theory II: $E_{7(7)}$

Coordinates (x^μ, Y^M), $\mu = 0, \ldots, 3$, M : fundamental $\mathbf{56}$ of $E_{7(7)}$

$$(t_\alpha)^{MN}\,\partial_M \otimes \partial_N \;=\; 0\;, \qquad \Omega^{MN}\,\partial_M \otimes \partial_N \;=\; 0$$

with Ω^{MN} symplectic form of $E_{7(7)} \subset Sp(56)$

Generalized Lie derivatives $\mathbb{L}_\Lambda \to E_{7(7)}$ bracket

$$\big[\Lambda_1, \Lambda_2\big]_E^M = 2\Lambda_{[1}^K \partial_K \Lambda_{2]}^M + 12\,(t_\alpha)^{MN}(t^\alpha)_{KL}\,\Lambda_{[1}^K \partial_N \Lambda_{2]}^L - \frac{1}{4}\Omega^{MN}\Omega_{KL}\partial_N\big(\Lambda_1^K \Lambda_2^L\big)$$

Jacobiator

$$J^M(\Lambda_1, \Lambda_2, \Lambda_3) \;=\; (t^\alpha)^{MN}\partial_N \chi_\alpha(\Lambda) + \Omega^{MN}\chi_N(\Lambda)\;,$$

where

$$\chi_\alpha = -\frac{1}{2}\,(t_\alpha)_{PQ}\Lambda_1^P[\Lambda_2, \Lambda_3]_E^Q + \cdots\;, \quad \chi_N = \frac{1}{12}\,\Omega_{PQ}\big(\Lambda_1^P \partial_N [\Lambda_2, \Lambda_3]_E^Q + [\Lambda_2, \Lambda_3]_E^P$$

χ_M ‘covariantly constrained’, satisfying the same constraints as ∂_M

→ covariant curvature involves two 2-forms

$$\mathcal{F}_{\mu\nu}{}^{M} \equiv F_{\mu\nu}{}^{M} - 12\,(t^{\alpha})^{MN}\partial_N B_{\mu\nu\,\alpha} - \frac{1}{2}\,\Omega^{MN} B_{\mu\nu\,N}$$

where $B_{\mu\nu N}$ is *covariantly constrained* compensator field

Twisted (electric-magnetic) self-duality relations

$$\mathcal{F}_{\mu\nu}{}^{M} = -\frac{1}{2}\,e\,\varepsilon_{\mu\nu\rho\sigma}\,\Omega^{MN}\mathcal{M}_{NK}\mathcal{F}^{\rho\sigma\,K}$$

gauge vectors $A_\mu{}^M$ include 7 Kaluza-Klein vectors from $D = 11$ metric

→ also 7 dual gauge vectors → dual graviton in *non-linear* duality relation

[no-go theorems: Bekaert, Boulanger, Henneaux (2002)]

'Resolution': dual graviton & compensating gauge field

[Boulanger, O.H. (2008)]

solving section constraint → only 7 $B_{\mu\nu N}$ survive
→ 7 'dual graviton' fields pure gauge

Exceptional Field Theory III: E$_{8(8)}$ and $3+n$ decomposition

'Toy model': $D = 3+1$ Einstein gravity $\rightarrow$ Ehlers group SL$(2,\mathbb{R})$

coordinates (x^μ, Y^M), $\mu = 0,1,2$, $\; Y^M$: adjoint $\mathbf{3}$ of SL$(2,\mathbb{R})$

Section constraint:

$$\eta^{MN}\partial_M \otimes \partial_N \;=\; 0 \qquad f^{MNK}\partial_N \otimes \partial_K \;=\; 0$$

with η_{MN} Cartan-Killing form, f^{MNK} structure constants.

Field content:

$$\{\; e_\mu{}^a \,,\; A_\mu{}^M \,,\; B_{\mu M} \,,\; \mathcal{M}_{MN} \;\}$$

with $B_{\mu M}$ covariantly constrained SL$(2,\mathbb{R})$ gauge field

$\rightarrow$ full covariant derivative

$$\mathcal{D}_\mu V_M \;=\; \partial_\mu V_M - \widehat{\mathcal{L}}_{A_\mu} V_M + B_\mu{}^K f_{KM}{}^L V_L$$

Gauge invariant action:

$$S = \int d^3x\, d^3Y \Big(e\, \widehat{R} - \frac{1}{2\sqrt{2}}\, \varepsilon^{\mu\nu\rho} B_{\mu M} F_{\nu\rho}{}^{M} + \frac{1}{16} e\, g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - e\, V(\mathcal{M}, g) \Big) ,$$

with ‘potential’:

$$V(\mathcal{M}, g) = -\frac{3}{16} \Big(\mathcal{M}^{KL} \partial_K \mathcal{M}^{MN} \partial_L \mathcal{M}_{MN} - 4 \mathcal{M}^{KL} \partial_K \mathcal{M}^{MN} \partial_N \mathcal{M}_{ML} \Big) - \frac{1}{2} g^{-1} \partial_M g\, \partial_N \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}^{MN} g^{-1} \partial_M g\, g^{-1} \partial_N g - \frac{1}{4} \mathcal{M}^{MN} \partial_M g^{\mu\nu}\, \partial_N g_{\mu\nu} \, .$$

Solving constraints: ‘dual graviton’ component $\varphi \subset \mathcal{M}$ pure gauge

$\rightarrow$ integrating out $B_\mu \subset B_{\mu M}$ yields EH in $3+1$ decomposition

For $\mathrm{E}_{8(8)}$ EFT: covariantly constrained $\mathrm{E}_{8(8)}$ gauge field

Outlook

- Further extension? $\mathsf{E}_{9(9)} = \widehat{\mathsf{E}}_{8(8)}$ for $2+9$ decomposition ?
  $\rightarrow$ Gauged supergravity in $D=2$ [Samtleben & Weidner (2007)]

- Making remaining (external) diffeomorphisms manifest?

$$\delta_\xi \mathcal{M}_{MN} \;=\; \xi^\mu \mathcal{D}_\mu \mathcal{M}_{MN} \;, \qquad \delta_\xi e_\mu{}^a \;=\; \xi^\rho D_\rho e_\mu{}^a + D_\mu \xi^\rho e_\rho{}^a \;,$$
$$\delta_\xi^{(0)} A_\mu{}^M \;=\; \xi^\nu \mathcal{F}_{\nu\mu}{}^M + \mathcal{M}^{MN} g_{\mu\nu} \partial_N \xi^\nu$$

  Combining into even larger vielbein?

$$E_{\widehat{M}}{}^{\widehat{A}} \;=\; \begin{pmatrix} e_\mu{}^a & A_\mu{}^M \mathcal{V}_M{}^A \\ 0 & \mathcal{V}_M{}^A \end{pmatrix}$$

  ‘non-covariant’ term from compensating Lorentz transformation?

  underlying group structure? Kac-Moody algebras E_{10}, E_{11}, Borcherds??

  [Aldazabal, Grana, Marques, Rosabal (December 2013)]

Problem: can't quite work naively,

1) $B_{\mu\nu N}$, $B_{\mu M}$, etc. missing from E$_{11}$ spectrum

2) δ_ξ on p-forms receives non-trivial 'on-shell modification':

$$\delta_\xi A_\mu{}^M \equiv \delta_\xi^{(0)} A_\mu{}^M - \xi^\nu \, \mathcal{E}_{\nu\mu}^{(A)M}$$
$$\delta_\xi B_{\mu\, M} \equiv \delta_\xi^{(0)} B_{\mu\, M} + f_{KM}{}^N \, \partial_N \xi^\nu \, \mathcal{E}_{\mu\nu}^{(A)K} - \xi^\nu \, \mathcal{E}^{(B)}_{\nu\mu\, M}$$

with field equations $\mathcal{E}_{\mu\nu}{}^M$ of A and B

- Novel algebraic sructure?
- Higher-derivative M-theory/type IIB corrections in a unified fashion?

$\rightarrow$ as in DFT deformed generalized geometric structures

[O.H., Siegel, Zwiebach (2013)]