Exotic Five-brane

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based on arXiv:1304.4061, 1305.4439, 1310.6163, 1402.5580, and more

with Shin SASAKI and Masaya YATA

well-known objects D-branes, NS-branes	String Dualities → in lower dimensions	less known objects Exotic Branes

well-known objects	String Dualities → in lower dimensions	less known objects
D-branes, NS-branes		Exotic Branes

Exotic Branes

- ✓ co-dimension 2 (“defect” branes), or less
- ✓ non–single-valued metric of spacetime

(Exotic) Branes are labelled as b_n^c-branes.

$$b_n^c$$

b : spatial dimensions

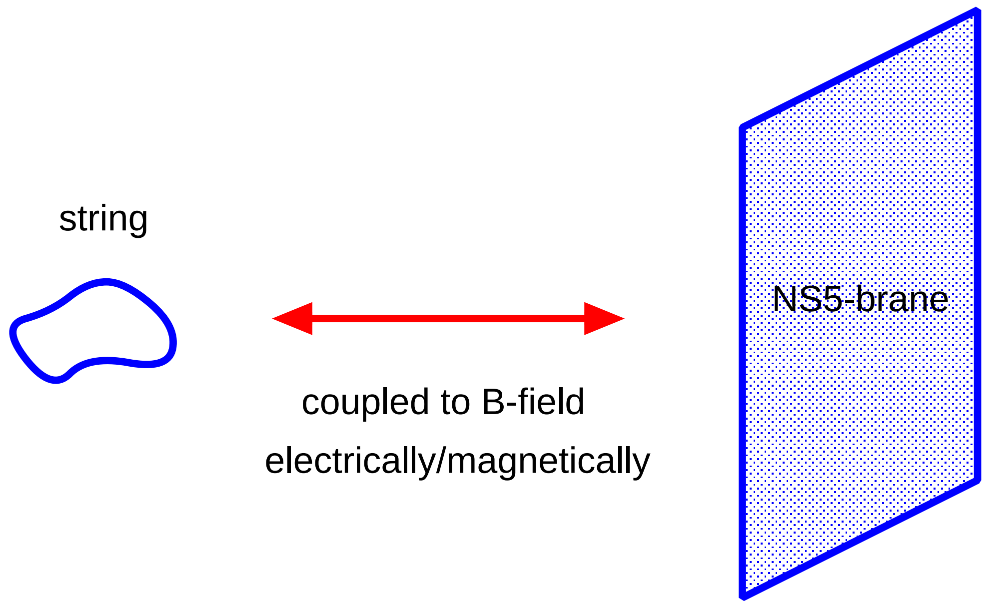
c : # of isometry directions

n : mass of brane $\sim g_s^{-n}$

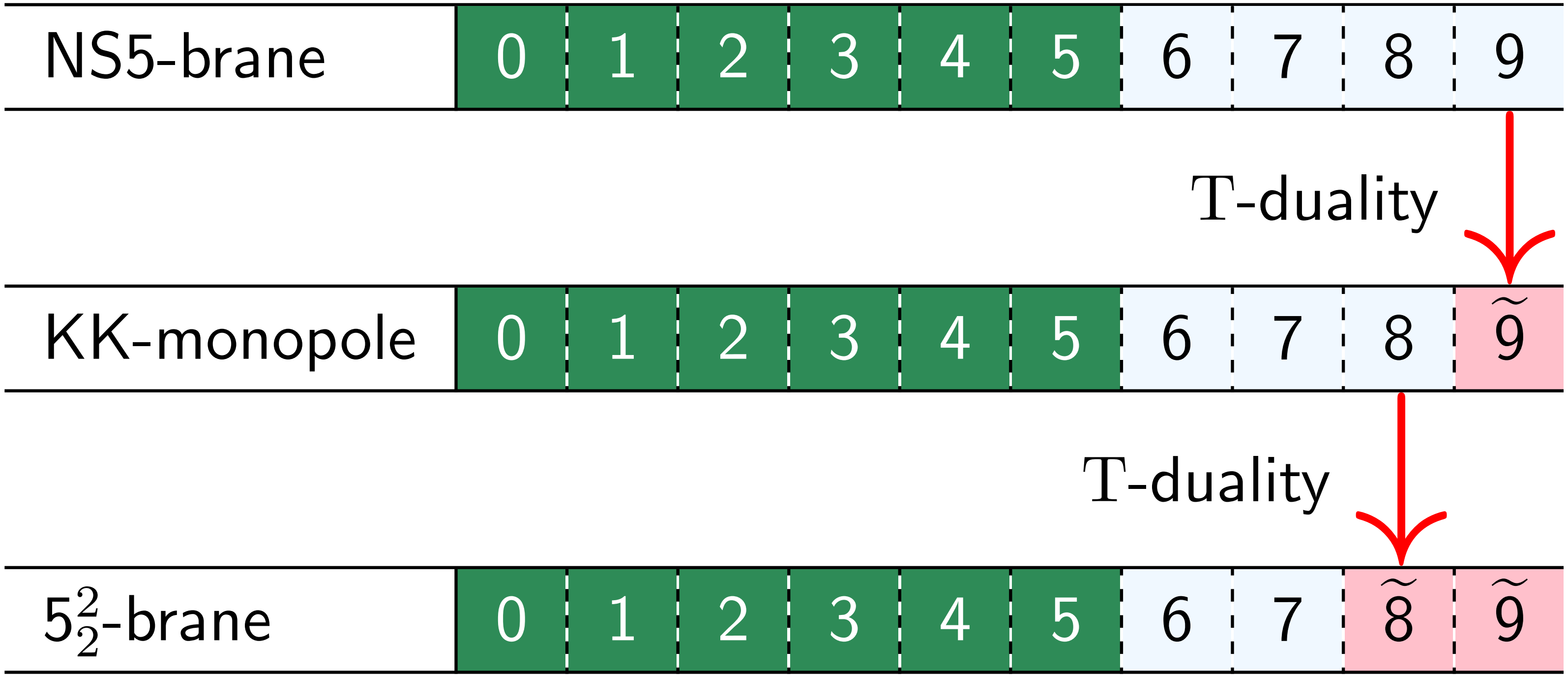
5^2_2-brane

We begin with NS5-brane.

Objects:



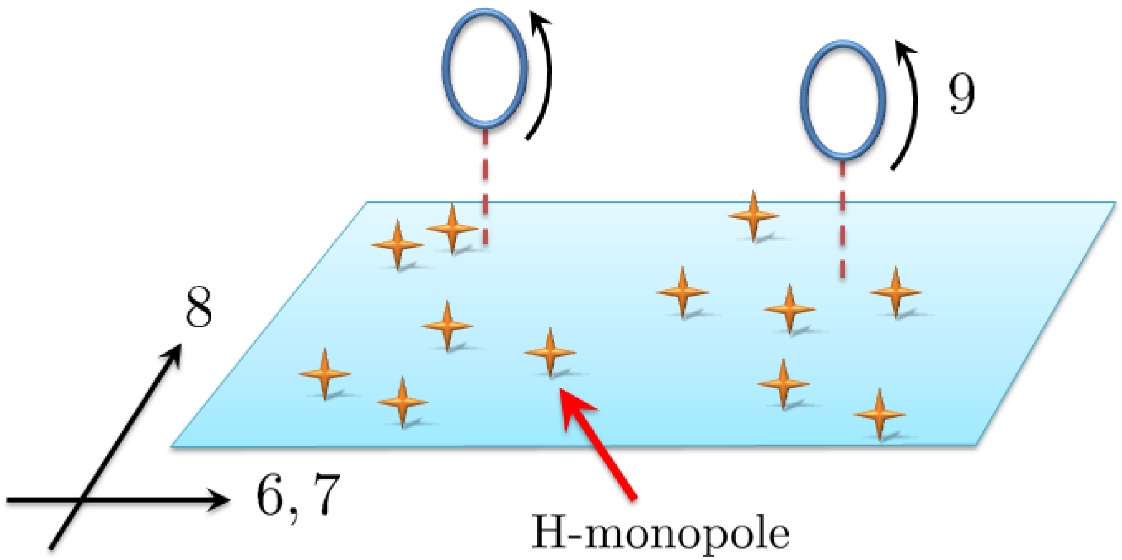
Exotic 5^2_2-brane is constructed from NS5-brane.



NS5-branes (smeared), or H-monopoles

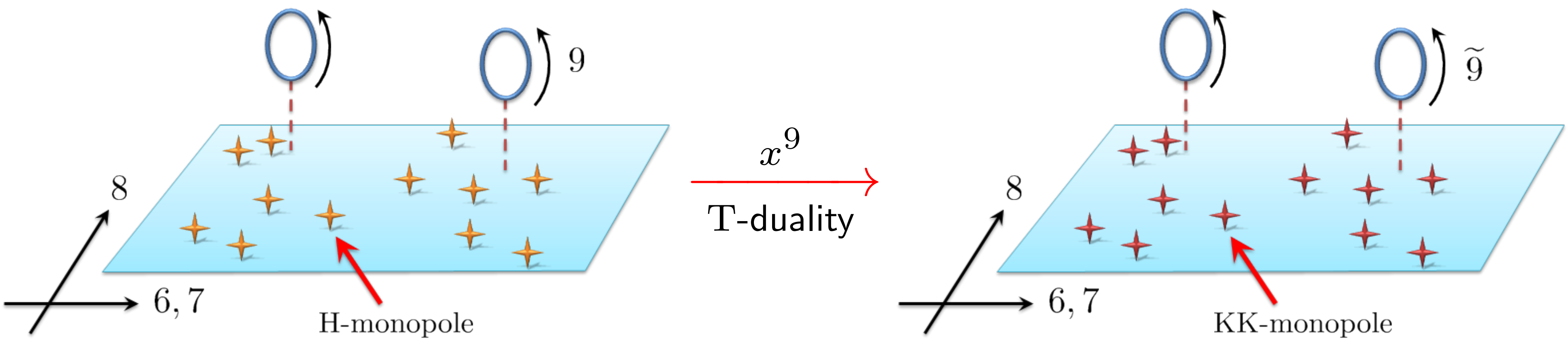
- co-dim. 3 $(\mathbb{R}^3 \times S^1\,, \quad \vec{r} \in \mathbb{R}^3)$
- $$\mathrm{d}s^2 \;=\; \mathrm{d}x^2_{012345} + H(x)\Big[(\mathrm{d}x^6)^2 + (\mathrm{d}x^7)^2 + (\mathrm{d}x^8)^2 + (\mathrm{d}x^9)^2\Big]$$
- $$H(x) = 1 + \sum_p \frac{Q'}{|\vec{r} - \vec{r}_p|}\,, \qquad H_{mnp} = \varepsilon_{mnp}{}^q\,\partial_q\,\log H(x)\,, \qquad \Phi = \frac{1}{2}\log H(x)$$

$\mathrm{d}x^2_{012345} = -\mathrm{d}t^2 + (\mathrm{d}x^1)^2 + \ldots + (\mathrm{d}x^5)^2$



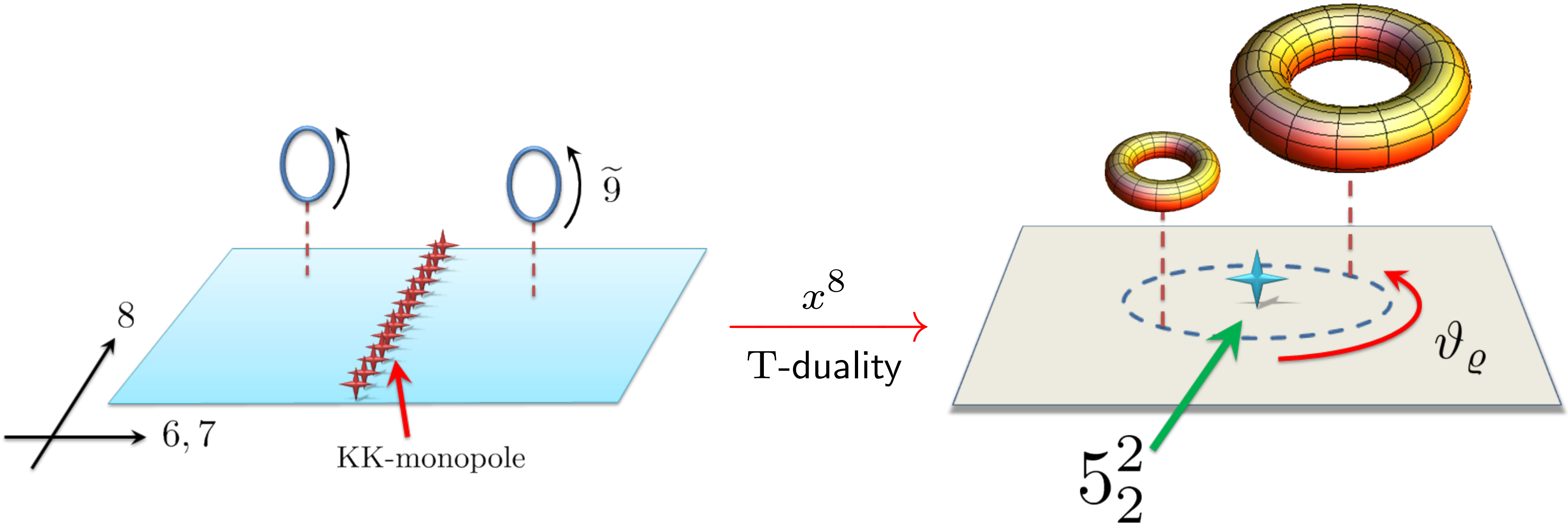
KK-monopoles

- co-dim. 3 ($\mathbb{R}^3 \times \widetilde{S}^1$: Taub-NUT space, $\vec{r} \in \mathbb{R}^3$)
- $\mathrm{d}s^2 \;=\; \mathrm{d}x^2_{012345} + H(x)\Big[(\mathrm{d}x^6)^2 + (\mathrm{d}x^7)^2 + (\mathrm{d}x^8)^2\Big] + \dfrac{1}{H(x)}\big(\mathrm{d}\widetilde{x}^9 + \omega\big)^2$
- $H(x) = 1 + \sum_p \dfrac{Q'}{|\vec{r} - \vec{r}_p|}\,, \qquad H_{mnp} = 0 = \Phi$



5^2_2-brane

- co-dim. 2 $(\mathbb{R}^2 \times T^2)$
- $H(x) = h + \sigma \log \left(\frac{\mu}{\varrho} \right) \ , \quad (\varrho, \vartheta_\varrho) \in \mathbb{R}^2$



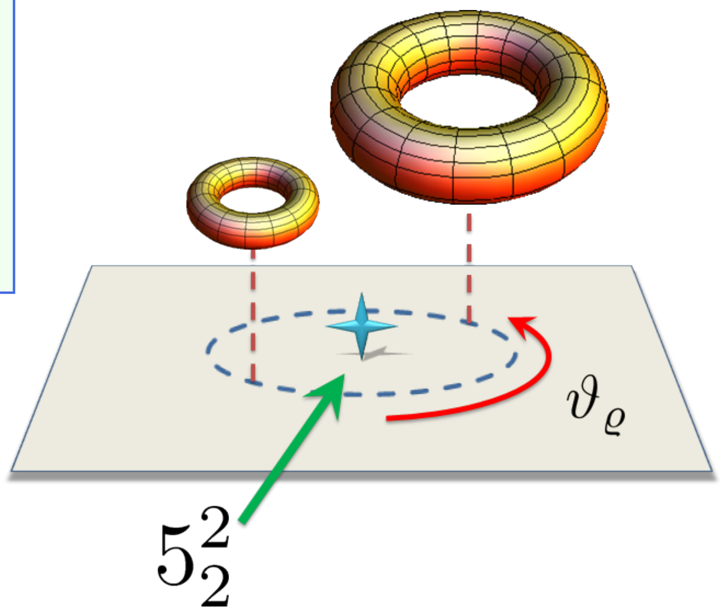
$$\mathrm{d}s^2 = \mathrm{d}x^2_{012345} + H\big[\mathrm{d}\varrho^2 + \varrho^2(\mathrm{d}\vartheta_\varrho)^2\big] + \frac{H}{K}\big[(\mathrm{d}\widetilde{x}^8)^2 + (\mathrm{d}\widetilde{x}^9)^2\big]$$

$$B_{89} = -\frac{\sigma\,\vartheta_\varrho}{K}, \quad \mathrm{e}^{2\Phi} = \frac{H}{K}, \quad K = H^2 + (\sigma\vartheta_\varrho)^2$$

$$H = h + \sigma \log\Big(\frac{\mu}{\varrho}\Big)$$

$$\vartheta_\varrho = 0 \quad : \quad G_{88} = G_{99} = \frac{1}{H}$$

$$\vartheta_\varrho = 2\pi \quad : \quad G_{88} = G_{99} = \frac{H}{H^2 + (2\pi\sigma)^2}$$



Exotic 5^2_2-brane has been analyzed in spacetime (SUGRA) picture.

Ready to study STRING THEORY pictures!

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Ready to study STRING THEORY pictures!

string theory pictures

- ☞ string worldsheet picture: NLSM, GLSM
- ☞ five-brane's worldvolume picture

Contents

- GLSM for (exotic) five-branes: 1st version

  Sasaki and TK: 1304.4061, 1305.4439, 1310.6163

- GLSM for (exotic) five-branes: 2nd version

  Yata and TK: 1402.5580 and more

- Worldvolume theories

  Sasaki, Yata and TK: to appear

GLSM: 1st ver.

- ✓ Duality transformation of neutral chiral superfield in D-term and F-term
- ✓ Exotic structure in NLSM is carried by non-dynamical field in GLSM

S. Sasaki and TK: arXiv:1304.4061, 1305.4439, 1310.6163

$$\mathscr{L}_{\mathsf{NS5}} \;=\; + \int \mathrm{d}^4\theta \, \frac{1}{g^2} \Big(- \overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

GLSM for NS5-brane

D. Tong hep-th/0204186

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$	role
neutral HM	chiral $\Psi = X^6 + \mathrm{i}\, X^8 + \ldots$ twisted chiral $\Theta = X^7 + \mathrm{i}\, X^9 + \ldots$	spacetime coord.

$$\mathscr{L}_{\mathsf{NS5}} = \int d^4\theta \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) \qquad \Big\} + \int d^4\theta \, \frac{1}{g^2} \Big(- \overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int d^2\theta \Big(\qquad + (s - \Psi)\Phi \Big) + (\text{h.c.})$$

$$+ \int d^2\widetilde{\theta} \, (t - \Theta)\Sigma + (\text{h.c.})$$

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VMs	twisted chiral $\Sigma = \overline{D}_+ D_- V$	chiral Φ	gauging isometry
FI parameters	$s = s^6 + \mathrm{i}\, s^8$	$t = t^7 + \mathrm{i}\, t^9$	position of five-branes

$$\mathscr{L}_{\mathsf{NS5}} = \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q} \, \mathrm{e}^{-2V} Q + \overline{\widetilde{Q}} \, \mathrm{e}^{+2V} \widetilde{Q} \Big\} + \int \mathrm{d}^4\theta \, \frac{1}{g^2} \Big(- \overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.})$$

$$+ \int \mathrm{d}^2\widetilde{\theta} \, (t - \Theta)\Sigma + (\text{h.c.})$$

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VMs	twisted chiral $\Sigma = \overline{D}_+ D_- V$	chiral Φ	gauging isometry
charged HM	chiral Q $(-)$	chiral $\widetilde{Q}$ $(+)$	curving geometry
FI parameters	$s = s^6 + \mathrm{i}\, s^8$	$t = t^7 + \mathrm{i}\, t^9$	position of five-branes

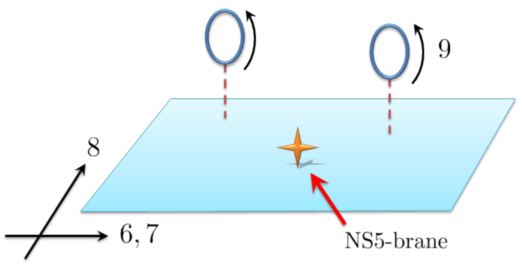
Bosonic Lagrangian after integrating-out auxiliary fields :

$$\begin{aligned}
\mathscr{L}^{\text{kin}}_{\text{NS5}} &= \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2\Big] - \Big[|D_m q|^2 + |D_m \widetilde{q}|^2\Big] \\
&\quad - \frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 + (\partial_m X^9)^2\Big] - (X^9 - t^9)\, F_{01} \\
\mathscr{L}^{\text{pot}}_{\text{NS5}} &= -2\big(|\sigma|^2 + |\phi|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big) \\
&\quad - \frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} - (X^6 - s^6) - \mathrm{i}(X^8 - s^8)\Big|^2
\end{aligned}$$

Steps to NLSM for NS5-brane

1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q, \widetilde{q})$
3. IR limit $e \to \infty$, and integrate out A_m

$\Longrightarrow$



$(X^6, X^7; X^8, X^9)$

$$\mathscr{L}^{\text{NLSM}}_{\text{NS5}} \;=\; -\frac{1}{2}\Big(\frac{1}{g^2} + \frac{1}{r}\Big)\Big[(\partial_m \vec{X})^2 + (\partial_m X^9)^2\Big] + \varepsilon^{mn}\,\Omega_i\,\partial_m X^i\,\partial_n X^9$$

Target space geometry is...

$$\boxed{\begin{array}{rclcrcl} G_{IJ} &=& H\,\delta_{IJ} & \qquad & H &=& \dfrac{1}{g^2} + \dfrac{1}{r} \\[2ex] B_{i9} &=& \Omega_i & & \nabla_i H &=& (\nabla \times \Omega)_i \end{array}}$$

T-duality transformations

(free) string	sign flip (parity) in right-mover	momentum $\leftrightarrow$ winding
spacetime	Buscher rule	$(G_{IJ}, B_{IJ}) \to (G'_{IJ}, B'_{IJ})$
SUSY sigma model	Roček-Verlinde formula	chiral $\leftrightarrow$ twisted chiral

$$\mathscr{L}_{\mathsf{NS5}} = \int d^4\theta \left\{ \frac{1}{e^2}\Big(-\overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{-2V} Q + \overline{\widetilde{Q}}\, e^{+2V} \widetilde{Q} \right\} + \int d^4\theta \, \frac{1}{g^2}\Big(-\overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int d^2\theta \, \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.})$$

$$+ \int d^2\widetilde{\theta} \, (t - \Theta)\Sigma + (\text{h.c.})$$

T-duality transformations to 5^2_2-brane

Duality (Roček-Verlinde) transformations

$$-\frac{1}{g^2}\big(\Theta + \overline{\Theta}\big) = \big(\Gamma + \overline{\Gamma}\big) + V \qquad \Sigma = \overline{D}_+ D_- V$$

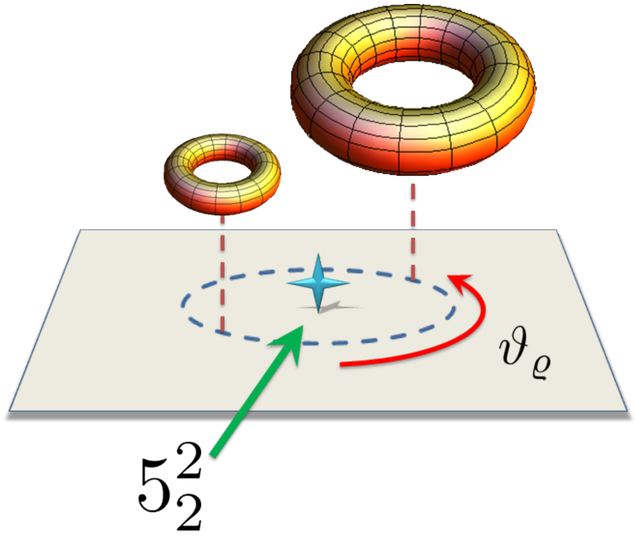
$$-\frac{1}{g^2}\big(\Psi + \overline{\Psi}\big) = \big(\Xi + \overline{\Xi}\big) - \big(C + \overline{C}\big) \qquad \Phi = \overline{D}_+ \overline{D}_- C$$

$$
\begin{aligned}
\mathscr{L}_{\mathsf{E}} \;&=\; \int \mathrm{d}^4\theta\,\Big\{\frac{1}{e^2}\Big(-\overline{\Sigma}\Sigma+\overline{\Phi}\Phi\Big)+\overline{Q}\,\mathrm{e}^{-2V}Q+\overline{\widetilde{Q}}\,\mathrm{e}^{+2V}\widetilde{Q}\Big\} \\
&\quad+\int \mathrm{d}^4\theta\,\frac{g^2}{2}\Big\{\Big(\Gamma+\overline{\Gamma}+V\Big)^2-\Big(\Xi+\overline{\Xi}-(C+\overline{C})\Big)^2\Big\}-\int \mathrm{d}^4\theta\,(\Psi-\overline{\Psi})(C-\overline{C}) \\
&\quad+\Big\{\int \mathrm{d}^2\theta\,\Big(\widetilde{Q}\Phi Q+s\Phi\Big)+(\text{h.c.})\Big\} \\
&\quad+\Big\{\int \mathrm{d}^2\widetilde{\theta}\,t\,\Sigma+(\text{h.c.})\Big\}-\varepsilon^{mn}\,\partial_m(X^9A_n)
\end{aligned}
$$

GLSM for exotic 5^2_2-brane

S. Sasaki and TK arXiv:1304.4061

$$\mathscr{L}_{\text{exotic}} = -\frac{H}{2}\Big[(\partial_m \varrho)^2 + \varrho^2(\partial_m \vartheta_\varrho)^2\Big] - \frac{H}{2K}\Big[(\partial_m \widetilde{X}^8)^2 + (\partial_m \widetilde{X}^9)^2\Big] - \frac{\sigma\vartheta_\varrho}{K}\,\varepsilon^{mn}(\partial_m \widetilde{X}^8)(\partial_n \widetilde{X}^9) - \varepsilon^{mn}\,\partial_m((X^9 - t^9)A_n)$$



We obtained GLSM/NLSM for Exotic Five-brane!

Remark 1:

Quantum corrections to the geometry can be traced by Vortices in GLSM:

$$\mathscr{L}^{\text{GLSM}}_{\text{topological}} = (X^9 - t)\, F_{01}$$

$$H = h + \sigma \log\left(\frac{\mu}{\varrho}\right) \quad \to \quad h + \sigma \log\left(\frac{\mu}{\varrho}\right) + \sum_{n \neq 0} K_0(|n|\, \varrho)\, \exp(\mathrm{i} n X^9)$$

X^9 is ...

X^9 is ...	physical coordinate in NS5-brane	Tong
	dual coordinate in KKM	Tong, Harvey-Jensen
	dual coordinate in 5^2_2-brane	Sasaki-TK 1305.4439

Remark 2:

5^2_2-brane geometry has two isometries (8- and 9-th)

Possible to gauge both directions?

Sasaki-TK 1310.6163 as a trial

Not completely understood yet...

We want to analyze 5^2_2-brane from a different viewpoint

$\downarrow$

Another GLSM

GLSM: 2nd ver.

- ✓ Toric structure is manifest
- ✓ Duality transformation of Charged chiral superfield in D-term and F-term
- ✓ Kähler potential for the system of "metric + B-field"

M. Yata and TK: arXiv:1402.5580, and more

A_N-type ALE Space

- ✓ hyper-Kähler
- ✓ transverse space of $N+1$ parallel KKMs
- ✓ toric

☞ T-duality along an isometry direction

$\rightarrow$ $N+1$ parallel NS5-branes with B-field

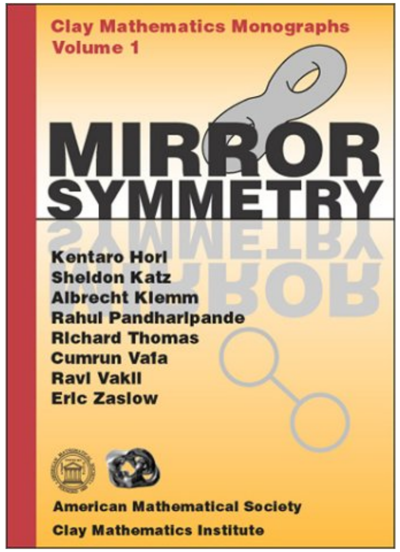
☞ large-N limit $+$ T-dualize along the singularity points

$\rightarrow$ exotic five-brane with B-field

Explicit form of GLSM for A$_N$-type ALE space associated with toric data?

field contents	A_1	A_2	A_3	A_4	$\ldots$	A_N	A_{N+1}	A_{N+2}
$U(1)_1 : V_1$	$+1$	-2	$+1$		$\ldots$			
$U(1)_2 : V_2$		$+1$	-2	$+1$				
$U(1)_3 : V_3$			$+1$	-2				
$\vdots$					$\vdots$			
$U(1)_{N-1} : V_{N-1}$						-2	$+1$	
$U(1)_N : V_N$						$+1$	-2	$+1$

in



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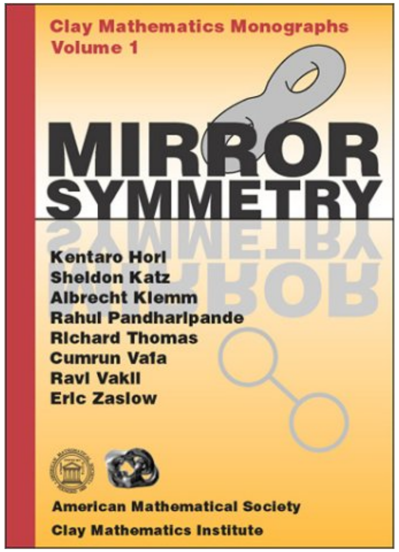
$U(1)_1 : V_1$	$+1$	-2	$+1$		$\dots$			
$U(1)_2 : V_2$		$+1$	-2	$+1$				
$U(1)_3 : V_3$			$+1$	-2				
$\vdots$					$\vdots$			
$U(1)_{N-1} : V_{N-1}$						-2	$+1$	
$U(1)_N : V_N$						$+1$	-2	$+1$

in



Neither `INSPIRE` nor `Google` tells us a good answer...

OK! Construct it by ourselves!

Notice:

D-term associated with toric data is not enough to derive correct NLSM

F-term is inevitable to fix the complex structure of the geometry

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D-term associated with toric data is not enough to derive correct NLSM

F-term is inevitable to fix the complex structure of the geometry

$\mathcal{N} = (4,4)$ SUSY ($SU(2)_R$ symmetry) helps everything!

- chiral multiplets $A_i \to \mathcal{N} = (4,4)$ HMs (A_i, B_i) as doublets
- vector multiplets $V_a \to \mathcal{N} = (4,4)$ VMs (V_a, Φ_a) as doublets
- F-term "$-\alpha \Phi_a A_a B_a$" as a pair of "$|A_a|^2 \, \mathrm{e}^{2\alpha V_a} + |B_a|^2 \, \mathrm{e}^{-2\alpha V_a}$"

$\mathcal{N}=(4,4)$	(A_1,B_1)	(A_2,B_2)	(A_3,B_3)	(A_4,B_4)	$\ldots(A_K,B_K)\ldots$	(A_{N+1},B_{N+1})	(A_{N+2},B_{N+2})
(V_1,Φ_1)	$(+1,-1)$	$(-2,+2)$	$(+1,-1)$		$\ldots$		
(V_2,Φ_2)		$(+1,-1)$	$(-2,+2)$	$(+1,-1)$			
(V_3,Φ_3)			$(+1,-1)$	$(-2,+2)$			
$\vdots$					$\vdots$		
(V_{N-1},Φ_{N-1})						$(+1,-1)$	
(V_N,Φ_N)						$(-2,+2)$	$(+1,-1)$
(V_0,Φ_0)					$(-\alpha,+\alpha)$		

$$
\begin{aligned}
\mathscr{L} \;&=\; \int \mathrm{d}^4\theta \sum_a \frac{1}{e_a^2}\Big(- |\Sigma_a|^2 + |\Phi_a|^2\Big) + \int \mathrm{d}^4\theta \sum_i \Big(|A_i|^2 \, \mathrm{e}^{2\sum_a Q_i^a V_a} + |B_i|^2 \, \mathrm{e}^{-2\sum_a Q_i^a V_a}\Big) \\
&\quad + \int \mathrm{d}^2\theta \sum_{a,i} \big(- Q_i^a \, \Phi_a A_i B_i\big) + \int \mathrm{d}^2\widetilde{\theta} \sum_a \big(- t_a \Sigma_a\big) + (\text{h.c.})
\end{aligned}
$$

Eguchi-Hanson (A_1-ALE):

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$$
\begin{aligned}
\mathscr{L} \;=\; & \int \mathrm{d}^4\theta \, \frac{1}{e_1^2}\Big(- |\Sigma_1|^2 + |\Phi_1|^2 \Big) + \int \mathrm{d}^4\theta \, \frac{1}{e_0^2}\Big(- |\Sigma_0|^2 + |\Phi_0|^2 \Big) \\
& + \int \mathrm{d}^4\theta \, \Big(|A_1|^2 \, \mathrm{e}^{2V_1} + |A_2|^2 \, \mathrm{e}^{-4V_1 - 2\alpha V_0} + |A_3|^2 \, \mathrm{e}^{2V_1} \Big) \\
& + \int \mathrm{d}^4\theta \, \Big(|B_1|^2 \, \mathrm{e}^{-2V_1} + |B_2|^2 \, \mathrm{e}^{4V_1 + 2\alpha V_0} + |B_3|^2 \, \mathrm{e}^{-2V_1} \Big) \\
& - \int \mathrm{d}^2\theta \, \Big(\Phi_1 (A_1 B_1 - 2 A_2 B_2 + A_3 B_3) - \alpha \, \Phi_0 (A_2 B_2) \Big) - \int \mathrm{d}^2\widetilde{\theta} \, t_1 \Sigma_1 + (\text{h.c.})
\end{aligned}
$$

Eguchi-Hanson (A_1-ALE):

$$
\begin{aligned}
\mathscr{L} \;=\; & \int d^4\theta \, \frac{1}{e_1^2}\Big(- |\Sigma_1|^2 + |\Phi_1|^2 \Big) + \int d^4\theta \, \frac{1}{e_0^2}\Big(- |\Sigma_0|^2 + |\Phi_0|^2 \Big) \\
& + \int d^4\theta \, \Big(|A_1|^2 \, e^{2V_1} + |A_2|^2 \, e^{-4V_1 - 2\alpha V_0} + |A_3|^2 \, e^{2V_1} \Big) \\
& + \int d^4\theta \, \Big(|B_1|^2 \, e^{-2V_1} + |B_2|^2 \, e^{4V_1 + 2\alpha V_0} + |B_3|^2 \, e^{-2V_1} \Big) \\
& - \int d^2\theta \, \Big(\Phi_1 (A_1 B_1 - 2 A_2 B_2 + A_3 B_3) - \alpha \, \Phi_0 (A_2 B_2) \Big) - \int d^2\widetilde{\theta} \, t_1 \Sigma_1 + (\text{h.c.})
\end{aligned}
$$

Integrate out $\{\Sigma_a, \Phi_a\}$ in the IR limit $e_a \to \infty$

$\downarrow$

$$
\mathscr{L}_{\text{IR}} \;=\; \int d^4\theta \, \Big\{ \sqrt{(t_1)^2 + 4(|A_1|^2 + |A_3|^2)^2} - t_1 \log \frac{t_1 + \sqrt{(t_1)^2 + 4(|A_1|^2 + |A_3|^2)^2}}{2(|A_1|^2 + |A_3|^2)} \Big\}
$$

with parametrization: $A_1 \;=\; \frac{r}{\sqrt{2}} \cos\frac{\vartheta}{2} \, e^{\frac{i}{2}(\psi + \varphi)}, \qquad A_2 \;=\; 0\,, \qquad A_3 \;=\; \frac{r}{\sqrt{2}} \sin\frac{\vartheta}{2} \, e^{\frac{i}{2}(\psi - \varphi)}$

Eguchi-Hanson (A_1-ALE):

$$
\begin{aligned}
\mathscr{L} \ = \ & \int \mathrm{d}^4\theta \, \frac{1}{e_1^2}\Big(- |\Sigma_1|^2 + |\Phi_1|^2 \Big) + \int \mathrm{d}^4\theta \, \frac{1}{e_0^2}\Big(- |\Sigma_0|^2 + |\Phi_0|^2 \Big) \\
& + \int \mathrm{d}^4\theta \, \Big(|A_1|^2 \, \mathrm{e}^{2V_1} + |A_2|^2 \, \mathrm{e}^{-4V_1 - 2\alpha V_0} + |A_3|^2 \, \mathrm{e}^{2V_1} \Big) \\
& + \int \mathrm{d}^4\theta \, \Big(|B_1|^2 \, \mathrm{e}^{-2V_1} + |B_2|^2 \, \mathrm{e}^{4V_1 + 2\alpha V_0} + |B_3|^2 \, \mathrm{e}^{-2V_1} \Big) \\
& - \int \mathrm{d}^2\theta \, \Big(\Phi_1 (A_1 B_1 - 2 A_2 B_2 + A_3 B_3) - \alpha \, \Phi_0 (A_2 B_2) \Big) - \int \mathrm{d}^2\widetilde{\theta} \, t_1 \Sigma_1 + \text{(h.c.)}
\end{aligned}
$$

T-duality along φ-direction

Duality transformations of $\{A_1, A_3\}$ in D-term and F-term

$$
\Phi_1 \ = \ \overline{D}_+ \overline{D}_- C
$$

Eguchi-Hanson (A_1-ALE):

$$\begin{aligned}
\mathscr{L} \;&=\; -\int \mathrm{d}^4\theta \Big((Y_1 + \overline{Y}_1) \log(Y_1 + \overline{Y}_1) + (Y_3 + \overline{Y}_3) \log(Y_1 + \overline{Y}_3) \Big) \\
&\quad - \int \mathrm{d}^4\theta \, \big((Y_1 + \overline{Y}_1) - (Y_3 + \overline{Y}_3) - t_1 \big) \log \big((Y_1 + \overline{Y}_1) - (Y_3 + \overline{Y}_3) - t_1 \big) \\
&\quad + \int \mathrm{d}^4\theta \, \mathcal{F}(C) \\
&\quad + \ldots
\end{aligned}$$

$$\begin{aligned}
|A_1|^2 \, \mathrm{e}^{2V_1} \;&=\; +(Y_1 + \overline{Y}_1) + T_1(C) \\
|A_3|^2 \, \mathrm{e}^{2V_1} \;&=\; -(Y_3 + \overline{Y}_3) + T_3(C)
\end{aligned}$$

GLSM for 5^2_2-brane:

GLSM for 5^2_2-brane:

sorry, under construction

Worldvolume

- ✓ covariant form of gauged isometries and Wess-Zumino terms
- ✓ type IIB, IIA, and Heterotics

S. Sasaki, M. Yata and TK: to appear

Worldvolume action of 5^2_2-brane could be obtained from

D5-brane's action via S- and T-dualities	(IIB)
KK6-brane action via reduction and T-duality	(IIB)
M5-brane action via reduction and T-dualities	(IIA)

Different point: Two isometries along transverse directions

How to describe its worldvolume theory with isometries?

Worldvolume action of 5^2_2-brane could be obtained from

D5-brane's action via S- and T-dualities	(IIB)
KK6-brane action via reduction and T-duality	(IIB)
M5-brane action via reduction and T-dualities	(IIA)

Different point: Two isometries along transverse directions

How to describe its worldvolume theory with isometries?

Already Known!

Bergshoeff et al hep-th/9706117

Worldvolume action with WZ-term is gauged with respect to isometries ex) gravity sector of KK6-brane action with gauged isometry in M-theory:

$$\begin{aligned}
\mathscr{L}^{\mathsf{M}}_{\mathsf{KK6}} &= -\frac{1}{2} T^{\mathsf{M}}_{\mathsf{KK6}} \sqrt{-\gamma} \Big(k^{\frac{4}{7}} \gamma^{ab} D_a X^{\mu} D_b X^{\nu} g_{\mu\nu} - 5 \Big) \\
&= -T^{\mathsf{M}}_{\mathsf{KK6}} \, k^2 \sqrt{- \det(D_a X^{\mu} D_b X^{\nu} g_{\mu\nu})} \\
&= -T^{\mathsf{M}}_{\mathsf{KK6}} \, k^2 \sqrt{- \det(\partial_a X^{\mu} \partial_b X^{\nu} \Pi_{\mu\nu})}
\end{aligned}$$

$$\begin{aligned}
D_a X^{\mu} &= \partial_a X^{\mu} + C_a k^{\mu} \qquad k^{\mu} \,:\, \text{Killing vector} \\
\Pi_{\mu\nu} &= g_{\mu\nu} - \frac{k^{\mu} k^{\nu}}{k^2}
\end{aligned}$$

✓ Construct the worldvolume action with WZ-term in a covariant way:

$$\text{KK6} \xrightarrow{\text{reduction}} \text{KK5 in IIA} \xrightarrow{\text{T-duality}} 5^2_2 \text{ in IIB}$$

$$\text{M5} \xrightarrow{\text{reduction}} \text{NS5 in IIA} \xrightarrow{\text{T-dualities}} 5^2_2 \text{ in IIA}$$

✓ 5^2_2-branes in heterotic theories can be obtained by truncations:

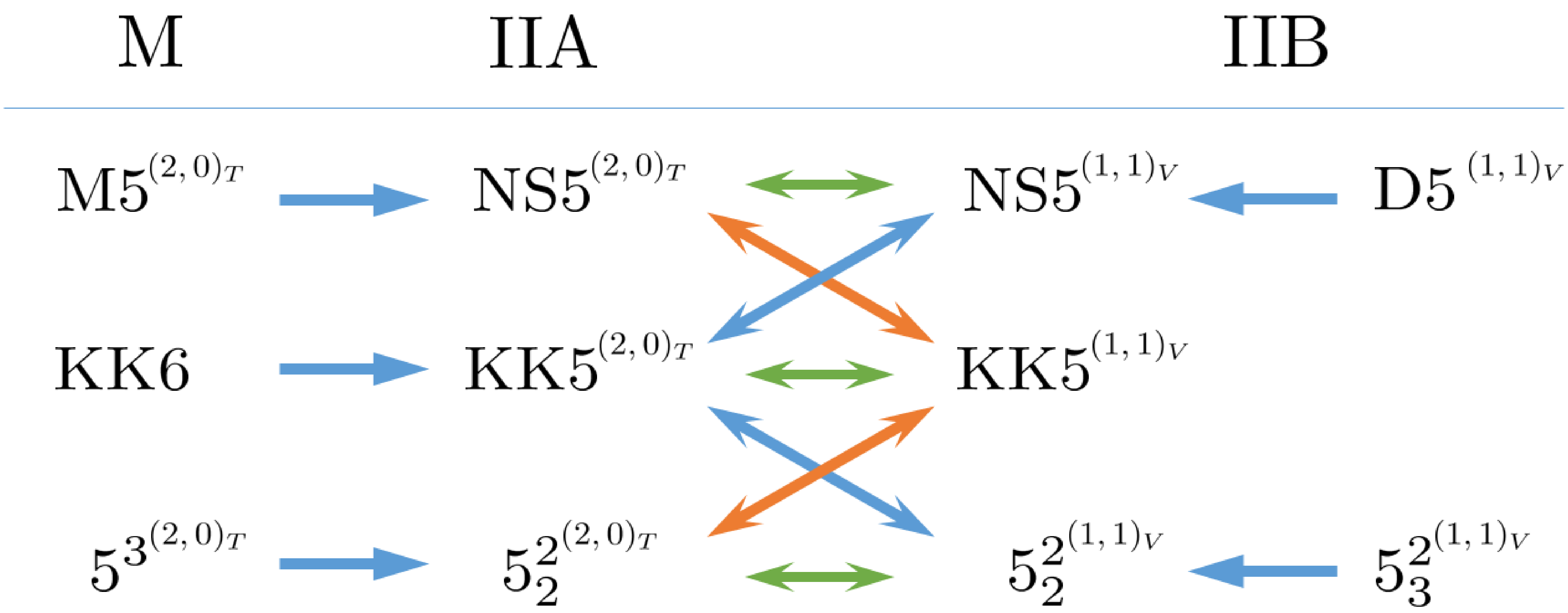
$$5^2_2 \text{ in IIB} \xrightarrow[\mathcal{N}=(1,1)\to\mathcal{N}=(1,0)]{\text{RR}=0} 5^2_2 \text{ in HSO}$$

$$5^2_2 \text{ in IIA} \xrightarrow[\mathcal{N}=(2,0)\to\mathcal{N}=(1,0)]{\text{RR}=0} 5^2_2 \text{ in HE8}$$

$$
\begin{aligned}
S^{\mathrm{IIB}}_{5^2_2} \;=\; & -T_{5^2_2} \int \mathrm{d}^6\xi\, \mathrm{e}^{-2\phi} (\det h_{IJ}) \sqrt{1 + \mathrm{e}^{+2\phi} (\det h_{IJ})^{-1} \big(i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B) \big)^2} \\
& \times \sqrt{-\det \Big[\partial_a X^\mu \partial_b X^\nu \Pi^{(2)}_{\mu\nu} + \frac{K^{(1)}_a K^{(1)}_b}{(k_2)^2} - (k_2)^2 \frac{K^{(2)}_a K^{(2)}_b + K^{(3)}_a K^{(3)}_b}{(\det h_{IJ})} + F_{ab} \Big]} \\
& -\mu_5 \int_{\mathcal{M}_6} \Big[P[i_{k_1} i_{k_2} {\color{red} B^{(8,2)}}] - \frac{1}{2} P[\check{B} \wedge \check{C}^{(2)} \wedge \check{C}^{(2)}] + P[\check{C}^{(4)} - \check{C}^{(2)} \wedge \check{B}] \wedge \check{F} \\
& \qquad - P[\check{B}] \wedge \check{F} \wedge \check{F} - \frac{i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B)}{(i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B))^2 + \mathrm{e}^{-2\phi} (\det h_{IJ})} \check{F} \wedge \check{F} \wedge \check{F} \Big]
\end{aligned}
$$

$$
h_{IJ} \;=\; k^\mu_I k^\nu_J (g_{\mu\nu} + B_{\mu\nu})
$$

For details, please wait for our coming paper!!


M
IIA
IIB
$M5^{(2,0)_T}$
$NS5^{(2,0)_T}$
$NS5^{(1,1)_V}$
$D5^{(1,1)_V}$
$KK6$
$KK5^{(2,0)_T}$
$KK5^{(1,1)_V}$
$5^{3(2,0)_T}$
$5^{2(2,0)_T}_2$
$5^{2(1,1)_V}_2$
$5^{2(1,1)_V}_3$

I	HSO	HE8	M
$\mathrm{D5}^{(1,0)_V}$	$\mathrm{NS5}^{(1,0)_V}$	$\mathrm{NS5}^{(1,0)_T}$	$\mathrm{M5}^{(2,0)_T}$
	$\mathrm{KK5}^{(1,0)_V}$	$\mathrm{KK5}^{(1,0)_T}$	
${5^2_3}^{(1,0)_V}$	${5^2_2}^{(1,0)_V}$	${5^2_2}^{(1,0)_T}$	${5^3}^{(2,0)_T}$

Summary

We established GLSM for exotic five-brane!

- ✓ Dualization of neutral chiral superfields in D-term and F-term (1st ver.)
- ✓ Dualization of Charged chiral superfields in D-term and F-term (2nd ver.)
- ✓ Gauge covariant isometries in worldvolume theories

We hope our five-branes' theories tell us more and more !

- ✓ modular invariance, dipole description ? — T. Kikuchi, T. Okada and Y. Sakatani
- ✓ more exotic as "multiple states of NS5 $+\ 5^2_2$" — de Boer and Shigemori

Thanks

Appendix

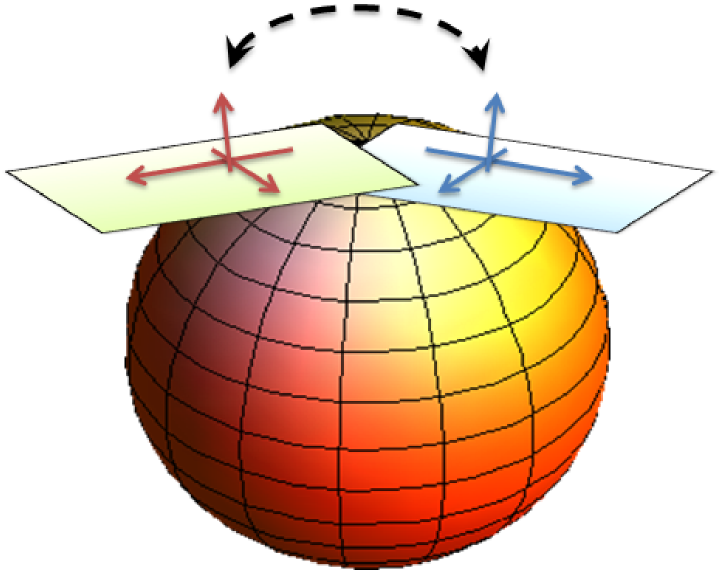
10D string theory $=$ D-dim spacetime $\otimes$ compact space $\mathcal{M}_d$

$\mathcal{M}_d$	geometry associated with G_{mn}	Conventional geometry (manifold) $O(d)$ global symmetry [Calabi-Yau, etc]	ordinary compactifications
	geometry associated with G_{mn}, B_{mn}	Generalized geometry $O(d, d; \mathbb{Z})$ T-duality symmetry [T-fold]	flux compactifications
	geometry associated with G_{mn}, $\tau = C_{(0)} + \mathrm{i}\,\mathrm{e}^{-\Phi}$	Generalized geometry $SL(2, \mathbb{Z})$ S-duality symmetry [S-fold]	F-theory
	geometry associated with G_{mn}, B_{mn}, Φ, $C_{(p)}$	Generalized geometry $E_{d+1(d+1)}(\mathbb{Z})$ U-duality symmetry [U-fold]	compactifications with non-abelian gauge

$\boxed{\text{Non-geometric structure}}$

$$\text{structure groups} \;=\; \text{diffeom.}\ (GL(d,\mathbb{R})) \quad \cup \quad \underbrace{\text{String duality groups}}_{\substack{\uparrow \\ \mathrm{T}\text{-duality, } \mathrm{U}\text{-duality, etc.}}}$$

$GL(d,\mathbb{R}) \cup$ duality transf.



Generalized Geometry (N. Hitchin)
Doubled Geometry (C. Hull)

5^2_2-brane is a concrete example (T-fold)

Exotic brane shows us a new insight of stringy spacetime

M-theory on $S^1(R_s)$	mass/tension ($l_{\mathsf{s}} \equiv 1$)	type IIA
longitudinal M2	1	F1
transverse M2	$\frac{1}{g_{\mathsf{s}}}$	D2
longitudinal M5	$\frac{1}{g_{\mathsf{s}}}$	D4
transverse M5	$\frac{1}{g_{\mathsf{s}}^2}$	NS5
longitudinal KK6	$\frac{R_{\mathsf{TN}}^2}{g_{\mathsf{s}}^2}$	KK5
KK6 with $R_{\mathsf{TN}} = R_{\mathsf{s}}$	$\frac{1}{g_{\mathsf{s}}}$	D6
transverse KK6	$\frac{R_{\mathsf{TN}}^2}{g_{\mathsf{s}}^3}$	6_3^1

0	1	2	3	4	5	6	7	8	9	M
✓	✓	✓	✓	✓	✓	✓	S^1	$\mathbb{R}^3$		
KK6 $\to$ 6_3^1							Taub-NUT			

$$b_n^c \, : \, M = \frac{(R_1 \cdots R_c)^2}{g_{\mathsf{s}}^n}$$

for review: N. Obers and B. Pioline `hep-th/9809039`

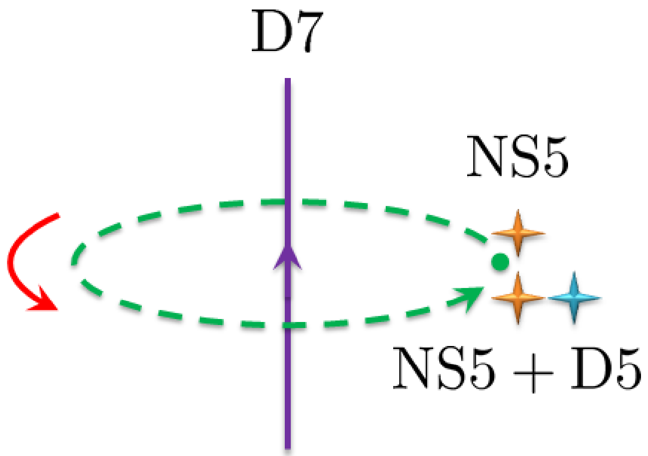
A very rough sketch on D7-brane (co-dim. 2 in 10D)

Moving around a D7-brane induces an $SL(2,\mathbb{Z})$ monodromy charge q

$$q : (C_2, B_2) \to (C_2 + B_2, B_2)$$

Since co-dim. 2, brane charges are not conserved but

$$(0,\ \text{NS5}) \to (\text{D5},\ \text{NS5})$$



an instructive discussion : J. de Boer and M. Shigemori arXiv:1209.6056

LESSON 1 : GLSM for NS5-brane

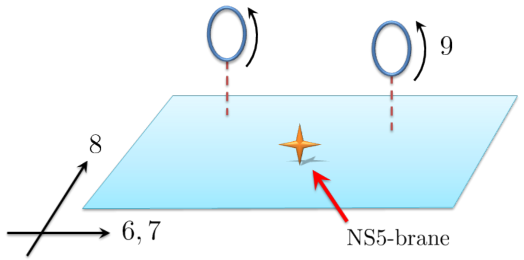
Bosonic Lagrangian after integrating-out auxiliary fields :

$$\begin{aligned}
\mathscr{L}^{\text{kin}}_{\text{NS5}} &= \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2\Big] - \Big[|D_m q|^2 + |D_m \widetilde{q}|^2\Big] \\
&\quad - \frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 + (\partial_m X^9)^2\Big] - (X^9 - t^9)\, F_{01} \\
\mathscr{L}^{\text{pot}}_{\text{NS5}} &= -2\big(|\sigma|^2 + |\phi|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big) \\
&\quad - \frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} - (X^6 - s^6) - \mathrm{i}(X^8 - s^8)\Big|^2
\end{aligned}$$

Steps to NLSM for NS5-brane

1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q, \widetilde{q})$
3. IR limit $e \to \infty$, and integrate out A_m

$\Longrightarrow$



$(X^6, X^7; X^8, X^9)$

1. SUSY vacua

$$\sigma = 0 = \phi \ , \qquad |q|^2 - |\widetilde{q}|^2 = X^7 - t^7 \ , \qquad q\widetilde{q} = (X^6 - s^6) + \mathrm{i}(X^8 - s^8)$$

2. solve constraints on $(q, \widetilde{q})$

$$q = -\mathrm{i}\,\mathrm{e}^{-\mathrm{i}\alpha}\sqrt{r + (X^7 - t^7)} \ , \qquad \widetilde{q} = \mathrm{i}\,\mathrm{e}^{+\mathrm{i}\alpha_a}\frac{(X^6 - s^6) + \mathrm{i}(X^8 - s^8)}{\sqrt{r + (X^7 - t^7)}}$$

$$|D_m q|^2 + |D_m \widetilde{q}|^2 = \frac{1}{2r}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] + r\Big(\partial_m \alpha + A_m - \Omega_i\,\partial_m X^i\Big)^2$$

$$r = \sqrt{(X^6 - s^6)^2 + (X^7 - t^7)^2 + (X^8 - s^8)^2}$$

$$\Omega_i\,\partial_m X^i = \frac{-(X^6 - s^6)\partial_m X^8 + (X^8 - s^8)\partial_m X^6}{r(r + (X^7 - t^7))}$$

3. IR limit $e \to \infty$, and integrate out A_m

$$A_m = \Omega_i\,\partial_m X^i + \frac{1}{r}\,\varepsilon_{mn}\,\partial^n X^9$$

$$\Longrightarrow \quad \mathscr{L}^{\mathsf{NLSM}}_{\mathsf{NS5}} \ = \ -\frac{1}{2}\Big(\frac{1}{g^2} + \frac{1}{r}\Big)\Big[(\partial_m \vec{X})^2 + (\partial_m X^9)^2\Big] + \varepsilon^{mn}\,\Omega_i\,\partial_m X^i\,\partial_n X^9$$

LESSON 2 : $\mathbb{T}$-duality

$\Theta \to \Gamma$:

$$\begin{aligned}
\mathscr{L}_{\mathsf{H}} \ni \mathscr{L}_{\Theta} &= \int \mathrm{d}^4\theta \Big(-\frac{1}{g^2}\overline{\Theta}\Theta \Big) + \Big\{ \int \mathrm{d}^2\widetilde{\theta}\, (-\Theta)\Sigma + (\mathsf{h.c.}) \Big\} \\
&= \int \mathrm{d}^4\theta \Big\{ -\frac{1}{2g^2}(\Theta + \overline{\Theta})^2 - (\Theta + \overline{\Theta})V \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)
\end{aligned}$$

$$\downarrow$$

$$\mathscr{L}_{B\Gamma} = \int \mathrm{d}^4\theta \Big\{ -\frac{1}{2g^2}B^2 - BV - B(\Gamma + \overline{\Gamma}) \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)$$

real $\overline{B} = B$; chiral $\overline{D}_{\pm}\Gamma = 0$

$\Theta \to \Gamma$:

$$\mathscr{L}_{B\Gamma} \;=\; \int \mathrm{d}^4\theta \Big\{ - \frac{1}{2g^2} B^2 - BV - (\Gamma + \overline{\Gamma}) B \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)$$

Integrating out Γ, $\overline{\Gamma}$: $\to$ GLSM for NS5-brane $\qquad B = \Theta + \overline{\Theta}$

or, Integrating out B : $\to$ GLSM for KK-monopole $\qquad \frac{1}{g^2} B = -(\Gamma + \overline{\Gamma}) - V$

duality relation : $\qquad \Theta = X^7 + \mathrm{i} X^9 + \ldots, \quad \Gamma = \widetilde{X}^7 + \mathrm{i}\widetilde{X}^9 + \ldots$

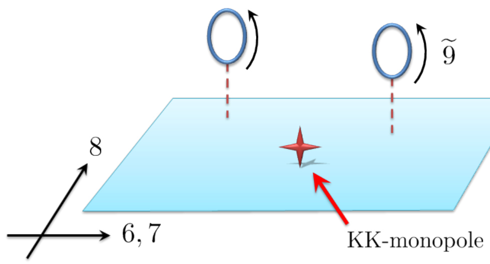
$$\Theta + \overline{\Theta} \;=\; -g^2 (\Gamma + \overline{\Gamma}) - g^2 V \quad \to \quad \boxed{\begin{array}{rcl} X^7 &=& -g^2\, \widetilde{X}^7 \\ \pm(\partial_0 \pm \partial_1) X^9 &=& -g^2 (D_0 \pm D_1) \widetilde{X}^9 \\ D_m \widetilde{X}^9 &=& \partial_m \widetilde{X}^9 + A_m \end{array}}$$

$$
\begin{aligned}
\mathscr{L}_{\mathsf{KK}} \;=\; & \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma} \Sigma + \overline{\Phi} \Phi \Big) + \overline{Q} \, \mathrm{e}^{-2V} Q + \overline{\widetilde{Q}} \, \mathrm{e}^{+2V} \widetilde{Q} \Big\} + \int \mathrm{d}^4\theta \, \Big\{ \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + V \Big)^2 + \frac{1}{g^2} \overline{\Psi} \Psi \Big\} \\
& + \int \mathrm{d}^2\theta \, \Big(\widetilde{Q} \Phi Q + (s - \Psi) \Phi \Big) + (\text{h.c.}) \\
& + \Big\{ \int \mathrm{d}^2 \widetilde{\theta} \, t \, \Sigma + (\text{h.c.}) \Big\} - \varepsilon^{mn} \, \partial_m (X^9 A_n)
\end{aligned}
$$

Steps to NLSM for KK-monopole

1. SUSY vacua $\mathscr{L}^{\mathsf{pot}} = 0$
2. solve constraints on $(q, \widetilde{q})$
3. IR limit $e \to \infty$, and integrate out A_m

$\Longrightarrow$



$(X^6, X^7; X^8, \widetilde{X}^9)$

1. SUSY vacua

$$\sigma = 0 = \phi \, , \qquad |q|^2 - |\widetilde{q}|^2 = X^7 - t^7 \, , \qquad q\widetilde{q} = (X^6 - s^6) + \mathrm{i}(X^8 - s^8)$$

2. solve constraints on $(q_a, \widetilde{q}_a)$

$$q = -\mathrm{i}\,\mathrm{e}^{-\mathrm{i}\alpha} \sqrt{r + (X^7 - t^7)} \, , \qquad \widetilde{q} = \mathrm{i}\,\mathrm{e}^{+\mathrm{i}\alpha} \frac{(X^6 - s^6) + \mathrm{i}(X^8 - s^8)}{\sqrt{r + (X^7 - t^7)}}$$

$$|D_m q|^2 + |D_m \widetilde{q}|^2 = \frac{1}{2r} \Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \Big] + r \Big(\partial_m \alpha + A_m - \Omega_i \, \partial_m X^i \Big)^2$$

3. IR limit $e \to \infty$, and integrate out A_m

$$A_m = \frac{1}{rH} \big(\partial_m \widetilde{X}^9 - \Omega_i \, \partial_m X^i \big) + \Omega_i \, \partial_m X^i \, , \qquad H = \frac{1}{g^2} + \frac{1}{r}$$

$$\mathscr{L}^{\mathsf{NLSM}}_{\mathsf{KK}} \;=\; -\frac{1}{2} H (\partial_m \vec{X})^2 - \frac{1}{2} H^{-1} (\partial_m \widetilde{X}^9 - \Omega_i \, \partial_m X^i)^2 - \varepsilon^{mn} \, \partial_m ((X^9 - t^9) A_n)$$

LESSON 3 : 5^2_2-brane

$\Psi \to \Xi$:

$$
\begin{aligned}
\mathscr{L}_{\mathsf{KK}} \ni \mathscr{L}_{\Psi} \;&=\; \int \mathrm{d}^4\theta \, \frac{1}{g^2} \overline{\Psi} \Psi + \Big\{ \int \mathrm{d}^2\theta \, (-\Psi) \Phi + (\mathsf{h.c.}) \Big\} \\
&=\; \int \mathrm{d}^4\theta \, \Big\{ \frac{a}{g^2} (\Psi + \overline{\Psi})^2 - (\Psi + \overline{\Psi})(C + \overline{C}) \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{2a-1}{2g^2} (\Psi - \overline{\Psi})^2 - (\Psi - \overline{\Psi})(C - \overline{C}) \Big\}
\end{aligned}
$$

$$\downarrow$$

$$
\begin{aligned}
\mathscr{L}_{RSX\Xi} \;&=\; \int \mathrm{d}^4\theta \, \Big\{ \frac{a}{g^2} R^2 - R(C + \overline{C}) + R(\Xi_1 + \overline{\Xi}_1) + R(X + \overline{X}) \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{2a-1}{2g^2} (\mathrm{i}S)^2 - (\mathrm{i}S)(C - \overline{C}) + \mathrm{i}S(\Xi_2 - \overline{\Xi}_2) + \mathrm{i}S(X - \overline{X}) \Big\}
\end{aligned}
$$

$\overline{R} = R, \;\; \overline{S} = S, \;\; \overline{D}_+ \Xi_{1,2} = 0 = D_- \Xi_{1,2}, \;\; \overline{D}_{\pm} X = 0, \;\; \Phi_a = \overline{D}_+ \overline{D}_- C_a$

$\Psi \to \Xi$:

$$\widetilde{\mathscr{L}} = \int d^4\theta \Big\{ \frac{a}{g^2} R^2 - R(C + \overline{C}) + R(\Xi_1 + \overline{\Xi}_1) + R(X + \overline{X}) \Big\} + \int d^4\theta \Big\{ \frac{2a-1}{2g^2} (\mathrm{i}S)^2 - (\mathrm{i}S)(C - \overline{C}) + \mathrm{i}S(\Xi_2 - \overline{\Xi}_2) + \mathrm{i}S(X - \overline{X}) \Big\}$$

Integrating out Ξ_1, Ξ_2, X : $\to$ GLSM for KK-monopole

or, Integrating out R, Ξ_2 : $\to$ GLSM for 5^2_2-brane

$$\frac{2a}{g^2} R = -(\Xi_1 + \overline{\Xi}_1) + (C + \overline{C})$$

duality relation at $a = \frac{1}{2}$:

$$\Psi = X^6 + \mathrm{i}\, X^8 + \ldots$$

$$\Psi + \overline{\Psi} = -g^2(\Xi_1 + \overline{\Xi}_1) + g^2(C + \overline{C})$$

$X^6 \sim$ real part of Ξ
$\partial X^8 \sim \partial$(imaginary part of Ξ) + "gauge" fields in C_a

$$
\begin{aligned}
\mathscr{L}_{\mathsf{E}} \;=\;& \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, \mathrm{e}^{-2V} Q + \overline{\widetilde{Q}}\, \mathrm{e}^{+2V} \widetilde{Q} \Big\} \\
&+ \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big\{ \Big(\Gamma + \overline{\Gamma} + V \Big)^2 - \Big(\Xi + \overline{\Xi} - (C + \overline{C}) \Big)^2 \Big\} \\
&+ \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}\Phi Q + s\Phi \Big) + (\text{h.c.}) \Big\} - \int \mathrm{d}^4\theta \, (\Psi - \overline{\Psi})(C - \overline{C}) \\
&+ \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, t\, \Sigma + (\text{h.c.}) \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)
\end{aligned}
$$

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$		role
neutral HM	twisted chiral $\Xi = X^6 + \mathrm{i}\, \widetilde{X}^8 + \ldots$	chiral $\Gamma = X^7 + \mathrm{i}\, \widetilde{X}^9 + \ldots$	spacetime coord.
VMs	twisted chiral $\Sigma = \overline{D}_+ D_- V$	chiral $\Phi = \overline{D}_+ \overline{D}_- C$	gauging isometry
charged HM	chiral Q $(-)$	chiral $\widetilde{Q}$ $(+)$	curving geometry
FI parameters	$s = s^6 + \mathrm{i}\, s^8$	$t = t^7 + \mathrm{i}\, t^9$	position of five-branes

$$
\begin{aligned}
\mathscr{L}_{\mathsf{E}}^{\mathsf{kin}} &= \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2\Big] - \Big[|D_m q|^2 + |D_m \widetilde{q}|^2\Big] \\
&\quad -\frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2\Big] - \frac{g^2}{2}\Big[(\partial_m \widetilde{X}^8)^2 + (D_m \widetilde{X}^9)^2\Big] - (X^9 - t^9)\, F_{01} \\
\mathscr{L}_{\mathsf{E}}^{\mathsf{pot}} &= -2\big(|\sigma|^2 + 4|M_c|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big) \\
&\quad -\frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} - (X^6 - s^6) - \mathrm{i}(X^8 - s^8)\Big|^2 \\
&\quad + \frac{g^2}{2}(A_{c=} + \overline{A}_{c=})(B_{c+\!\!+} + \overline{B}_{c+\!\!+})
\end{aligned}
$$

$$
\begin{aligned}
(\partial_0 + \partial_1)X^8 &= -g^2(\partial_0 + \partial_1)\widetilde{X}^8 + g^2(B_{c+\!\!+} + \overline{B}_{c+\!\!+}) \\
(\partial_0 - \partial_1)X^8 &= +g^2(\partial_0 - \partial_1)\widetilde{X}^8 + g^2(A_{c=} + \overline{A}_{c=}) \\
+\frac{g^2}{2}(A_{c=} + \overline{A}_{c=})(B_{c+\!\!+} + \overline{B}_{c+\!\!+}) &= -\frac{1}{2g^2}(\partial_m X^8)^2 + \frac{g^2}{2}(\partial_m \widetilde{X}^8)^2 + \varepsilon^{mn}\,(\partial_m X^8)(\partial_n \widetilde{X}^8)
\end{aligned}
$$

Steps to NLSM for 5^2_2-brane

1. search SUSY vacua $\mathscr{L}^{\text{potential}} = 0$
2. solve constraints of charged HM $(Q, \widetilde{Q})$
3. integrate out VM (V, Φ) in IR

★4. integrate s^8, and solve EOM for X^8 in $\Psi - \overline{\Psi}$

Step 1.,2.,3. :

$$\begin{aligned}\mathscr{L} \;=\;& -\frac{1}{2}H\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] - \frac{1}{2H}(\partial_m \widetilde{X}^9)^2 \\ & -\frac{(\Omega_8)^2}{2H}(\partial_m X^8)^2 + \frac{\Omega_8}{H}(\partial_m X^8)(\partial^m \widetilde{X}^9) \\ & -\frac{(\Omega_6)^2}{2H}(\partial_m X^6)^2 - \frac{\Omega_6\Omega_8}{H}(\partial_m X^6)(\partial^m X^8) + \frac{\Omega_6}{H}(\partial_m X^6)(\partial^m \widetilde{X}^9) \\ & -\varepsilon^{mn}\partial_m((X^9 - t^9)A_n) + \varepsilon^{mn}(\partial_m X^8)(\partial_n \widetilde{X}^8)\end{aligned}$$

$$H \;=\; \frac{1}{g^2} + \frac{1}{r}, \qquad \Omega_6 \;=\; \frac{X^8 - s^8}{r(r + (X^7 - t^7))}, \quad \Omega_8 \;=\; -\frac{X^6 - s^6}{r(r + (X^7 - t^7))}$$

$$A_m = \frac{1}{rH}\big(\partial_m \widetilde{X}^9 - \Omega_i\, \partial_m X^i\big) + \Omega_i\, \partial_m X^i$$

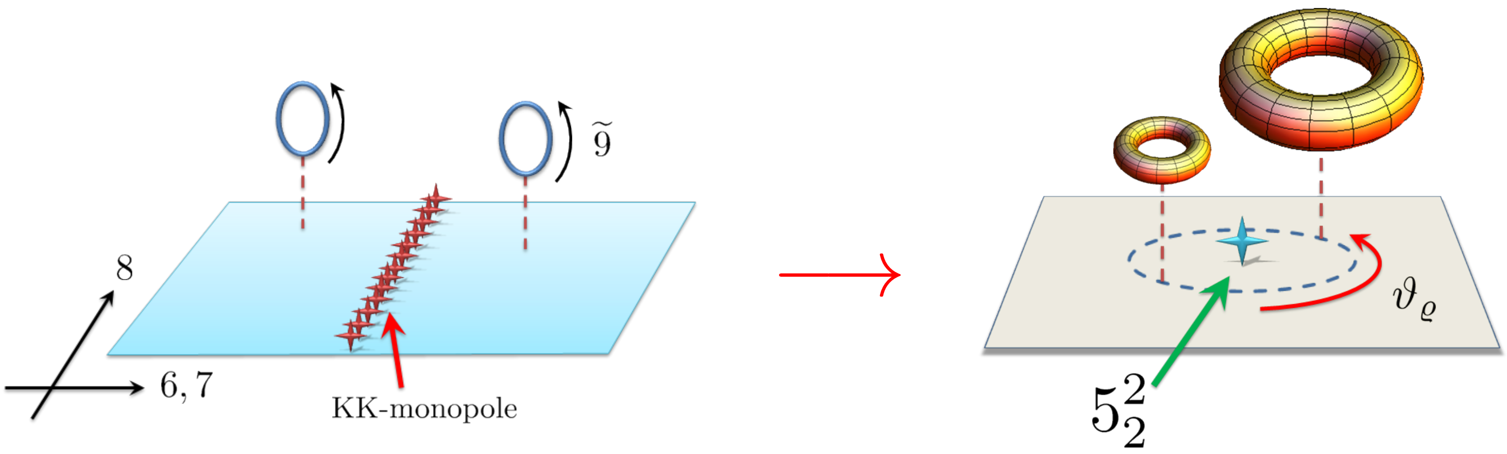
Step 4. : $s^8 = 2\pi \mathcal{R}_8\, s \xrightarrow{\text{integral of } s}$ emerge isometry

$$\begin{cases} H \to h_0 + \sigma \log(\mu/\varrho) & : \text{ co-dim. 2} \\ \Omega_6 \to 0 & : \text{ isometry along } X^8 \\ \Omega_8 \to \sigma \arctan\left(\dfrac{X^7 - t^7}{X^6 - s^6}\right) \equiv \sigma\, \vartheta_\varrho & : \text{ “non–single-valued” metric} \end{cases}$$

$\varrho = \sqrt{(X^6 - s^6)^2 + (X^7 - t^7)^2}$

EOM for X^8 : $\partial_m X^8 = \dfrac{H}{K}\Big[\dfrac{\sigma\vartheta_\varrho}{H}(\partial_m \widetilde{X}^9) + \varepsilon_{mn}(\partial^n \widetilde{X}^8)\Big]$

$K = H^2 + (\sigma\vartheta_\varrho)^2$



GLSM is a powerful tool, also in this stage :

Worldsheet instantons in NLSM can be captured by
vortex solution in gauge theory

ex.) GLSM for NS5-brane :

Take the configuration : $\phi = 0 = \sigma$ with $g^2 \to 0$ and finite e^2

$$\begin{aligned} \mathscr{L}_{\mathsf{E}} &= \frac{1}{2e^2}(F_{12})^2 + |D_m q|^2 + \frac{e^2}{2}\big(|q|^2 - \zeta\big)^2 + \mathrm{i}\, X^9\, F_{12} \\ F_{12} &= \mp e^2\big(|q|^2 - \zeta\big)\,, \qquad 0 = (D_1 \pm \mathrm{i} D_2) q \end{aligned}$$

Abrikosov-Nielsen-Olesen vortex eq.

then, $S_{\mathsf{E}} = \frac{1}{2\pi}\int \mathrm{d}^2 x\, \mathscr{L}_{\mathsf{E}} = \zeta|n| - \mathrm{i}\, X^9\, n$ $\qquad n = -\frac{1}{2\pi}\int \mathrm{d}^2 x\, F_{12}$

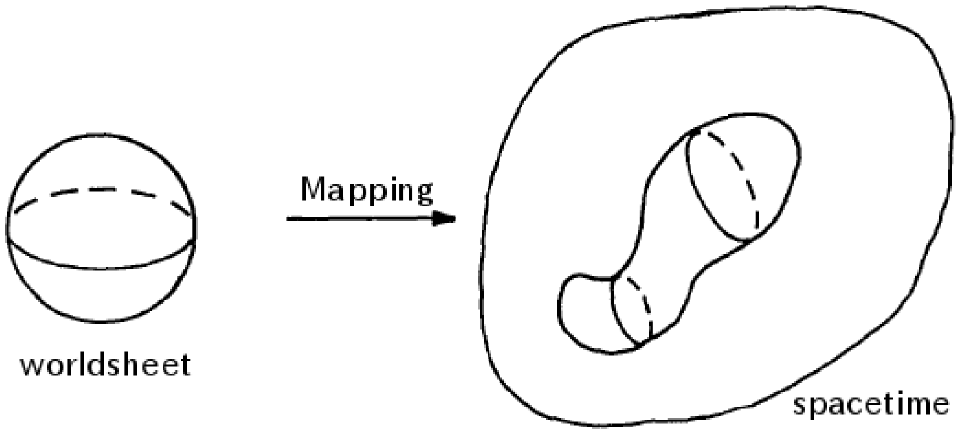
Quantum Corrections

S. Sasaki and TK arXiv:1305.4439

String Worldsheet Instanton Corrections

$\downarrow$

Deform target space geometry by momentum and/or winding effects



GLSM is a powerful tool in this stage :

Worldsheet instantons in NLSM can be captured by vortex solution in gauge theory

- NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{r} \ : \ \text{radius of } X^9$$

$g \to 0$ case	general r
radius of X^9	∞
KK-modes	light
winding modes	heavy

$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of X^9	∞	0
KK-modes	light	heavy
winding modes	heavy	light

- NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{r} \,:\ \text{radius of } X^9$$

$g \to 0$ case	general r
radius of X^9	∞
KK-modes	light
winding modes	heavy

$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of X^9	∞	0
KK-modes	light	heavy
winding modes	heavy	light

GLSM for NS5-brane has $X^9 F_{01}$

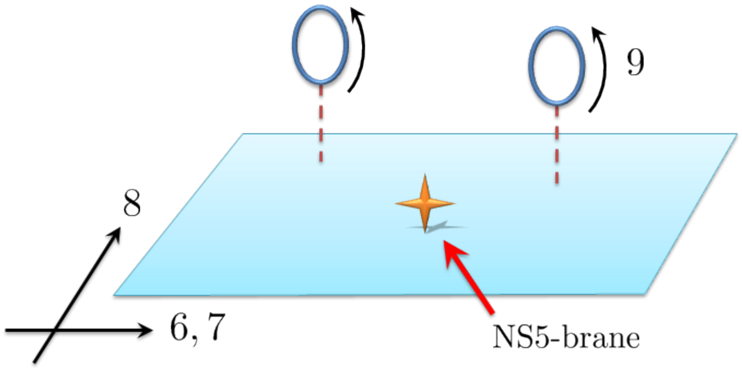
$\downarrow$

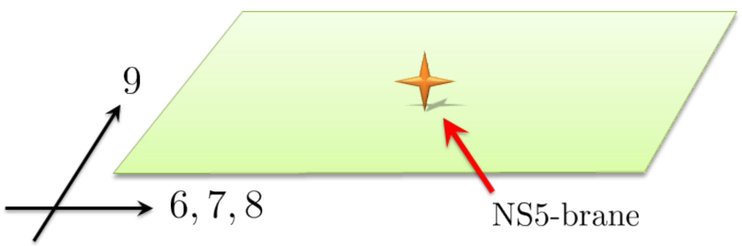
KK-mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to NS5-brane :

$X^9 F_{01}$ in GLSM

$\rightarrow$ unfolding effect on compactified circle X^9

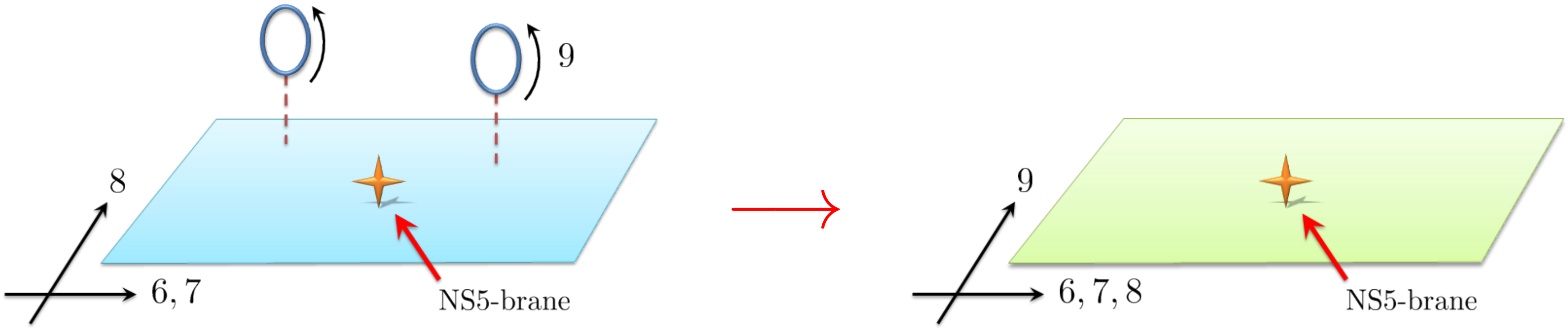




Worldsheet instanton corrections to NS5-brane :

$X^9 F_{01}$ in GLSM

$\rightarrow$ unfolding effect on compactified circle X^9



$$H = \frac{1}{g^2} + \frac{1}{r} \rightarrow \frac{1}{g^2} + \frac{1}{r}\sum_{n=1}^{\infty} \mathrm{e}^{-nr}\Big[\mathrm{e}^{+\mathrm{i}\,nX^9} + \mathrm{e}^{-\mathrm{i}\,nX^9}\Big]$$

$$= \frac{1}{g^2} + \frac{1}{r}\frac{\sinh(r)}{\cosh(r) - \cos(X^9)}$$

D. Tong hep-th/0204186

- KK-monopole

$$H^{-1} = \left(\frac{1}{g^2} + \frac{1}{r}\right)^{-1} \ : \ \text{radius of } \widetilde{X}^9$$

$g \to 0$ case	general r
radius of $\widetilde{X}^9$	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of $\widetilde{X}^9$	0	∞
KK-modes	heavy	light
winding modes	light	heavy

- KK-monopole

$$H^{-1} = \left(\frac{1}{g^2} + \frac{1}{r}\right)^{-1} \; : \text{ radius of } \widetilde{X}^9$$

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$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of $\widetilde{X}^9$	0	∞
KK-modes	heavy	light
winding modes	light	heavy

GLSM for KK-monopole has $\varepsilon^{mn}\partial_m(X^9 A_n)$

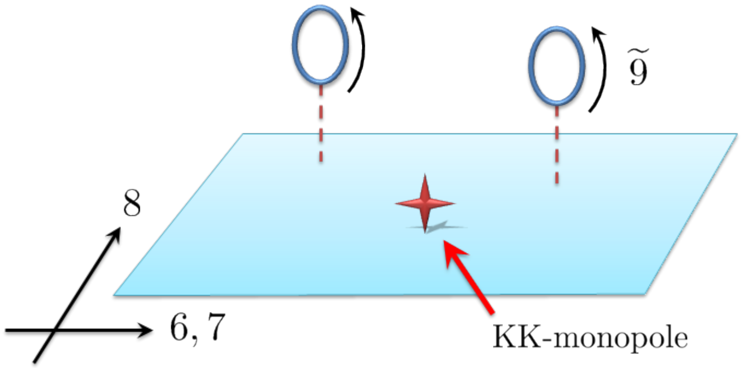
$\downarrow$

Winding mode corrections can be captured by vortex solution in gauge theory

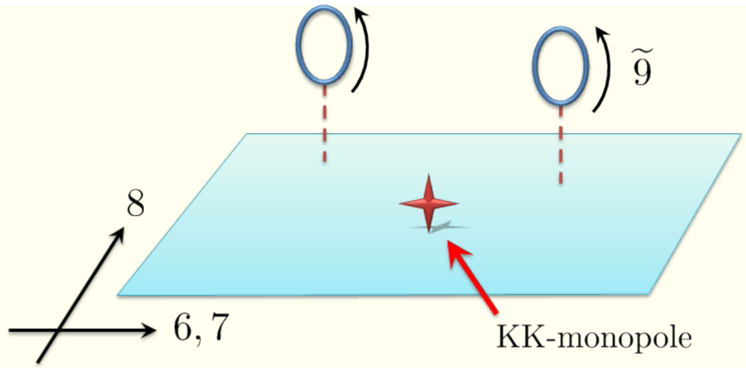
Worldsheet instanton corrections to KK-monopole :

$\varepsilon^{mn} \partial_m (X^9 A_n)$ in GLSM

$\rightarrow$ string winding modes along X^9



$\longrightarrow$



$$H \rightarrow \frac{1}{g^2} + \frac{1}{r} \frac{\sinh(r)}{\cosh(r) - \cos(X^9)}$$

$+$

string winding along dual circle X^9

J. Harvey and S. Jensen hep-th/0507204; K. Okuyama hep-th/0508097

- 5^2_2-brane

$$\frac{H}{K} : \text{radius of } \widetilde{X}^9 \qquad H = \frac{1}{g^2} + \sigma \log\left(\frac{\Lambda}{\varrho}\right),\ K = H^2 + (\sigma\vartheta_\varrho)^2$$

$g \to 0$ case	general ϱ
radius of $\widetilde{X}^9$	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$\varrho \to 0$	$\varrho \to \Lambda$	
radius of $\widetilde{X}^9$	0	0	∞ at $\vartheta_\varrho = 0$
KK-modes	heavy	heavy	light at $\vartheta_\varrho = 0$
winding modes	light	light	heavy at $\vartheta_\varrho = 0$

- 5^2_2-brane

$$\frac{H}{K} : \text{radius of } \widetilde{X}^9 \qquad H = \frac{1}{g^2} + \sigma \log\left(\frac{\Lambda}{\varrho}\right), \; K = H^2 + (\sigma\vartheta_\varrho)^2$$

$g \to 0$ case	general ϱ
radius of $\widetilde{X}^9$	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$\varrho \to 0$	$\varrho \to \Lambda$	
radius of $\widetilde{X}^9$	0	0	∞ at $\vartheta_\varrho = 0$
KK-modes	heavy	heavy	light at $\vartheta_\varrho = 0$
winding modes	light	light	heavy at $\vartheta_\varrho = 0$

GLSM for 5^2_2 with one gauged isometry has $\varepsilon^{mn}\partial_m(X^9 A_n)$

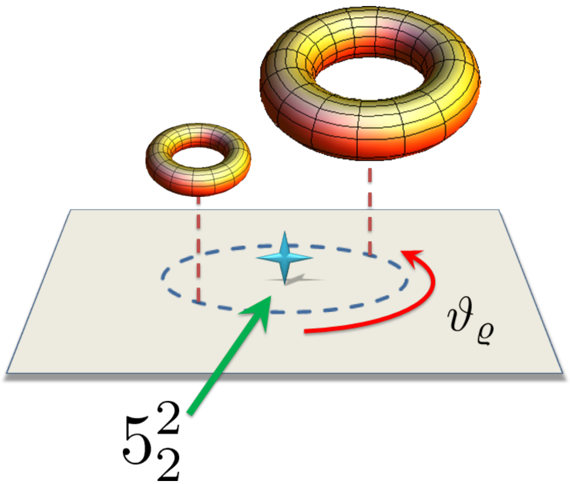
$\downarrow$

Winding mode corrections can be captured by vortex solution in gauge theory

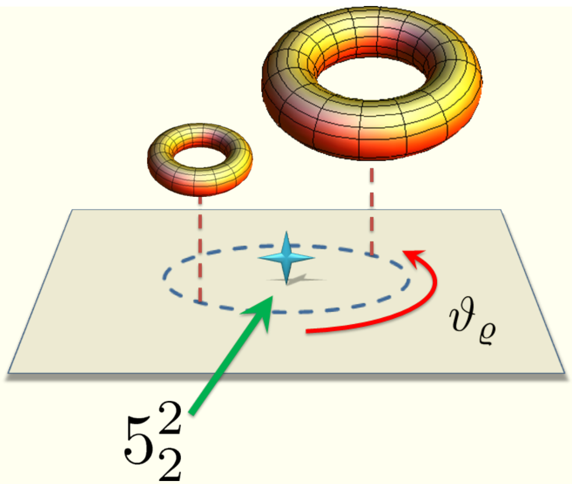
Worldsheet instanton corrections to 5^2_2-brane with one gauged isometry :

$\varepsilon^{mn}\,\partial_m(X^9 A_n)$ in GLSM

$\rightarrow$ string winding modes along X^9



$\longrightarrow$



$+$

string winding along
dual circle X^9

$$H \;\rightarrow\; h_0 + \sigma \log(\mu/\varrho) + \sigma \sum_{n \neq 0} \exp(\mathrm{i}\, n\, X^9) K_0(|n|\varrho)$$

S. Sasaki and TK arXiv:1305.4439

$$
\begin{aligned}
\Psi &= (X^6 + \mathrm{i} X^8) + \mathrm{i}\,\theta^+ \chi_+ + \mathrm{i}\,\theta^- \chi_- + \mathrm{i}\,\theta^+ \theta^- G + \ldots \\
\Xi &= (\widetilde{X}^6 + \mathrm{i}\,\widetilde{X}^8) + \mathrm{i}\,\theta^+ \overline{\xi}_+ + \mathrm{i}\,\overline{\theta}^- \xi_- + \mathrm{i}\,\theta^+ \overline{\theta}^- G_\Xi + \ldots \\
\Theta &= (X^7 + \mathrm{i} X^9) + \mathrm{i}\,\theta^+ \overline{\widetilde{\chi}}_+ + \mathrm{i}\,\overline{\theta}^- \widetilde{\chi}_- + \mathrm{i}\,\theta^+ \overline{\theta}^- \widetilde{G} + \ldots \\
\Gamma &= (\widetilde{X}^7 + \mathrm{i}\,\widetilde{X}^9) + \mathrm{i}\,\theta^+ \zeta_+ + \mathrm{i}\,\theta^- \zeta_- + \mathrm{i}\,\theta^+ \theta^- G_\Gamma + \ldots \\
Q_a &= q + \mathrm{i}\,\theta^+ \psi_+ + \mathrm{i}\,\theta^- \psi_- + \mathrm{i}\,\theta^+ \theta^- F + \ldots \\
\widetilde{Q}_a &= \widetilde{q} + \mathrm{i}\,\theta^+ \widetilde{\psi}_+ + \mathrm{i}\,\theta^- \widetilde{\psi}_- + \mathrm{i}\,\theta^+ \theta^- \widetilde{F} + \ldots \\
V &= \theta^+ \overline{\theta}^+ (A_0 + A_1) + \theta^- \overline{\theta}^- (A_0 - A_1) - \theta^- \overline{\theta}^+ \sigma - \theta^+ \overline{\theta}^- \overline{\sigma} \\
&\quad -\mathrm{i}\,\theta^+ \theta^- \big(\overline{\theta}^+ \overline{\lambda}_+ + \overline{\theta}^- \overline{\lambda}_- \big) + \mathrm{i}\,\overline{\theta}^+ \overline{\theta}^- \big(\theta^+ \lambda_+ + \theta^- \lambda_- \big) - \theta^+ \theta^- \overline{\theta}^+ \overline{\theta}^- D_V \\
\Phi_a &= \phi + \mathrm{i}\,\theta^+ \widetilde{\lambda}_+ + \mathrm{i}\,\theta^- \widetilde{\lambda}_- + \mathrm{i}\,\theta^+ \theta^- D_\Phi + \ldots \qquad = \overline{D}_+ \overline{D}_- C \\
C &= \phi_c + \mathrm{i}\,\theta^+ \psi_{c+} + \mathrm{i}\,\theta^- \psi_{c-} + \mathrm{i}\,\overline{\theta}^+ \chi_{c+} + \mathrm{i}\,\overline{\theta}^- \chi_{c-} \\
&\quad + \mathrm{i}\,\theta^+ \theta^- F_c + \mathrm{i}\,\overline{\theta}^+ \overline{\theta}^- M_c + \mathrm{i}\,\theta^+ \overline{\theta}^- G_c + \mathrm{i}\,\overline{\theta}^+ \theta^- N_c + \theta^- \overline{\theta}^- A_{c=} + \theta^+ \overline{\theta}^+ B_{c\#} \\
&\quad - \mathrm{i}\,\theta^+ \theta^- \overline{\theta}^+ \zeta_{c+} - \mathrm{i}\,\theta^+ \theta^- \overline{\theta}^- \zeta_{c-} + \mathrm{i}\,\overline{\theta}^+ \overline{\theta}^- \theta^+ \lambda_{c+} + \mathrm{i}\,\overline{\theta}^+ \overline{\theta}^- \theta^- \lambda_{c-} - \theta^+ \theta^- \overline{\theta}^+ \overline{\theta}^- D_c
\end{aligned}
$$

with $\overline{D}_\pm = -\dfrac{\partial}{\partial \overline{\theta}^\pm} + \mathrm{i}\,\theta^\pm (\partial_0 \pm \partial_1)$, $\;D_\pm = \dfrac{\partial}{\partial \theta^\pm} - \mathrm{i}\,\overline{\theta}^\pm (\partial_0 \pm \partial_1)$