On branes in $\mathcal{N} = 2$ supergravities

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Outline

1. Branes in maximal supergravities

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2. Supergravities with less supersymmetry

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2. Supergravities with less supersymmetry
3. Real forms and Tits-Satake diagrams

Outline

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Branes in maximal sugras

IIA/IIB string theory on $T^d \to O(d,d;\mathbb{Z})$ T-duality

Consider $d=1$: one defines

$$p_L = \frac{m}{2R} + \frac{nR}{2\alpha'} \qquad p_R = \frac{m}{2R} - \frac{nR}{2\alpha'}$$

One finds that the spectrum contains 1/2-BPS states that are purely momentum or purely winding states

These states are charged under the graviphoton $g_{\mu 9}$ with charge $p_L + p_R$, and under $B_{\mu 9}$ with charge $p_L - p_R$

The invariant quantity under $O(1,1;\mathbb{Z})$ is $p_L^2 - p_R^2 \sim mn$, which vanishes for 1/2-BPS states

This generalises to any d

Branes in maximal sugras

From the low-energy supergravity view-point, one has a continuous $SO(d,d)$ symmetry, and one finds 1/2-BPS black hole solutions with charge Q_A such that $Q^A Q_A = 0$

But maximal supergravity theories have bigger symmetries:

$$E_{d+1(d+1)} \supset SO(d,d) \times \mathbb{R}^+$$

e.g.:

$$D = 7: \quad SL(5,\mathbb{R}) \supset SO(3,3) \times \mathbb{R}^+$$

$$D = 6: \quad SO(5,5) \supset SO(4,4) \times \mathbb{R}^+$$

$$D = 5: \quad E_{6(6)} \supset SO(5,5) \times \mathbb{R}^+$$

In the quantum theory these are conjectured to be broken to discrete U-duality symmetries

Hull, Townsend

Branes in maximal sugras

1/2-BPS branes in string theory correspond to 1/2-BPS brane solutions of the supergravity theory

These include D-branes, but also more non-perturbative branes

Supersymmetry and the global symmetries allow a complete classification of these branes

Such classification includes *non-standard* branes, i.e. domain walls and space-filling branes, for which one can write a supersymmetric (and κ-symmetric) effective action

Wess-Zumino term for a p-brane: $Q \int A_{p+1} + \ldots$

IIA: $Q \int A_9 + \ldots$ D8-brane

IIB: $Q \int A_{10} + \ldots$ D9-brane

Branes in maximal sugras

10-forms of IIB:

$A_{10}^{\alpha\beta\gamma}$ 4-plet of $SL(2,\mathbb{R})$

Bergshoeff, de Roo, Kerstan, FR

Corresponding charge:

$Q_{\alpha\beta\gamma}$

1/2-BPS constraint:

$Q_{\alpha\beta\gamma}\epsilon^{\gamma\delta}Q_{\delta\epsilon\tau}=0$

Q_{111} Q_{112} Q_{122} Q_{222}

Constraint selects Q_{111} and Q_{222} $\rightarrow$ longest weights

Branes in maximal sugras

In any dimensions, one classifies all the fields A_{p+1}, belonging to representations of $E_{d+1(d+1)}$

FR, West

Bergshoeff, De Baetselier, Nutma

The 1/2-BPS condition gives a constraint on the charge. This is always the highest-weight constraint

Ferrara, Maldacena

Bergshoeff, Marrani, FR

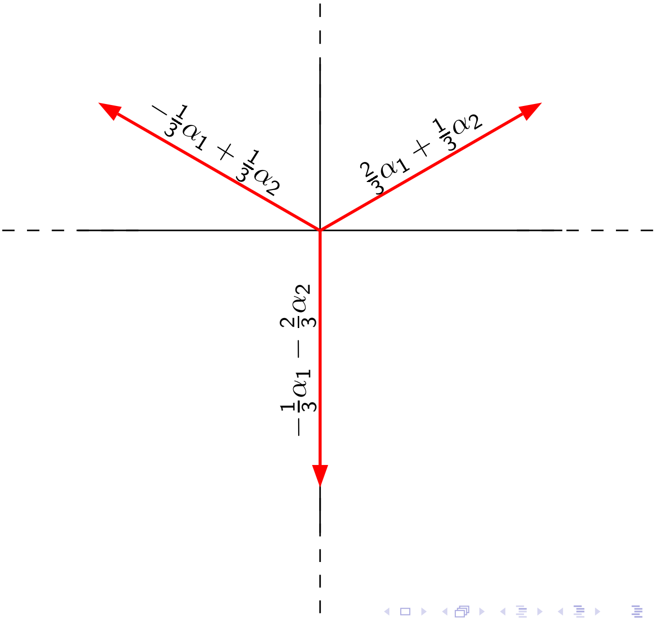
The number of independent branes inside a representation is the number of longest weights of that representation

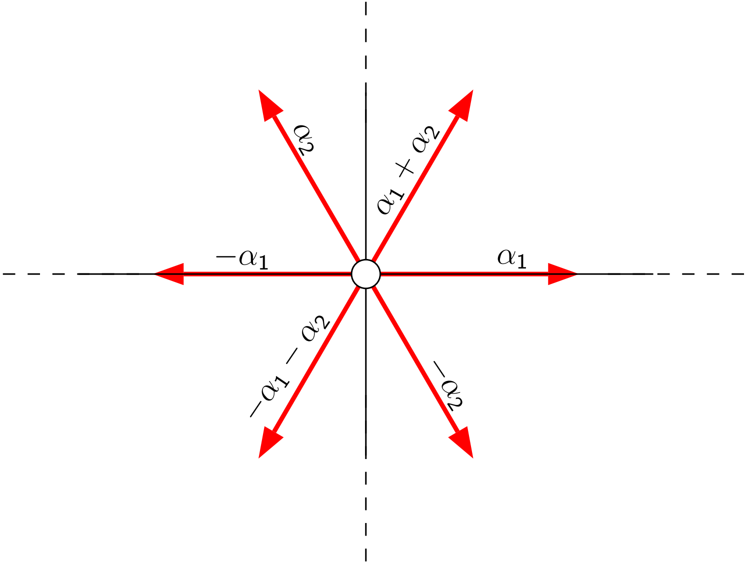
Bergshoeff, FR, Romano

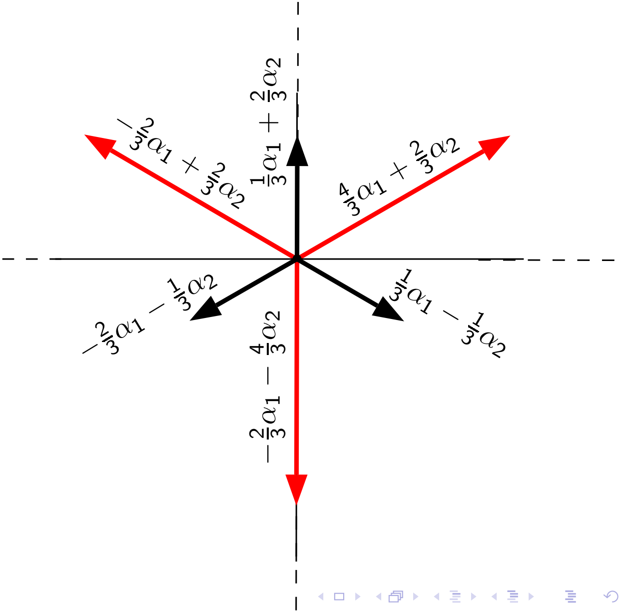
Branes in maximal sugras

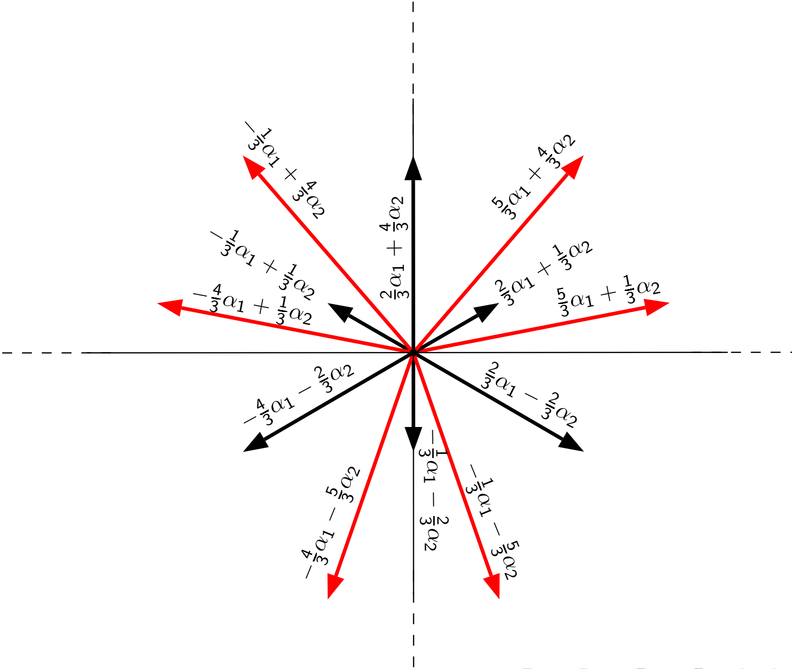
$D = 8$ Global symmetry $G = \mathrm{SL}(3, \mathbb{R}) \times \mathrm{SL}(2, \mathbb{R})$

field	representation	# of branes
1-forms	$(\overline{\mathbf{3}}, \mathbf{2})$	6
2-forms	$(\mathbf{3}, \mathbf{1})$	3
3-forms	$(\mathbf{1}, \mathbf{2})$	2
4-forms	$(\overline{\mathbf{3}}, \mathbf{1})$	3
5-forms	$(\mathbf{3}, \mathbf{2})$	6
6-forms	$(\mathbf{8}, \mathbf{1})$	6
	$(\mathbf{1}, \mathbf{3})$	2
7-forms	$(\mathbf{6}, \mathbf{2})$	6
8-forms	$(\mathbf{15}, \mathbf{1})$	6


$-\frac{1}{3}\alpha_1 + \frac{1}{3}\alpha_2$
$\frac{2}{3}\alpha_1 + \frac{1}{3}\alpha_2$
$-\frac{1}{3}\alpha_1 - \frac{2}{3}\alpha_2$


α_2
$\alpha_1 + \alpha_2$
$-\alpha_1$
α_1
$-\alpha_1 - \alpha_2$
$-\alpha_2$


$\frac{1}{3}\alpha_1 + \frac{2}{3}\alpha_2$
$-\frac{2}{3}\alpha_1 + \frac{2}{3}\alpha_2$
$\frac{4}{3}\alpha_1 + \frac{2}{3}\alpha_2$
$-\frac{2}{3}\alpha_1 - \frac{1}{3}\alpha_2$
$\frac{1}{3}\alpha_1 - \frac{1}{3}\alpha_2$
$-\frac{2}{3}\alpha_1 - \frac{4}{3}\alpha_2$


$-\frac{1}{3}\alpha_1 + \frac{4}{3}\alpha_2$
$\frac{2}{3}\alpha_1 + \frac{4}{3}\alpha_2$
$\frac{5}{3}\alpha_1 + \frac{4}{3}\alpha_2$
$-\frac{1}{3}\alpha_1 + \frac{1}{3}\alpha_2$
$\frac{2}{3}\alpha_1 + \frac{1}{3}\alpha_2$
$-\frac{4}{3}\alpha_1 + \frac{1}{3}\alpha_2$
$\frac{5}{3}\alpha_1 + \frac{1}{3}\alpha_2$
$-\frac{4}{3}\alpha_1 - \frac{2}{3}\alpha_2$
$\frac{2}{3}\alpha_1 - \frac{2}{3}\alpha_2$
$-\frac{4}{3}\alpha_1 - \frac{5}{3}\alpha_2$
$-\frac{1}{3}\alpha_1 - \frac{2}{3}\alpha_2$
$-\frac{1}{3}\alpha_1 - \frac{5}{3}\alpha_2$

Sugras with less susy

Half-maximal sugra theories coupled to n_V vector multiplets in $10-d$ dimensions:

$SO(d, n_V)/[SO(d) \times SO(n_V)]$

We do not consider gaugings

String theory: $n_V = d + 16$

Real form not maximally non-compact

1/2-BPS branes: same constraint as before: highest-weight constraint

Counting 1/2-BPS single branes → lightcone rules

Bergshoeff, FR

Sugras with less susy

Quarter-maximal sugra theories exist in six dimensions and below. They couple sugra to tensor multiplets (six dim), vector multiplets and hyper-multiplets.

Scalars in the vector multiplets parametrise:

- a very special real manifold in five dimensions
- a special Kähler manifold in four dimensions
- a quaternionic manifold in three dimensions

We will consider theories that give rise to symmetric spaces upon reduction to three dimensions

Such theories contain no hypers in dimension higher than three

In general the global symmetry is not maximally non-compact

Sugras with less susy

Symmetric quaternionic manifolds:

$SO(4, n_H)/[SO(4) \times SO(n_H)]$

$F_{4(4)}/[USp(6) \times SU(2)] \qquad (n_H = 7)$

$E_{6(2)}/[\,SU(6) \times SU(2)] \qquad (n_H = 10)$

$E_{7(-5)}/[SO(12) \times SU(2)] \qquad (n_H = 16)$

$E_{8(-24)}/[E_7 \times SU(2)] \qquad (n_H = 28)$

$G_{2(2)}/SO(4) \qquad (n_H = 2)$

$SU(n_H, 2)/[SU(n_H) \times SU(2) \times U(1)]$

$USp(2n_H, 2)/[USp(2n_H) \times USp(2)]$

$D=3$	$D=4$	$D=5$	$D=6$
$\frac{SO(4,n)}{SO(4)\times SO(n)}$ $n_H=n$	$\frac{SO(2,n-2)}{SO(2)\times SO(n-2)}\times\frac{SU(1,1)}{U(1)}$ $n_V=n-1$	$\frac{SO(1,n-3)}{SO(n-3)}\times\mathbb{R}^+$ $n_V=n-2$	$SO(1,1)$ $n_T=1, n_V=n-4$ $\frac{SO(1,n-3)}{SO(n-3)}$ $n_T=n-3, n_V=0$
$\frac{F_{4(4)}}{USp(6)\times SU(2)}$ $n_H=7$	$\frac{Sp(6,\mathbb{R})}{U(3)}$ $n_V=6$	$\frac{SL(3,\mathbb{R})}{SO(3)}$ $n_V=5$	$\frac{SO(1,2)}{SO(2)}$ $n_T=2, n_V=2$
$\frac{E_{6(2)}}{SU(6)\times SU(2)}$ $n_H=10$	$\frac{SU(3,3)}{SU(3)\times SU(3)}$ $n_V=9$	$\frac{SL(3,\mathbb{C})}{SU(3)}$ $n_V=8$	$\frac{SO(1,3)}{SO(3)}$ $n_T=3, n_V=4$
$\frac{E_{7(-5)}}{SO(12)\times SU(2)}$ $n_H=16$	$\frac{SO^*(12)}{U(6)}$ $n_V=15$	$\frac{SU^*(6)}{USp(6)}$ $n_V=14$	$\frac{SO(1,5)}{SO(5)}$ $n_T=5, n_V=8$
$\frac{E_{8(-24)}}{E_7\times SU(2)}$ $n_H=28$	$\frac{E_{7(-25)}}{E_6\times SO(2)}$ $n_V=27$	$\frac{E_{6(-26)}}{F_4}$ $n_V=26$	$\frac{SO(1,9)}{SO(9)}$ $n_T=9, n_V=16$
$\frac{G_{2(2)}}{SO(4)}$ $n_H=2$	$\frac{SU(1,1)}{U(1)}$ $n_V=1$	1 $n_V=0$	-
$\frac{SU(n,2)}{SU(n)\times SU(2)\times U(1)}$ $n_H=n$	$\frac{SU(n-1,1)}{SU(n-1)\times U(1)}$ $n_V=n-1$	-	-
$\frac{USp(2n,2)}{USp(2n)\times USp(2)}$ $n_H=n$	-	-	-

Sugras with less susy

1/2-BPS branes $\rightarrow$ highest-weight constraints

We want to count the branes

SO(4, n) theories: lightcone rule

dim.	type of brane	field	# of branes
$D = 3$	0-brane	A_{1,A_1A_2}	24
	1-brane	$A_{2,AB}$	8
		$A_{2,A_1\ldots A_4}$	16
$D = 4$	0-brane	$A_{1,Aa}$	8
	1-brane	$A_{2,ab}$	2
		A_{2,A_1A_2}	4
$D = 5$	0-brane	$A_{1,A}$ $(\alpha = 0)$	2
		A_1 $(\alpha = -2)$	1
	1-brane	A_2 $(\alpha = 0)$	1
		$A_{2,A}$ $(\alpha = -2)$	2
$D = 6$	1-brane	$A_{2,A}$	2

How do we count the branes in the other theories?

Real forms and TS diagrams

A real form exists if the algebra admits a basis such that all the structure constants are real

Killing form:

$B(X, Y) = \mathrm{Tr}(\mathrm{ad}X \mathrm{ad}Y)$

- Positive eigenvectors: non-compact generators
- Negative eigenvectors: compact generators

Cartan involution:

$B_\theta(X, Y) = B(X, \theta Y)$

such that $B_\theta(X, Y)$ is negative-definite. Diagonalise it:

- $\theta = 1$ eigenvectors are compact
- $\theta = -1$ eigenvectors are non-compact

Real forms and TS diagrams

Generators of $SL(2,\mathbb{R})$: $\sigma_1 \quad i\sigma_2 \quad \sigma_3$

Generators of $SU(2)$: $i\sigma_1 \quad i\sigma_2 \quad i\sigma_3$

Killing forms:

$$B_{\mathfrak{sl}(2,\mathbb{R})} = \begin{pmatrix} 8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 8 \end{pmatrix} \qquad B_{\mathfrak{su}(2)} = \begin{pmatrix} -8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -8 \end{pmatrix}$$

Define $H = \sigma_3 \quad E_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad E_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

Cartan involution of $SL(2,\mathbb{R})$ is

$$\theta H = -H \qquad \theta E_+ = -E_-$$

Real forms and TS diagrams

From

$\mathrm{ad}(\theta X) = -(\mathrm{ad} X)^\dagger$

and

$[H, E_\alpha] = \alpha(H) E_\alpha$

one derives the reality properties of the roots $\alpha(H)$.

- real root: vanishes for compact $H \rightarrow \theta\alpha = -\alpha$
- imaginary root: vanishes for non-compact $H \rightarrow \theta\alpha = \alpha$
- complex root: otherwise

Cartan involution on the roots: $\theta H_\alpha = H_{\theta\alpha}$

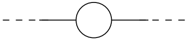
Implies $\theta E_\alpha = \pm E_{\theta\alpha}$

Real forms and TS diagrams

$\theta\alpha = \alpha$ (fixed root)

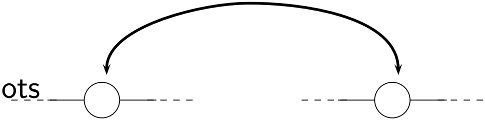
imaginary (compact generator)

$\theta\alpha = -\alpha$ up to fixed roots



complex

$\theta\alpha_i = -\alpha_j$ up to fixed roots



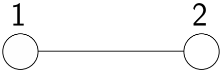
complex

From the TS diagram one derives the action of the Cartan involution on the generators

Real forms and TS diagrams

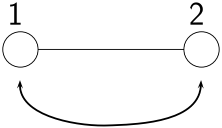
Example: real forms of $SL(3,\mathbb{C})$

$SL(3,\mathbb{R})$: $\theta\alpha_i = -\alpha_i$

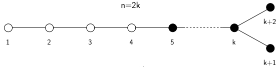
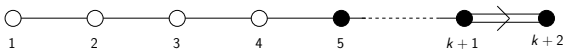
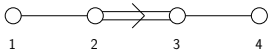
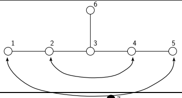
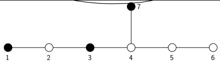
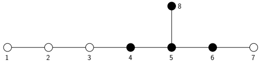
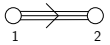
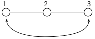
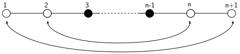
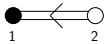
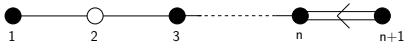


$SU(3)$: $\theta\alpha_i = \alpha_i$

$SU(2,1)$: $\theta\alpha_1 = -\alpha_2$



real form	generators	compact	non-compact
$\mathfrak{sl}(3,\mathbb{R})$	Cartan		H_{α_1} H_{α_2}
	root generators	$E_{\alpha_1}-E_{-\alpha_1}$ $E_{\alpha_2}-E_{-\alpha_2}$ $E_{\alpha_1+\alpha_2}-E_{-\alpha_1-\alpha_2}$	$E_{\alpha_1}+E_{-\alpha_1}$ $E_{\alpha_2}+E_{-\alpha_2}$ $E_{\alpha_1+\alpha_2}+E_{-\alpha_1-\alpha_2}$
$\mathfrak{su}(3)$	Cartan	iH_{α_1} iH_{α_2}	
	root generators	$E_{\alpha_1}-E_{-\alpha_1}$ $i(E_{\alpha_1}+E_{-\alpha_1})$ $E_{\alpha_2}-E_{-\alpha_2}$ $i(E_{\alpha_2}+E_{-\alpha_2})$ $E_{\alpha_1+\alpha_2}-E_{-\alpha_1-\alpha_2}$ $i(E_{\alpha_1+\alpha_2}+E_{-\alpha_1-\alpha_2})$	
$\mathfrak{su}(2,1)$	Cartan	$i(H_{\alpha_1}-H_{\alpha_2})$	$H_{\alpha_1}+H_{\alpha_2}$
	root generators	$E_{\alpha_1}+E_{\alpha_2}-E_{-\alpha_1}-E_{-\alpha_2}$ $i(E_{\alpha_1}-E_{\alpha_2}+E_{-\alpha_1}-E_{-\alpha_2})$ $i(E_{\alpha_1+\alpha_2}+E_{-\alpha_1-\alpha_2})$	$E_{\alpha_1}+E_{\alpha_2}+E_{-\alpha_1}+E_{-\alpha_2}$ $i(E_{\alpha_1}-E_{\alpha_2}-E_{-\alpha_1}+E_{-\alpha_2})$ $i(E_{\alpha_1+\alpha_2}-E_{-\alpha_1-\alpha_2})$

G/H	Tits-Satake diagram of G	Cartan involution
$\frac{SO(4,n)}{SO(4)\times SO(n)}$	n=2k  $n=2k+1$ 	$\theta(\alpha_1)=-\alpha_1$ $\theta(\alpha_2)=-\alpha_2$ $\theta(\alpha_3)=-\alpha_3$ $\theta(\alpha_4)=-\alpha_4-2\sum_{i=5}^{k}\beta_i$ $-2^{n \bmod 2}(\beta_{k+1}+\beta_{k+2})$ $\theta(\beta_i)=\beta_i \quad i=5,\ldots,k+2$
$\frac{F_{4(4)}}{USp(6)\times SU(2)}$		$\theta(\alpha_i)=-\alpha_i \quad i=1,2,3,4$
$\frac{E_{6(2)}}{SU(6)\times SU(2)}$		$\theta(\alpha_1)=-\alpha_5$ $\theta(\alpha_2)=-\alpha_4$ $\theta(\alpha_3)=-\alpha_3$ $\theta(\alpha_4)=-\alpha_2$ $\theta(\alpha_5)=-\alpha_1$ $\theta(\alpha_6)=-\alpha_6$
$\frac{E_{7(-5)}}{SO(12)\times SU(2)}$		$\theta(\alpha_2)=-\alpha_2-\beta_1-\beta_3$ $\theta(\alpha_4)=-\alpha_4-\beta_3-\beta_7$ $\theta(\alpha_5)=-\alpha_5$ $\theta(\alpha_6)=-\alpha_6$ $\theta(\beta_i)=\beta_i \quad i=1,3,7$
$\frac{E_{8(-24)}}{E_7\times SU(2)}$		$\theta(\alpha_1)=-\alpha_1$ $\theta(\alpha_2)=-\alpha_2$ $\theta(\alpha_3)=-\alpha_3-2\beta_4-2\beta_5-\beta_6-\beta_8$ $\theta(\alpha_7)=-\alpha_7-\beta_4-2\beta_5-2\beta_6-\beta_8$ $\theta(\beta_i)=\beta_i \quad i=4,5,6,8$
$\frac{G_{2(2)}}{SO(4)}$		$\theta(\alpha_i)=-\alpha_i \quad i=1,2$
$\frac{SU(n,2)}{SU(n)\times SU(2)\times U(1)}$	$n=2$ 	$\theta(\alpha_1)=-\alpha_3$ $\theta(\alpha_2)=-\alpha_2$ $\theta(\alpha_3)=-\alpha_1$
	$n>2$ 	$\theta(\alpha_1)=-\alpha_{n+1}$ $\theta(\alpha_2)=-\alpha_n-\sum_{i=3}^{n-1}\beta_i$ $\theta(\beta_i)=\beta_i \quad i=3,..,n-1$ $\theta(\alpha_n)=-\alpha_2-\sum_{i=3}^{n-1}\beta_i$ $\theta(\alpha_{n+1})=-\alpha_1$
$\frac{USp(2n,2)}{USp(2n)\times USp(2)}$	$n=1$ 	$\theta(\beta_1)=\beta_1$ $\theta(\alpha_2)=-\alpha_2-2\beta_1$
	$n>1$ 	$\theta(\beta_i)=\beta_i \quad i\neq 2$ $\theta(\alpha_2)=-\alpha_2-\beta_1$ $-2\sum_{i=3}^{n}\beta_i-\beta_{n+1}$

Branes in $\mathcal{N} = 2$ sugras

Project roots under $P = \frac{1}{2}(1 - \theta)$

Projected roots define the restricted-root algebra

Take the highest weight of the representation of the field

If highest weight is not fixed under P: no branes

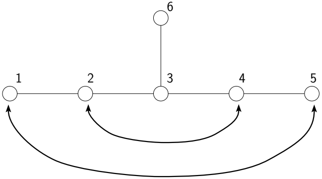
If it is fixed: write it as highest weight of a representation of the restricted-root algebra

Count the longest weights of that representation: this is the number of real longest weights, i.e. the number of branes

dim.	type of brane	repr.	highest weight	# of branes
$D=3$	0-brane	**52** (1 0 0 0)	$2\alpha_1+3\alpha_2+4\alpha_3+2\alpha_4$	24
	1-brane	**324** (0 0 0 2)	$2\alpha_1+4\alpha_2+6\alpha_3+4\alpha_4$	24
$D=4$	0-brane	**14** (1 0 0)	$\frac{3}{2}\alpha_2+2\alpha_3+\alpha_4$	8
	1-brane	**21** (0 0 2)	$\alpha_2+2\alpha_3+2\alpha_4$	6
$D=5$	0-brane	**6** (2 0)	$\frac{4}{3}\alpha_3+\frac{2}{3}\alpha_4$	3
	1-brane	$\overline{\mathbf{6}}$ (0 2)	$\frac{2}{3}\alpha_3+\frac{4}{3}\alpha_4$	3
$D=6$	1-brane	**3** (2)	α_4	2

Branes in $\mathcal{N} = 2$ sugras

Consider $E_{6(2)}$:



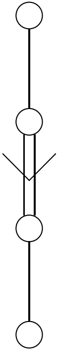
Restricted-root algebra:

$$P\alpha_6 = \alpha_6$$

$$P\alpha_3 = \alpha_3$$

$$P\alpha_2 = \frac{1}{2}(\alpha_2 + \alpha_4)$$

$$P\alpha_1 = \frac{1}{2}(\alpha_1 + \alpha_5)$$



dim.	brane	repr.	highest weight	restricted repr.	#
$D=3$	0-brane	$\mathbf{78}$ (0 0 0 0 0 1)	$\alpha_1+2\alpha_2+3\alpha_3+2\alpha_4+\alpha_5+2\alpha_6$	$\mathbf{52}$ (1 0 0 0)	24
	1-brane	$\mathbf{650}$ (1 0 0 0 1 0)	$2\alpha_1+3\alpha_2+4\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6$	$\mathbf{324}$ (0 0 0 2)	24
	2-brane	$\mathbf{5824}$ (1 1 0 0 0 0)	$3\alpha_1+5\alpha_2+6\alpha_3+4\alpha_4+2\alpha_5+3\alpha_6$	-	-
		$\overline{\mathbf{5824}}$ (0 0 0 1 1 0)	$2\alpha_1+4\alpha_2+6\alpha_3+5\alpha_4+3\alpha_5+3\alpha_6$	-	-
$D=4$	0-brane	$\mathbf{20}$ (0 0 1 0 0)	$\frac{1}{2}\alpha_1+\alpha_2+\frac{3}{2}\alpha_3+\alpha_4+\frac{1}{2}\alpha_5$	$\mathbf{14}$ (1 0 0)	8
	1-brane	$\mathbf{35}$ (1 0 0 0 1)	$\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5$	$\mathbf{21}$ (0 0 2)	6
	2-brane	$\mathbf{70}$ (1 1 0 0 0)	$\frac{3}{2}\alpha_1+2\alpha_2+\frac{3}{2}\alpha_3+\alpha_4+\frac{1}{2}\alpha_5$	-	-
		$\overline{\mathbf{70}}$ (0 0 0 1 1)	$\frac{1}{2}\alpha_1+\alpha_2+\frac{3}{2}\alpha_3+2\alpha_4+\frac{3}{2}\alpha_5$	-	-
	3-brane	$\mathbf{280}$ (2 0 0 1 0)	$2\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4+\alpha_5$	-	-
		$\overline{\mathbf{280}}$ (0 1 0 0 2)	$\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5$	-	-
$D=5$	0-brane	$(\overline{\mathbf{3}},\mathbf{3})$ (0 1 1 0)	$\frac{1}{3}\alpha_1+\frac{2}{3}\alpha_2+\frac{2}{3}\alpha_4+\frac{1}{3}\alpha_5$	$\mathbf{6}$ (2 0)	3
	1-brane	$(\mathbf{3},\overline{\mathbf{3}})$ (1 0 0 1)	$\frac{2}{3}\alpha_1+\frac{1}{3}\alpha_2+\frac{1}{3}\alpha_4+\frac{2}{3}\alpha_5$	$\overline{\mathbf{6}}$ (0 2)	3
	2-brane	$(\mathbf{8},\mathbf{1})$ (1 1 0 0)	$\alpha_1+\alpha_2$	-	-
		$(\mathbf{1},\mathbf{8})$ (0 0 1 1)	$\alpha_4+\alpha_5$	-	-
	3-brane	$(\mathbf{6},\mathbf{3})$ (2 0 1 0)	$\frac{4}{3}\alpha_1+\frac{2}{3}\alpha_2+\frac{2}{3}\alpha_4+\frac{1}{3}\alpha_5$	-	-
		$(\overline{\mathbf{3}},\overline{\mathbf{6}})$ (0 1 0 2)	$\frac{1}{3}\alpha_1+\frac{2}{3}\alpha_2+\frac{2}{3}\alpha_4+\frac{4}{3}\alpha_5$	-	-
	4-brane	$(\mathbf{15},\overline{\mathbf{3}})$ (2 1 0 1)	$\frac{5}{3}\alpha_1+\frac{4}{3}\alpha_2+\frac{1}{3}\alpha_4+\frac{2}{3}\alpha_5$	-	-
		$(\mathbf{3},\overline{\mathbf{15}})$ (1 0 1 2)	$\frac{2}{3}\alpha_1+\frac{1}{3}\alpha_2+\frac{4}{3}\alpha_4+\frac{5}{3}\alpha_5$	-	-
$D=6$	0-brane	$(\mathbf{2},\mathbf{1})$ (1 0)	$\frac{1}{2}\alpha_1$	-	-
		$(\mathbf{1},\mathbf{2})$ (0 1)	$\frac{1}{2}\alpha_5$	-	-
	1-brane	$(\mathbf{2},\mathbf{2})$ (1 1)	$\frac{1}{2}\alpha_1+\frac{1}{2}\alpha_5$	$\mathbf{3}$ (2)	2
	2-brane	$(\mathbf{2},\mathbf{1})$ (1 0)	$\frac{1}{2}\alpha_1$	-	-
		$(\mathbf{1},\mathbf{2})$ (0 1)	$\frac{1}{2}\alpha_5$	-	-
	3-brane	$(\mathbf{1},\mathbf{3})$ (0 2)	α_5	-	-
		$(\mathbf{3},\mathbf{1})$ (2 0)	α_1	-	-
	4-brane	$(\mathbf{2},\mathbf{3})$ (1 2)	$\frac{1}{2}\alpha_1+\alpha_5$	-	-
		$(\mathbf{3},\mathbf{2})$ (2 1)	$\alpha_1+\frac{1}{2}\alpha_5$	-	-
	5-brane	$(\mathbf{4},\mathbf{2})$ (3 1)	$\frac{3}{2}\alpha_1+\frac{1}{2}\alpha_5$	-	-
		$(\mathbf{2},\mathbf{4})$ (1 3)	$\frac{1}{2}\alpha_1+\frac{3}{2}\alpha_5$	-	-

dim.	brane	repr.	highest weight	restricted repr.	#
$D=3$	0-brane	**133** (0 0 0 0 0 1 0)	$\beta_1+2\alpha_2+3\beta_3+4\alpha_4+3\alpha_5+2\alpha_6+2\beta_7$	**52** (1 0 0 0)	24
	1-brane	**1539** (0 1 0 0 0 0 0)	$2\beta_1+4\alpha_2+5\beta_3+6\alpha_4+4\alpha_5+2\alpha_6+3\beta_7$	**324** (0 0 0 2)	24
	2-brane	**40755** (1 0 0 0 0 0 1)	$3\beta_1+5\alpha_2+7\beta_3+9\alpha_4+6\alpha_5+3\alpha_6+5\beta_7$	-	-
$D=4$	0-brane	**32** (0 0 0 0 1 0)	$\frac{1}{2}\beta_1+\alpha_2+\frac{3}{2}\beta_3+2\alpha_4+\frac{3}{2}\alpha_5+\beta_7$	**14** (1 0 0)	8
	1-brane	**66** (0 1 0 0 0 0)	$\beta_1+2\alpha_2+2\beta_3+2\alpha_4+\alpha_5+\beta_7$	**21** (0 0 2)	6
	2-brane	**352** (1 0 0 0 0 1)	$\frac{3}{2}\beta_1+2\alpha_2+\frac{5}{2}\beta_3+3\alpha_4+\frac{3}{2}\alpha_5+2\beta_7$	-	-
	3-brane	**462** (0 0 0 0 0 2)	$\beta_1+2\alpha_2+3\beta_3+4\alpha_4+2\alpha_5+3\beta_7$	-	-
		2079 (1 0 1 0 0 0)	$2\beta_1+3\alpha_2+4\beta_3+4\alpha_4+2\alpha_5+2\beta_7$	-	-
$D=5$	0-brane	$\overline{\mathbf{15}}$ (0 0 0 1 0)	$\frac{1}{3}\beta_1+\frac{2}{3}\alpha_2+\beta_3+\frac{4}{3}\alpha_4+\frac{2}{3}\beta_7$	**6** (2 0)	3
	1-brane	**15** (0 1 0 0 0)	$\frac{2}{3}\beta_1+\frac{4}{3}\alpha_2+\beta_3+\frac{2}{3}\alpha_4+\frac{1}{3}\beta_7$	$\overline{\mathbf{6}}$ (0 2)	3
	2-brane	**35** (1 0 0 0 1)	$\beta_1+\alpha_2+\beta_3+\alpha_4+\beta_7$	-	-
	3-brane	$\overline{\mathbf{21}}$ (0 0 0 0 2)	$\frac{1}{3}\beta_1+\frac{2}{3}\alpha_2+\beta_3+\frac{4}{3}\alpha_4+\frac{5}{3}\beta_7$	-	-
		105 (1 0 1 0 0)	$\frac{4}{3}\beta_1+\frac{5}{3}\alpha_2+2\beta_3+\frac{4}{3}\alpha_4+\frac{2}{3}\beta_7$	-	-
	4-brane	**384** (1 1 0 0 1)	$\frac{5}{3}\beta_1+\frac{7}{3}\alpha_2+2\beta_3+\frac{5}{3}\alpha_4+\frac{4}{3}\beta_7$	-	-
$D=6$	0-brane	$(\overline{\mathbf{4}},\mathbf{2})$ (0 0 1 1)	$\frac{1}{4}\beta_1+\frac{1}{2}\alpha_2+\frac{3}{4}\beta_3+\frac{1}{2}\beta_7$	-	-
	1-brane	$(\mathbf{6},\mathbf{1})$ (0 1 0 0)	$\frac{1}{2}\beta_1+\alpha_2+\frac{1}{2}\beta_3$	**3** (2)	2
	2-brane	$(\mathbf{4},\mathbf{2})$ (1 0 0 1)	$\frac{3}{4}\beta_1+\frac{1}{2}\alpha_2+\frac{1}{4}\beta_3+\frac{1}{2}\beta_7$	-	-
	3-brane	$(\mathbf{1},\mathbf{3})$ (0 0 0 2)	β_7	-	-
		$(\mathbf{15},\mathbf{1})$ (1 0 1 0)	$\beta_1+\alpha_2+\beta_3$	-	-
	4-brane	$(\mathbf{20},\mathbf{2})$ (1 1 0 1)	$\frac{5}{4}\beta_1+\frac{3}{2}\alpha_2+\frac{3}{4}\beta_3+\frac{1}{2}\beta_7$	-	-
	5-brane	$(\mathbf{10},\mathbf{3})$ (2 0 0 2)	$\frac{3}{2}\beta_1+\alpha_2+\frac{1}{2}\beta_3+\beta_7$	-	-
		$(\mathbf{64},\mathbf{1})$ (1 1 1 0)	$\frac{3}{2}\beta_1+2\alpha_2+\frac{3}{2}\beta_3$	-	-

dim.	brane	repr.	highest weight	restricted repr.	#
$D=3$	0-brane	**248** (1 0 0 0 0 0 0 0)	$2\alpha_1+3\alpha_2+4\alpha_3+5\beta_4+6\beta_5+4\beta_6+2\alpha_7+3\beta_8$	**52** (1 0 0 0)	24
	1-brane	**3875** (0 0 0 0 0 0 1 0)	$2\alpha_1+4\alpha_2+6\alpha_3+8\beta_4+10\beta_5+7\beta_6+4\alpha_7+5\beta_8$	**324** (0 0 0 2)	24
	2-brane	**147250** (0 0 0 0 0 0 0 1)	$3\alpha_1+6\alpha_2+9\alpha_3+12\beta_4+15\beta_5+10\beta_6+5\alpha_7+8\beta_8$	-	-
$D=4$	0-brane	**56** (1 0 0 0 0 0 0)	$\frac{3}{2}\alpha_2+2\alpha_3+\frac{5}{2}\beta_4+3\beta_5+2\beta_6+\alpha_7+\frac{3}{2}\beta_8$	**14** (1 0 0)	8
	1-brane	**133** (0 0 0 0 0 1 0)	$\alpha_2+2\alpha_3+3\beta_4+4\beta_5+3\beta_6+2\alpha_7+2\beta_8$	**21** (0 0 2)	6
	2-brane	**912** (0 0 0 0 0 0 1)	$\frac{3}{2}\alpha_2+3\alpha_3+\frac{9}{2}\beta_4+6\beta_5+4\beta_6+2\alpha_7+\frac{7}{2}\beta_8$	-	-
	3-brane	**8645** (0 0 0 0 1 0 0)	$2\alpha_2+4\alpha_3+6\beta_4+8\beta_5+6\beta_6+3\alpha_7+4\beta_8$	-	-
$D=5$	0-brane	**27** (1 0 0 0 0 0)	$\frac{4}{3}\alpha_3+\frac{5}{3}\beta_4+2\beta_5+\frac{4}{3}\beta_6+\frac{2}{3}\alpha_7+\beta_8$	**6** (2 0)	3
	1-brane	$\overline{\mathbf{27}}$ (0 0 0 0 1 0)	$\frac{2}{3}\alpha_3+\frac{4}{3}\beta_4+2\beta_5+\frac{5}{3}\beta_6+\frac{4}{3}\alpha_7+\beta_8$	$\overline{\mathbf{6}}$ (0 2)	3
	2-brane	**78** (0 0 0 0 0 1)	$\alpha_3+2\beta_4+3\beta_5+2\beta_6+\alpha_7+2\beta_8$	-	-
	3-brane	**351** (0 0 0 1 0 0)	$\frac{4}{3}\alpha_3+\frac{8}{3}\beta_4+4\beta_5+\frac{10}{3}\beta_6+\frac{5}{3}\alpha_7+2\beta_8$	-	-
	4-brane	**1728** (0 0 0 0 1 1)	$\frac{5}{3}\alpha_3+\frac{10}{3}\beta_4+5\beta_5+\frac{11}{3}\beta_6+\frac{7}{3}\alpha_7+3\beta_8$	-	-
$D=6$	0-brane	**16** (1 0 0 0 0)	$\frac{5}{4}\beta_4+\frac{3}{2}\beta_5+\beta_6+\frac{1}{2}\alpha_7+\frac{3}{4}\beta_8$	-	-
	1-brane	**10** (0 0 0 1 0)	$\frac{1}{2}\beta_4+\beta_5+\beta_6+\alpha_7+\frac{1}{2}\beta_8$	**3** (2)	2
	2-brane	$\overline{\mathbf{16}}$ (0 0 0 0 1)	$\frac{3}{4}\beta_4+\frac{3}{2}\beta_5+\beta_6+\frac{1}{2}\alpha_7+\frac{5}{4}\beta_8$	-	-
	3-brane	**45** (0 0 1 0 0)	$\beta_4+2\beta_5+2\beta_6+\alpha_7+\beta_8$	-	-
	4-brane	**144** (0 0 0 1 1)	$\frac{5}{4}\beta_4+\frac{5}{2}\beta_5+2\beta_6+\frac{3}{2}\alpha_7+\frac{7}{4}\beta_8$	-	-
	5-brane	**320** (0 0 1 1 0)	$\frac{3}{2}\beta_4+3\beta_5+3\beta_6+2\alpha_7+\frac{3}{2}\beta_8$	-	-

Branes in $\mathcal{N}=2$ sugras

Hypermultiplet sector: we consider in particular the $\mathrm{SO}(4, n_H)$ case

We obtain the brane content (including domain walls and space-filling branes) by requiring that the gauge algebra is a truncation of the gauge algebra of the half-maximal theory

e.g. in three dimensions: $\mathrm{SO}(8, n+m) \supset \mathrm{SO}(4, n) \times \mathrm{SO}(4, m)$

By analogy with the previous analysis, we expect the number of branes to be the same as for the $\mathrm{F}_{4(4)}$, $\mathrm{E}_{6(2)}$, $\mathrm{E}_{7(-5)}$ and $\mathrm{E}_{8(-24)}$

dim.	type of brane	field	# of branes
$D=3$	0-brane	A_{1,M_1M_2}	24
	1-brane	$A_{2,MN}$	8
		$A_{2,M_1\dots M_4}$	16
		$A_{2,A_1A_2M_1M_2}$	576
	2-brane	$A_{3,MN_1N_2N_3A_1A_2}$	2304
		$A_{3,AB_1B_2B_3M_1M_2}$	2304
$D=4$	1-brane	A_{2,M_1M_2}	24
	2-brane	A_{3,M_1M_2Aa}	192
	3-brane	$A_{4,M_1M_2A_1A_2ab}$	192
		$A_{4,MN_1N_2N_3}$	96
		A_{4,ABM_1M_2}	96
$D=5$	2-brane	A_{3,M_1M_2} $(\alpha=-2)$	24
	3-brane	A_{4,AM_1M_2} $(\alpha=-2)$	48
		A_{4,M_1M_2} $(\alpha=-4)$	24
$D=6$	3-brane	A_{4,M_1M_2} $(\alpha=-2)$	24

Conclusions

- Our analysis reveals a universal structure underlying the various theories with eight supercharges that can be uplifted to six dimensions
- Low-energy of heterotic theory: wrapping rules are satisfied as in the maximal and half-maximal theories
- Central charges and degeneracies
- We find vector 3-branes in four dimensions
- Many of the branes we find are exotic!