D-brane in Poisson manifold and 1-vector gauge potential

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based on

T. Asakawa, H. Muraki, S.W. arXiv:1402.0942 ,

T. Asakawa, S. Sasa, S.W. JHEP 1210(2012)064,

Contents

1. Introduction

T-duality is one of the characteristic symmetry of the string, and distinguishes it from the theory of particles.

Thus to understand the stringy geometry, we need a framework where such a symmetry is manifest.

We take “Generalized Geometry” for this purpose.

alternative: Double Field Theory,

There appear various EXOTC geometries as string background such as

1. For the open string with B-field background：

The NCG in effective theory of D-brane

2. For the closed string with various fluxes such as non-geometric flux：It seems there appears also NCG, even NAG in the effective theory

We want to understand these structures in a unified way, using Generalized Geometry

- Generalized Geometry is proposed by N. Hitchin:

Generalized Complex Geometry, is used for example, for classifying compactification , CY, Flux …

mainly to analyze closed string sector

However, it is also possible to use it for characterization of D-brane effective theory (Open string sector)

DBI-action, Seiberg-Witten map, effect of H-flux,…

Today we show a result from these attempts:

There is a new type of representation of gauge theory as an effective theory of D-brane. This may give also a hint to understand "non-geometric flux"

2. Generalized Geometry

briefly

- Ge. Geom. is a generalization of Diff. Geom. with the properties

① Over the target space M, consider generalized tangent bundle

$$\mathbb{T}M = TM \oplus T^*M \ \to \ M$$

② Its section is generalized vector（1-vector＋1-form）

$$v + \xi \in \Gamma(\mathbb{T}M), \ v \in \Gamma(TM), \ \xi \in \Gamma(T^*M)$$
$$v + \xi = v^M \partial_M + \xi_M dx^M$$

③ canonical inner prodduct

$$\langle u+\xi, v+\eta \rangle = \frac{1}{2}(\imath_u \eta + \imath_v \xi) = \frac{1}{2} \begin{pmatrix} u \\ \xi \end{pmatrix}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} v \\ \eta \end{pmatrix},$$

Symmetry: $O(D,D)$

④ anchor map $\pi : \Gamma(\mathbb{T}M) \to \Gamma(TM)$, $\pi(v+\xi) = v$

It generalizes the following properties of tangent bundle:

- Closed under Lie bracket

  Tangent vectors $\Gamma(TM)$ is Lie algebra
- Anchor map (trivial): $a \; : \; E = TM \; \to \; TM$

$$[u, fv] = f[u, v] + (a(u)f)v$$

which are the properties of a Lie algebroid

For generalized tangent bundle

- There is a Dorfman bracket, instead of Lie bracket

$$[u+\xi, v+\eta] = [u, v] + \mathcal{L}_u\eta - \imath_v d\xi, \qquad \Gamma(\mathbb{T}M)$$

- Anchor map $\pi : \Gamma(\mathbb{T}M) \to \Gamma(TM), \; \pi(v+\xi) = v$

We get, as a generalization of Lie algebroid, a Courant algebroid

Symmetry of $\mathbb{T}M$ (Courant algebroid)

- Diffeomorphism

  Diff(M): For a diffeo. $f : M \to M$,

$$u + \xi \mapsto f_*(u) + f^{*-1}(\xi),$$

- B-transformation: For $B \in \Omega^2_{\text{closed}}(M)$,

$$e^B(u+\xi) \mapsto u + \xi + \imath_u B,$$

Shift of 1-form

- For generic B: H-twisted Courant algebroid

They are a generalization of the symmetry of TM (diffeo)

Generalized diffeomorphism

Summary

Differential geometry		Generalized Geometry
TM		$TM \oplus T^*M$
$v = v^M \partial_M$		$v + \xi = v^M \partial_M + \xi_M dx^M$
Lie bracket $[\cdot, \cdot]$	$\Rightarrow$	Dorfman bracket $[\cdot, \cdot]$
Lie algebroid		Courant algebroid
symmetry $\mathsf{Diff}(M)$		$\mathsf{Diff}(M) \ltimes \Omega^2_{\mathsf{closed}}(M)$
generator Lie derivative		Generalized Lie derivative
$\mathcal{L}_v$		$\mathcal{L}_{v+\xi}$

3. D-brane as a Dirac structure

Before we start to discuss the effective theory of D-brane, let us first explain how we can characterize the D-brane in Generalized Geometry.

It is Dirac structure that gives a geometrical characterization of D-brane

D-brane as a Dirac structure

- Standard description of D-brane:

D-brane:

- Embedding $\varphi : \Sigma \ni \sigma^a \hookrightarrow x^M(\sigma) \in M$

Fluctuations is

- Scalar fields Φ^i (in static gauge): transverse displacements
- Gauge fields A_a:

In static gauge: they are function of $x^a = \sigma^a$

Dirac structure

What is the Dirac structure:
Dirac structure is a subbundle L with properties:

- Isotropic: $\forall X, Y \in \Gamma(L) \ \ \langle X, Y \rangle = 0$
- Involutive: sections of L, i.e. generalized vectors are closed under Dorfman bracket

- Example:

  $L = \mathrm{Span}\{\partial_0, \ldots, \partial_p, dx^{p+1} \cdots dx^{D-1}\}$ ~flat Dp-brane

  i) generalized vector of L $X = u^a \partial_a + \xi_i dx^i \in L$

  ii) generalized 1-form (connection) of dual of L

$$\alpha = \Phi^i \partial_i + A_a dx^a \in L^*$$

(this represents just a fluctuation)

- Dirac structure : maximally isotropic, involutive

$$\langle X, Y \rangle = 0$$

1. Isotropic

$$\begin{aligned} [u+\xi, v+\eta] &= [u,v] + \mathcal{L}_u\eta - \imath_v d\xi \\ &= [u,v] + \mathcal{L}_u\eta - \mathcal{L}_v\xi - d\langle u+\xi, v+\eta\rangle \end{aligned}$$

Last term vanishes and Dorfman becomes Lie bracket

2. Maximal subbundle dimension = D
3. Involutive

generalized vectors closed under Dorf. bracket

Courant algebroid becomes Lie algebroid on Dirac structure. (Lie (anti) + anchor+Jacobi)

generalization of TM

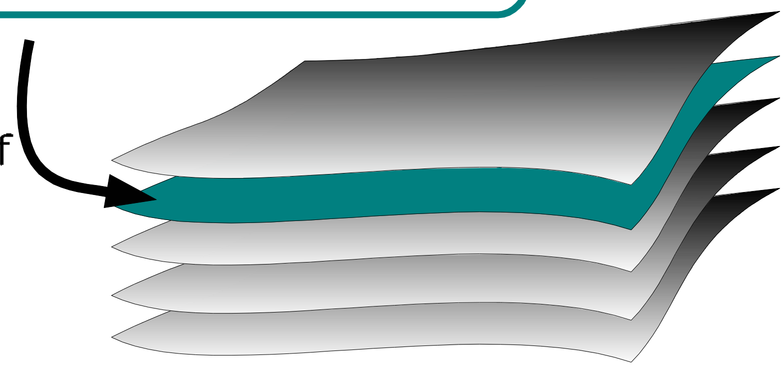
Geometrical image of D-brane

Static gauge: $\Phi^i(x^a)$ x^i independent

a D-brane = a leaf

Lie algebroid defines a foliation

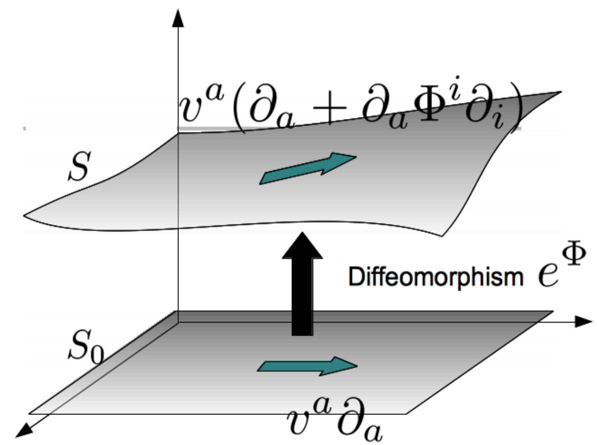
Mathematically, a Lie algebroid (Lie bracket of $v \in \Gamma(TS)$) defines a foliation.



Identify a leaf in foliation as a D-brane

Scalar field defines a deformation of a leaf by diffeo.

We may consider deformation of foliation in static gauge.



Fluctuation of D-brane in Ge. Ge.

We identify a D-brane with a sub-bundle defined by a Dirac structure

Furthermore, D-brane with fluctuation is also a Dirac structure

It means that the fluctuation of D-brane

is a deformation of Dirac structure

Subbundle $L = \mathrm{span}\{\partial_a, dx^i\} \subset \mathbb{T}M$

corresponds to D-brane $x^i = const$

Section is $V_L = v^a(x)\partial_a + \xi_i(x)dx^i \quad \in L.$

Fluctuation is identified with an element of a dual Dirac structure:

Generalized 1-form $\alpha = \Phi + A = \Phi^i\partial_i + A_a dx^a \in L^*$

Fluctuation of D-brane in Ge. Ge.

a generalized 1-form generates a deformation of Dirac structure L

$$\alpha = \Phi + A = \Phi^i \partial_i + A_a dx^a \in L^*$$

Fluctuation $\Phi^i \partial_i$ defines diffeo.,
in general α defines a generalized diffeo.
and corresponding deformed Dirac structure is

$$L_{\mathcal{F}} = e^{-\mathcal{L}_{\Phi+A}} L \subset \mathbb{T}M,$$

In static gauge, it is easy to show : fields are function of x^a

$$L_{\mathcal{F}} \ni V + \mathcal{F}(V) = v^a(x)(\partial_a + \partial_a \Phi^i \partial_i + F_{ab} dx^b) + \xi_i(x)(dx^i - \partial_a \Phi^i dx^a),$$

$$\mathcal{F} = F_{ab} dx^a \wedge dx^b + \partial_a \Phi^i dx^a \wedge \partial_i \quad \in \Gamma(\wedge^2 L^*),$$

And this "field strength" is defined by using the differential of Lie algebroid as $\mathcal{F} = d_L \alpha = d_L (A + \Phi)$

it is a generalized 2-form

Definition:

Note for explanation

The exterior derivative of Lie algebroid

For a given Lie algebroid $(A, \rho, [\cdot,\cdot]_A)$, an associated A-differential form $(\Gamma(\wedge^\bullet A^*), \wedge, d_A)$ can be defined. The exterior differential d_A is defined by

$$
\begin{aligned}
& d_A\omega(X_1, \cdots, X_{k+1}) \\
= & \sum_i (-)^{i+1} \rho(X_i) \cdot \omega(X_1, \cdots, \widehat{X}_i, \cdots, X_{k+1}) \\
& + \sum_{i<j} (-)^{i+j} \omega([X_i, X_j]_A, X_1, \cdots, \widehat{X}_i, \cdots, \widehat{X}_j, \cdots, X_{k+1}).
\end{aligned}
$$

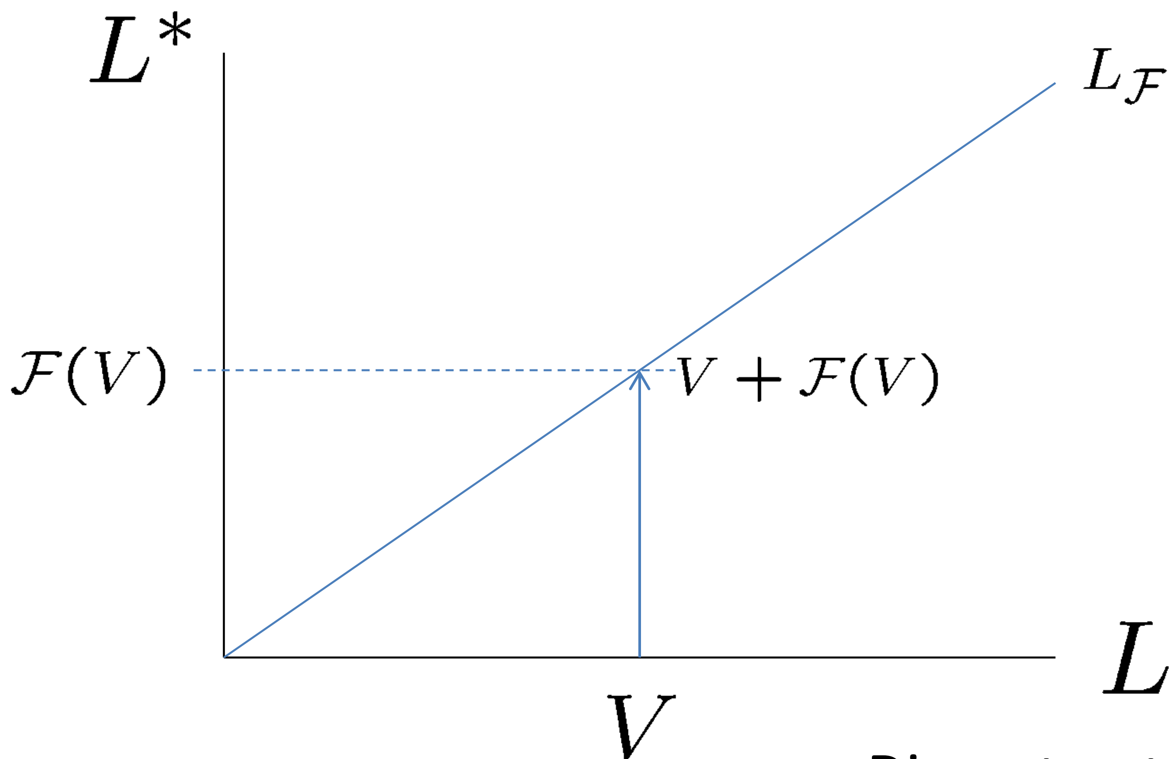
d_A is a graded derivation on a wedge product, and $d_A^2 = 0$.

Graphical representation of Dirac structure

Deformed Dirac structure can be represented by a graph of the map from L to L*

$$L_{\mathcal{F}} \ni V + \mathcal{F}(V) = v^a(x)(\partial_a + \partial_a\Phi^i\partial_i + F_{ab}dx^b) + \xi_i(x)(dx^i - \partial_a\Phi^i dx^a),$$

Dp-brane with fluctuation



Dirac structure of Dp-brane

Symmetry of the Dirac structure

Generalized diffeo.$\mathcal{L}_{\epsilon+\lambda}$, Diffeo.$\ltimes$ B-field gauge transformation:

$$\epsilon = \epsilon_{||} + \epsilon_{\perp} = \epsilon^M \partial_M = \epsilon^a \partial_a + \epsilon^i \partial_i$$

$$\Lambda = \Lambda_{\perp} + \Lambda_{||} = \Lambda_M dx^M = \Lambda_i dx^i + \Lambda_a dx^a.$$

Note that $\epsilon_{||} + \Lambda_{\perp} \in L$ and $\epsilon_{\perp} + \Lambda_{||} \in L^*$.

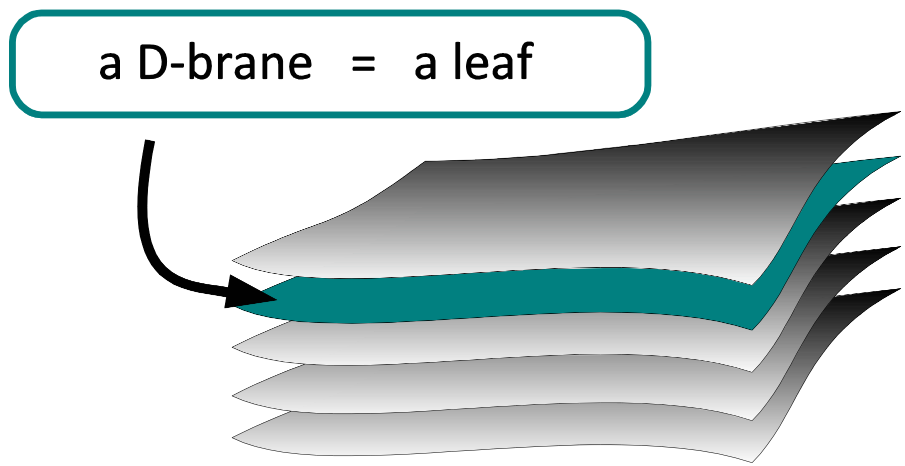
Symmetry: (in static gauge)

1. Generalized diffeo which preserve foliation

$$\partial_b \epsilon^i = 0 \ , \quad \partial_{[a}\Lambda_{b]} = 0$$

2. Generalized diffeo which preserve leaf:

$$L\text{-}Diff \subset F\text{-}Diff$$



Low energy theorem

F-Diff is total symmetry of D-brane keeping the foliation strucuture. A D-brane chooses a leaf and induces SSB

F-Diff $\Rightarrow$ L-Diff

broken symmetry direction becomes NG boson and generalized diffeo. symmetry is realized nonlinearly in DBI action. This is a generalization of the low energy theorem based on Lie algebra to Courant/Lie algebroid

Note: Inportant result here is that we could identify a vector field as a kind of NG boson. We discuss this later again.

[T.Asakawa,S.Sasa,S.W.] JHEP10(2012)064,

Transf. of broken direction is

$$\delta A_a = \Lambda_a - \epsilon^c F_{ca} + \Lambda_k \partial_a \Phi^k,$$
$$\delta \Phi^i = \epsilon^i - \epsilon^c \partial_c \Phi^i.$$

4. Altenative representation of Fluctuation of D-brane

- Using this formulation we can define a new type of representation of the fluctuation:

  Field strength $\widehat{F}$ is 2-vector

  Potential is 1-vector

  Bianchi identity is Maurer-Cartan type relation

$$d_\theta \widehat{F} + \frac{1}{2}[\widehat{F}, \widehat{F}]_S = 0$$

Gauge symmetry is generated by Hamiltonian vector field

[T.Asakawa,S.Muraki,S.W]

What is the alternative representation of Dirac structure?

From a graph, one can see that there is always another representation of the same Dirac structure

In the following, we only consider the D9-brane

thus $L = TM$

then 2-form $\omega \in \Lambda^2 T^* M$ defines a deformation

$$L_\omega = \{X + \omega(X) | X \in TM\}$$

This becomes Dirac structure if ω is symplectic.

Then the same Dirac structure can be represented by a 2-vector θ as

Dirac structure can be represented by

B-transformation

$$L_\omega = \{ e^\omega(X) = X + \omega(X) \big| X \in TM, \omega \in \wedge^2 T^*M \}.$$

symplectic form $d\omega = 0$

β-transformation

$$L_\theta = \{ e^\theta(\xi) = \xi + \theta(\xi) \big| \xi \in T^*M, \theta \in \wedge^2 TM \}.$$

Poisson bivector $[\theta, \theta]_S = 0$

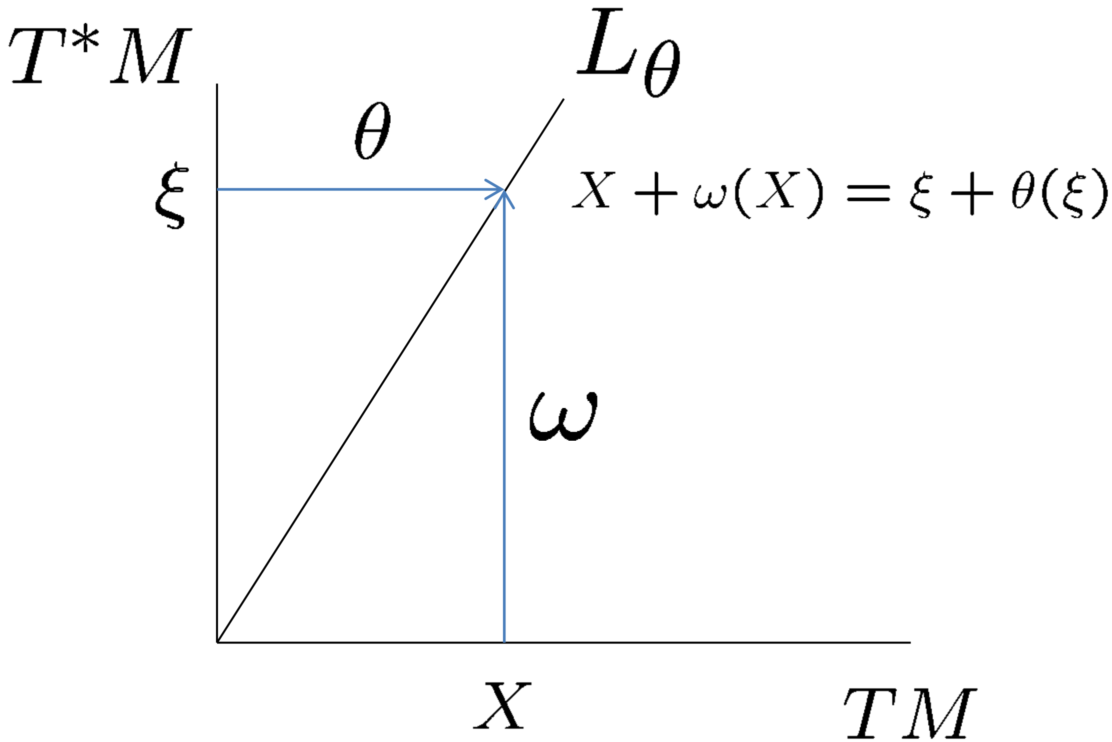
In general, take a pair (L,L*), deformation can be

L by $\mathcal{F} \in \Lambda^2 L^*$ or L* by $\mathcal{F}' \in \Lambda^2 L$

with Maurer-Cartan relation $d_L \mathcal{F} + \frac{1}{2}[\mathcal{F}, \mathcal{F}]_L = 0$

2 ways to represent a Dirac structure:

$$\theta = \frac{1}{2}\theta^{ij}\partial_i \wedge \partial_j$$



$$\xi = \omega(X) = \omega_{ij}X^j dx^i = -i_X\omega$$

$$X = \theta(\xi) = \theta^{ij}\xi_j\partial_i$$

$$\xi = \omega(\theta(\xi)) \qquad \theta = \omega^{-1}$$

- Now we consider the fluctuation of the bound state of D-branes given by a background ω

The fluctuation is given by 1-form A

$$\alpha = A = A_a dx^a \in T^*M$$

The D-brane with fluctuation is also Dirac struc.

$$e^{\tilde{F}}(X + \omega(X)) = X + (\omega + \tilde{F})(X) \ , \ \tilde{F} = dA.$$

This is a natural way to add the fluctuation

for L_ω , since $d(\omega + \tilde{F}) = 0$

$L_{\omega + \tilde{F}}$ becomes Dirac structure again.

This is the standard description of fluctuation.

Defining the Dirac structure L_θ by β-transformation, a natural way to describe a fluctuation is also by β-transformation

The fluctuation is given by 2-vector $\widehat{F}$

$$e^{\widehat{F}}(X+\theta(X)) = X + (\theta+\widehat{F})(X)\ ,\ \ \widehat{F} \in \Lambda^2 L_\theta^*.$$

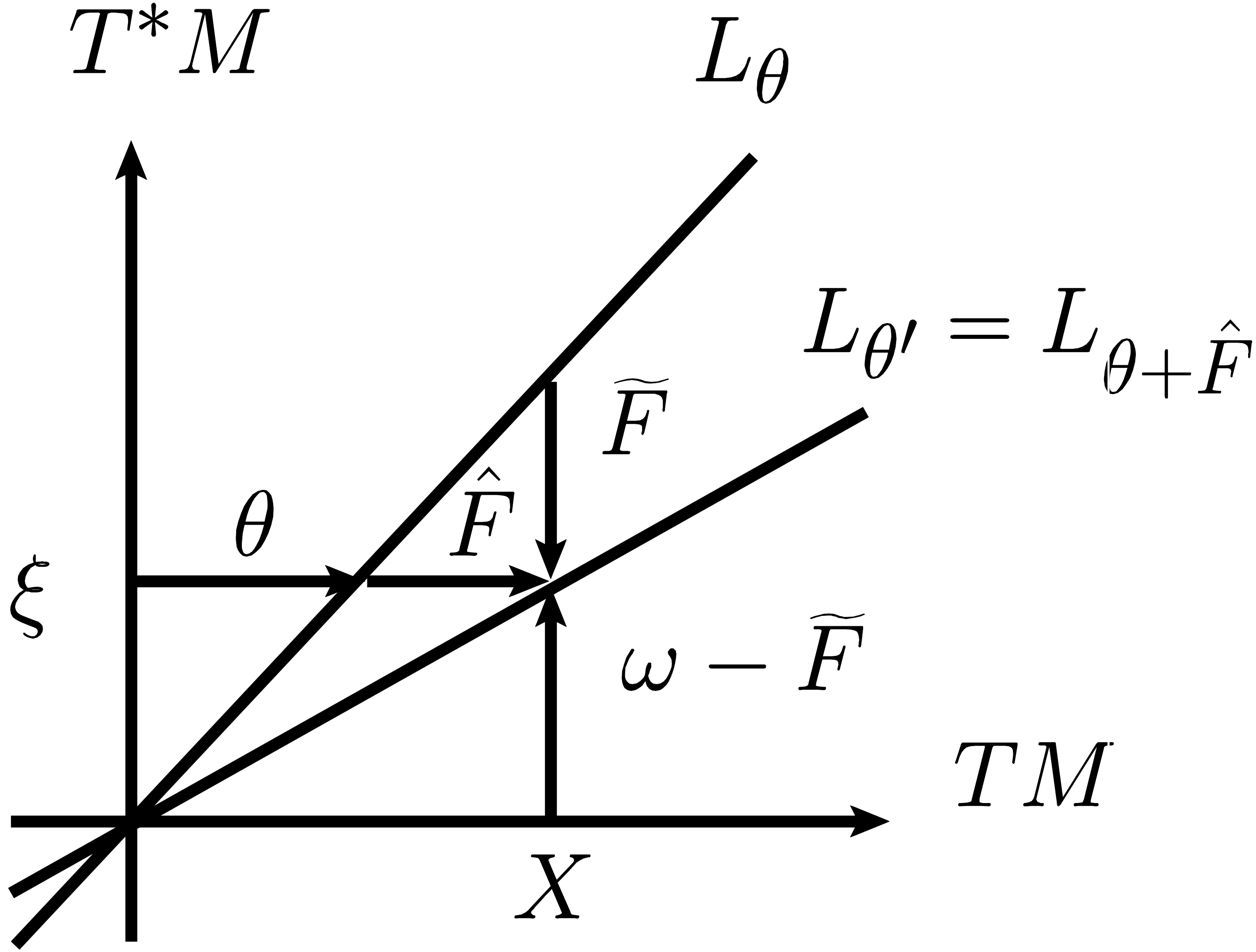
The deformed Dirac structure $L_{\theta+\widehat{F}}$ becomes Dirac structure again if

$$[\theta+\widehat{F}, \theta+\widehat{F}]_S = 0$$

Or equivalently Maurer-Cartan type relation:

$$\begin{aligned} \frac{1}{2}[\theta+\widehat{F},\theta+\widehat{F}]_S &= [\theta,\widehat{F}]_S + \frac{1}{2}[\widehat{F},\widehat{F}]_S \\ &= d_\theta \widehat{F} + \frac{1}{2}[\widehat{F},\widehat{F}]_S = 0 \end{aligned}$$

- Graphically, we can represent the relation as



Therefore, the fluctuation can be represented in two ways, and their equivalence can be written by the condition:

$$\xi + (\theta + \hat{F})(\xi) = X + (\omega + \tilde{F})(X)$$

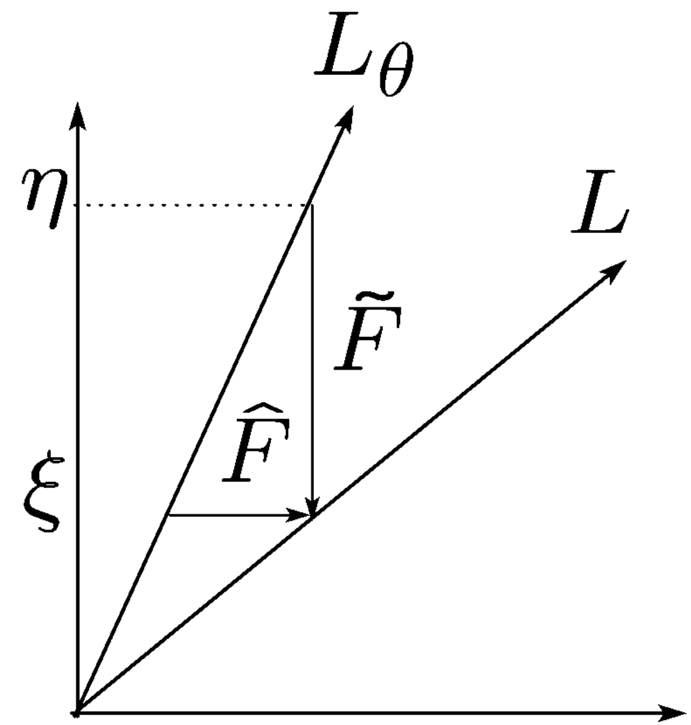
$$\theta + \hat{F} = (\omega + \tilde{F})^{-1} = (1 + \theta\tilde{F})^{-1}\theta$$

$$\hat{F} = [(1 + \theta\tilde{F})^{-1} - 1]\theta = -(1 + \theta\tilde{F})^{-1}\theta\tilde{F}\theta$$

$$\hat{F} = \theta F'\theta$$

For constant $\theta, \tilde{F}, F'$

This is the relation given by Seiberg-Witten as a relation between the commutative and noncommutative field strengths.



- This relation to NCG is interesting : discussion
- Here, I want to concentrate on the fluctuations and its gauge symmetry.

Our previous result tells us :[ASW]

Among the total symmetry of generalized tangent bundle, the generalized diffeo. which preserves the Dirac structure is not fluctuation, since we identified D-brane and Dirac structure, and the broken direction can be considered as a Nambu-Goldstone mode and thus as a fluctuation.

- For L_ω case, the generalized diffeo. transformation of vector $X+\omega(X)\in L_\omega$ can be written:

$$\mathcal{L}_{\epsilon+\omega(\epsilon)+\Lambda}(X+\omega(X)) = \mathcal{L}_\epsilon X + \omega(\mathcal{L}_\epsilon X) - i_X d\Lambda.$$

where ε is 1-vector and Λ is 1-form parameters of symmetry.

From this we can read that the symmetry is generalized diffeo. by $\epsilon+\omega(\epsilon)+d\lambda$

But if Λ is not dλ, then symmetry is broken.

Such 1-form Λ can be identified with NG boson corresponding to the gauge potential: $A\in L_\omega^* = T^*M$

$$e^{-\mathcal{L}_A}(X+\omega(X)) = X + (\omega+\tilde{F})(X),$$

- For L_θ case, the generalized diffeo. transformation of vector $\xi + \theta(\xi) \in L_\theta$ is

$$\mathcal{L}_{\epsilon+a+\theta(a)}(\xi + \theta(\xi)) = (\mathcal{L}_{\epsilon+\theta(a)}\xi - i_{\theta(\xi)}da) + \theta(\mathcal{L}_{\epsilon+\theta(a)}\xi - i_{\theta(\xi)}da) + (\mathcal{L}_\epsilon\theta)(\xi).$$

where ε is 1-vector and a is 1-form parameters of symmetry.

From this we can read that the symmetry is

general. Diffeo. by $\epsilon + \theta(a) + a$ where $\mathcal{L}_\epsilon \theta = 0$

But if ε is not θ preserving diffeo., then symmetry is broken, and such 1-vector ε can be identified with NG boson corresponding to the gauge potential: $\Phi \in L_\theta^* = TM$

Infinitesimal action of the 1-vector is

$$\mathcal{L}_\Phi(\xi + \theta(\xi)) = \mathcal{L}_\Phi\xi + \theta(\mathcal{L}_\Phi\xi) + (\mathcal{L}_\Phi\theta)(\xi),$$

and finite version is

$$\begin{aligned} e^{-\mathcal{L}_\Phi}(\xi + \theta(\xi)) &= e^{-\mathcal{L}_\Phi}\xi + (e^{-\mathcal{L}_\Phi}\theta)(e^{-\mathcal{L}_\Phi}\xi). \\ &= \xi' + \theta'(\xi') \end{aligned}$$

where $\theta' = e^{-\mathcal{L}_\Phi}\theta$

and ϕ defines a map $L_\theta \to L_{\theta'}$

$$\theta' = \theta + \hat{F} \quad \text{where } \hat{F} = e^{-\mathcal{L}_\Phi}\theta - \theta.$$

- We can prove that

$$\widehat{F} = e^{-\mathcal{L}_\Phi}\theta - \theta.$$

can be considered as field strength and ϕ as gauge potential in a sense that by considering

Maurer-Cartan type relation as a Bianchi identity

and the $\widehat{F}$ written by ϕ guarantees

$$d_\theta \widehat{F} + \frac{1}{2}[\widehat{F}, \widehat{F}]_S = 0$$

However, the relation with field strength and potential is nontrivial as

$$\widehat{F} = d_\theta \Phi - \frac{1}{2}[\Phi, d_\theta \Phi]_S + \frac{1}{3!}[\Phi, [\Phi, d_\theta \Phi]_S]_S \cdots .$$

- Finally the gauge transformation of ϕ is given by the Hamiltonian vector field

$$d_\theta h = \{h, \cdot\}_\theta$$

since

$$\mathcal{L}_{d_\theta h}\theta = -d_\theta^2 h = 0$$

5. Summary and discussion

1. We have constructed the 1-vector gauge potential:

- Field strength is 2-vector $\widehat{F}$
- Bianchi identity is Maurer-Cartan type relation which guarantees that the β-transformation by $\widehat{F}$ gives Dirac structure again so that it can be identified with the D-brane fluctuation.
- Gauge symmetry is generated by Hamiltonian vector field.

・We have analyzed the relation between the field A and ϕ and also the relation between the gauge parameters.

・The explicit relations can be derived by using Moser’s Lemma and Magnus expansion.[AMW]

2. Dual description of the same subbundle from the dual Dirac structure works also for metric structure:

In Generalized Geometry , metric is also characterized by a subbundle, and can be represented by a graph:

$$E = g + B \; : \; TM \to T^*M \; : \; X^j \mapsto (g+B)_{ij} X^j$$

This subbundle is a set of generalized tangent vectors of positive/negative length

$$C_+ = \{V_+ = v + (g+B)(v) \,|\, v \in TM\}, \; , \; \langle V_+, V_+ \rangle = g(v,v) > 0$$

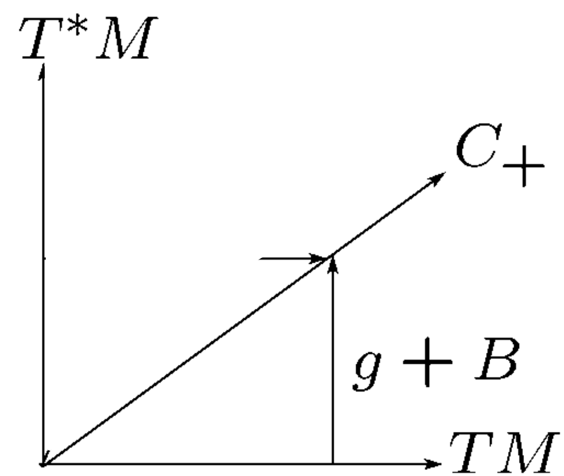
$$C_- = \{V_- = v + (-g+B)(v) \,|\, v \in TM\}, \; , \; \langle V_-, V_- \rangle = -g(v,v) < 0$$

Or equivalently

$$GV_\pm = \pm V_\pm \; , \quad \text{for} \quad V_\pm \in \Gamma(C_\pm).$$

$$G = \begin{pmatrix} -g^{-1}B & g^{-1} \\ g - Bg^{-1}B & Bg^{-1} \end{pmatrix}$$

$$O(D,D) \;\to\; O(D) \times O(D)$$



We see the metric from L_θ

Similarly to the fluctuation case

$$\xi + (\theta + t)(\xi) = X + (g + B)(X)$$

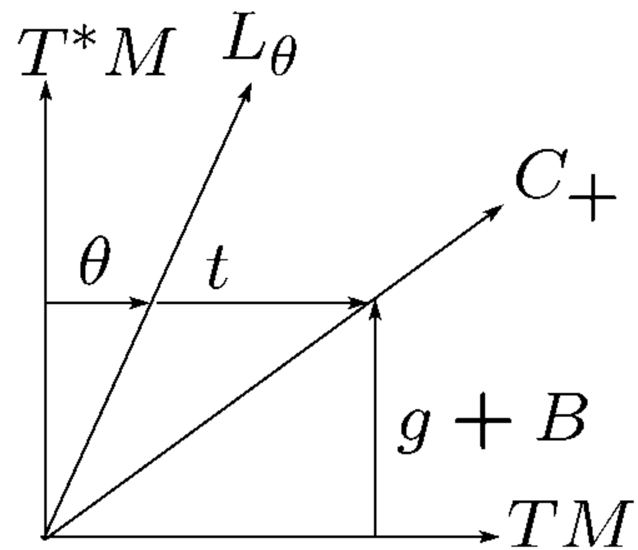
thus

$$\theta + t = (g + B)^{-1}$$

Here, t is the metric seen from L_θ

$$\theta + \frac{1}{G+\Phi} = \frac{1}{g+B}$$

We get Seiberg relation or
so-called open-closed relation



3. Discussion （H, B, A, λ） (gerbe structure)

3-form flux, 2-form potential

closed 2-form with 1-form potential

(R, F, ϕ, h)

3-vector flux, 2-vector potential MC-nontrivial

MC trivial 2-vector with 1-vector gauge pot.

(this part is now identified)