Granular friction in a wide rage of shear rates

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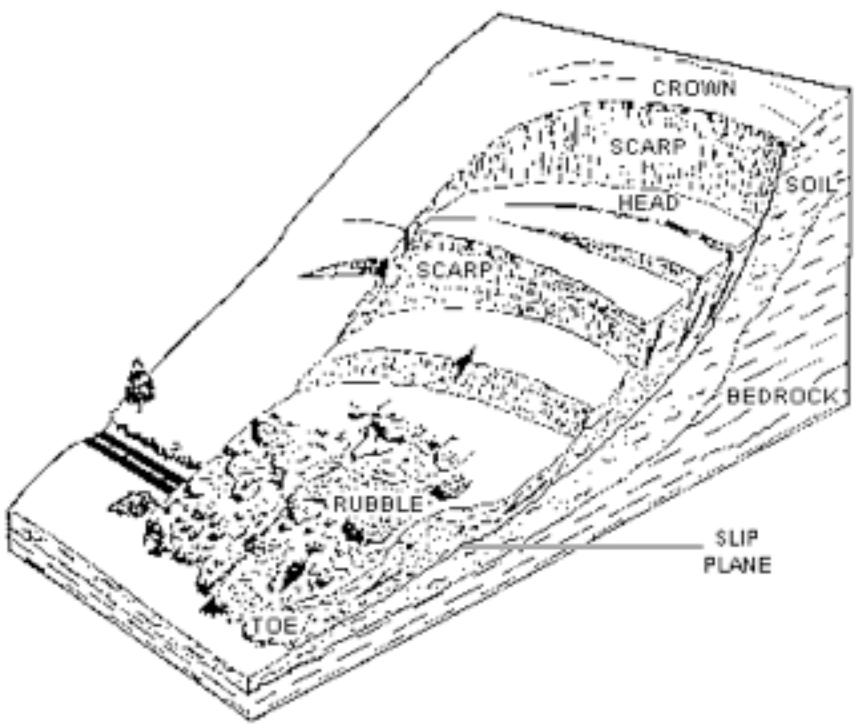
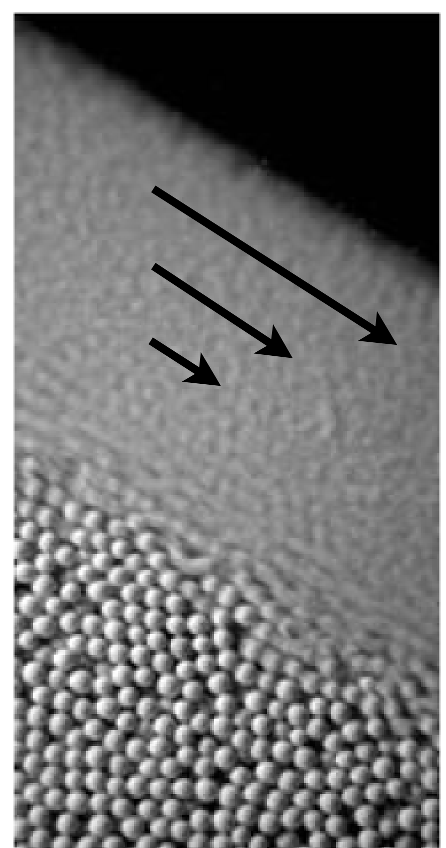
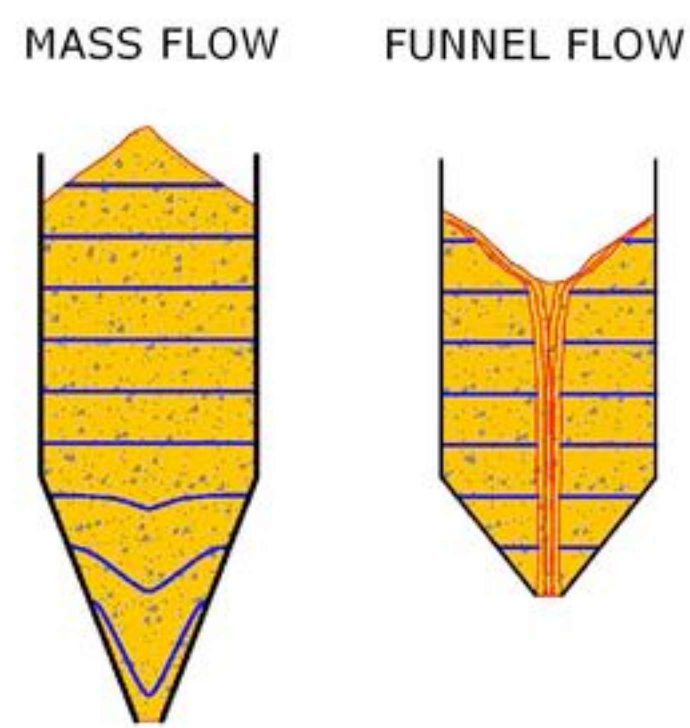
06/27/2013 Physics of Granular Flow (Kyoto)

granular friction: examples

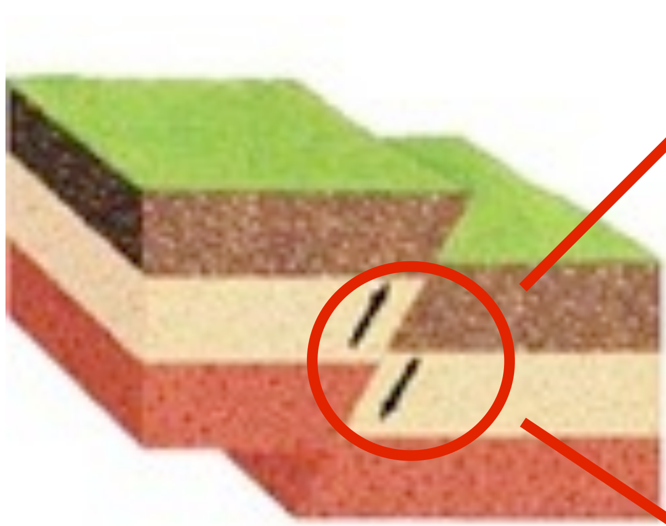






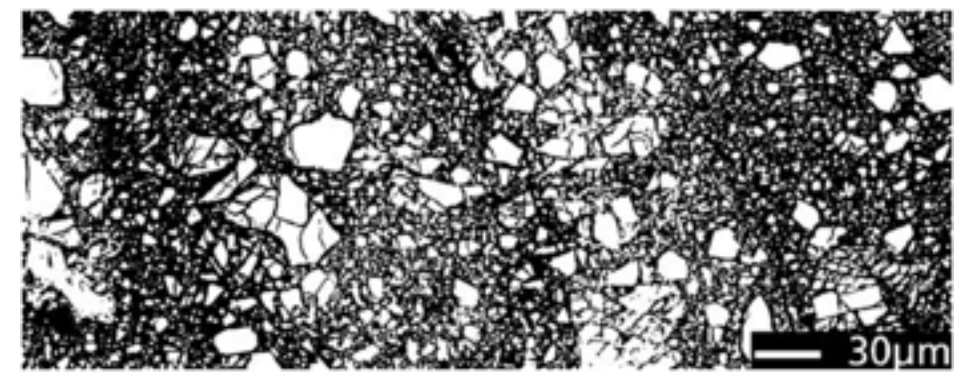
friction of fault

10^3 m



10^0 m

10^{-4}m



fine rock powder

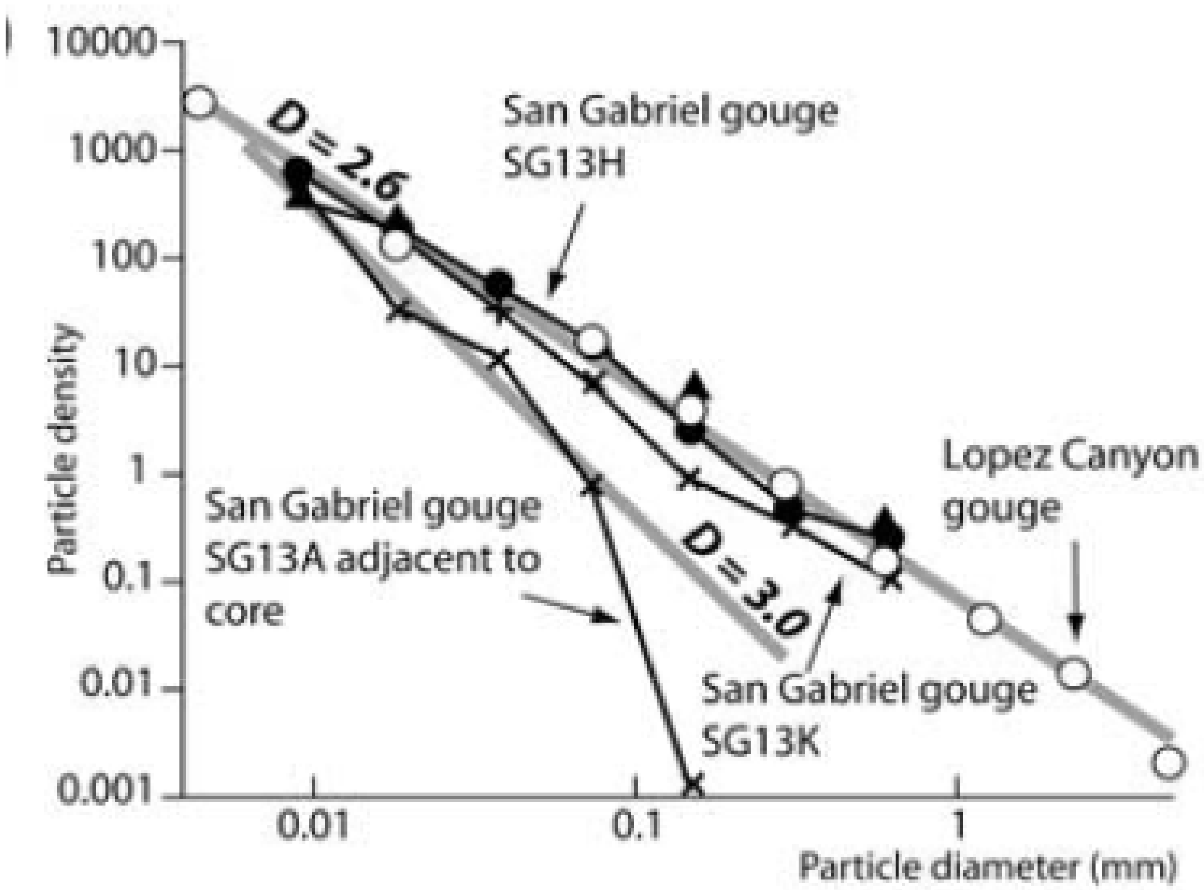
“fault gouge”

friction of fault ← friction of granular matter

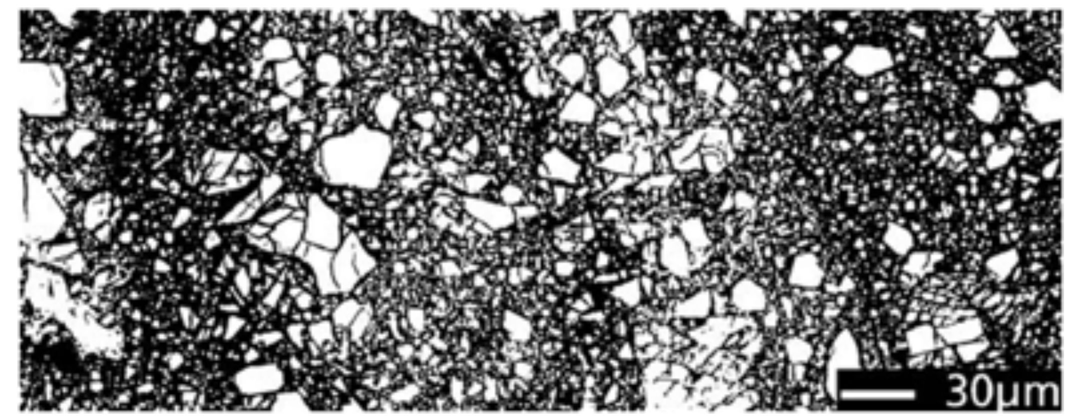
(microscopic basis)

character of “fault gouge”

particle size distribution is power-law (fractal)



(Chester & Chester 1993)



(Heilbronner & Keulen 2006)

exponent 2.5 to 3.0

numerous sub-micron particles

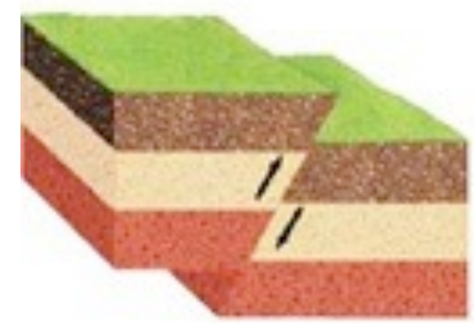
very different from industrial situation!

velocity range of fault motion

must investigate a very wide range

slip velocity [m/s]

10^{0} earthquakes



10^{-2} "slow earthquakes"

(no seismic waves)

10^{-10} plate motion

note:
shear rate depends on
shear-band thickness

(thickness < 10cm)

$\longrightarrow 10^{-9} \leq \dot{\gamma} \leq 10^{1}$

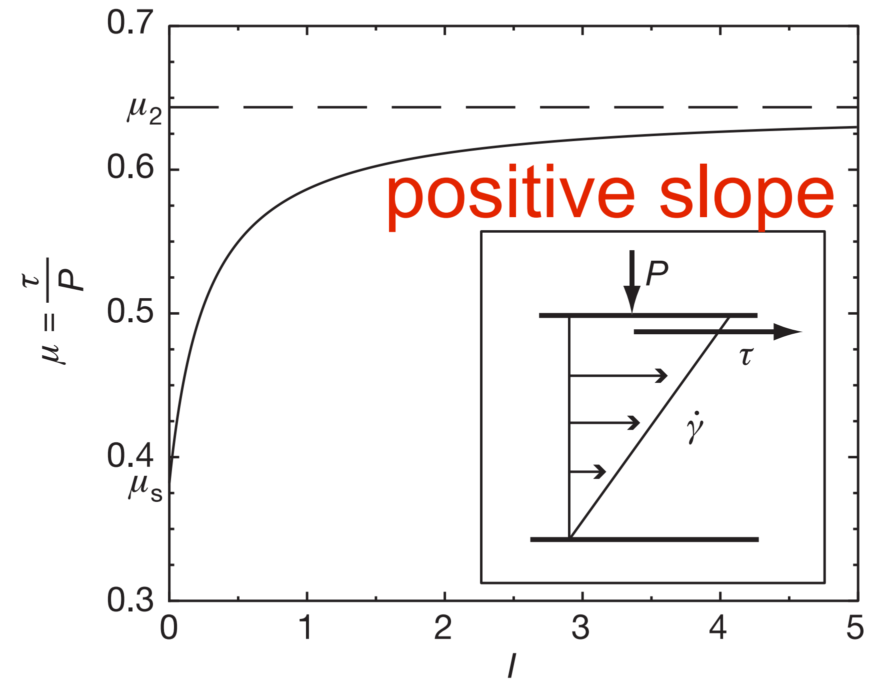
?

granular friction: an empirical law

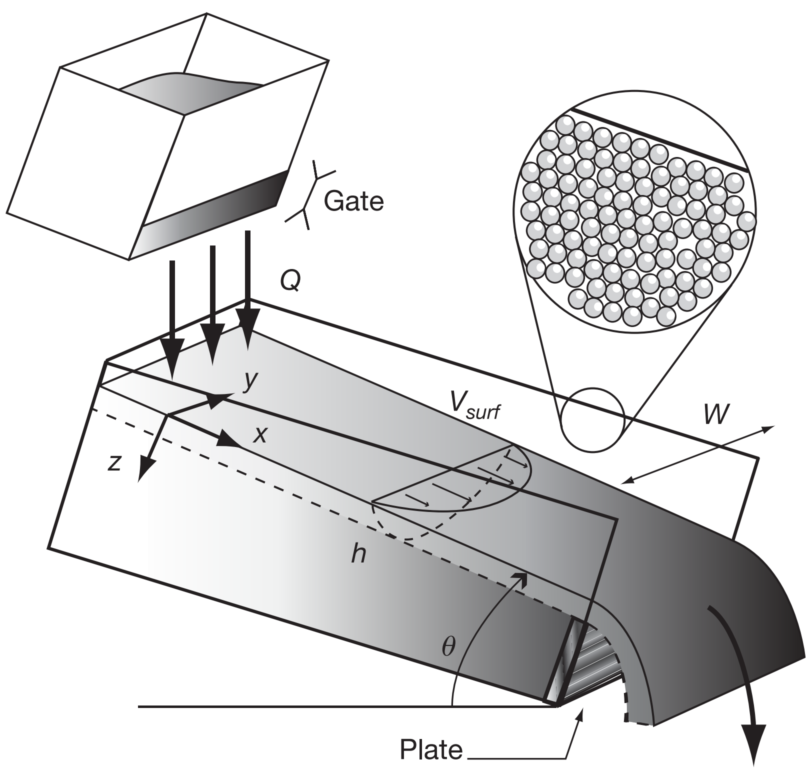
$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0/I + 1}$$

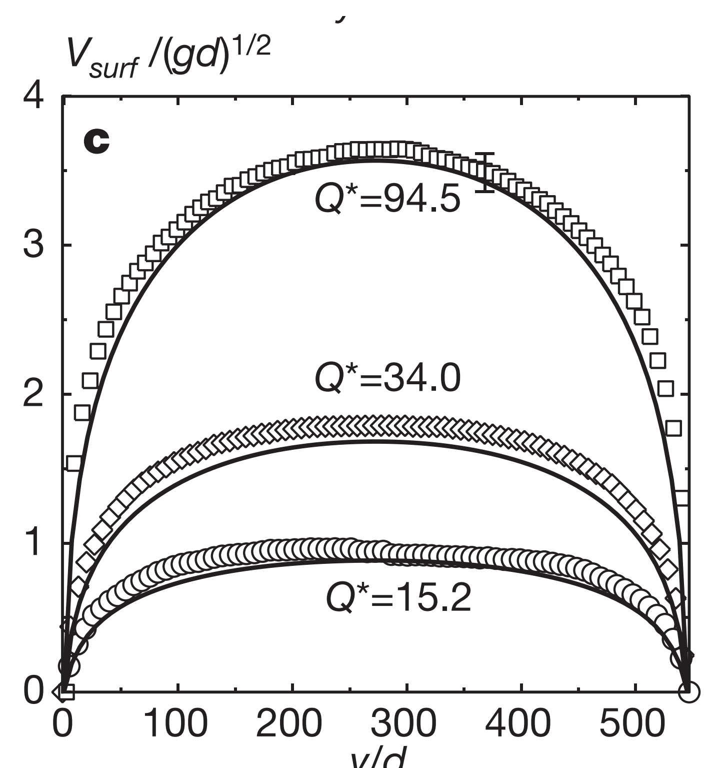
$$I \equiv \dot{\gamma}\sqrt{\frac{m}{Pd}}$$ “inertial number”

Jop et al. JFM 2005; Jop et al. Nature 2006

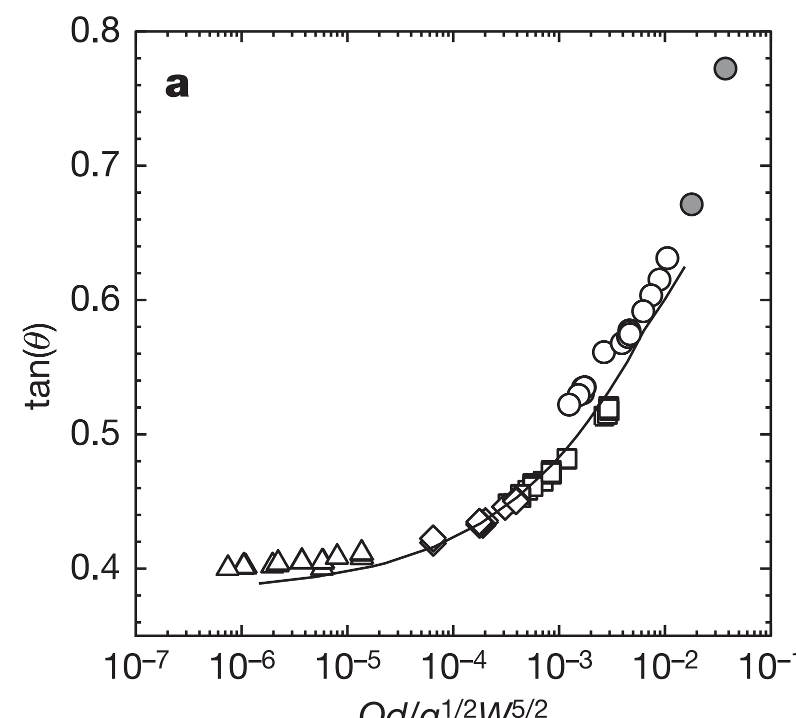


... works well for large inertial number; I=O(1)



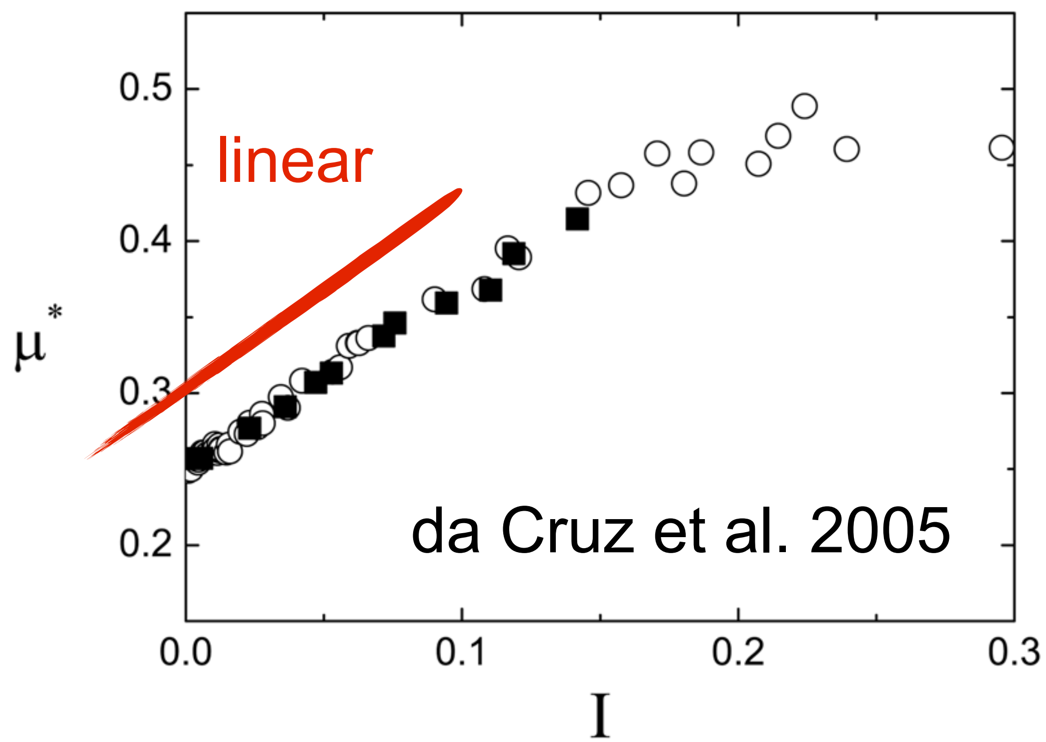


surface velocity profile



flow rate vs tanθ

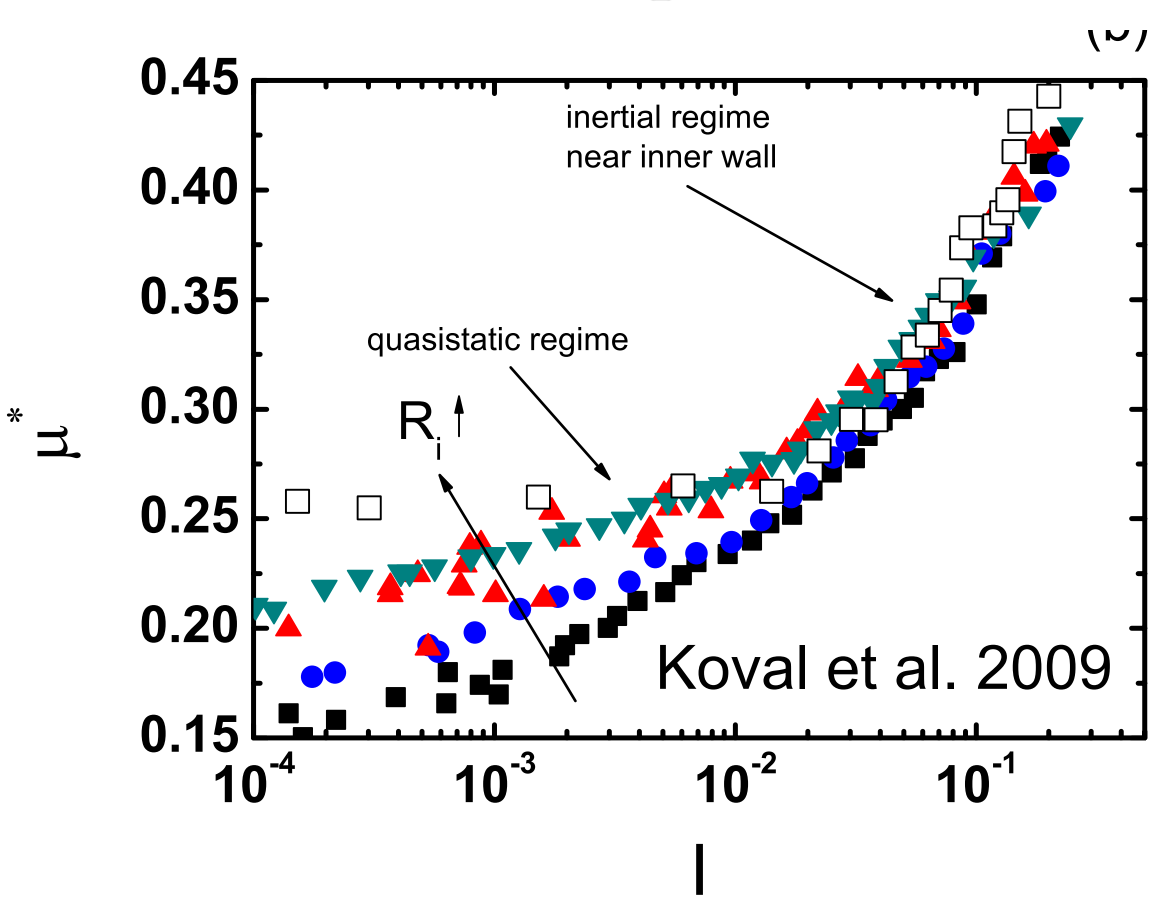
granular friction: numerical experiment



consistent with Pouliquen's law

$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0/I + 1}$$

$$\xrightarrow[I \ll 1]{} \quad \mu(I) \simeq \mu_s + \frac{\mu_2 - \mu_s}{I_0} I$$



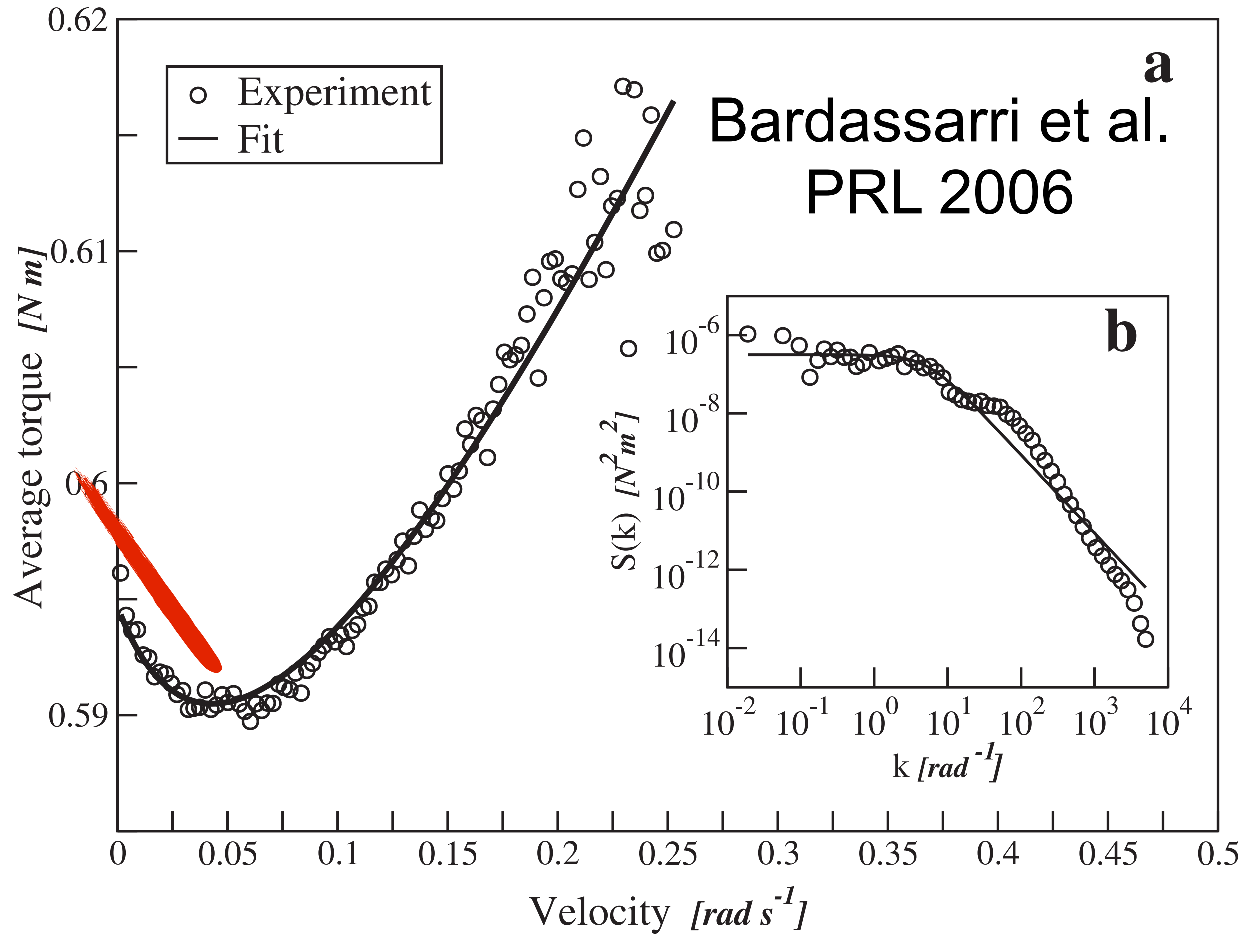
size-dependence <-- nonlocal effect

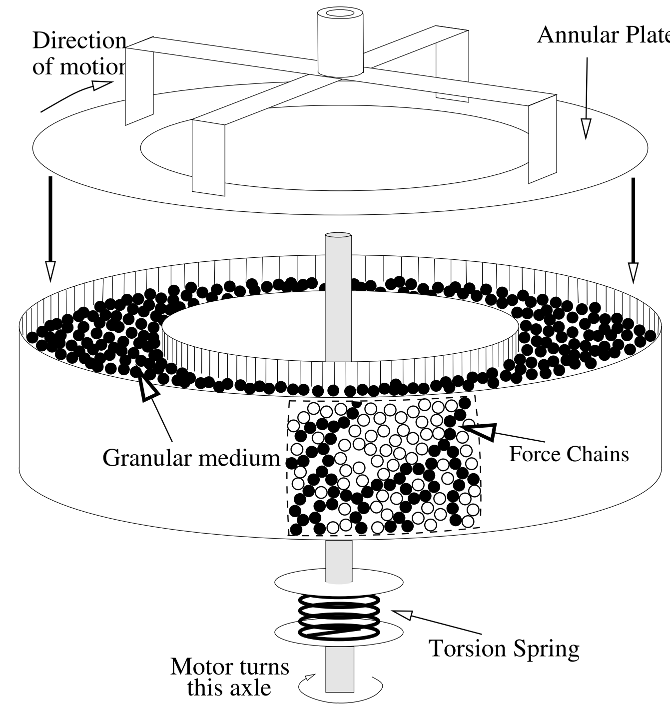
(Kamrin's talk)

positive slopes only $\quad I \geq 10^{-4}$

da Cruz et al. Phys. Rev. E (2005)
TH, Phys. Rev. E (2007)
Peyneau & Roux, Phys. Rev. E (2008)
Koval et al. Phys. Rev. E (2009)

granular friction: physical experiments

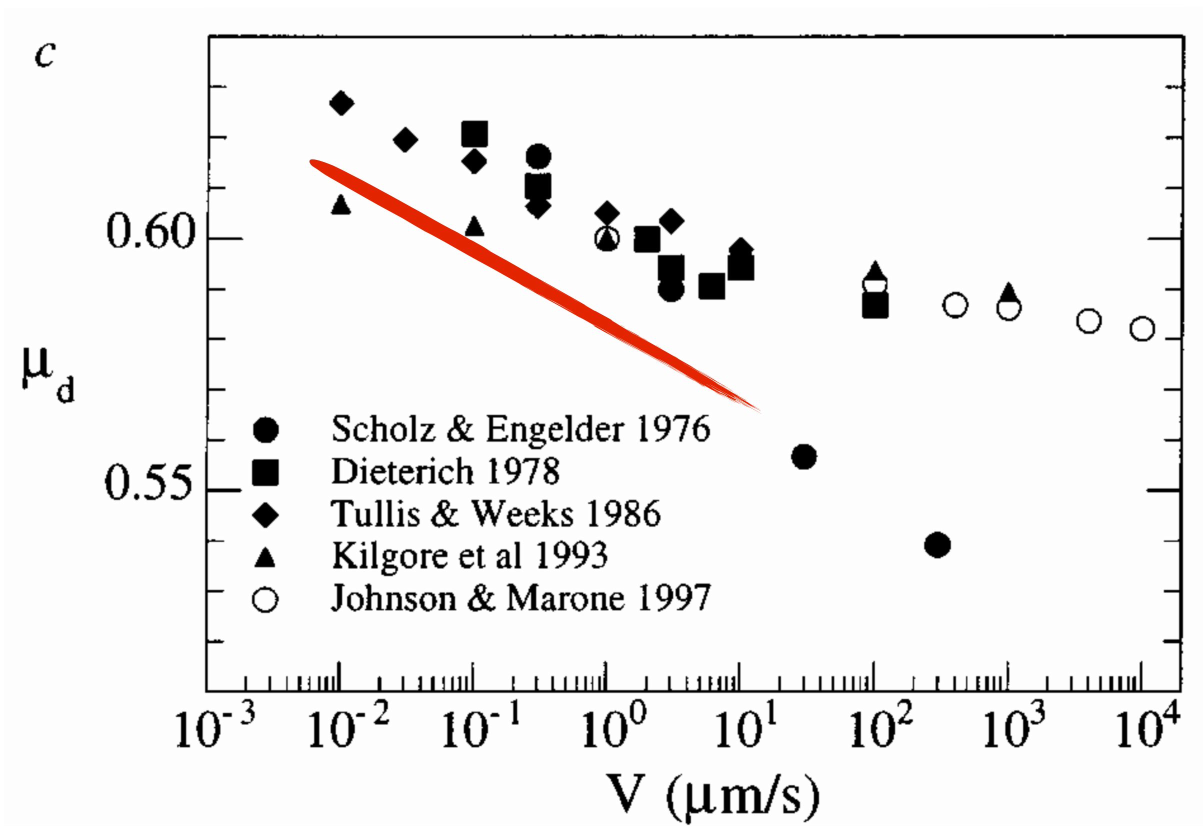




negative slope --> positive slope

see also:
Lu et al. J. Fluid Mech. 2007
Petri et al. EPJB 2008

in earthquake physics....



(e.g. Marone, Ann. Rev. Earth Planet. Sci. 1998)

for steady states

$$\mu(V) = \mu(V_*) + \alpha \log \frac{V}{V_*}$$

constant α is generally negative!

Dieterich 1979

$$\alpha \sim -10^{-2} \text{ to } -10^{-3}$$

(up to ~ mm/sec)

normal pressure ~ 100 MPa

negative slope is rather ubiquitous!

current status

At very low shear rates, constitutive law is still not established

$$\mu(V) = \mu(V_*) + \alpha \log \frac{V}{V_*} \quad \longleftrightarrow \quad \mu(I) \simeq \mu_s + \frac{\mu_2 - \mu_s}{I_0} I$$

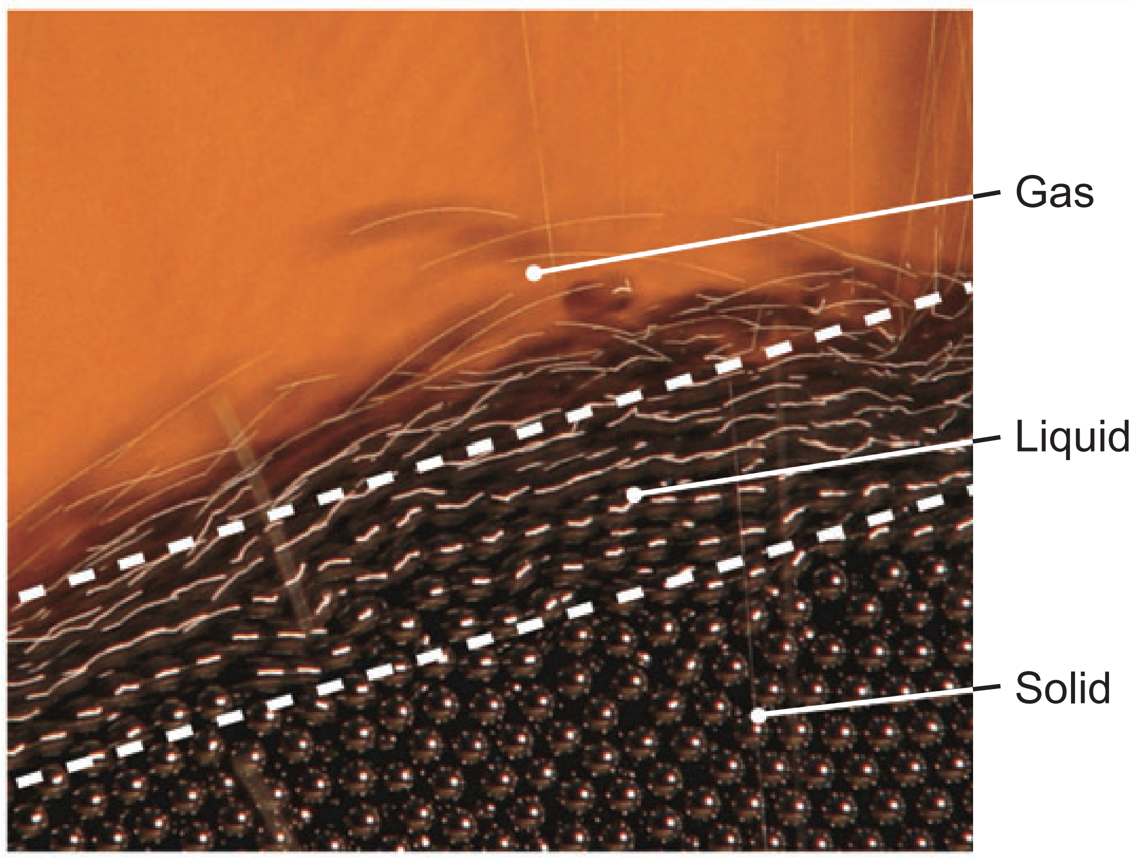
?

physical experiment

numerical experiment

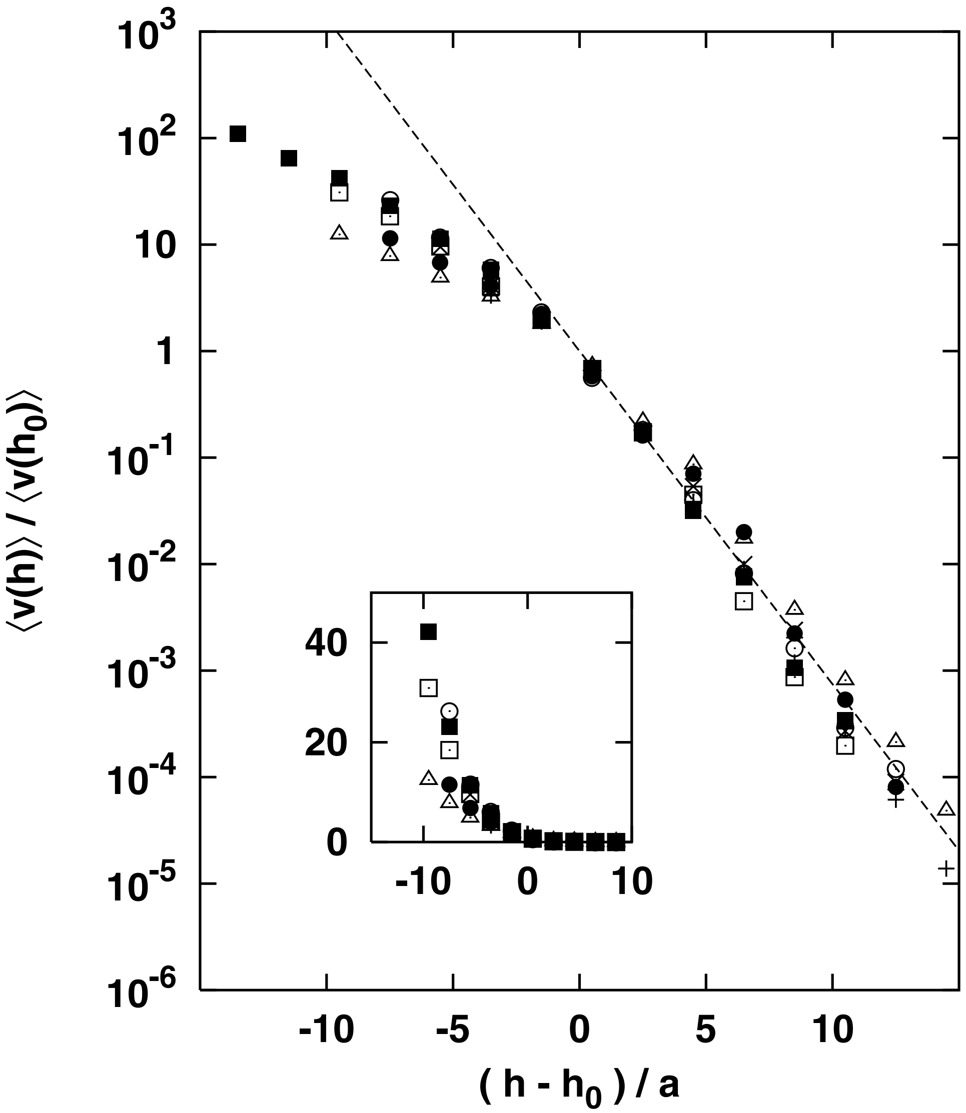
and an example is …

exponential velocity profile in inclined plane flow



Forterre and Pouliquen, Ann. Rev. Fluid. Mech. 2008

Komatsu et al. PRL 2001



what kind of constitutive law can explain this?

questions

1. At very low shear rates, constitutive law is still not established

$\longrightarrow$ A. nonlocal effect (e.g. Kamrin & Koval 2012)

B. physics of negative slope? (Dieterich 1979)

2. If negative slope is true, how is it compatible with Pouliquen's law?

$$\mu(V) = \mu(V_*) + \alpha \log \frac{V}{V_*} \quad \longleftrightarrow \quad \mu(I) \simeq \mu_s + \frac{\mu_2 - \mu_s}{I_0} I$$

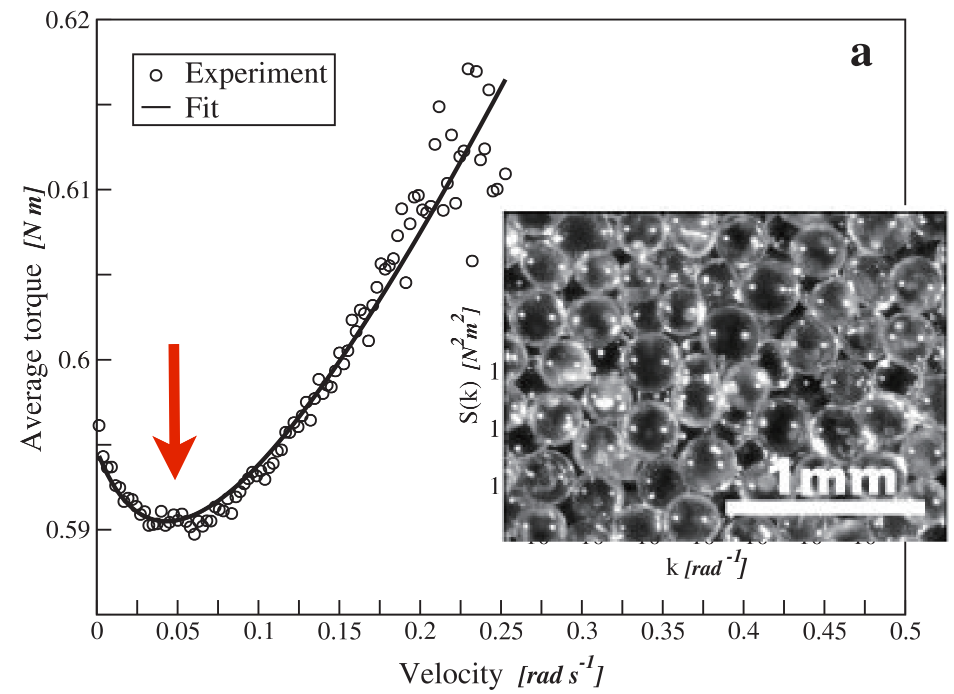
OUR GOAL

1. Negative slope for glass beads?

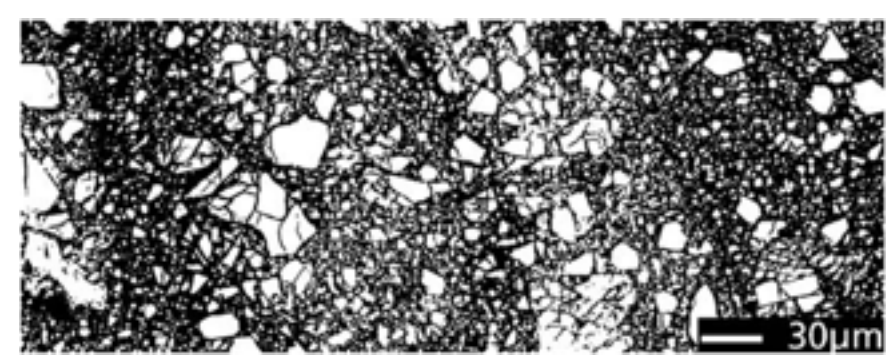
2. Negative to positive crossover? How?

$$I = I_c$$

$\mu(I_c)$ minimum

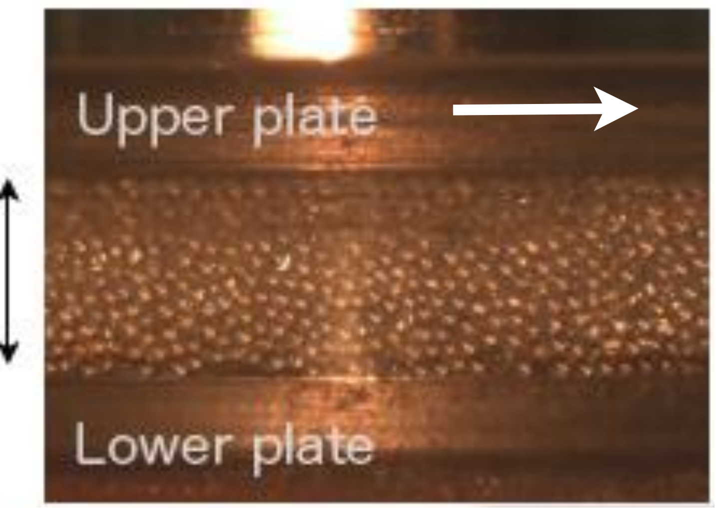


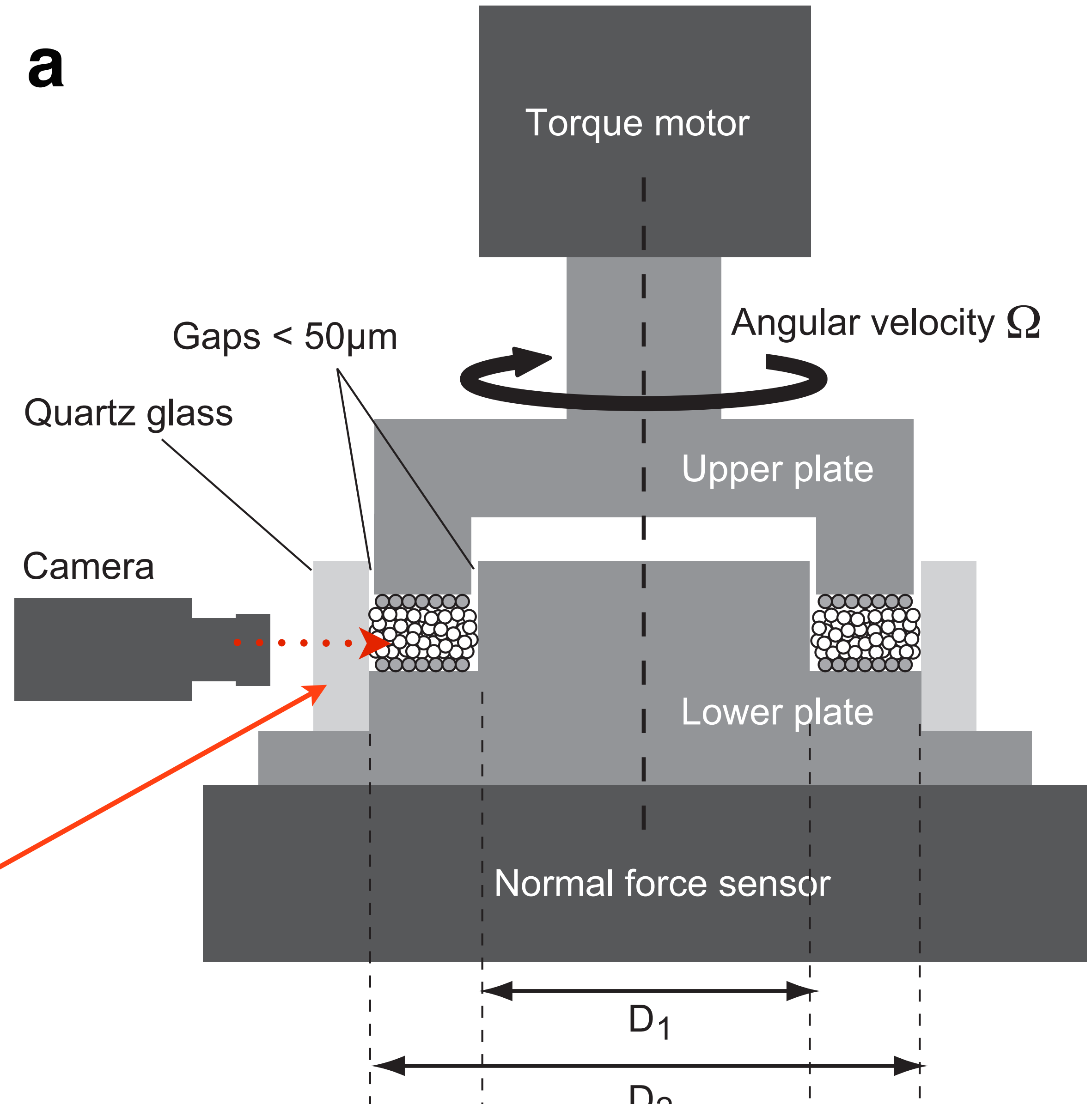
3. What if fault gouge?



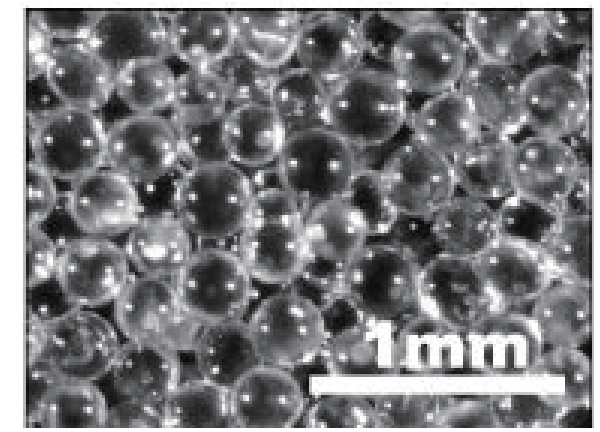
experimental

A commercial rheometer (AR2000ex, TA Instruments)





transparent sidewall



glass beads

mean diameter 270 μm (dispersity ~ 10 %)

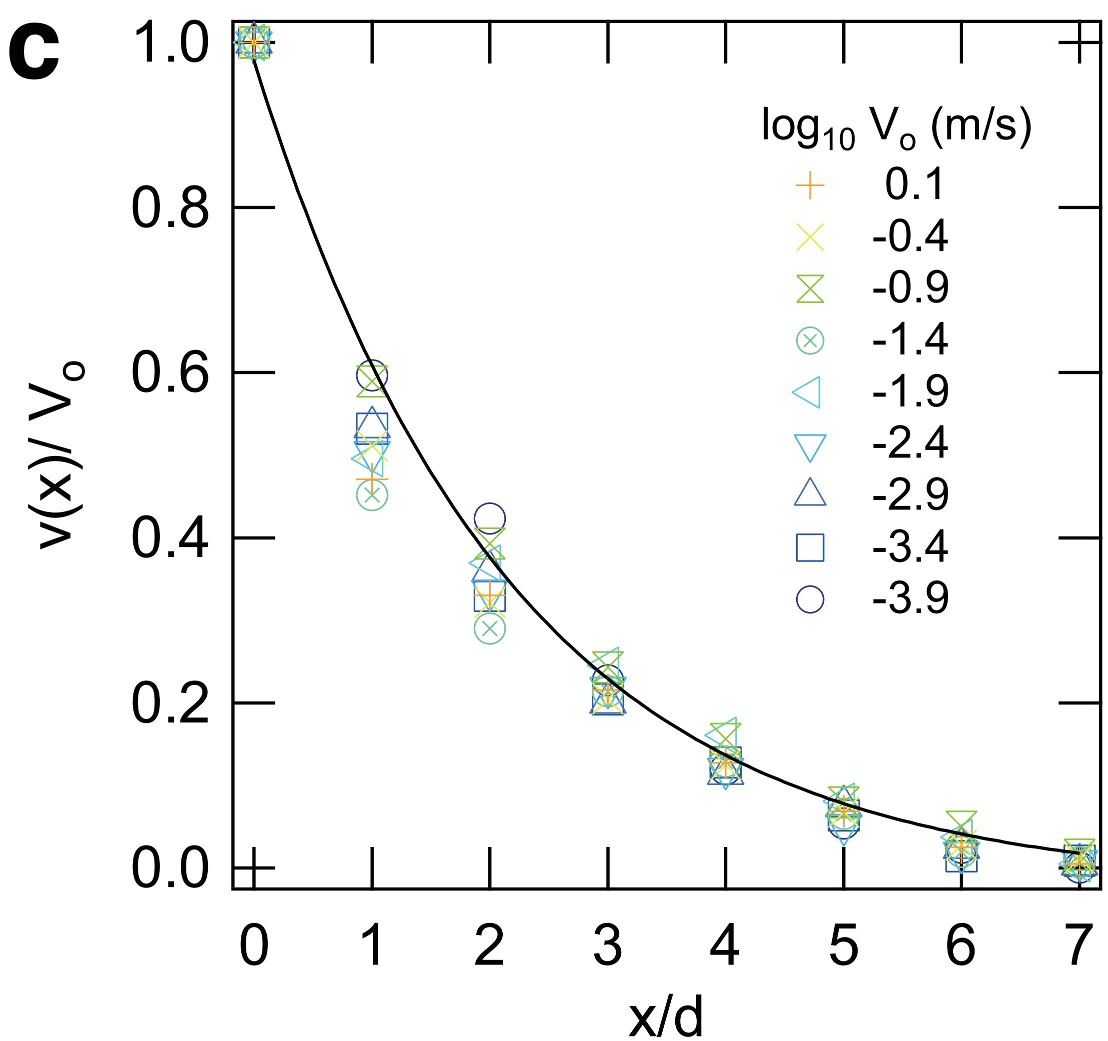
room humidity ~ 50%

annular channel (D1=15mm, D2=25mm)

sliding velocity $\Omega D_2/2 = 10^{-4}$ to 3 [m/sec]

normal stress 10 to 30 kPa (constant pressure)

velocity profile



(depth)/(mean diameter)

normalized by upper plate velocity V_0

$\longrightarrow$ collapse to a master curve

$$V(x) \simeq V_0 10^{-x/W}$$

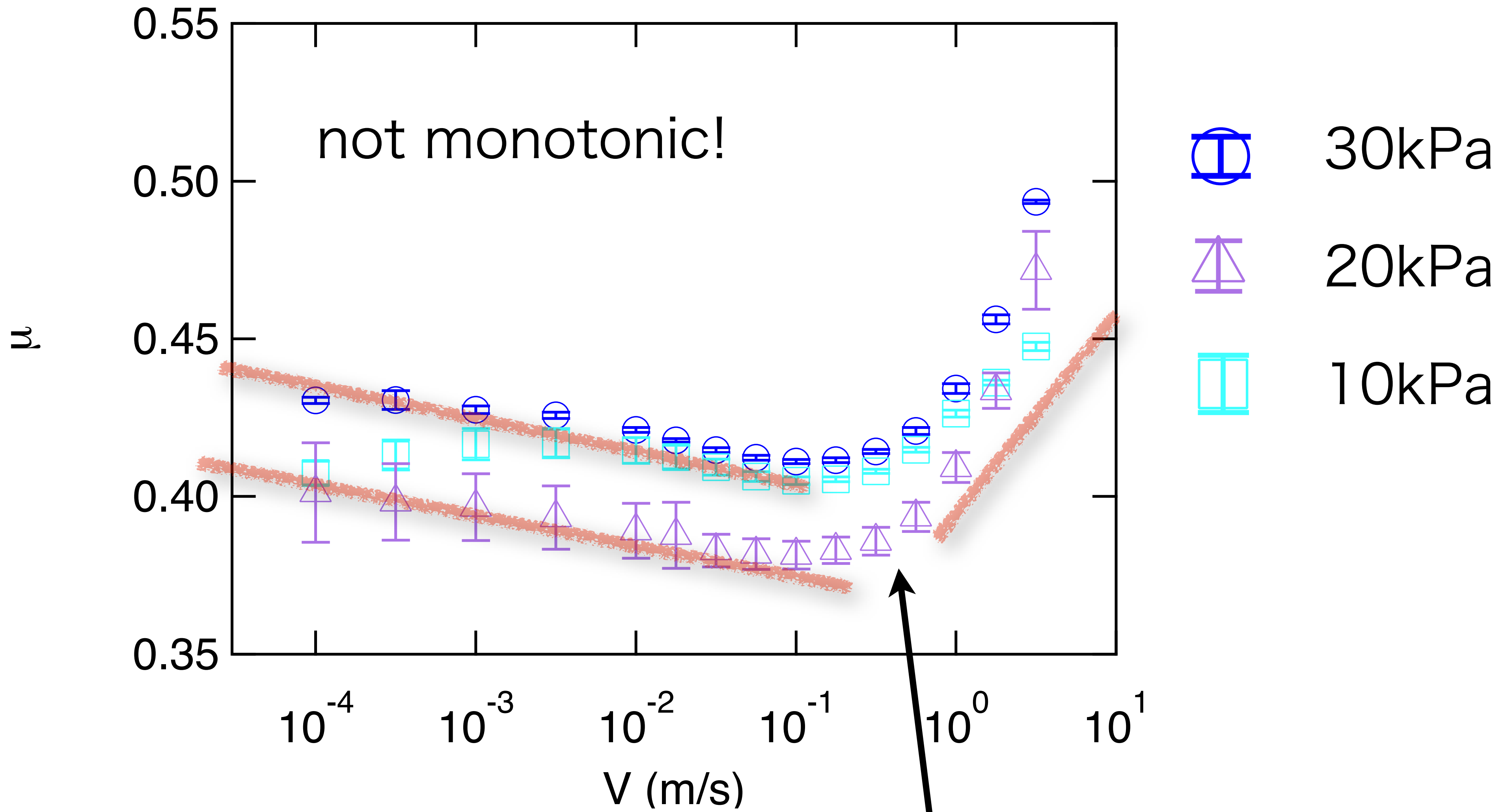
$\longrightarrow$ $W \simeq 5d$

(effective flow width)

shear rate $\dot{\gamma} \equiv V_0/W$

$$\longrightarrow \quad I = \frac{V_0}{W}\sqrt{\frac{m}{Pd}}$$

rate dependence of friction coefficient



At lower velocities,

$$\mu = \mu_* + \alpha \log \frac{V}{V_*}$$

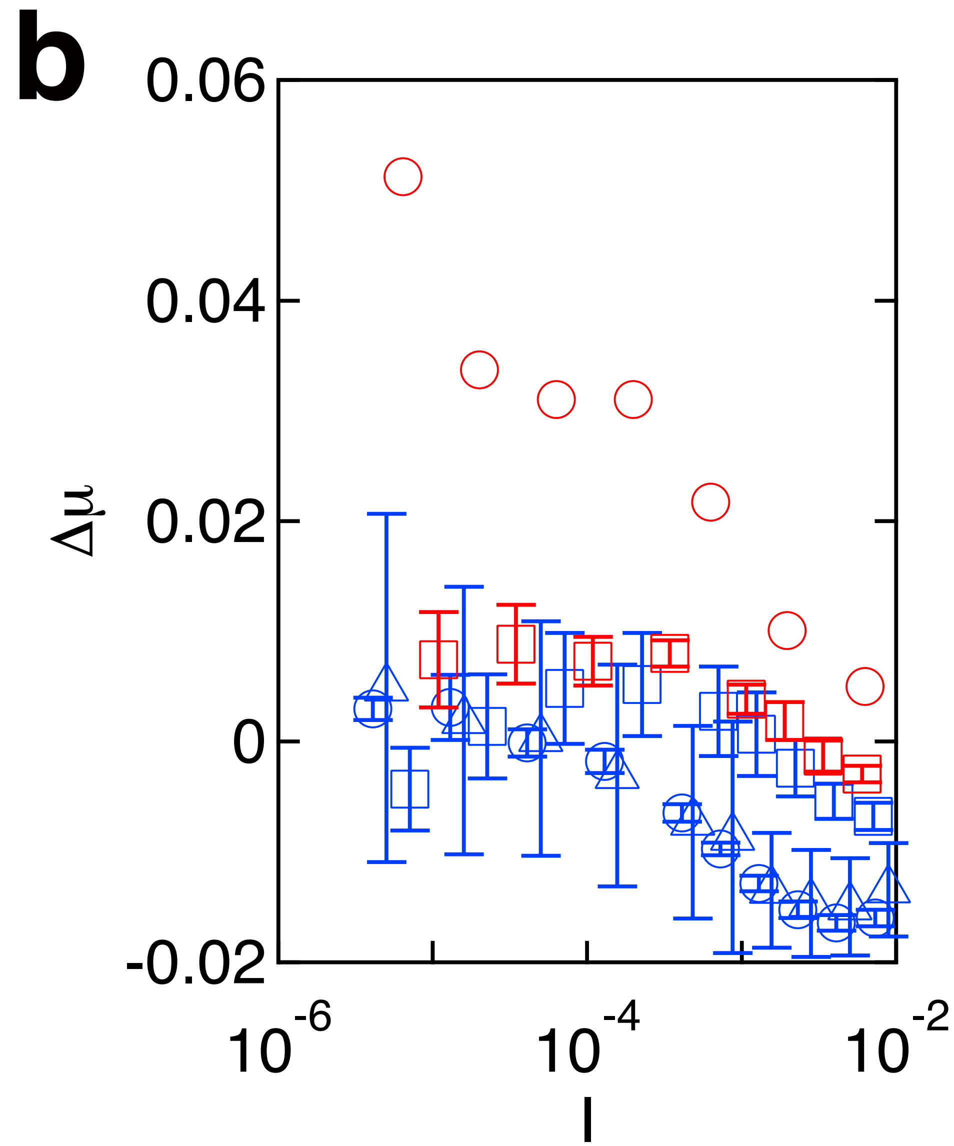
minimum value $\mu_{\min}$

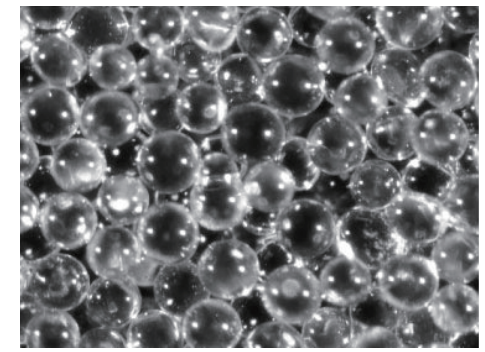
$$\alpha = -0.003 \sim -0.004$$

comparable to fault gouge

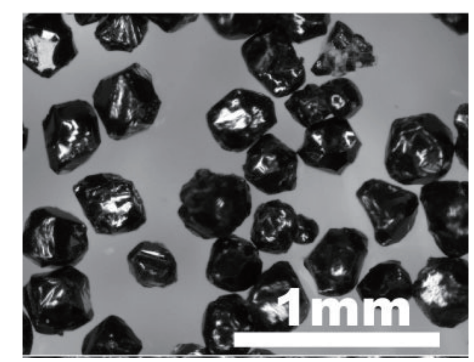
negative slope apparent for $I \leq 10^{-2}$

$$\Delta\mu \equiv \mu - \mu_{\min}$$





□ 10kPa
△ 20kPa
○ 30kPa



□ 10kPa
○ 30kPa

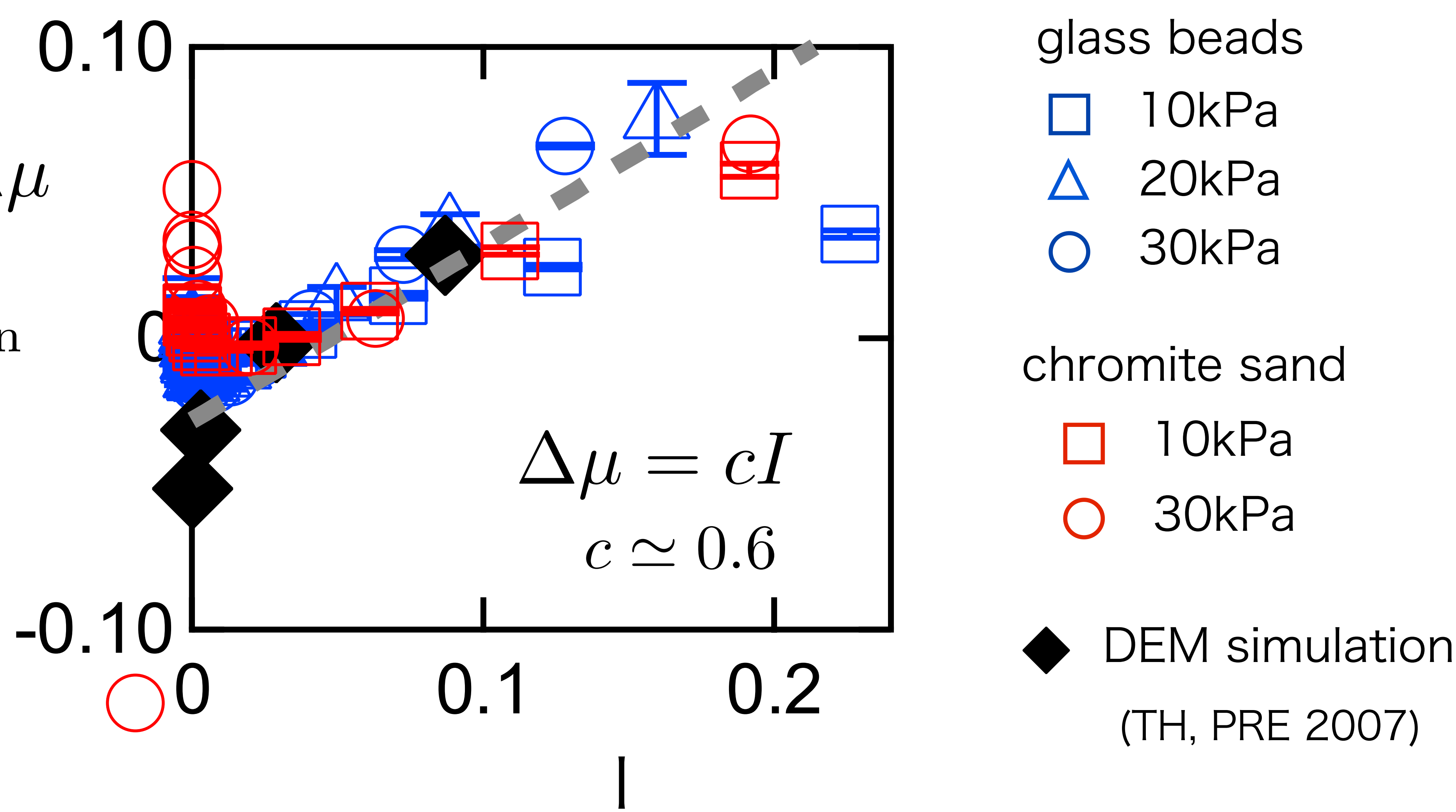
a certain range of α

$$\alpha \sim -10^{-2} \text{ to } -10^{-3}$$

what sets α ? (open question)

constitutive law at high velocities

$$\Delta\mu \equiv \mu - \mu_{\min}$$

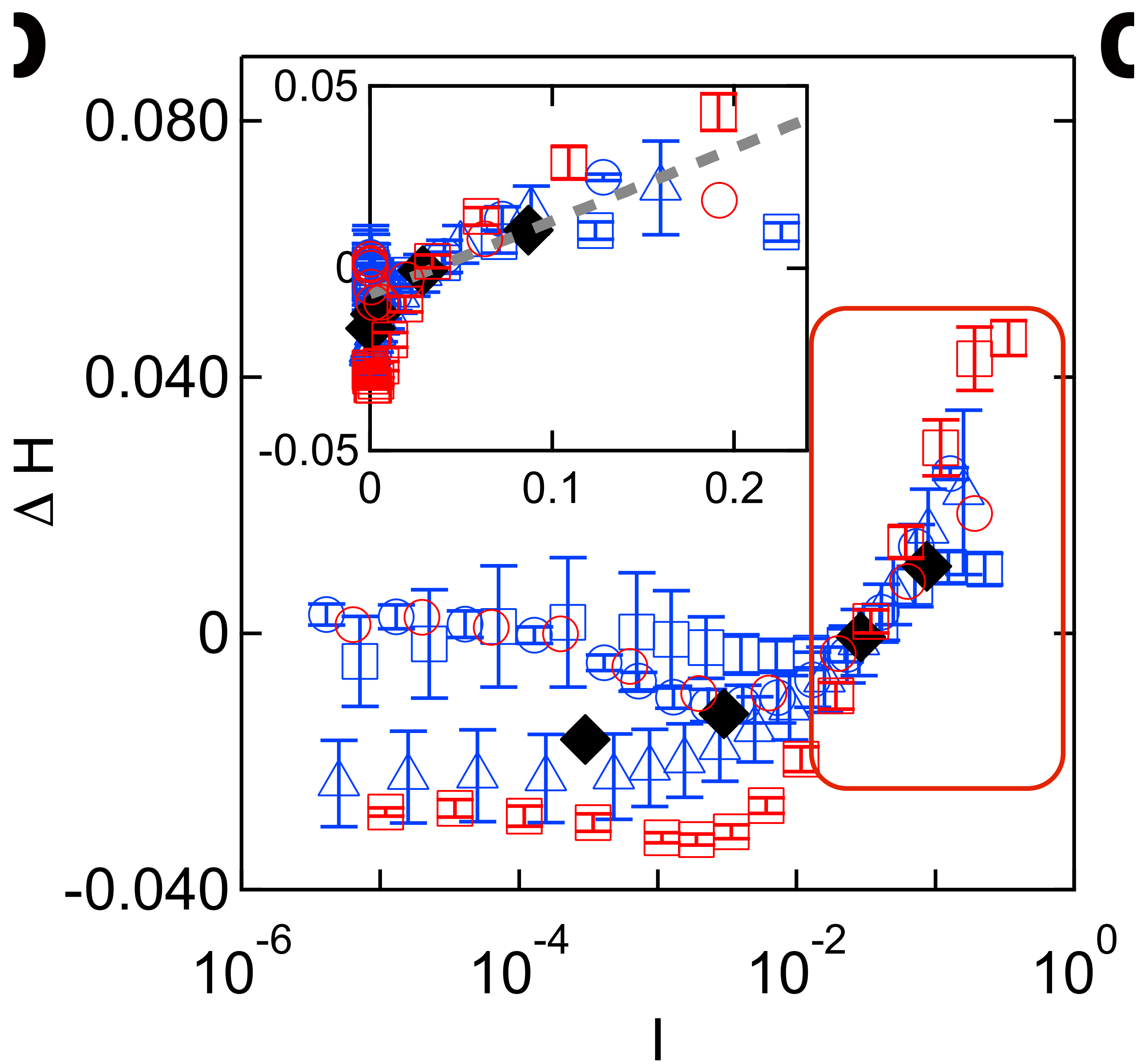


$$I > I_c \longrightarrow \mu = \mu_{\min} + cI \quad \text{(c=0.6)}$$

agrees with simulations (including numerical factor!)

e.g., da Cruz et al. PRE 2005

dilation at high velocities



$$\Delta H \equiv \frac{H(I) - H(I_c)}{W_s}$$ obeys a master curve $$\Delta H(I) = c' I$$

$$c' \simeq 0.2$$

agrees with simulations (including numerical factor!)

1. At higher shear rates, constitutive law agrees with DEM simulation.

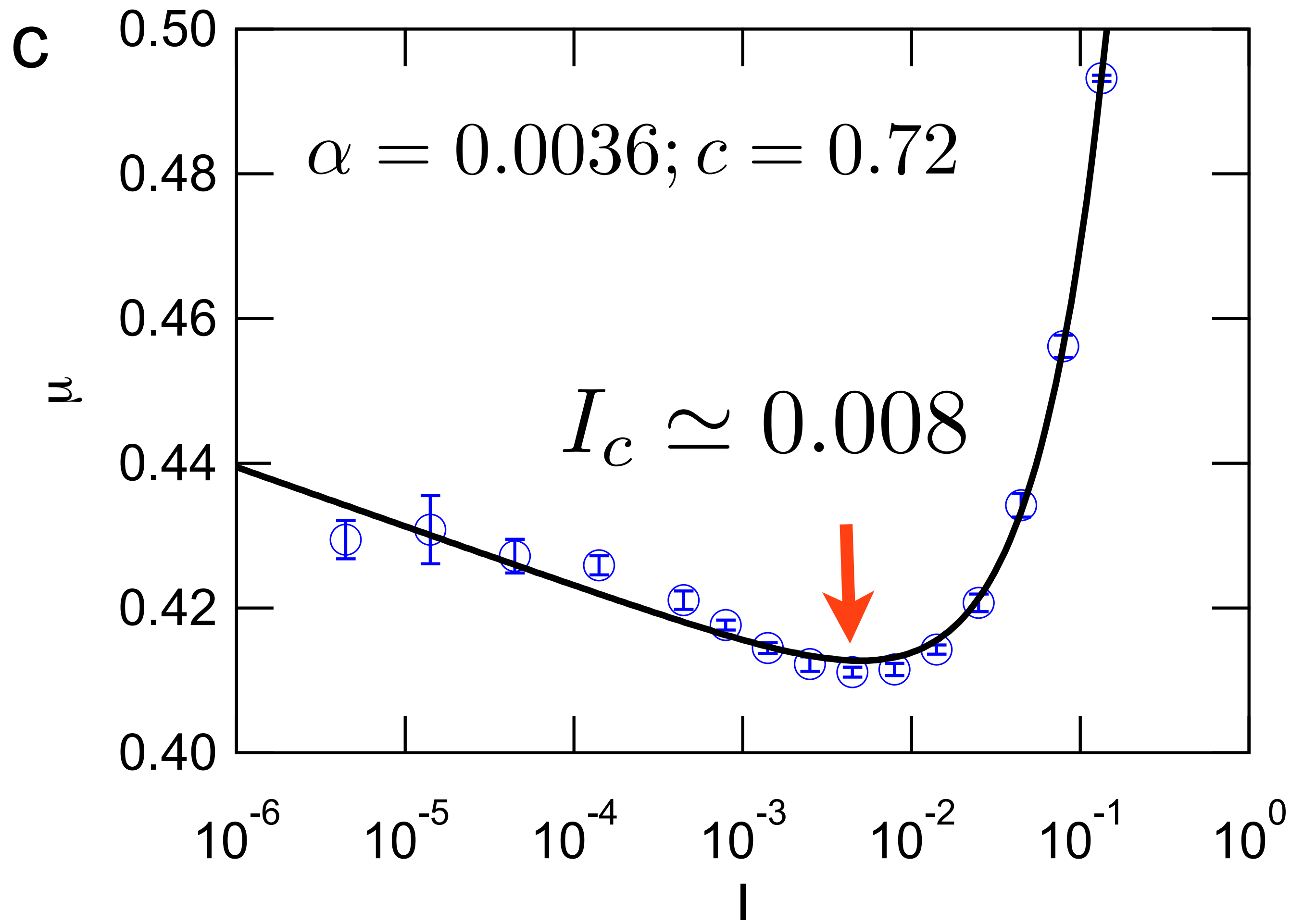
2. Collapse to a master curve using I --> Bagnold's regime.

3. No master curves at sufficiently lower inertial number $I \leq 10^{-2}$

Instead, $$\mu = \mu_* + \alpha \log \frac{V}{V_*}$$

$$\alpha \sim -10^{-2} \text{ to } -10^{-3}$$

where is crossover point?



data:
glass beads
30 kPa

constitutive law

$$\mu = \mu_0 + \alpha \log(\dot{\gamma}/\dot{\gamma}_0) + c\dot{\gamma}\sqrt{m/Pd}$$

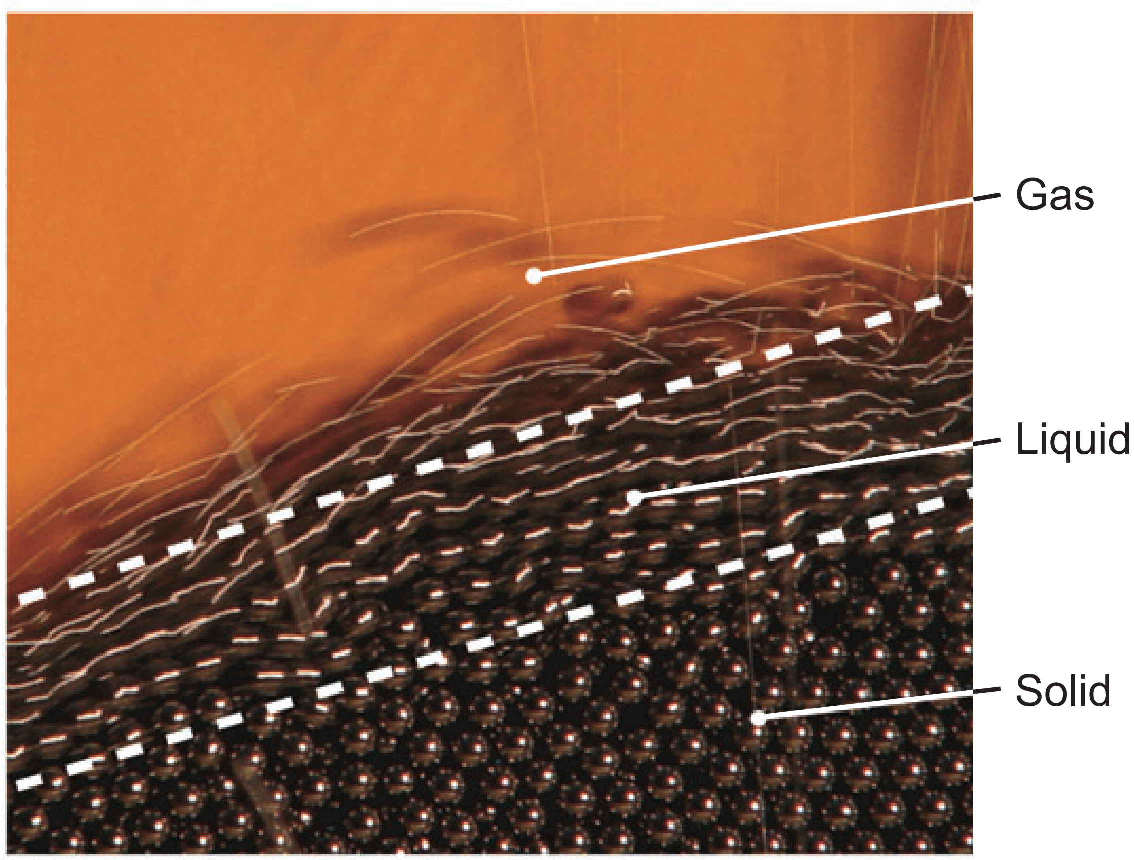
$$\longrightarrow \quad I_c = -\alpha/c \quad = O(10^{-3})$$

$$\mu = \mu_0 + \alpha \log(\dot\gamma/\dot\gamma_0) + c\dot\gamma\sqrt{m/Pd}$$

an application:

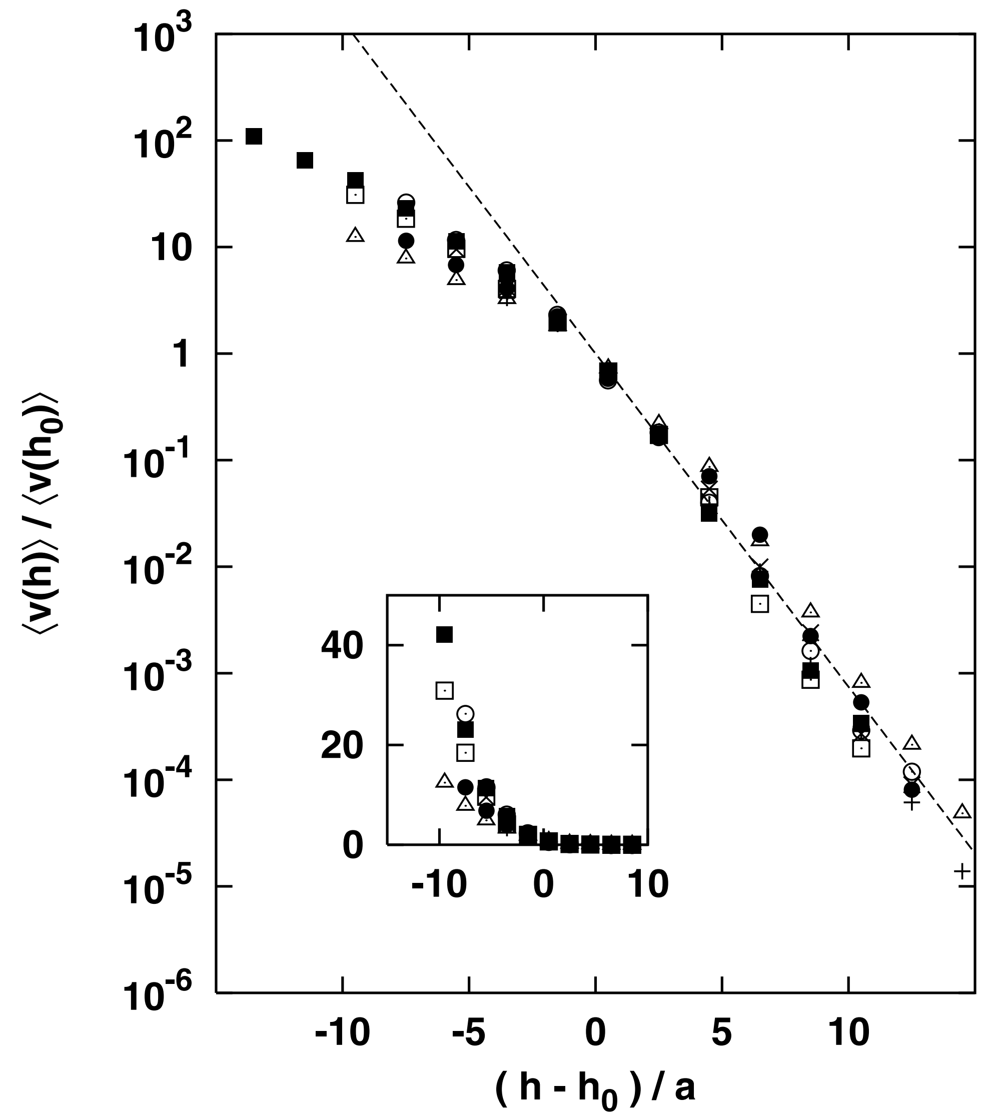
exponential velocity profile for "creep" deformation of solid regime

velocity profile in inclined plane flow



Forterre and Pouliquen, Ann. Rev. Fluid. Mech. 2008

Komatsu et al. PRL 2001



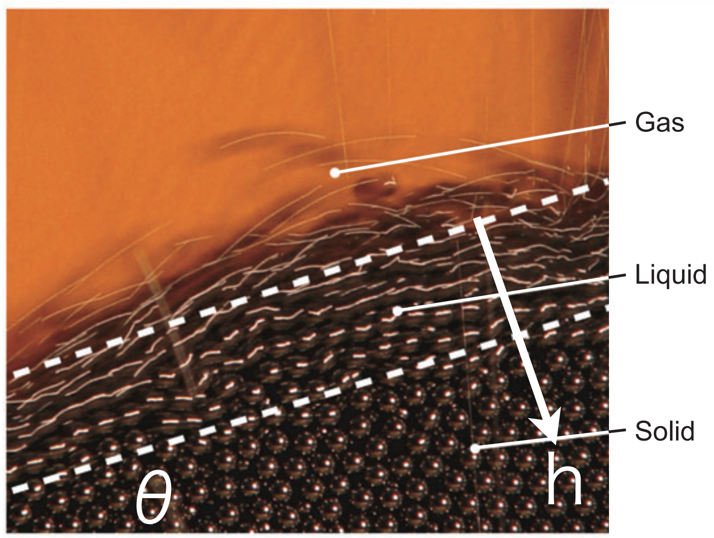
Pouliquen's law cannot reproduce this.

--> Other laws may come into play

can reproduce exponential flow profile

$$\mu = \mu_0 - \alpha \log(V/V_0) + c\dot{\gamma}\sqrt{m/Pd} \qquad (1)$$

force balance eq. for heap flow (along flow direction)



$$\frac{d\sigma}{dh} = \rho g \sin\theta,$$

$$\sigma = \mu P$$

ρ : mass density
θ : angle of slope
σ : shear stress
h: depth
P: normal pressure

if P is independent of h (Janssen's law),

$$\longrightarrow \quad \frac{d\dot{\gamma}}{dh}\frac{d\mu}{d\dot{\gamma}} = \frac{\rho g \sin\theta}{P}$$

use (1) $\longrightarrow$ $\dot{\gamma}(h) \simeq \dot{\gamma}_0 e^{-h/h_0} \quad h_0 \equiv \alpha P/\rho g \sin\theta$

underlying physics of weakening?

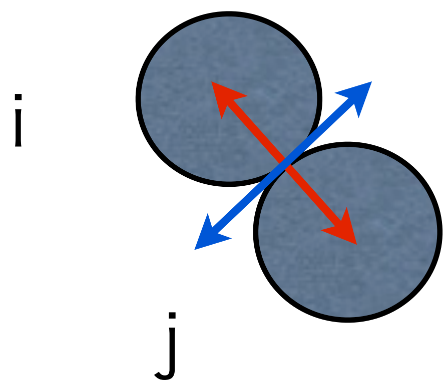
$$\mu = \mu_0 + \alpha \log(\dot{\gamma}/\dot{\gamma}_0) + c\dot{\gamma}\sqrt{m/Pd}$$

underlying physics?

$$\dot{\gamma}\sigma = D$$ (power input) = (energy dissipation rate)

$$D = \mathbf{F}_{ij}^{\mathrm{dis}} \cdot \mathbf{v}_{ij}$$

$$\mathbf{F}_{ij}^{\mathrm{dis}} = \mathbf{F}_1 + \mathbf{F}_2$$ non-conservative force



1. friction $\mathbf{F}_1$

2. damping $\mathbf{F}_2$

$$\mathbf{F}_1 \cdot \mathbf{F}_2 = 0$$

$$|\mathbf{F}_1| \propto \mu_p$$

$$D = (\mathbf{F}_1 + \mathbf{F}_2) \cdot \mathbf{v}$$
$$= |\mathbf{F}_1| v^{(t)} + |\mathbf{F}_2| v^{(n)}$$

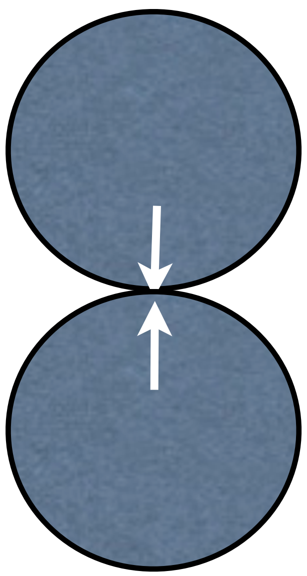
frictional normal

$$\mu = \frac{D}{\dot{\gamma} P} = \cdots \simeq \mu_p + cI$$

first term is particle-level friction

aging of grain contact

particle-level friction is not a constant (but time-dependent)



increase of contact area due to plasticity

(Brechet & Estrin 1994)

$$A(t) = A_0(1 + a \log \frac{t}{t_0})$$

t: duration of contact

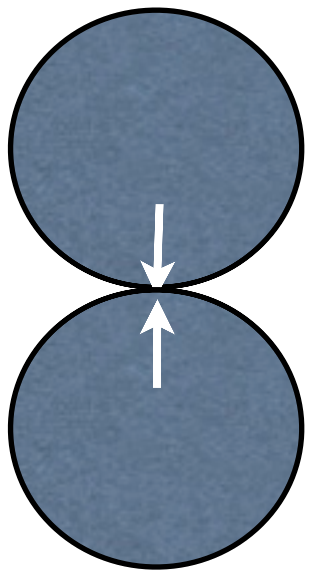
a, t_0: constants

in sheared systems,

$$t \simeq \dot{\gamma}^{-1} \longrightarrow A(t) = A_0(1 - a \log(\dot{\gamma} t_0))$$

$$\longrightarrow \mu_p(\dot{\gamma}) = \mu_0(1 - a' \log(\dot{\gamma} t_0))$$

aging of grain contact



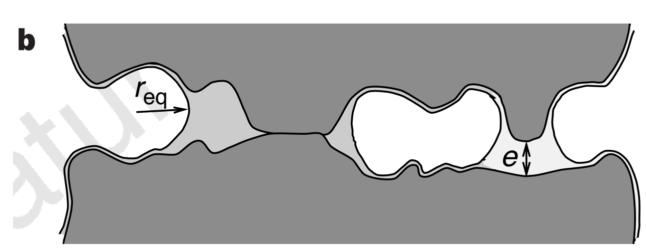
$$\mu_p(t) = \mu_0(1 - a\log(\dot{\gamma}t/t_0))$$

$$\mu_p(\dot{\gamma}) = \mu_0(1 - a'\log(\dot{\gamma}t_0))$$

particle-level friction is time-dependent!

(in DEM, it is constant)

cf. Bocquet et al. Nature 1998



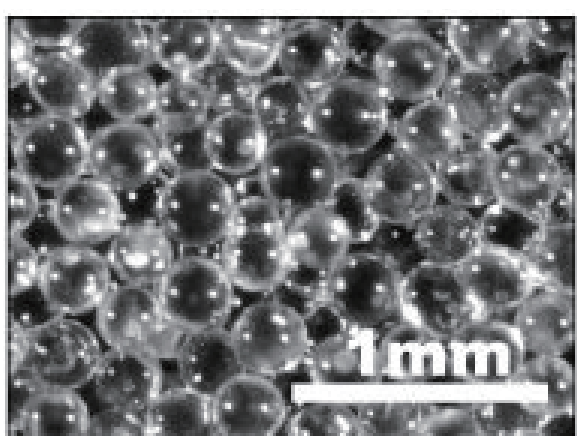
aging due to moisture

OUR GOAL

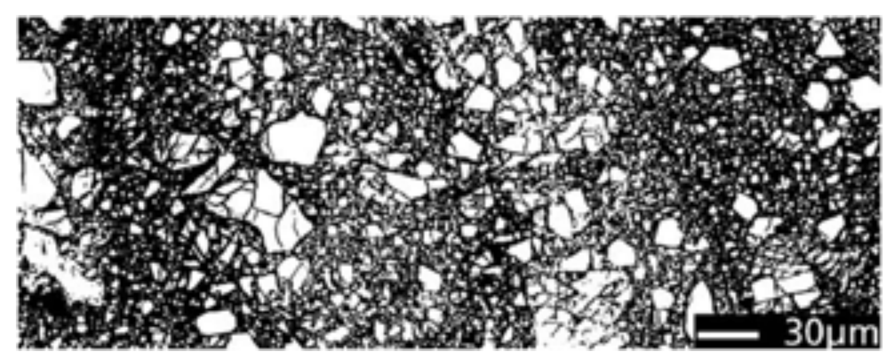
1. How does this crossover occur?

identify $I = I_c$ done $\mu(I_c)$ minimum

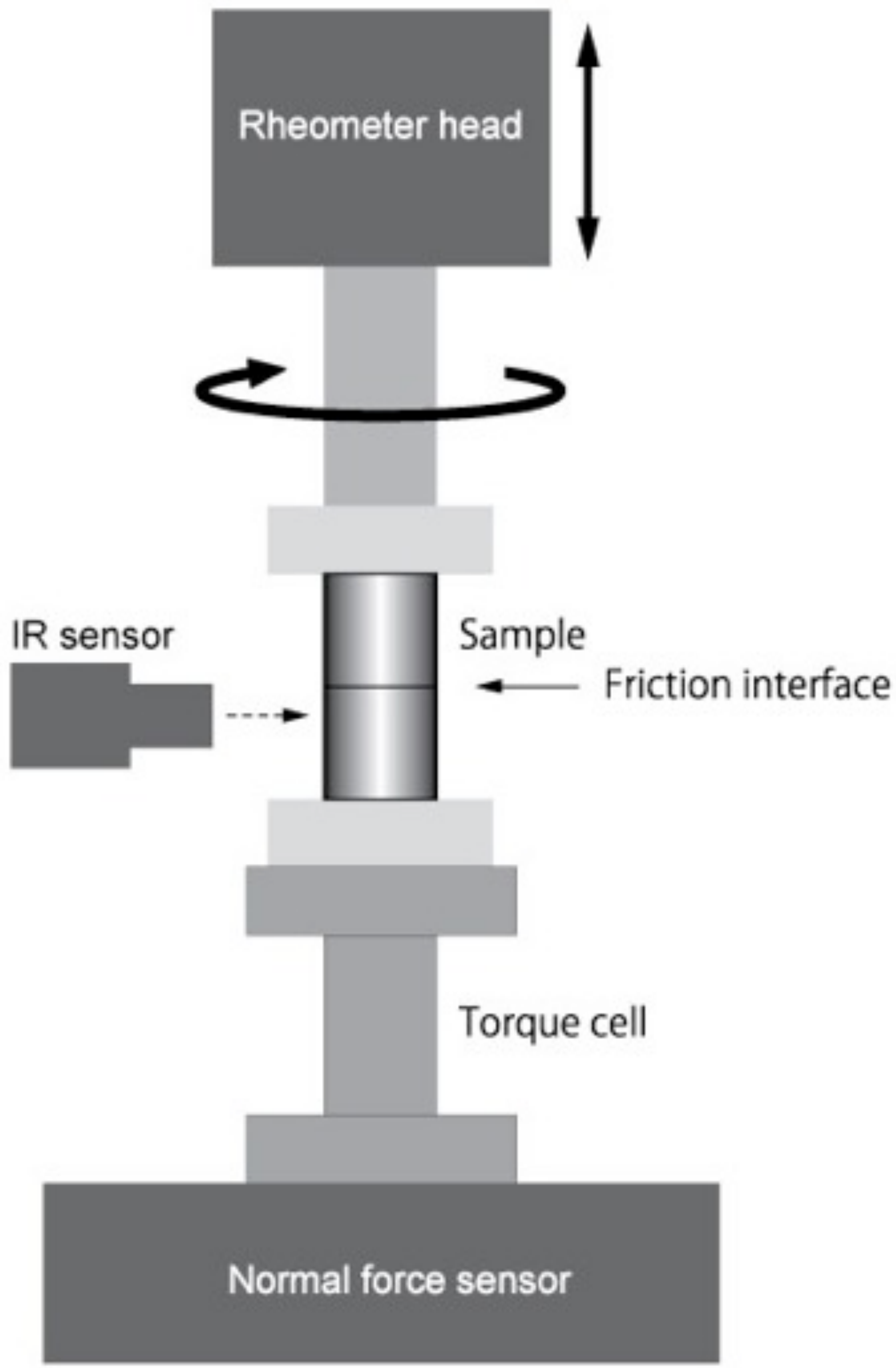
2. Inertial-number description valid for gouge?

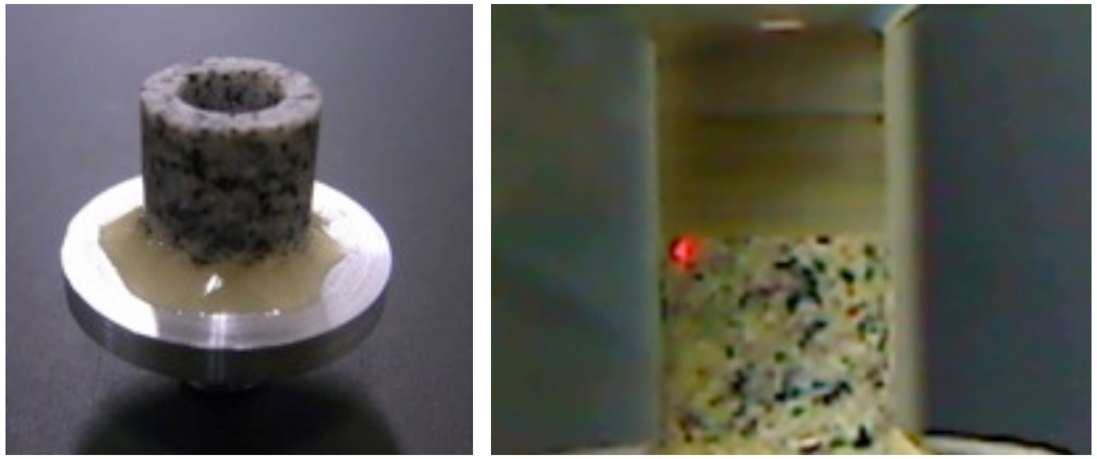


versus



experimental

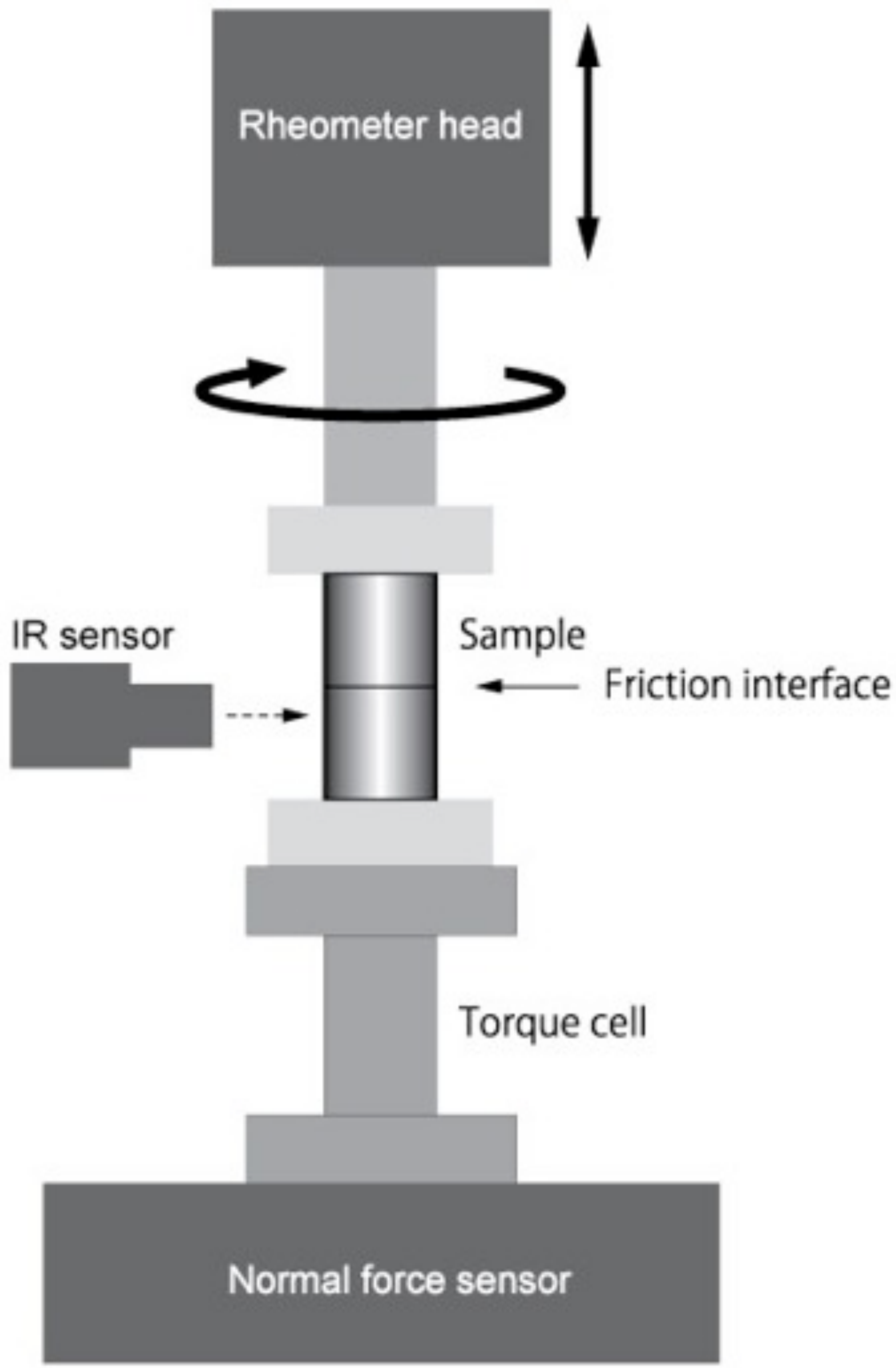


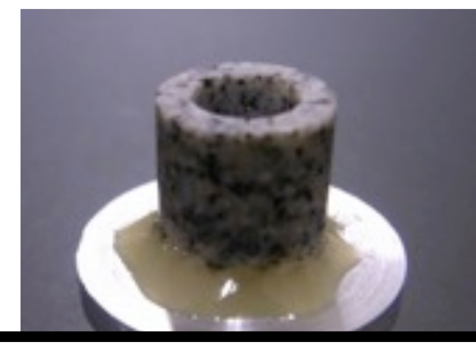


- Slip rate : 100μm/s to 0.3m/s
- Normal stress: 0.1-0.9 MPa
- Room temperature and humidity
- Cylindrical Specimen
  - Westerly granite
  - Inner/outer diameters : 6mm/10mm.
- Temperature measurement with an IR sensor
- Gouge layer formed by preshearing.

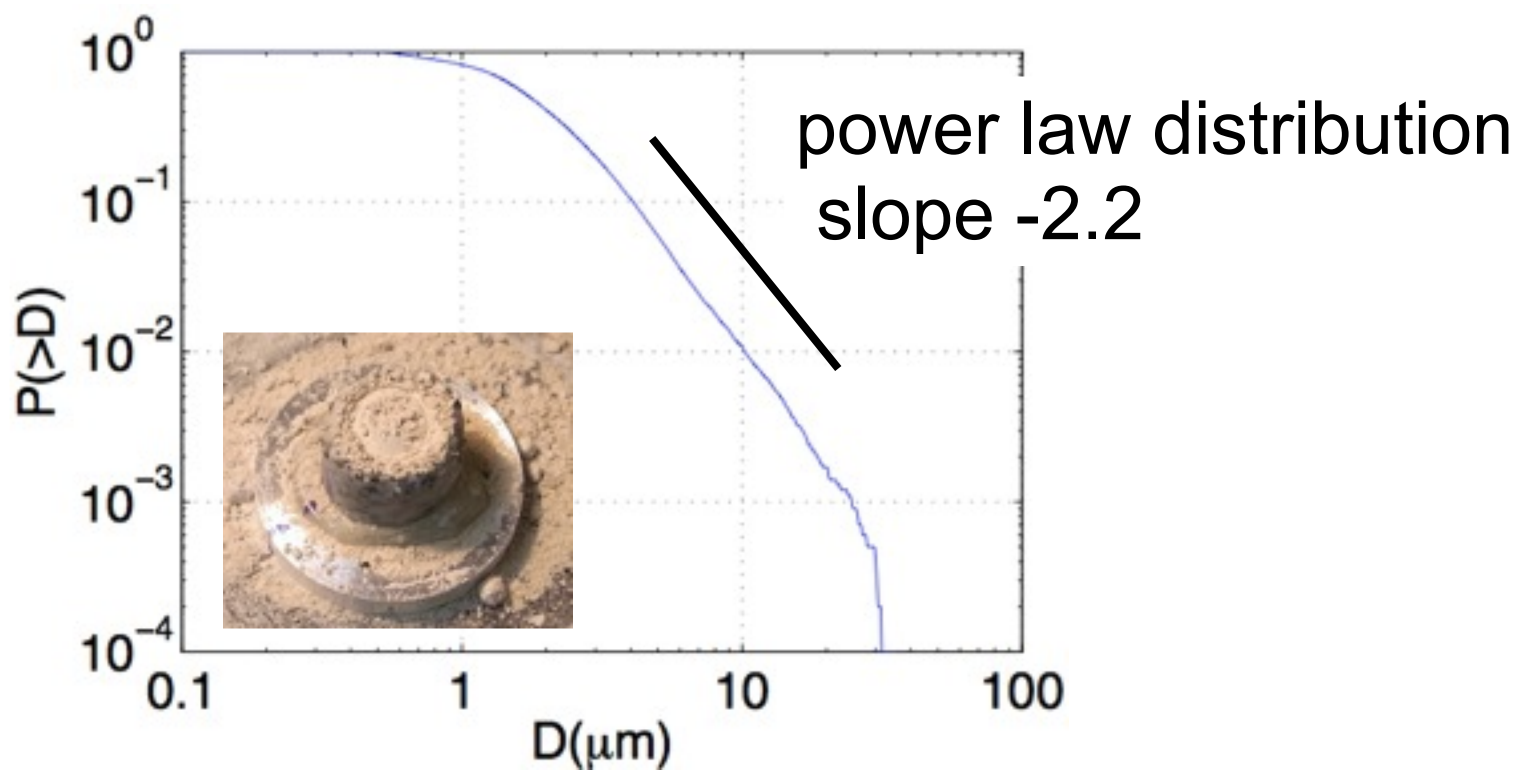
(sample not sealed; an open system)

experimental



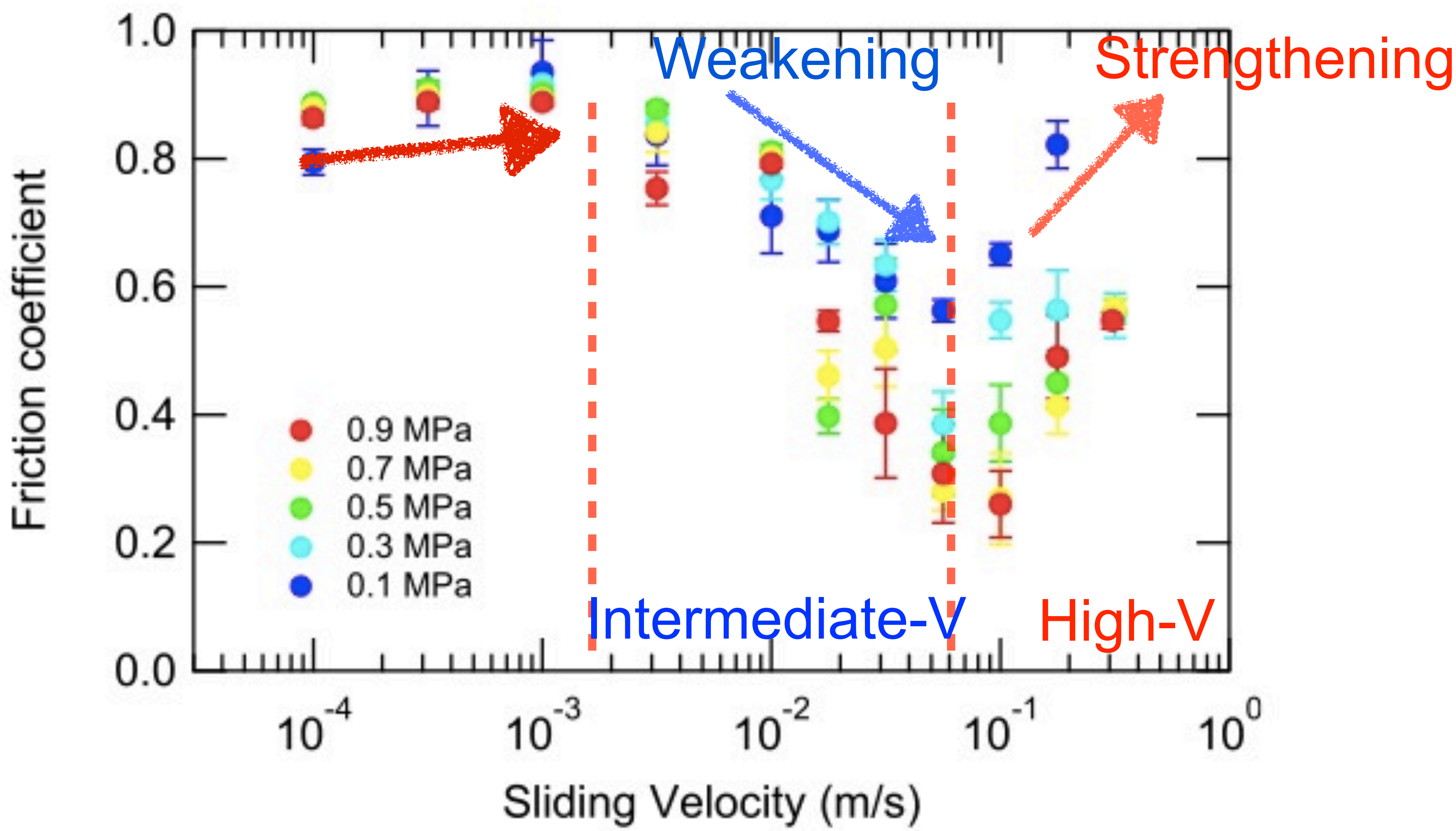


Cumulative particle-size distribution



Mean particle size: 2.4 μm
Gouge layer thickness : ~10μm (4-5 particles)

velocity dependence of friction coefficient

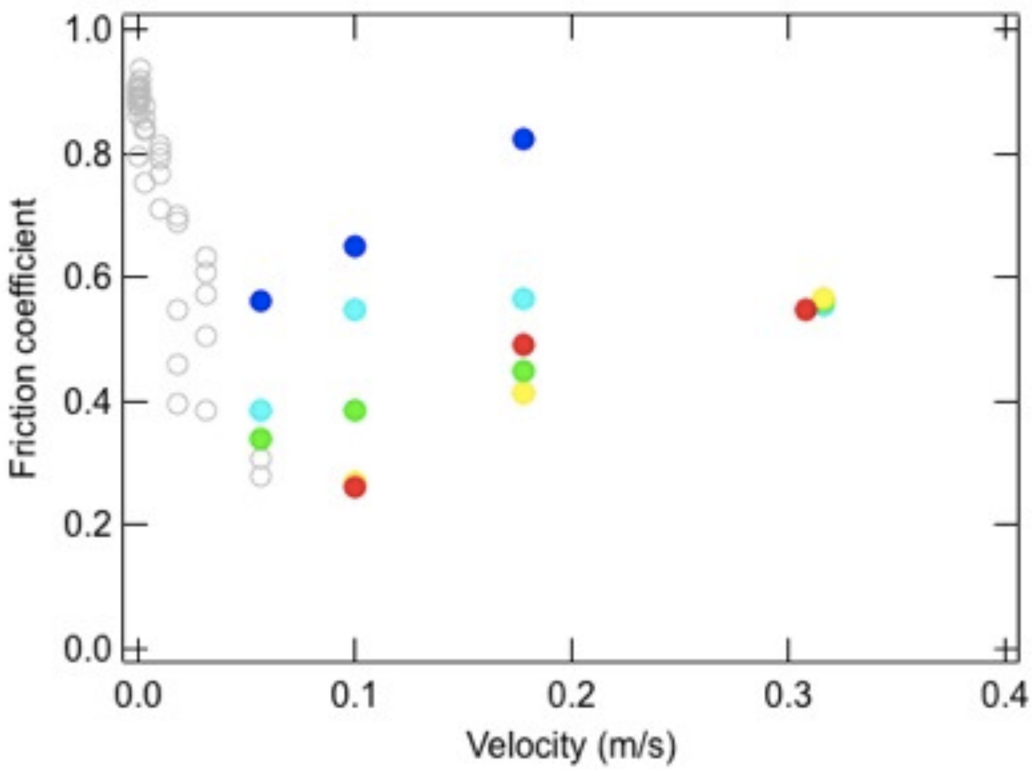


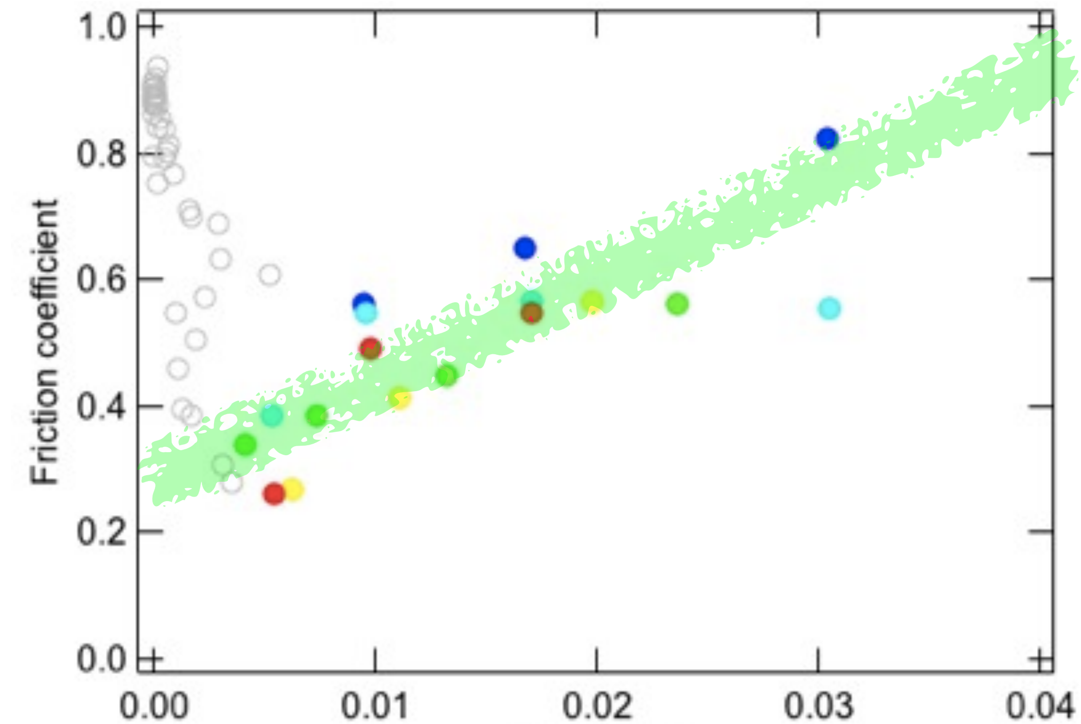
remarkable weakening in intermediate regime

$$\mu = \mu_* + \alpha \log \frac{V}{V_*} \quad ? \qquad \alpha \simeq -0.2 \quad \text{too large!}$$

e.g. Goldsby & Tullis 2002; di Toro et al. 2004, etc...

inertial number description?





$V\sqrt{\rho/P}$

Velocity

$$\mu = \mu_* + cI$$

Inertial number

$$c \simeq 20$$

Inertial number description verified for gouge!

★ data do not collapse completely due to fluctuation in gouge layer thickness

★ constant c is much larger than glass beads (wide size dispersity?)

conclusions

1. Negative to positive rate dependence of friction

$$\mu = \mu_0 + \alpha \log(\dot{\gamma}/\dot{\gamma}_0) + c\dot{\gamma}\sqrt{m/Pd}$$

$$I_c = \alpha/c$$

$I_c = O(10^{-3})$ in this system

2. Inertial-number description valid for gouge

(with power-law size distribution)

3. Anomalous weakening in intermediate regime?

<u>References:</u>

Kuwano, Ando, Hatano, Geophys. Res. Lett. 40, 1295 (2013)

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Kuwano, Hatano, Geophys. Res. Lett. 38, L17305 (2011)