Physics of Granular flows, YITP, Kyoto, 26 June 2013

Memory of plastic fluid and its application to control pattern formation of desiccation cracks

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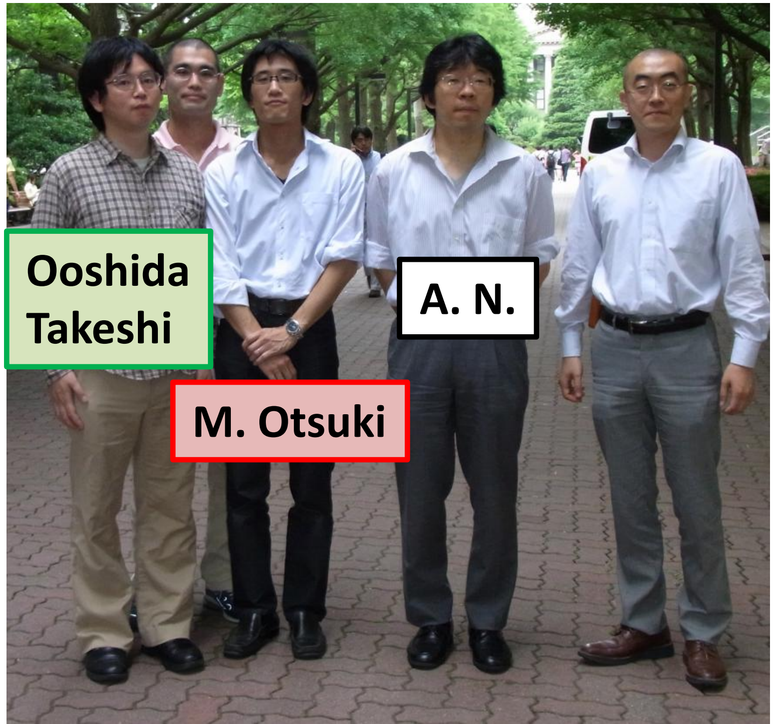
Nara Women's Univ.	S. Kitsunezaki

Nihon Univ.

Tottori Univ.

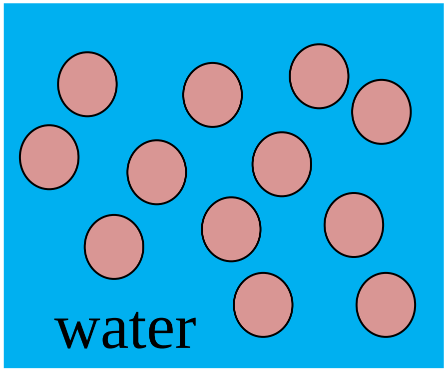
Shimane Univ.

Nara Women’s Univ.



Desiccation cracks of colloidal suspension

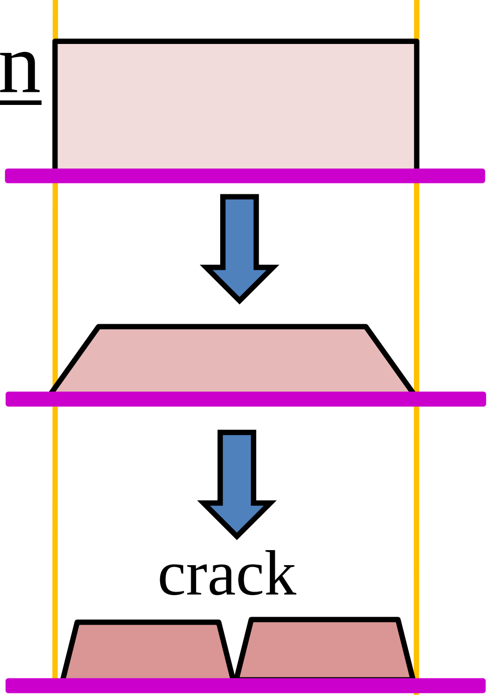
Colloidal particles
(~μm)



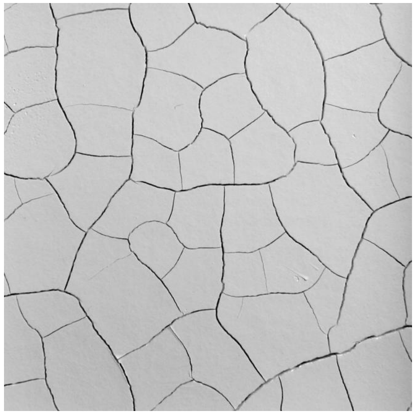
Contraction due to drying
+
Adhesion at the bottom

To release ↓ stress horizontally

Formation of cracks
spacing ~ depth of paste

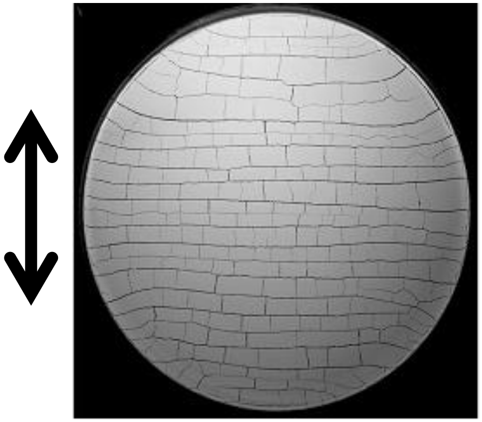


Usually cellular isotropic cracks



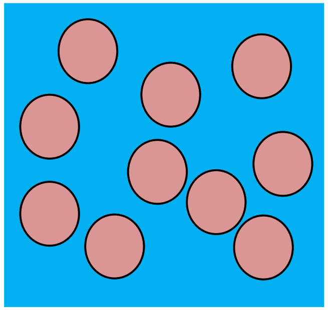
Crack patterns produced by memory effect

Memory is induced by plasticity



vibration

=

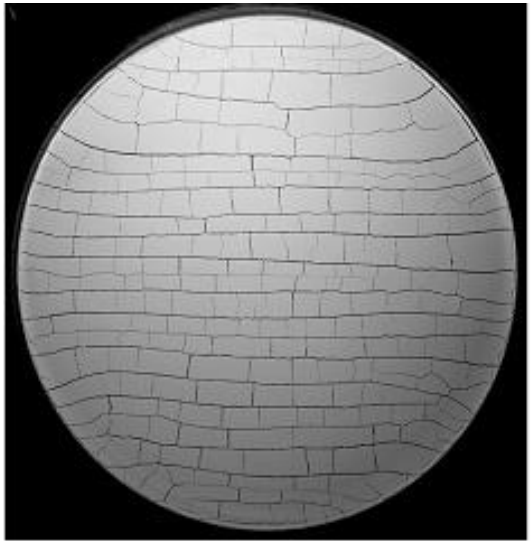


Random (jamming)

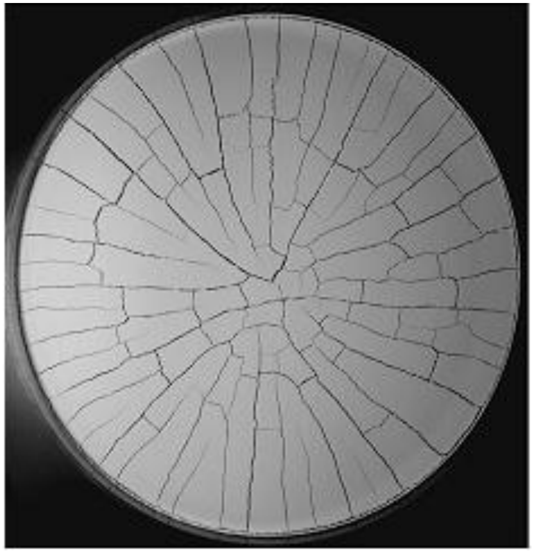
+

Structure? (memory)

Desiccation crack patterns produced by memory effect



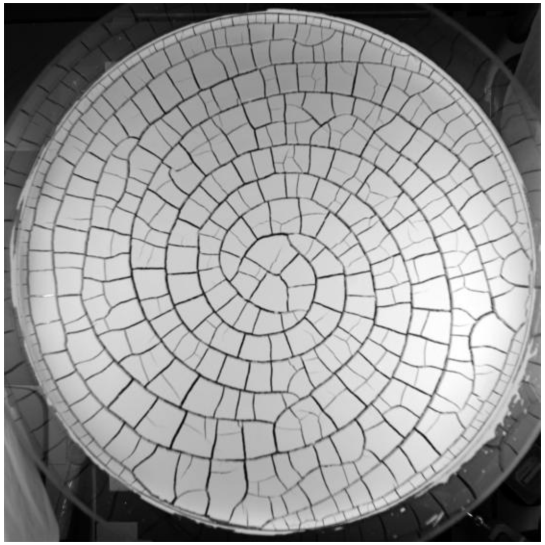
Lamellar



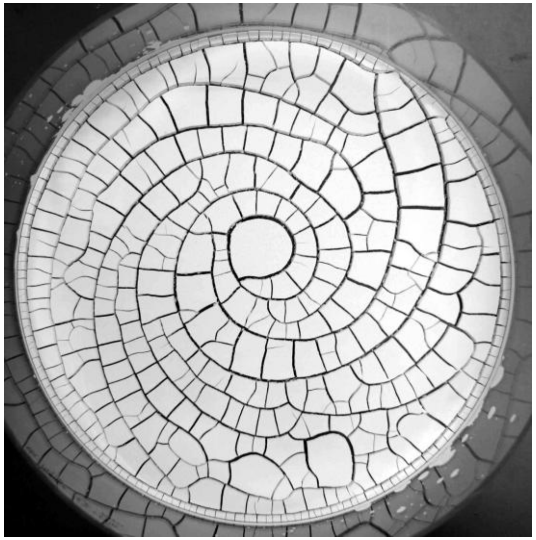
Radial

lattice

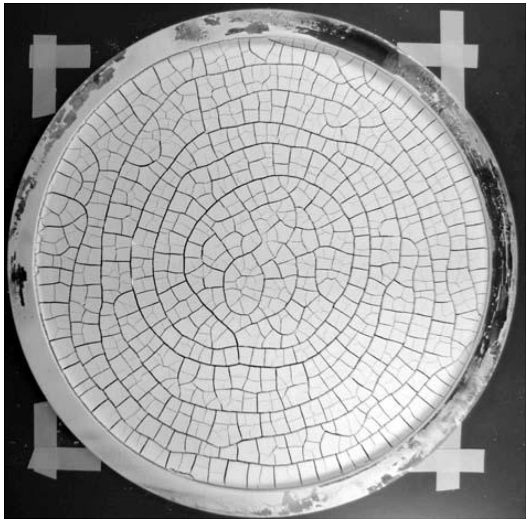
500mm



Spiral



Spiral

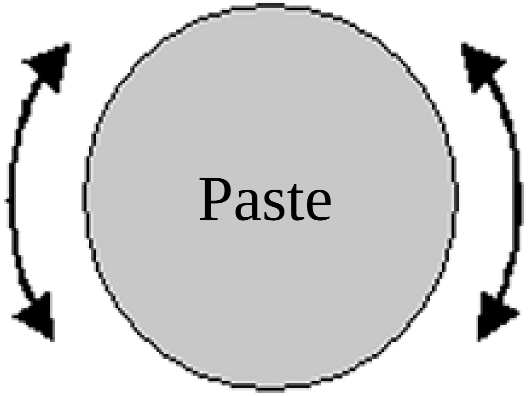


Ring

Memory of vibration that water-poor $CaCO_3$ paste has

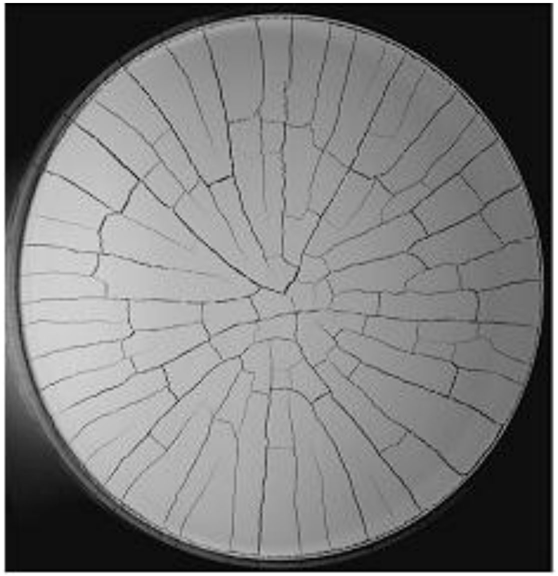
Water-poor $CaCO_3$ paste

In angular direction



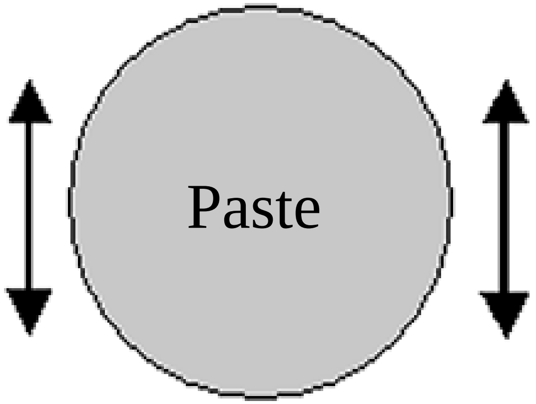
Stop vibration & dry

movie



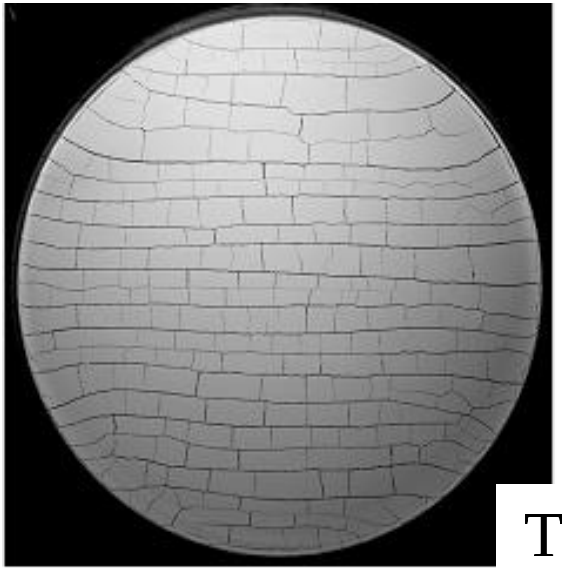
radial

In one direction



Stop vibration & dry

movie



lamellar

Memory of vibration

Perpendicular to vibration

Not by boundary effect

Thickness 10mm
Diameter 500mm

Memory of vibration

A. Nakahara and Y. Matsuo
J. Phys. Soc. Jpn. 74 (2005) 1362.

⊕ charged

Initial vibration (horizontally)

Stop vibration & dry

Crack formation (top view)

Water-poor CaCO$_3$ paste

Paste is vibrated

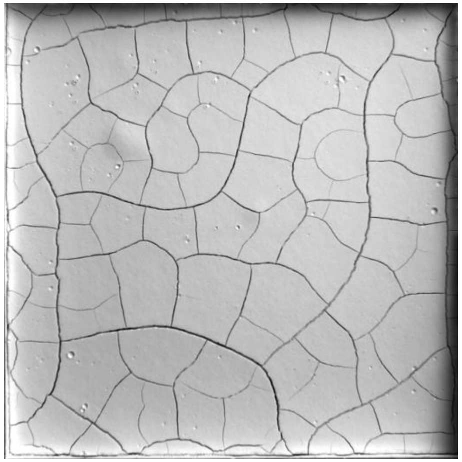
Lamellar cracks
(perpendicular to vibration)

Memory of vibration

Water-rich CaCO$_3$ paste

Paste is vibrated and fluidized

Cellular cracks
(isotropic and homogeneous)



No memory

Uncharged Pastes remember vibration and flow !

Magnesium Carbonate Hydroxide, Carbon, Kaolin, etc.

A. Nakahara and Y. Matsuo
PRE. 74 (2006) 045102.

Initial vibration

Stop & dry

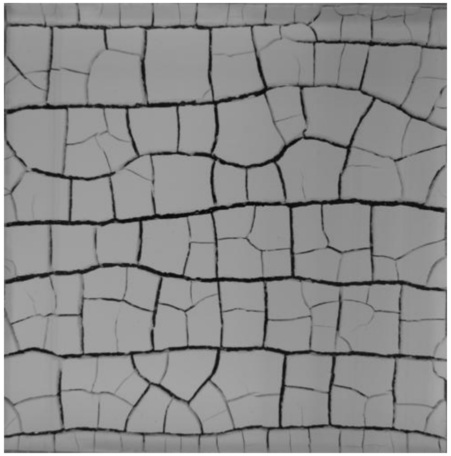
Crack formation

Uncharged

Water-poor

paste

vibration

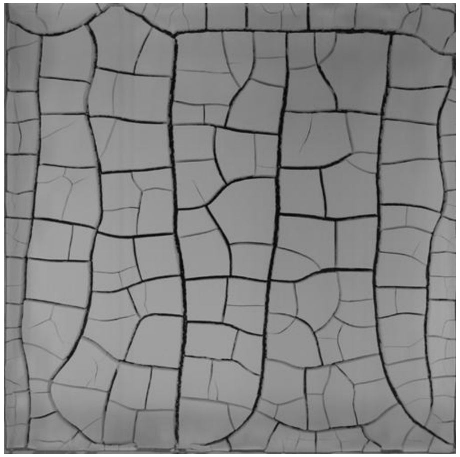


Memory of vibration

Water-rich

paste

flow

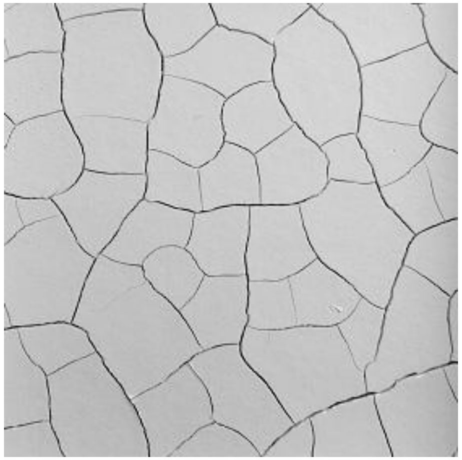


Memory of flow

Water-too-much

paste

Turbulence



No memory

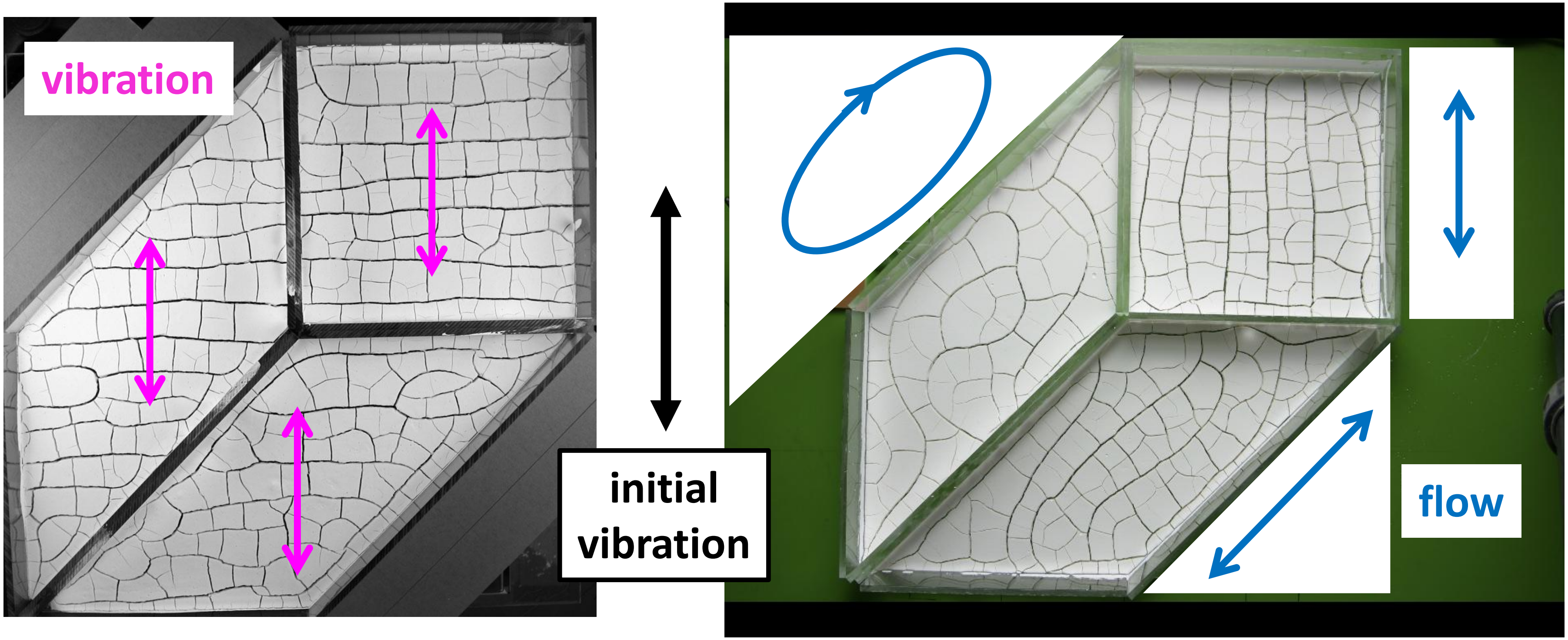
Difference between two memories

Water-poor paste

Lamellar cracks
(perpendicular to vibration)
Memory of vibration

Water-rich & uncharged paste

Lamellar cracks
(parallel to flow)
Memory of flow



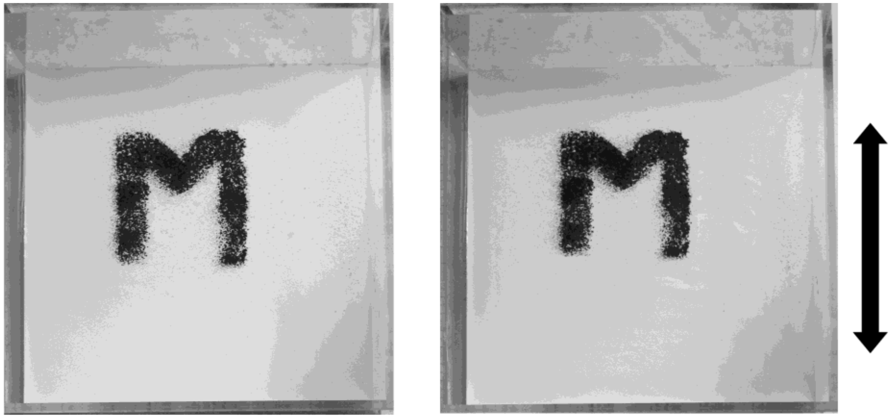
all perpendicular to vibration

parallel to local flow direction

movie

Quantitative difference between vibration and flow

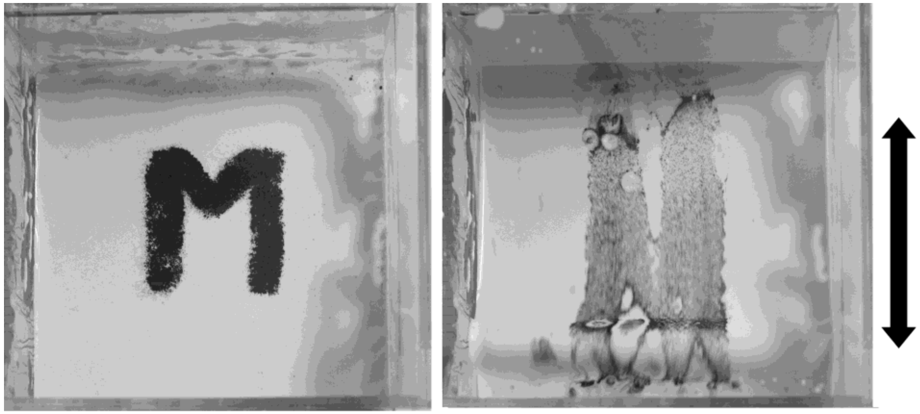
Water-poor paste



(a) before vibration (b) under vibration

before vibration under vibration

Water-rich paste



(a) before flow (b) under flow

before flow under flow

Check deformation of letter **M** (written by carbon powder)

Faint plastic deformation under vibration

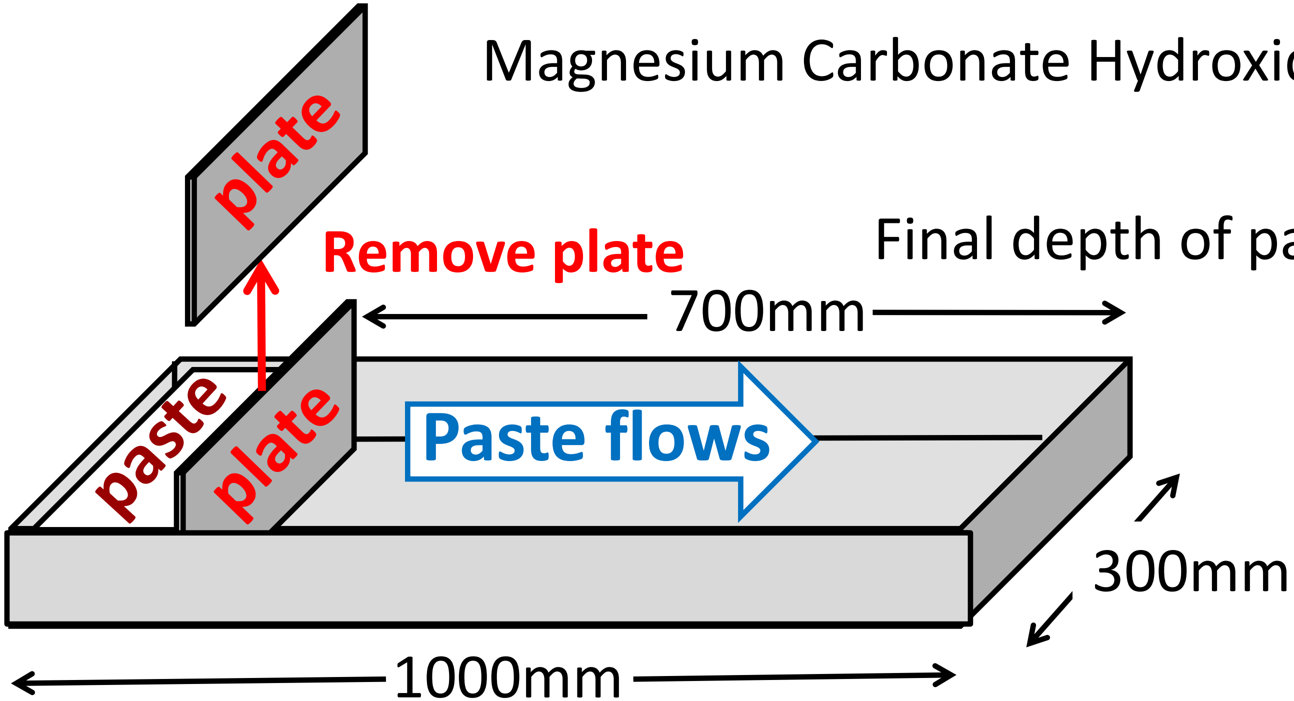
Large scale deformation (elongation) under flow and shear

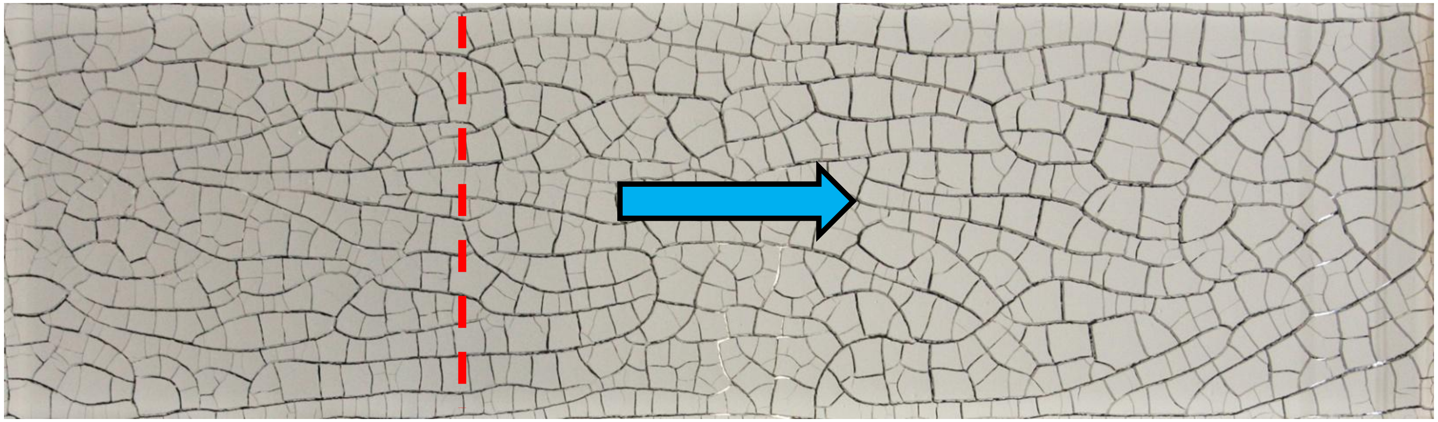
We need to control Strain and Shear rate in future experiments

Only flow experiment (without vibration)

Magnesium Carbonate Hydroxide, ρ=7.7%

Final depth of paste = 2cm

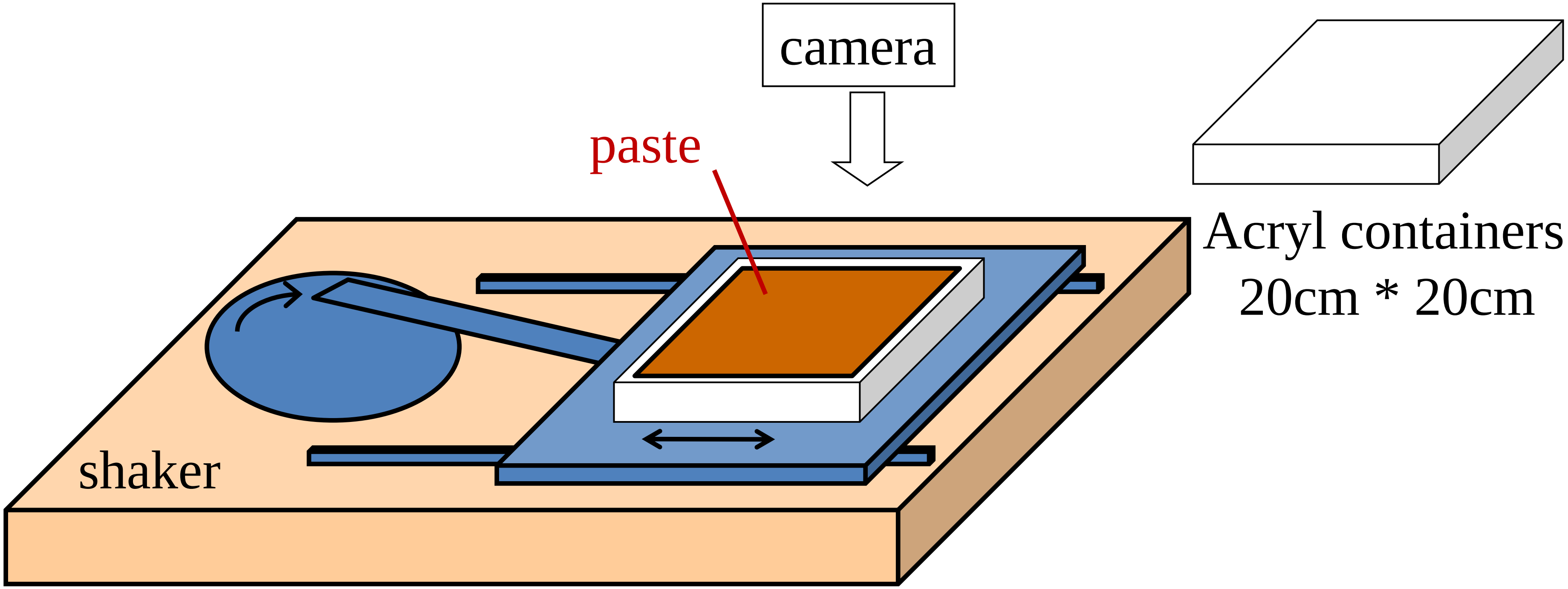




Just a Flow

Memory of flow (cracks parallel to flow)

Experiment to make morphological phase diagrams



Initial external vibration

for 60 sec, amplitude 15 mm, frequency 30~150 rpm

Controlling parameter in experiments

- solid volume fraction ρ
- strength of the initial vibration

  amplitude r , frequency f

Change systematically

Morphological Phase Diagram of **uncharged** paste

magnesium
carbonate
hydroxide

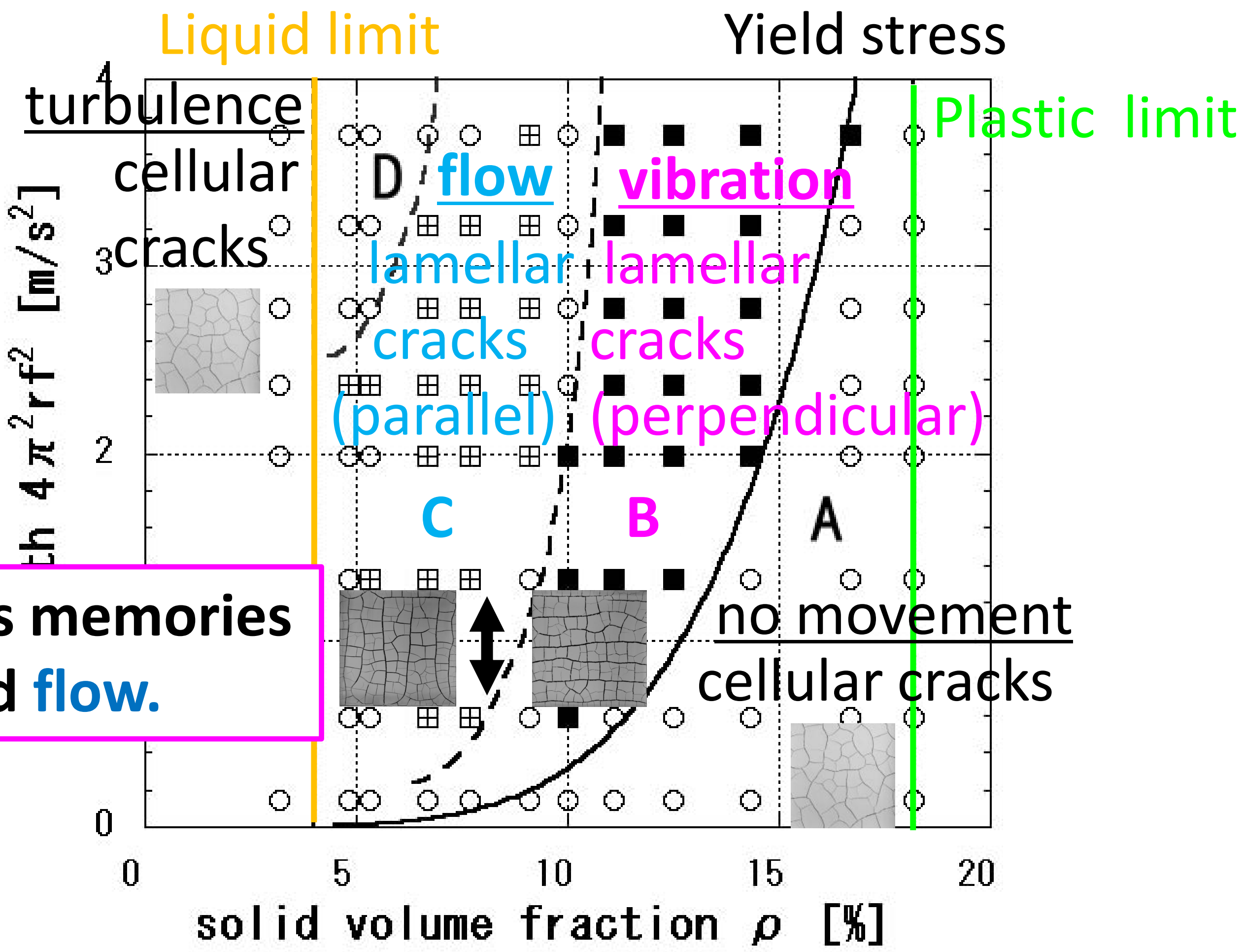
Movies of

no movement

vibration

flow

t



Uncharged paste has memories of vibration and flow.

A no movement → no experience → cellular cracks (no movement)

B vibration → remember → Lamellar cracks (perpendicular)

C flow → remember → **Lamellar cracks** (parallel)

D turbulence → forget → Cellular cracks (no memory)

Morphological Phase Diagram of **$CaCO_3$** paste

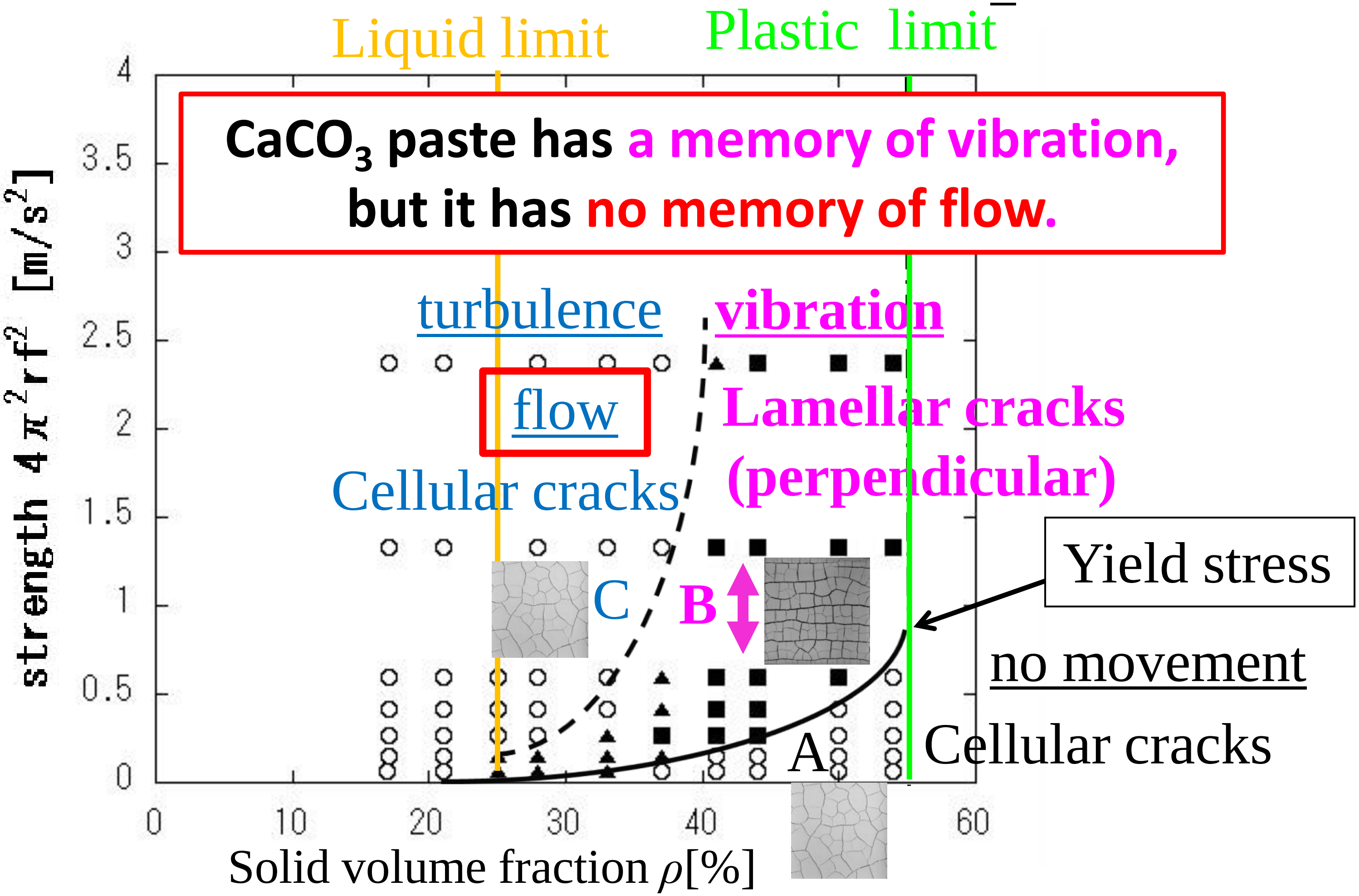
Movies of

[no movement]

[vibration]

[flow]

[turbulence]


$CaCO_3$ paste has a memory of vibration, but it has no memory of flow.

A	no movement	→ no experience	→	cellular cracks (no movement)
B	**vibration**	→ **remember**	→	**Lamellar cracks** (perpendicular)
C	flow	→ forget	→	Cellular cracks (no memory)
	turbulence	→ forget	→	Cellular cracks (no memory)

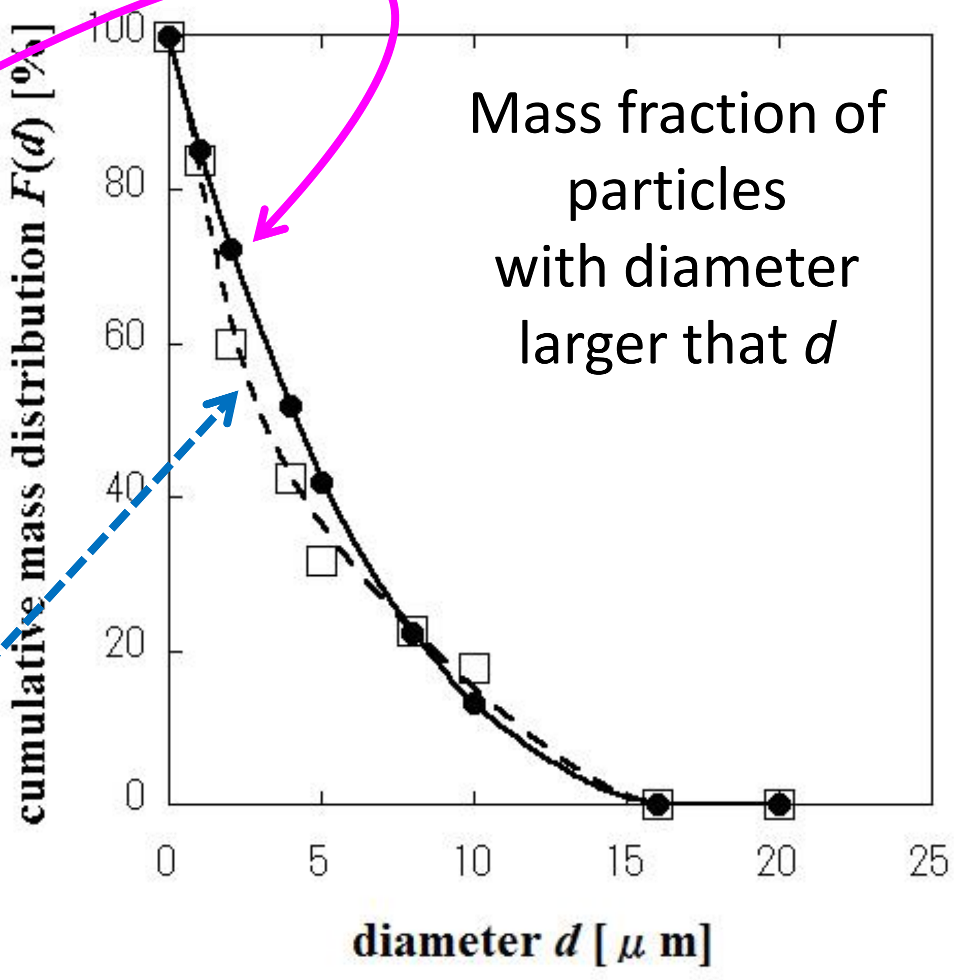
Why does $CaCO_3$ paste have no memory of flow?

$CaCO_3$

Only memory of **vibration**

magnesium carbonate hydroxide

Memory of **vibration** and **flow**



Mass fraction of particles with diameter larger that *d*

Similar size distribution

Shape of particles

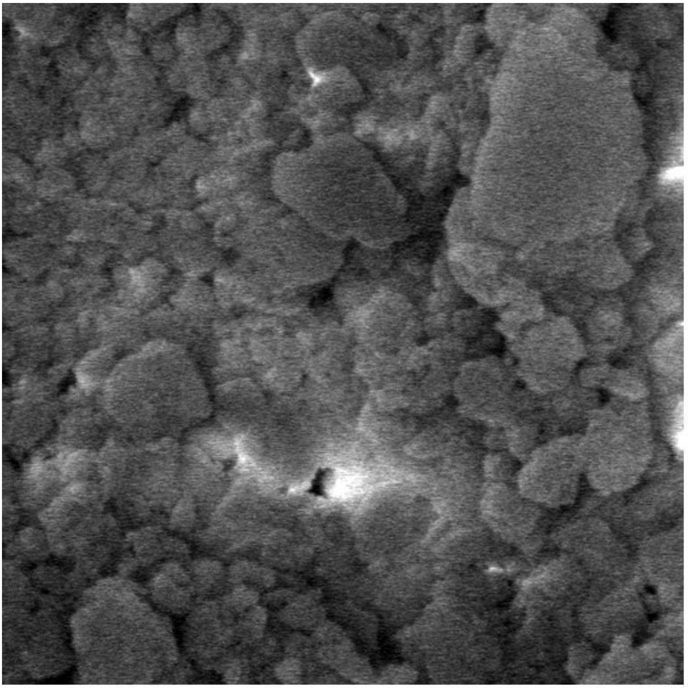
$CaCO_3$ paste has
only memory of
vibration

magnesium carbonate hydroxide
kaolin, carbon, …
who have
memory of **vibration**
+ memory of **flow**

Rough

isotropic

1µm



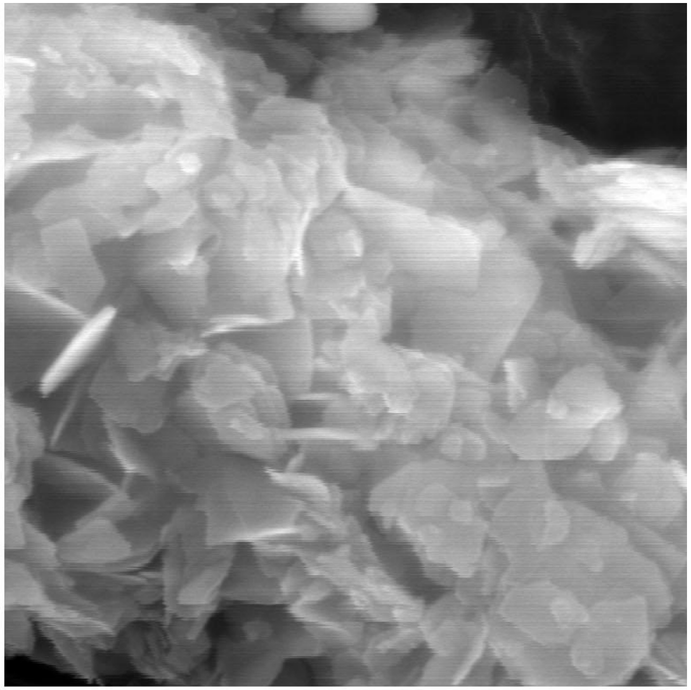


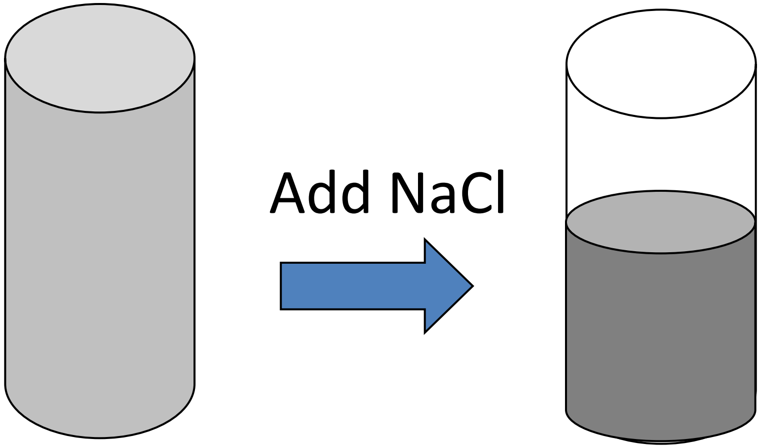
Plate-
like

1µm

For memory of **flow**

Shape dependence ?

Coulombic repulsion between $CaCO_3$ particles



Dispersion of $CaCO_3$ particles by Coulombic repulsion

Add NaCl

Screening effect by Na^+ ion

↓

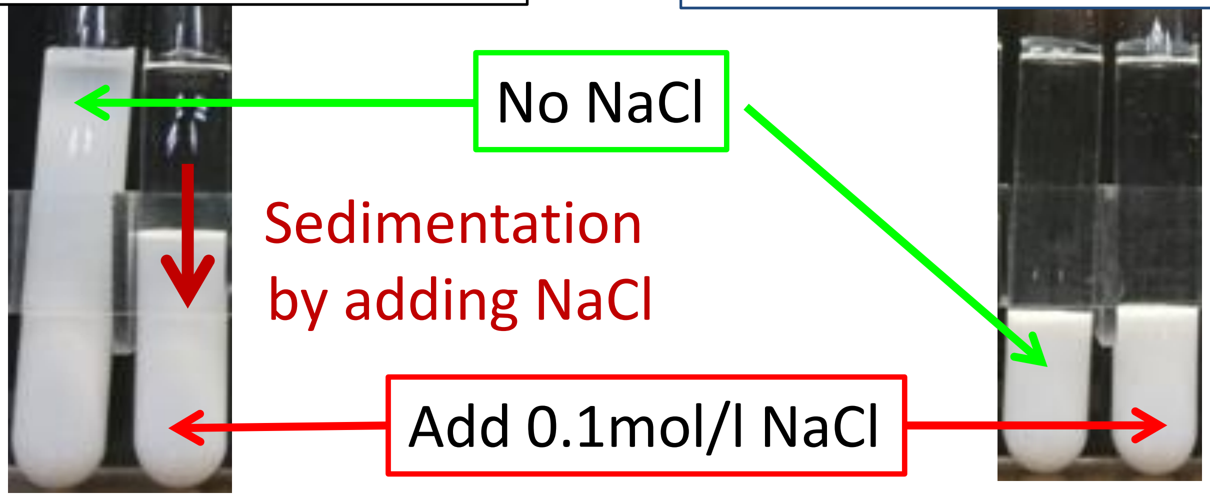
Aggregation

↓

Sedimentation

$CaCO_3$
(memory of only **vibration**)

magnesium carbonate hydroxide
(memory of **vibration** and **flow**)



No NaCl

Sedimentation by adding NaCl

Charged

Add 0.1mol/l NaCl

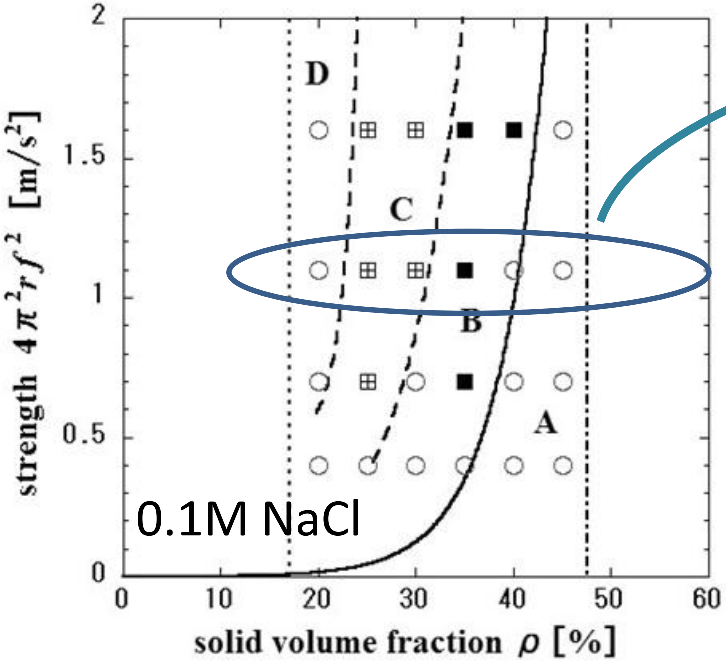
Sedimentation any case

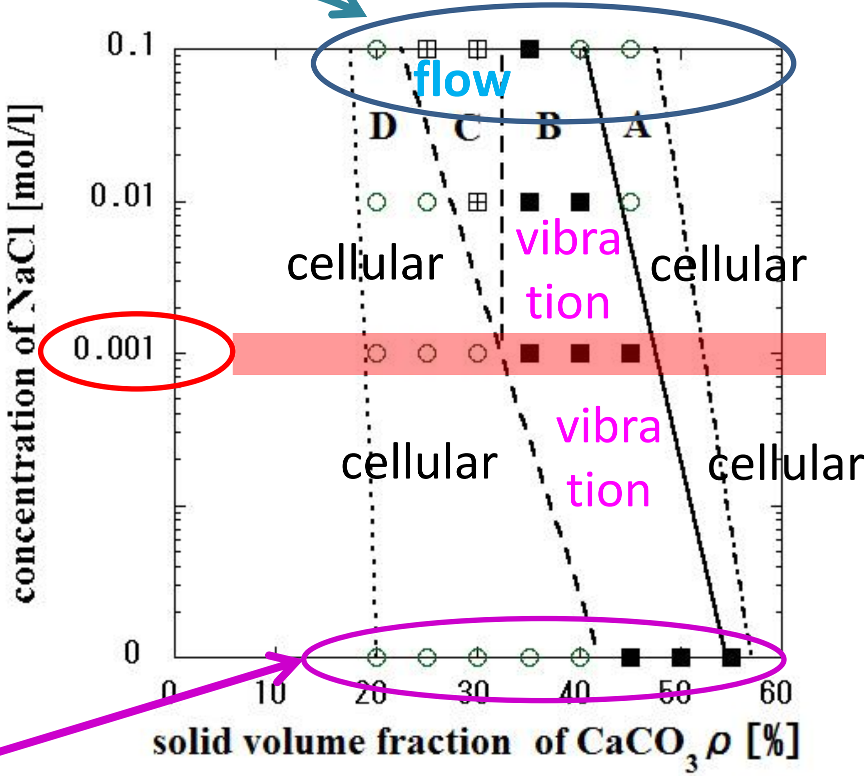
Uncharged

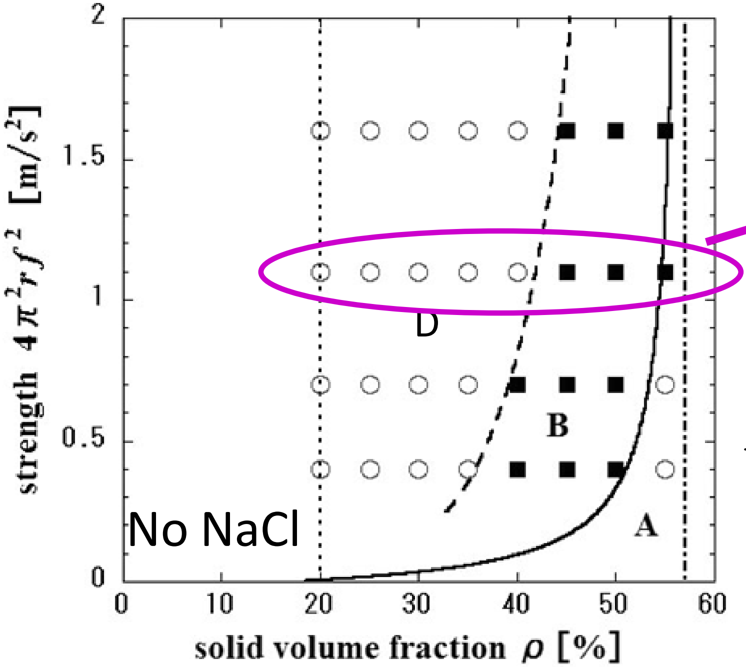
Particles interact with Coulombic repulsion

Particles interact without Coulombic repulsion

Add NaCl and CaCO$_3$ paste can remember flow direction





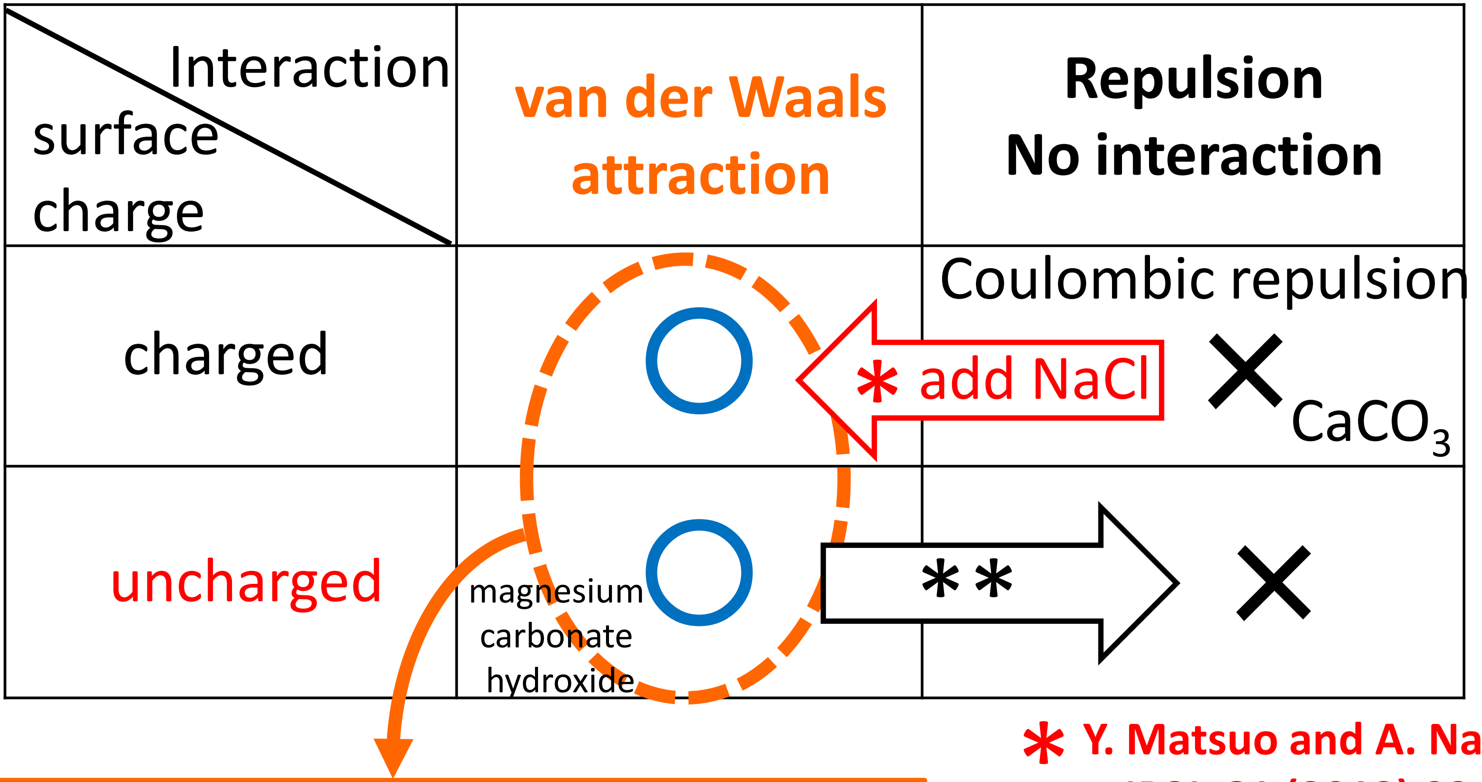


What determines the value of this threshold?

- Debye length = $0.304/\sqrt{[\text{NaCl}]}$ (0.96nm for 0.1M)
- Mean particle radius = 2μm
- Mean interparticle distance = 6μm (for ρ=30%)
- Distance between surfaces of particles = ?

Non-dimensional parameter ? Dynamics?

Mechanism for memory of flow…Attractive interaction

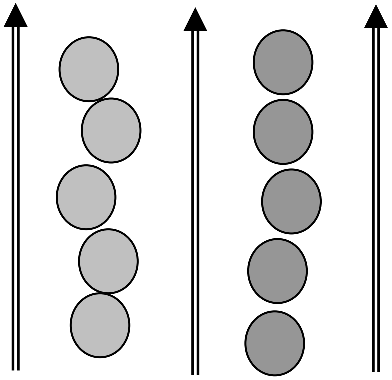


Interaction surface charge	van der Waals attraction	Repulsion No interaction
charged	○	Coulombic repulsion ✕ $CaCO_3$
uncharged	magnesium carbonate hydroxide ○	✕

Dilute network is formed
by van der Waals attraction

* Y. Matsuo and A. Nakahara
JPSJ. 81 (2012) 024801.

** In preparation

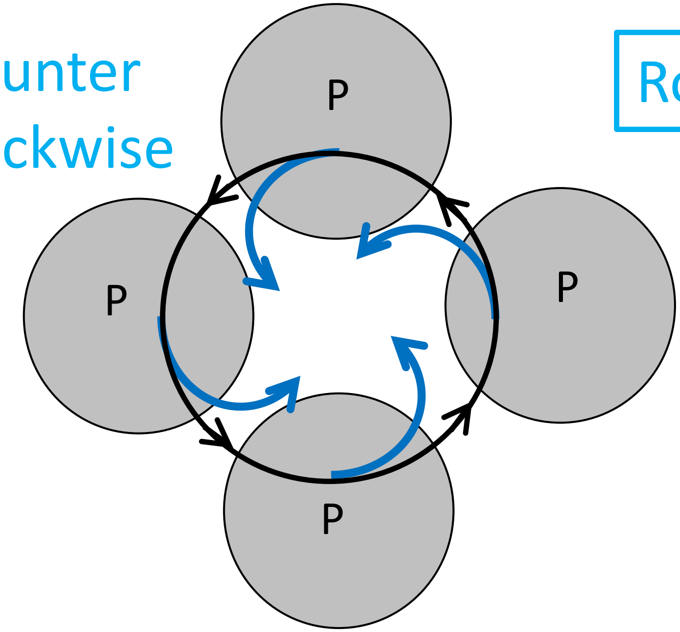


Attractive interaction is needed
for the memory of flow.

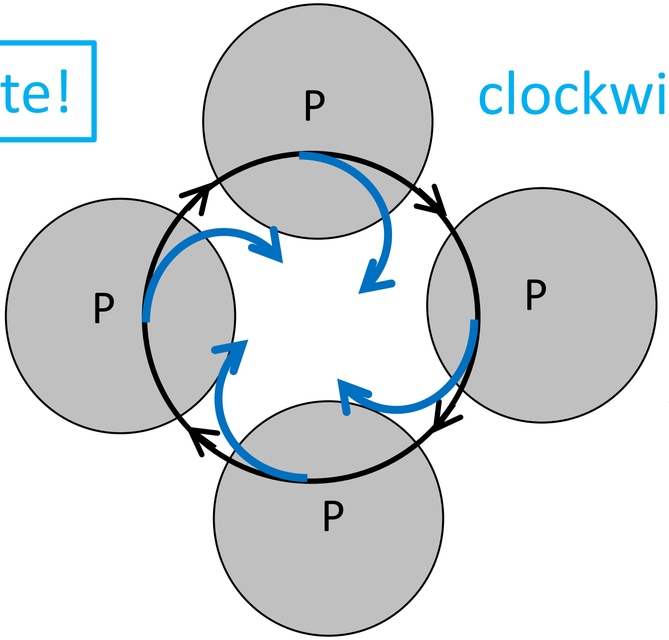
Flow ⇒ elongation of dilute network

Spiral crack patterns made by memory of flow

counter
-clockwise



Rotate!



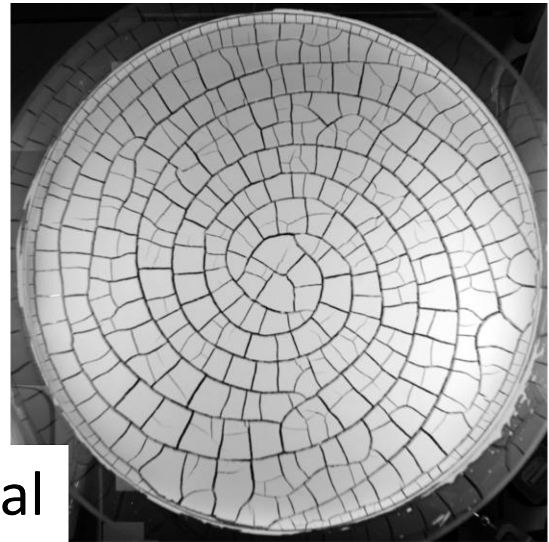
clockwise

Stop rotation & dry

Stop rotation & dry

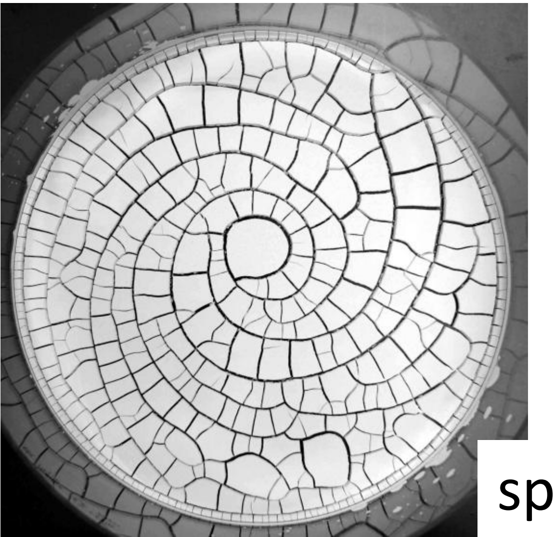
Parallel to final flow

movie



spiral

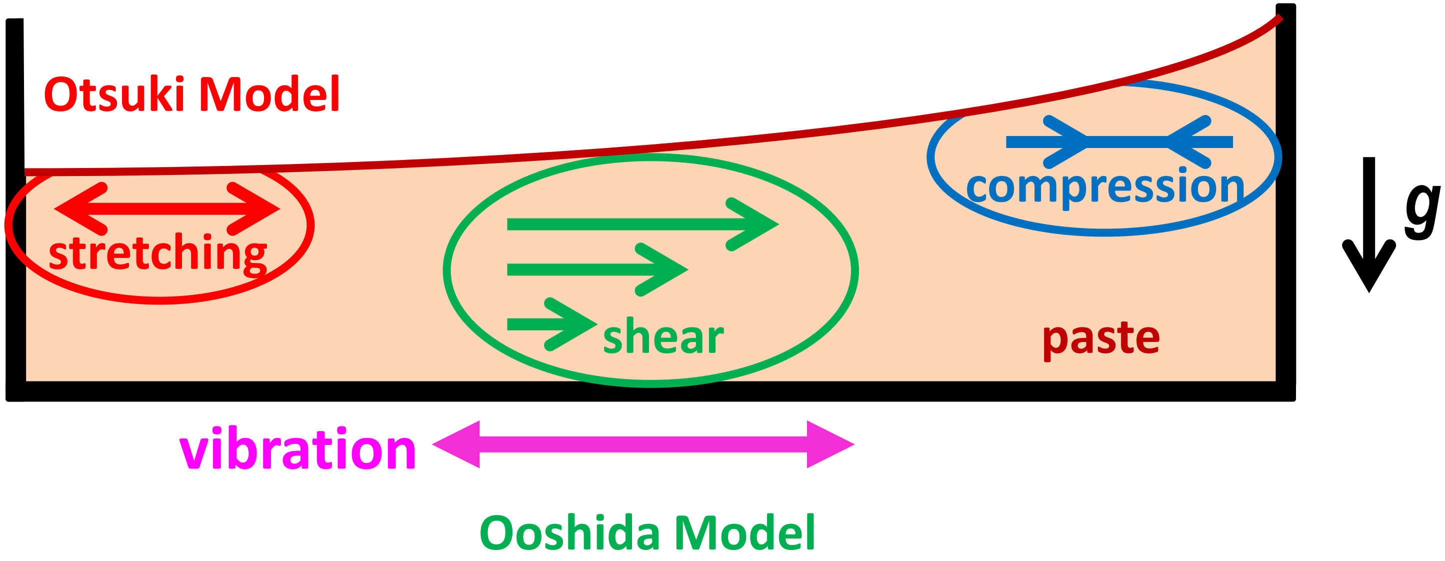
diameter
500mm



spiral

Memory
of
flow

Mechanism for memory of vibration…Residual tension



Two Residual Tension Theories

Model A: horizontal stretching mechanism

Non-uniform horizontal stretching, global model

M. Otsuki, PRE 72 (2005) 046114.

Model B: shear deformation mechanism

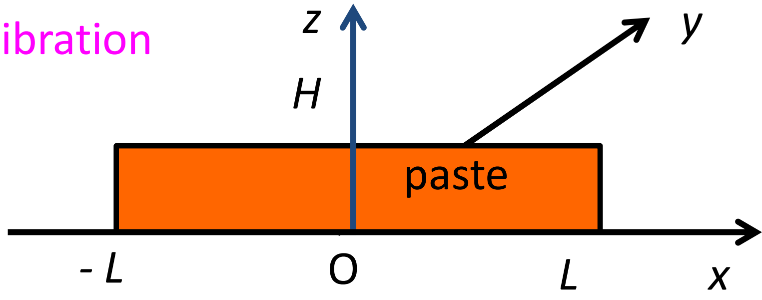
Longitudinal tension due to nonlinear elasticity,

local model

Ooshida T., PRE77 (2008) 061501, JPSJ 78 (2009) 104801.

Model A: Otsuki Model on memory of vibration

by M. Otsuki, PRE. **72** (2005) 046114.



Displacement in x direction, *u*(*x*, *z*, *t*)

$$\gamma \frac{\partial u(x,z,t)}{\partial t} = \frac{\partial \sigma_{xx}(x,z,t)}{\partial x} + \frac{\partial \sigma_{xz}(x,z,t)}{\partial z} + \alpha(t)$$

vibration

Plastic shear strain, *s*(*x*, *z*, *t*) (Bingham type)

$$B\frac{\partial s(x,z,t)}{\partial t} = \begin{cases} 0 \quad , & |\sigma_{xz}| < \sigma_Y(t) \\ \left[\, |\sigma_{xz}| - \sigma_Y(t)\, \right] \dfrac{\sigma_{xz}}{|\sigma_{xz}|}, & |\sigma_{xz}| \geq \sigma_Y(t) \end{cases}$$

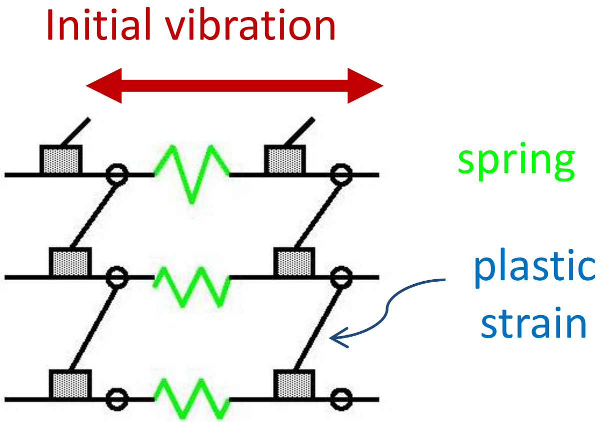
Normal stress, $\sigma_{xx}(x,z,t)$ (λ, μ; Lame coefficient)

$$\sigma_{xx}(x,z,t) = (\lambda + 2\mu)\left(\frac{\partial u(x,z,t)}{\partial x} + c(t)\right)$$

shrinkage

Shear stress, $\sigma_{xz}(x,z,t)$

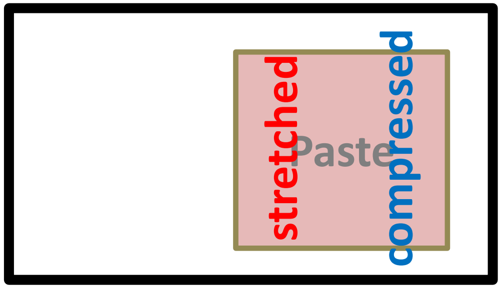
$$\sigma_{xz}(x,z,t) = \mu\left(\frac{\partial u(x,z,t)}{\partial z} - s(x,z,t)\right)$$



0 T_1 T_2 T_3 t

	vibration	relaxation	relaxation	dry
External force	$\alpha(t) = \alpha_M \sin(\pi t / T_1)$	$\alpha(t) = 0$	$\alpha(t) = 0$	$\alpha(t) = 0$
	← plastic deformation →		← no plasticity →	
Yield stress	$\sigma_Y(t) = \sigma_{Y0}$	$\sigma_Y(t) = \sigma_{Y0}$	$\sigma_Y(t) = \infty$	$\sigma_Y(t) = \infty$
Boundary cond.	$u(\pm L, z, t) = 0$	$u(\pm L, z, t) = 0$	$\sigma_{xx}(\pm L, z, t) = 0$	$\sigma_{xx}(\pm L, z, t) = 0$
Reference strain rate	$c(t) = 0$	$c(t) = 0$	$c(t) = 0$	$c(t) = b(t - T_3)$

Model A: Otsuki Model on memory of vibration



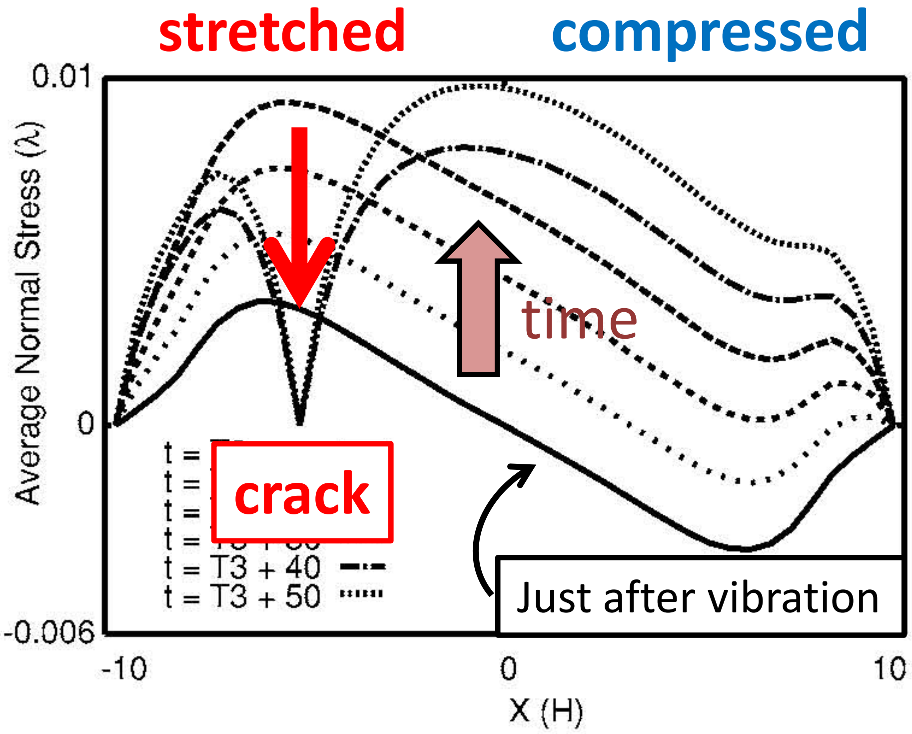
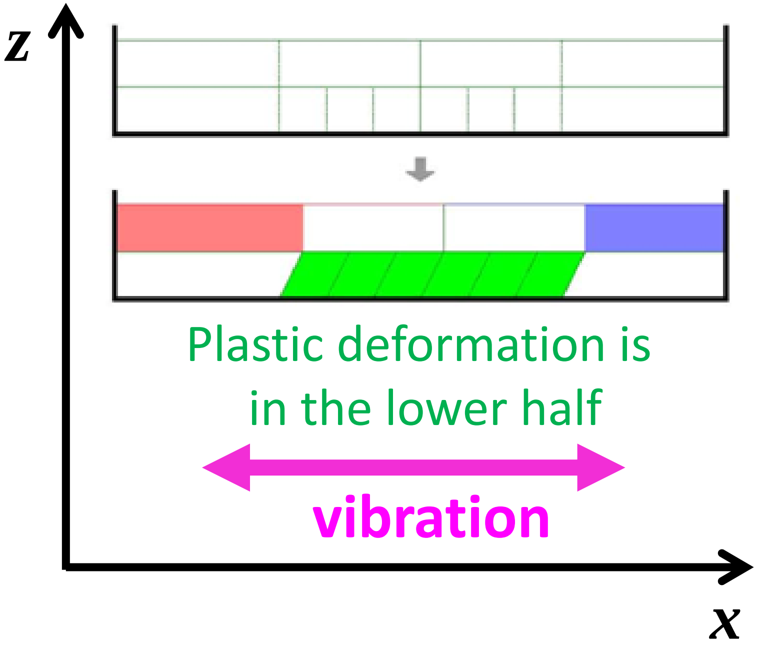


FIG. 6. The average normal stress $\langle \sigma_{xx}(x,t) \rangle$ in the case α_M=0.08 as a function of x.

Model B: Ooshida model

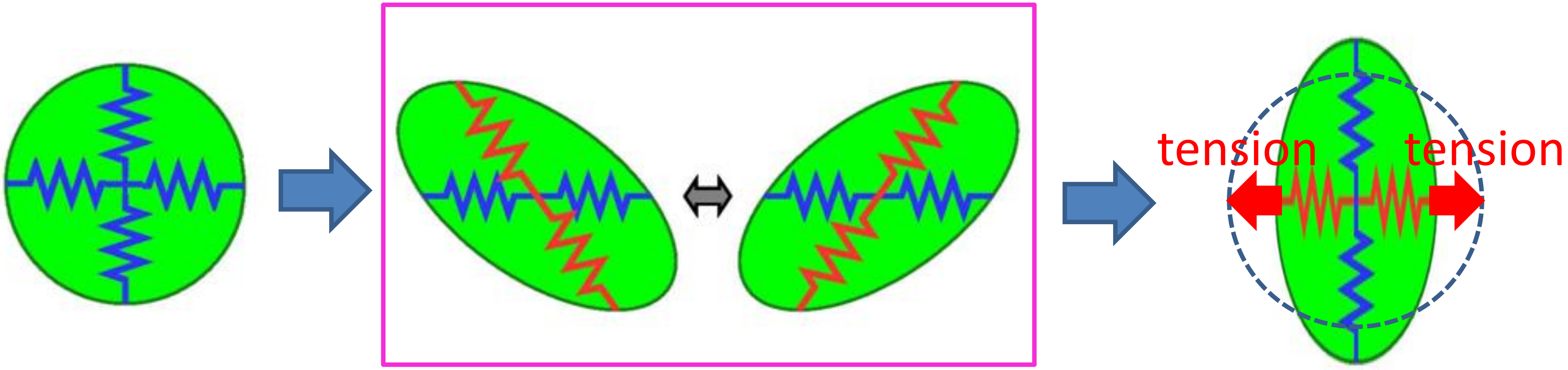
shear deformation mechanism

Longitudinal tension due to nonlinear elasticity



Plastic deformation is in the lower half

vibration



Vibration

||

Repeated shear
(oblique stretching)

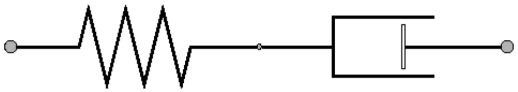
Horizontal
x-directional
tension remains

Model B: Ooshida Model on memory of vibration

Continuum model of a paste

Ooshida (2008) PRE **77**; Ooshida (2009) JPSJ **78**

- a kind of nonlinear Maxwell model of viscoelasticity



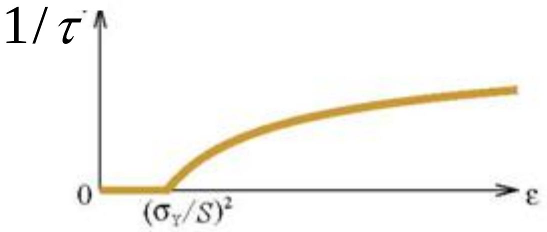
- formulation with label variable $\boldsymbol{\xi} = (\xi, \zeta)$ and natural metric $\mathsf{g}^\natural = (g^\natural_{ij}) = (g_\natural^{ij})^{-1}$ (cf. $T = \kappa\,(x - x^\natural)$)

$$\underset{\text{stress}}{\mathsf{P}} = P^{ij}\underset{\text{convected basis}}{(\partial_i \mathbf{r}) \otimes (\partial_j \mathbf{r})}, \quad P^{ij} = \tilde{p} g^{ij} + S\left(g^{ij} - g_\natural^{ij}\right) \quad (1)$$

$$\partial_t g_\natural^{ij} = -\nu g_\natural^{ij} + \nu_* g^{ij} \quad (2)$$

$\mathbf{r} = \mathbf{r}(\boldsymbol{\xi}, t)$, $g_\natural^{ij} = g_\natural^{ij}(\boldsymbol{\xi}, t)$; $g_{\xi\zeta} = (\partial_\xi \mathbf{r}) \cdot (\partial_\zeta \mathbf{r})$ etc.; $(g^{ij}) = (g_{ij})^{-1}$

- plasticity:
  relaxation time τ can diverge
  depending on $\varepsilon = \underset{\text{elastic energy}}{g_{ij} g_\natural^{ij}} - n_{\mathrm{d}}$



Model B: Ooshida Model on memory of vibration

Equation of motion

For momentum $\rho\mathbf{v}$ $(i = \xi, \eta;\ \mathbf{F} = F_x(t)\mathrm{e}_x - G\mathrm{e}_z)$

$$\boxed{\rho\left(\partial_t v^i + v^j \nabla_j v^i\right) = -\underset{\text{stress}}{\nabla_j P^{ij}} + \underset{\text{forcing}}{F^i}} \qquad (3)$$

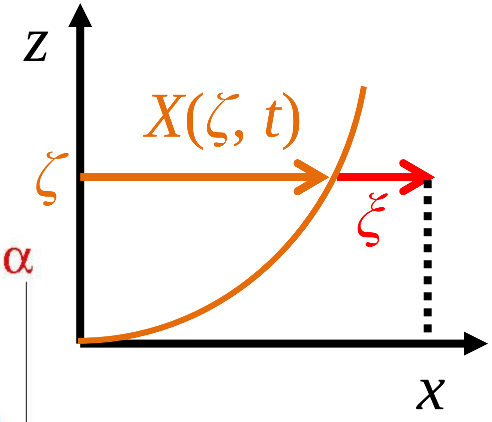
All vectors/tensors in **convected coordinate system**

$$\underbrace{\mathbf{v} = \sum v^i \partial_i \mathbf{r}}_{\text{contravariant}} = \underbrace{\sum v_i \nabla \xi^i}_{\text{covariant}} \quad \text{etc.} \qquad \underbrace{\nabla_j v^i = \partial_j v^i + \Gamma^i_{jk} v^k}_{\text{covariant derivative}}$$

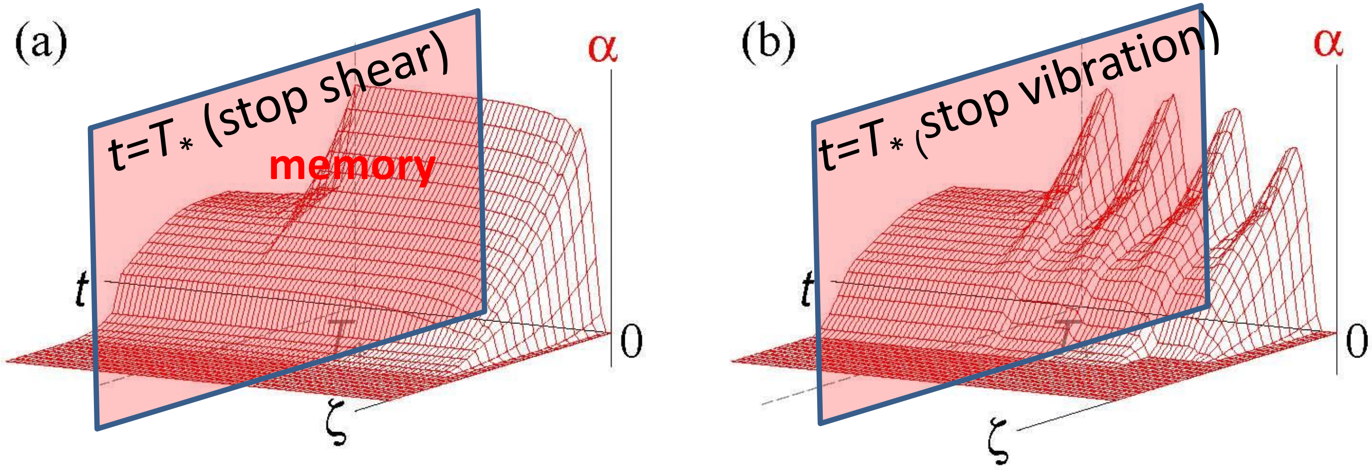
Boundary conditions

- free surface at $\zeta = \zeta_{\mathrm{max}}$
- no-slip condition at $\zeta = 0$

Model B: Ooshida Model on memory of vibration



Residual tension theory in parallel flow setup



simple shear

vibration

$\alpha = (\xi\text{-directional contraction of natural metric}) > 0$

- memory written $(0 < t < T_*$, flowing state)
- memory retained $(t \to +\infty$, solidified state)

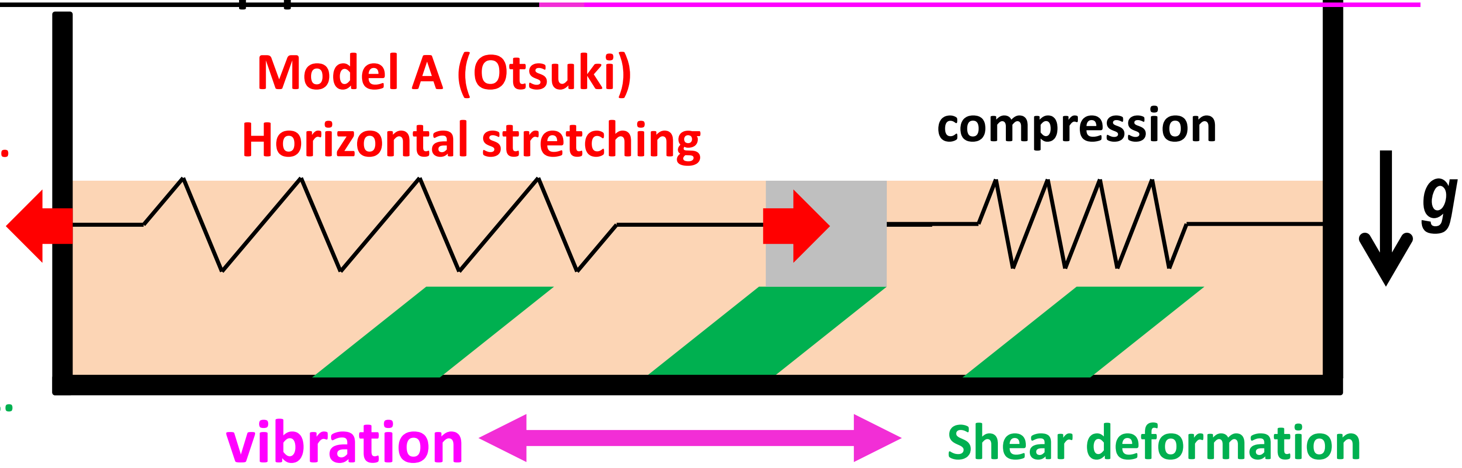
Movie!

$$g^{\natural} = \begin{pmatrix} e^{-\alpha} & \beta \\ \beta & (1+\beta^2)\, e^{\alpha} \end{pmatrix}$$

Where will cracks appear in Residual tension models?

M. Otsuki,
PRE 72 (2005) 046114.

Ooshida T.,
PRE77 (2008) 061501,
JPSJ 78 (2009) 104801.



Model A (Otsuki)
Horizontal stretching

compression

vibration

Shear deformation

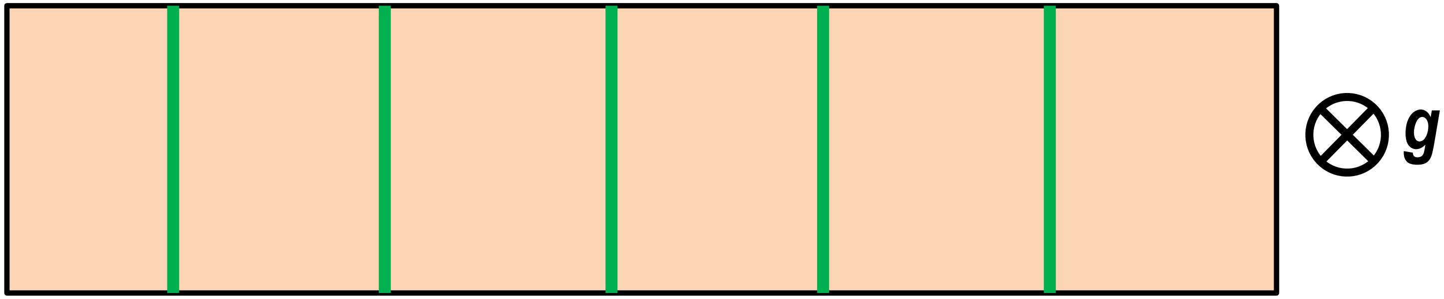
Model B (Ooshida)

Crack here

Otsuki Model
horizontal stretching

Cracks anywhere

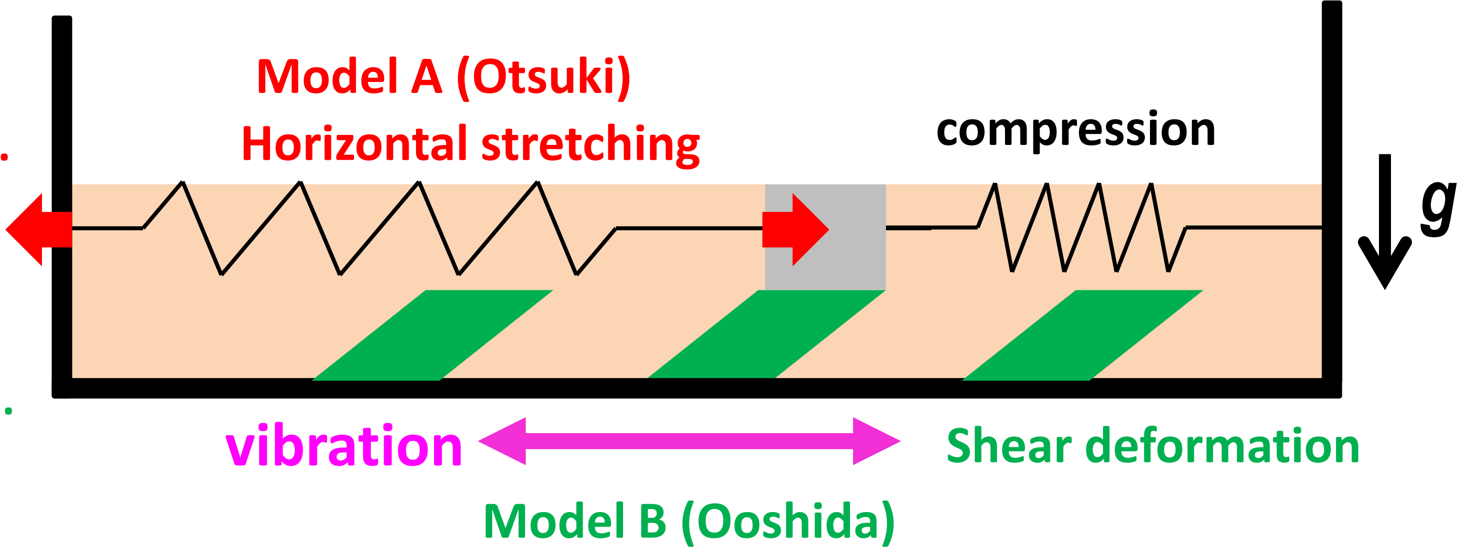
Ooshida Model
shear deformation



Experimental verification of two residual tension models

M. Otsuki,
PRE 72 (2005) 046114.

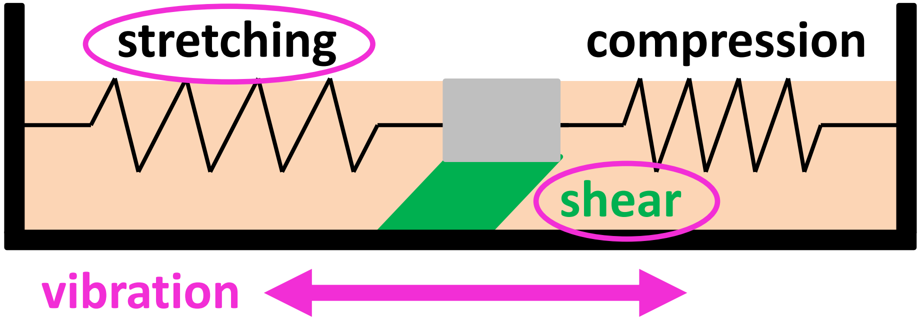
Ooshida T.,
PRE77 (2008) 061501,
JPSJ 78 (2009) 104801.



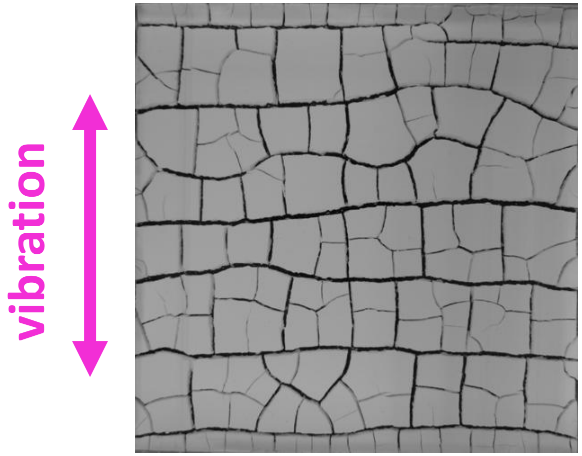
Overwriting	Where do the perpendicular cracks appear?
½ shake	Stretched part (Model A by Otsuki)
2 shakes	Sheared part (Model B by Ooshida)

Conclusion 1

- Memory of vibration = residual tension

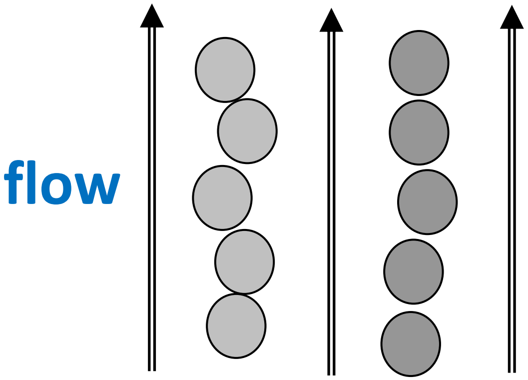


Water-poor

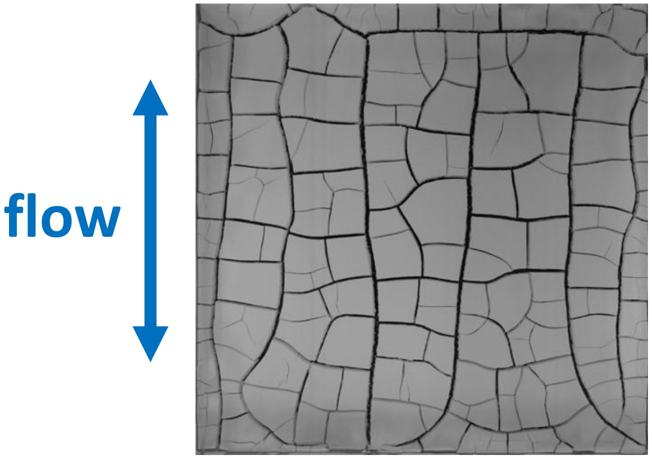


- Memory of flow = elongation of dilute network under flow

Attractive interaction is needed.



Uncharged
Water-rich



Can we control position of cracks?

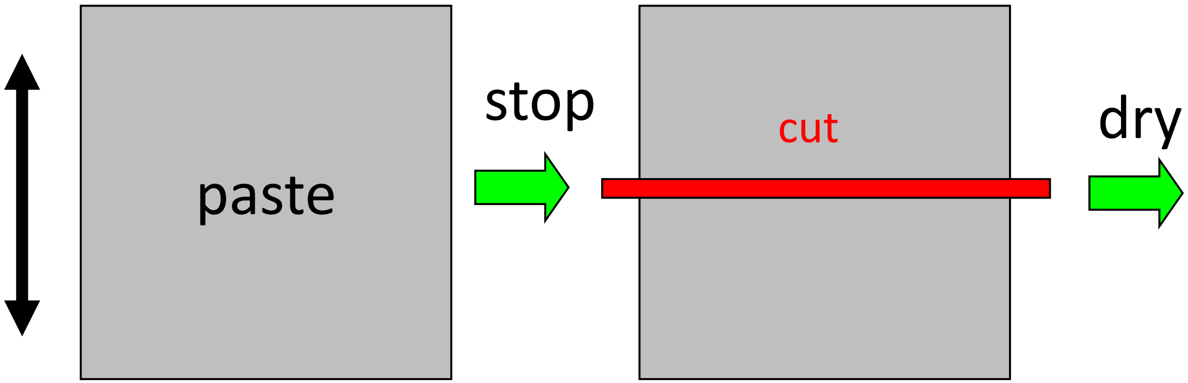
By horizontal vibration… Only Direction!

direction・・・by same memory

Direction: imprinted by memory effect

Position : stochastically formed

ρ=6.7%



paste

stop

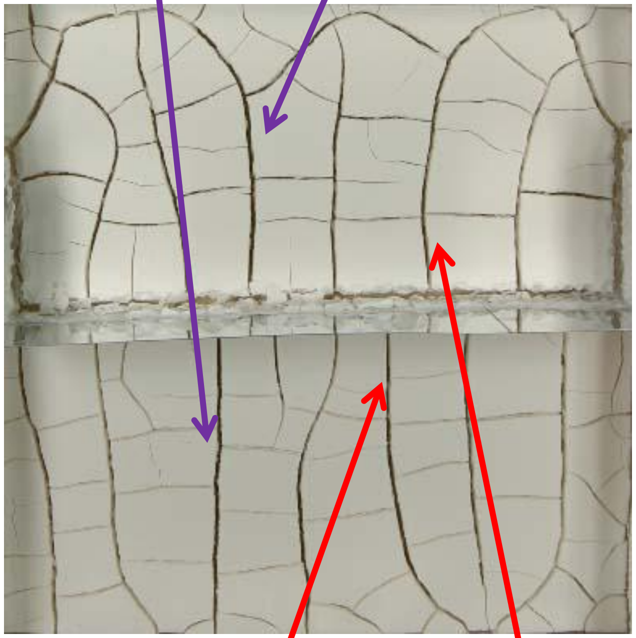
cut

dry

Horizontal vibration

Memory of flow

(memory of vibration)



position ・・・stochastic

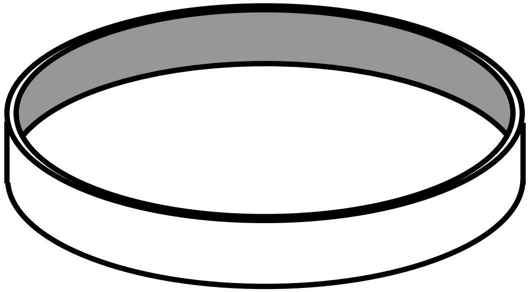
Memories are formed
not by macroscopic hydrodynamic structure
like convection.

Position control of cracks using Faraday waves

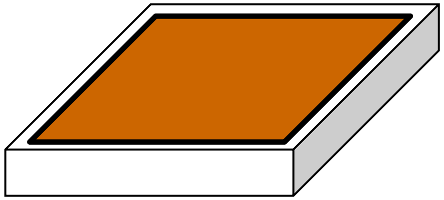
Experimental setup for vertical vibration

Paste: Water-poor $CaCO_3$ paste (only memory of vibration)

vertical vibration

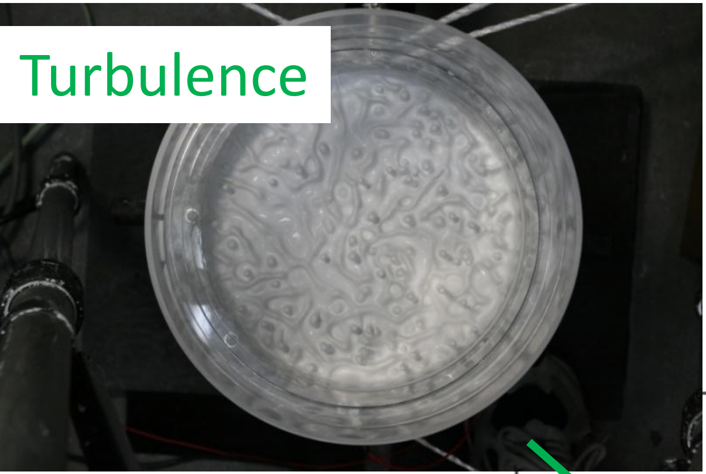


Circular container
diameter d = 200 mm

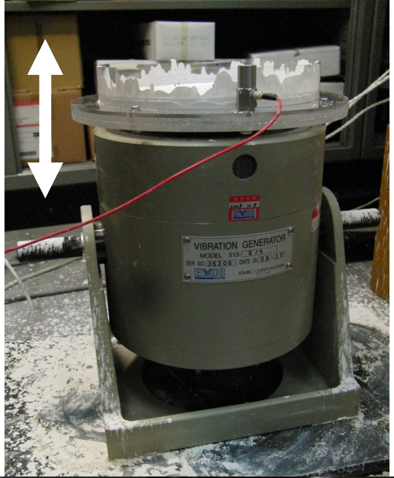


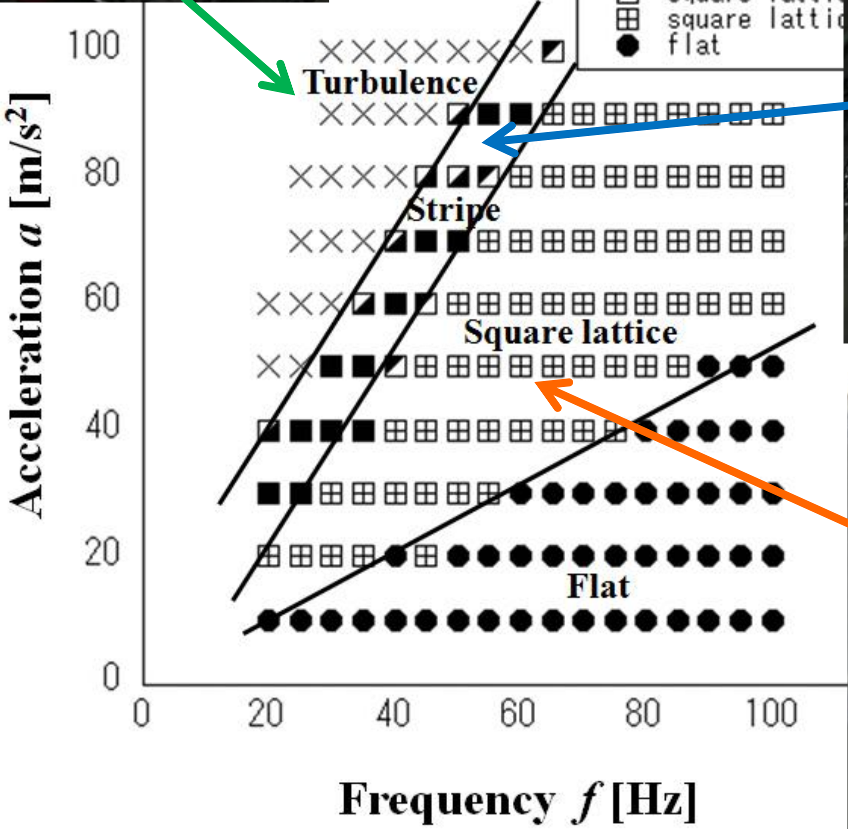
Square container
150mm side

Position control by vertical vibration



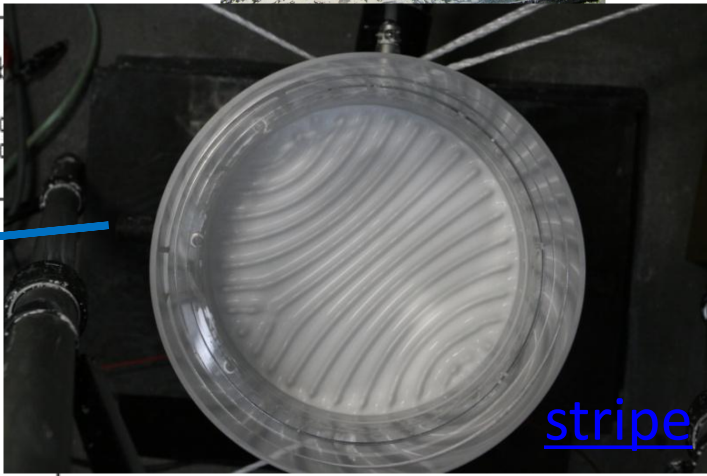
Vertical vibration
(Faraday waves)
↓
Stop and dry!





diameter
d = 200 cm

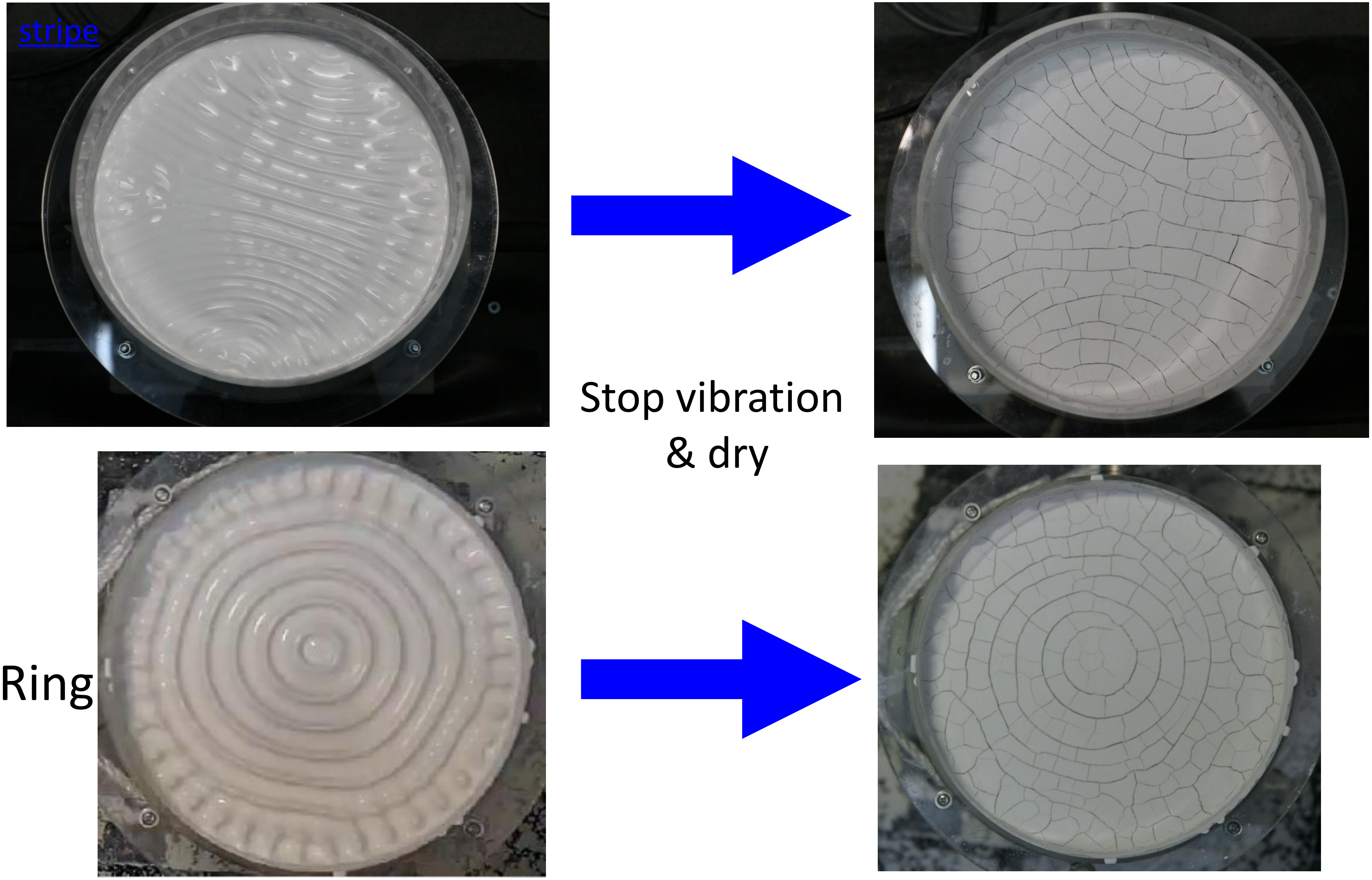
Water-poor
$CaCO_3$ paste
which has
only
memory of
vibration



Stripe and ring cracks produced by vertical vibration

Vertical vibration→ Faraday waves

desiccation crack patterns



stripe

Stop vibration & dry

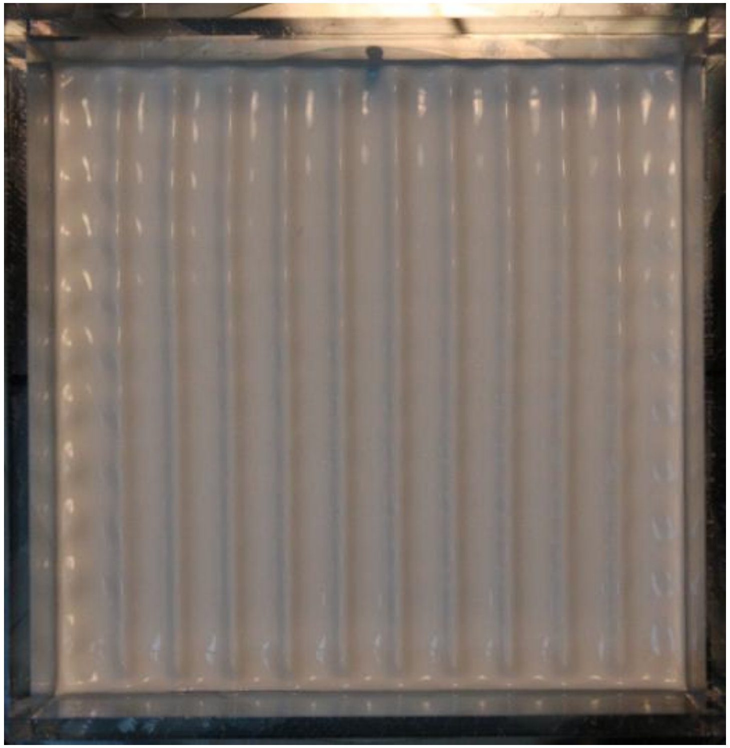
Ring

Spatial correlation between Faraday waves and cracks

Where do cracks appear?

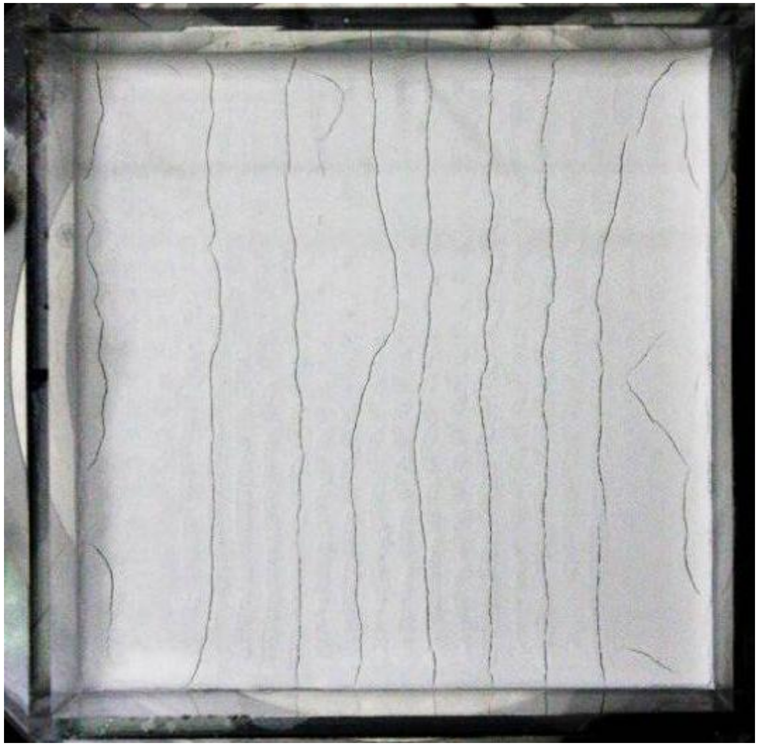
At Anti-Node regions or at Node regions?

Faraday waves



overlap

Desiccation crack pattern



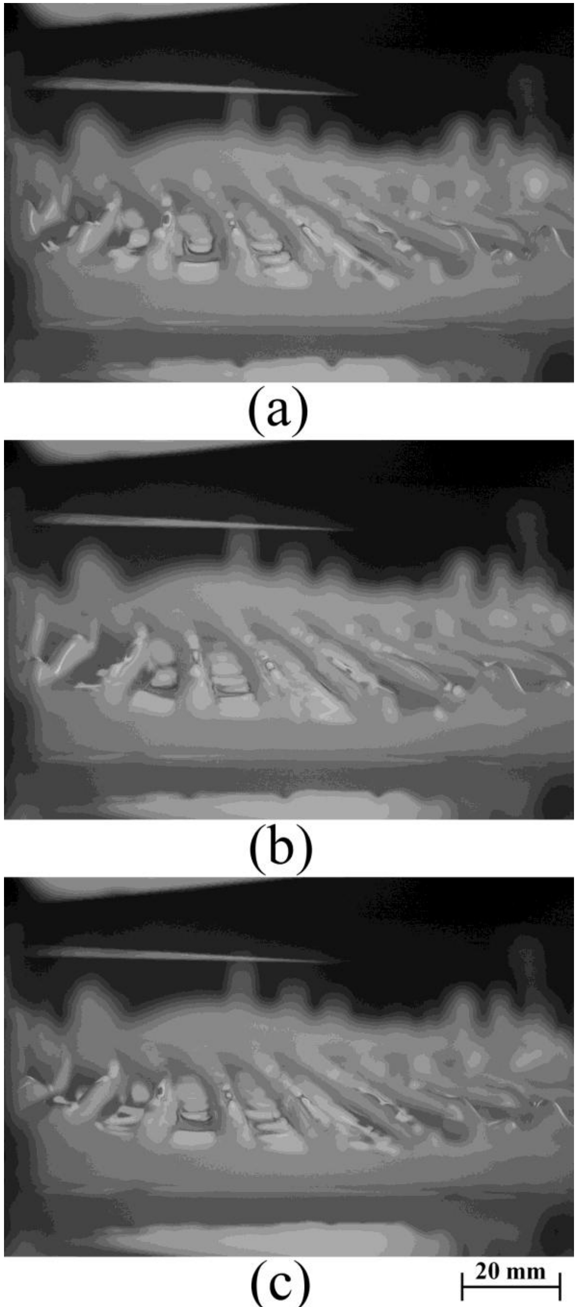
Structure of Faraday waves of $CaCO_3$ paste

t

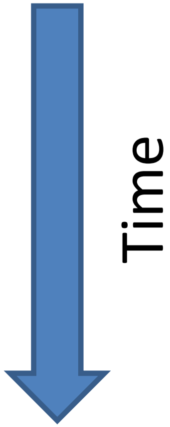
$t + T_V$

$t + 2T_V$

T_F



Faraday waves at each snapshot



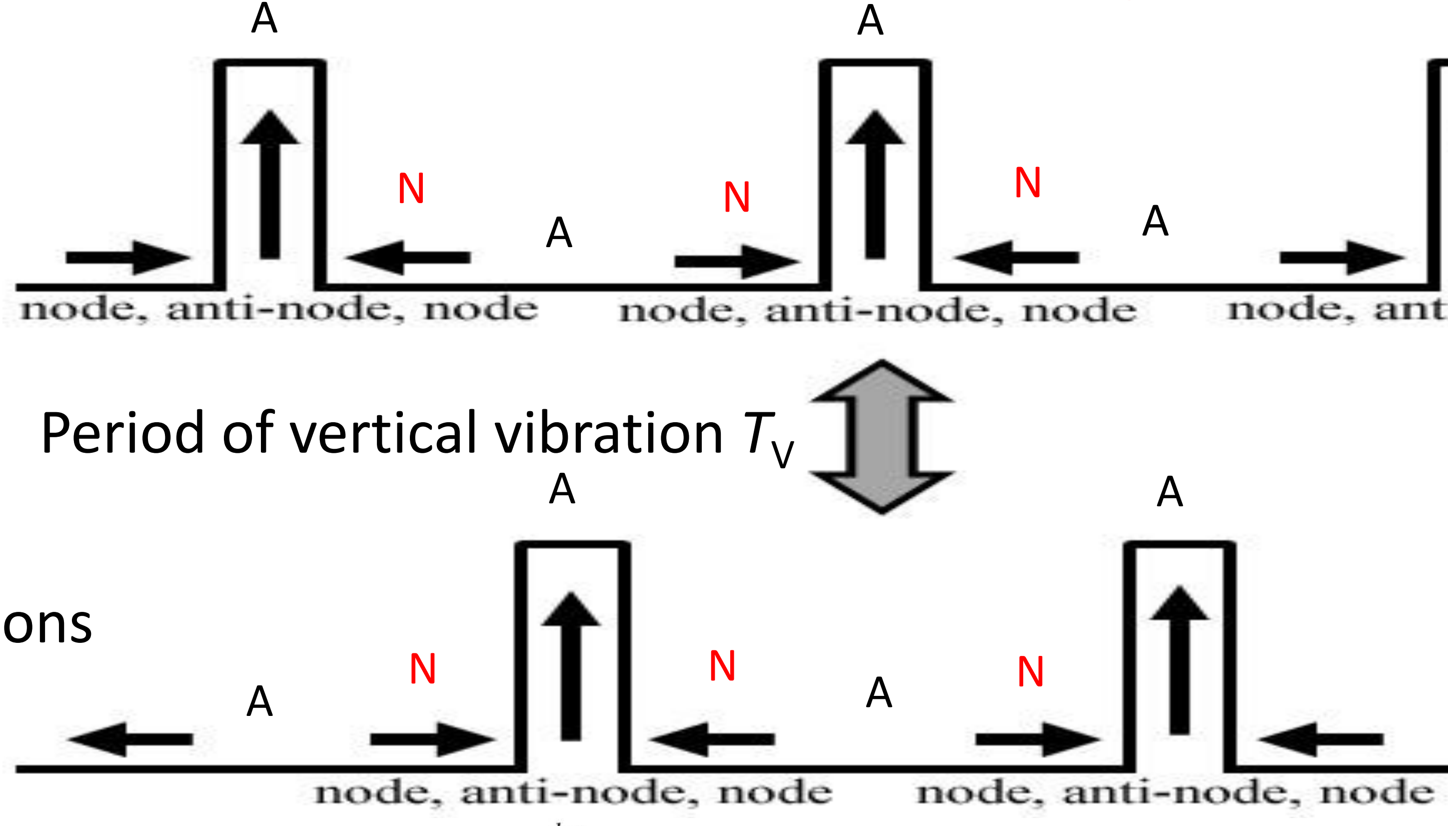
Period of Faraday waves T_F
$= 2 \times$ Period of vertical vibration T_V

Side view : Square waves

Prediction of Horizontal oscillation in node regions

Schematic

Faraday waves at each snapshot

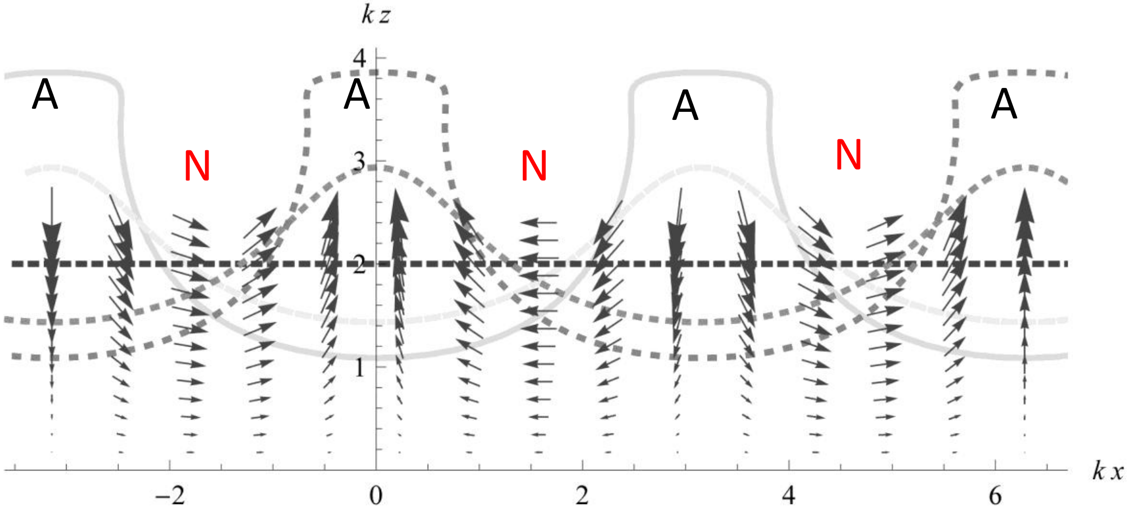


Period of vertical vibration T_V

N : Node regions

A : Anti-node regions

Analytic

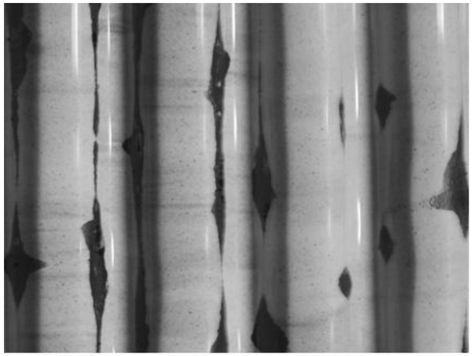


Experimental observation of horizontal oscillation in node regions

Faraday waves at each snapshot

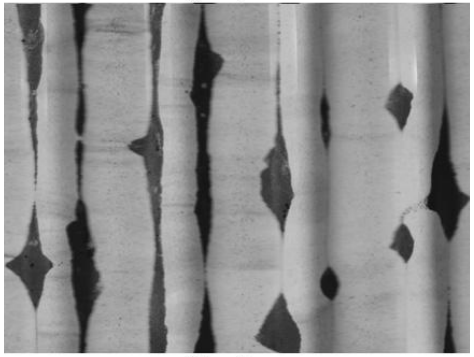
Time

t



(a)

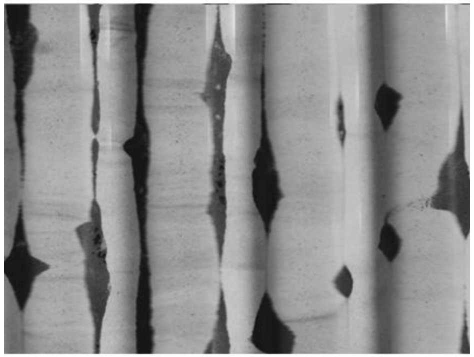
$t + T_V$



(b)

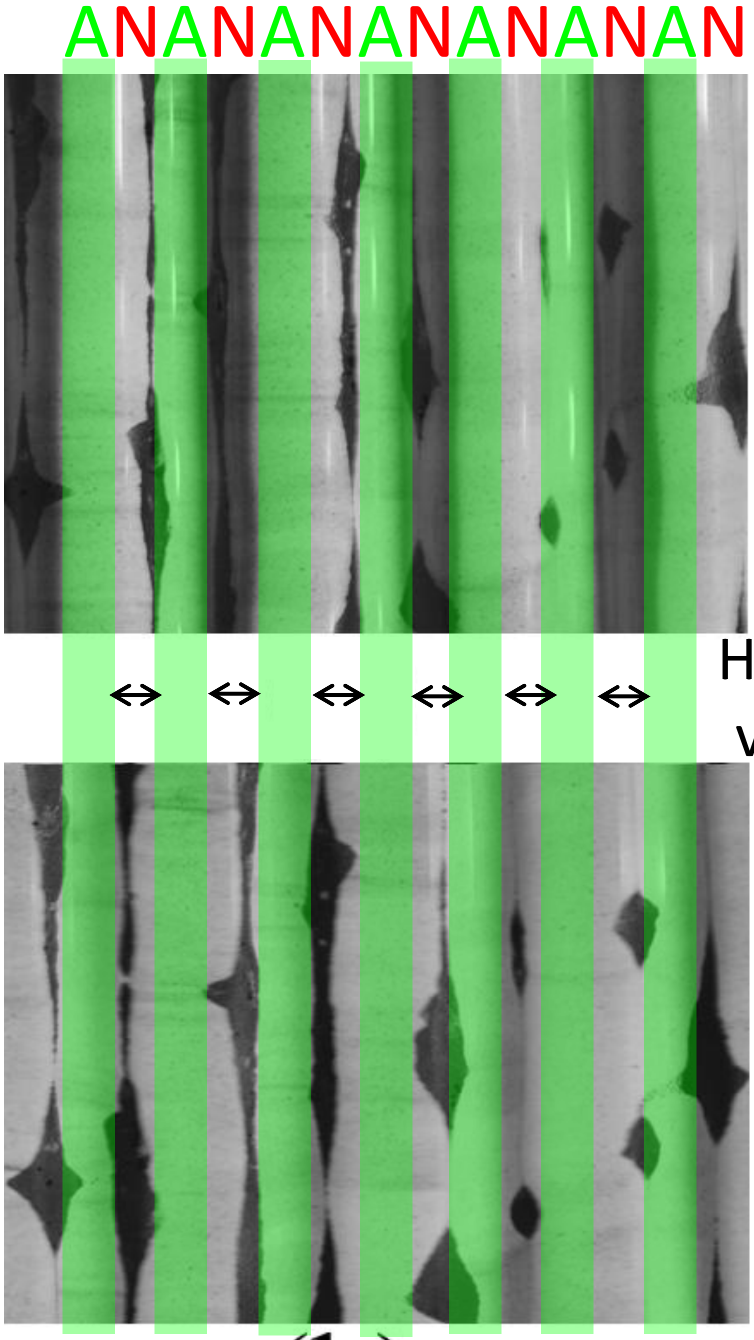
$t + 2T_V$

T_F



(c)

10 mm

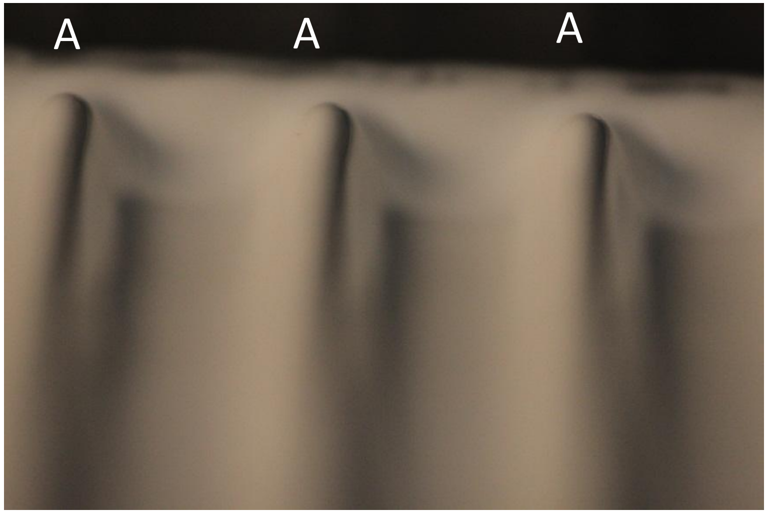


Black carbon powder on $CaCO_3$ paste

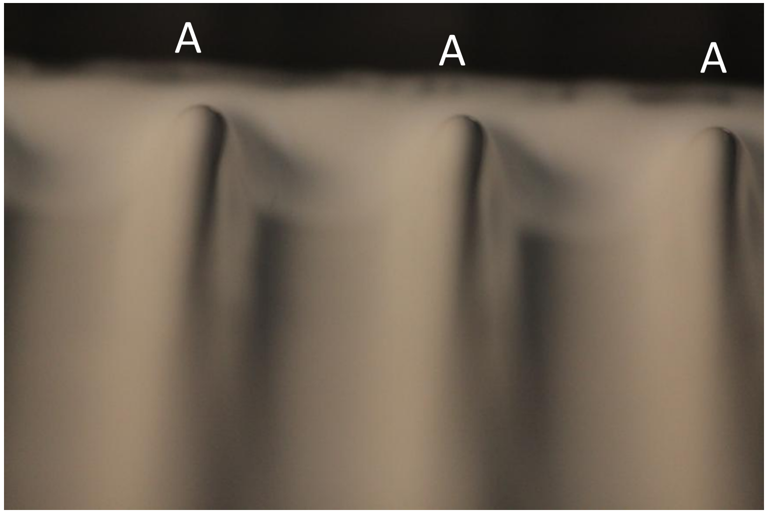
Horizontal vibration

Visualization of node regions (N) of Faraday waves

Faraday waves at each snapshot



Period of vertical vibration T_V

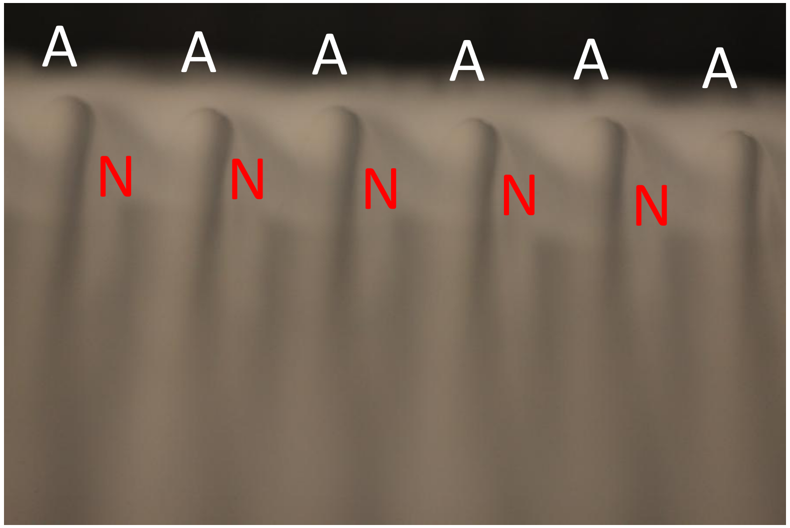


A : Anti-node regions

(Regions where anti-nodes stand up alternatively)

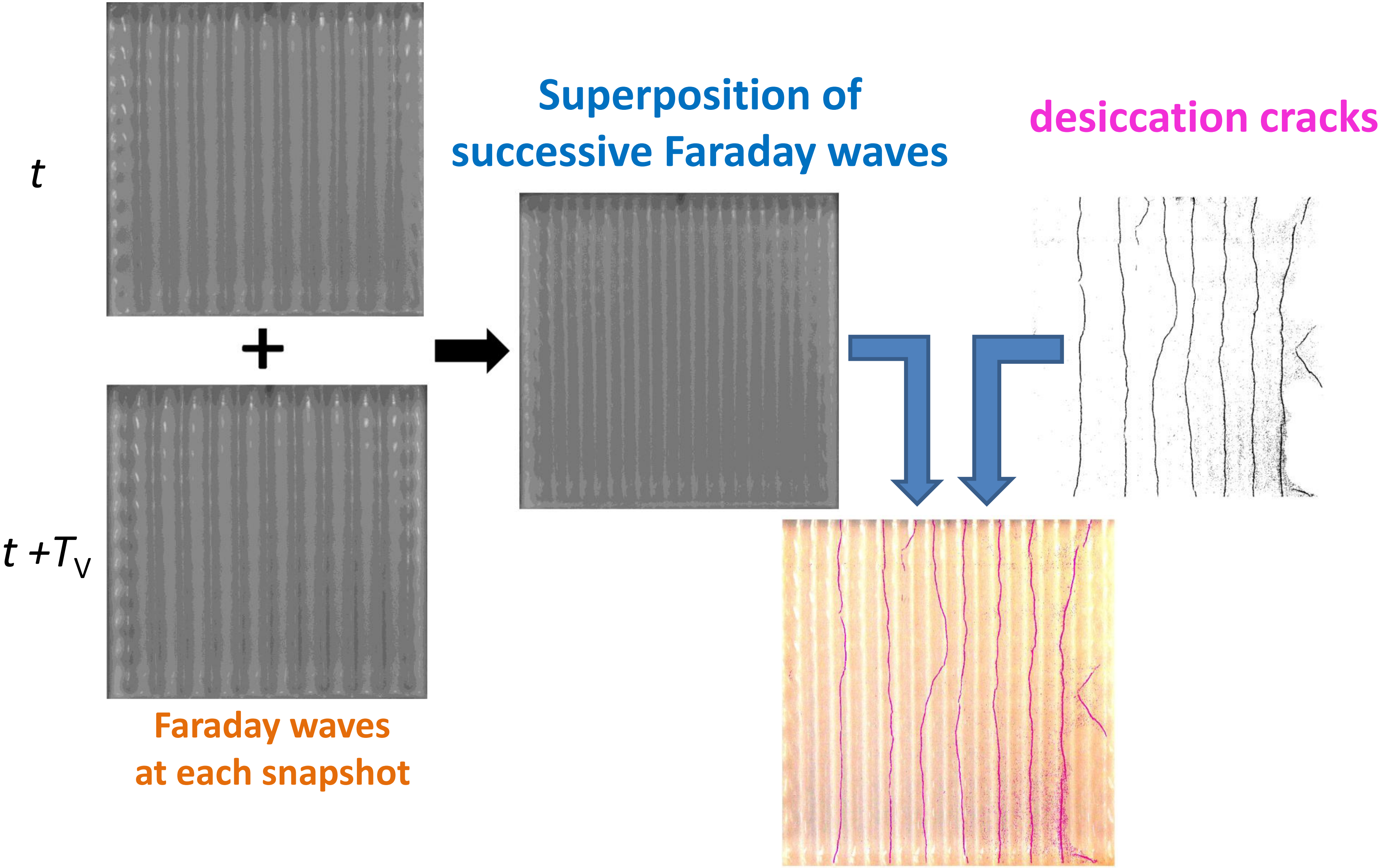
N : Node regions

(Always flat regions)



Superposition of successive Faraday waves

Superposition of successive Faraday waves and desiccation cracks



t

+

$t + T_V$

Faraday waves
at each snapshot

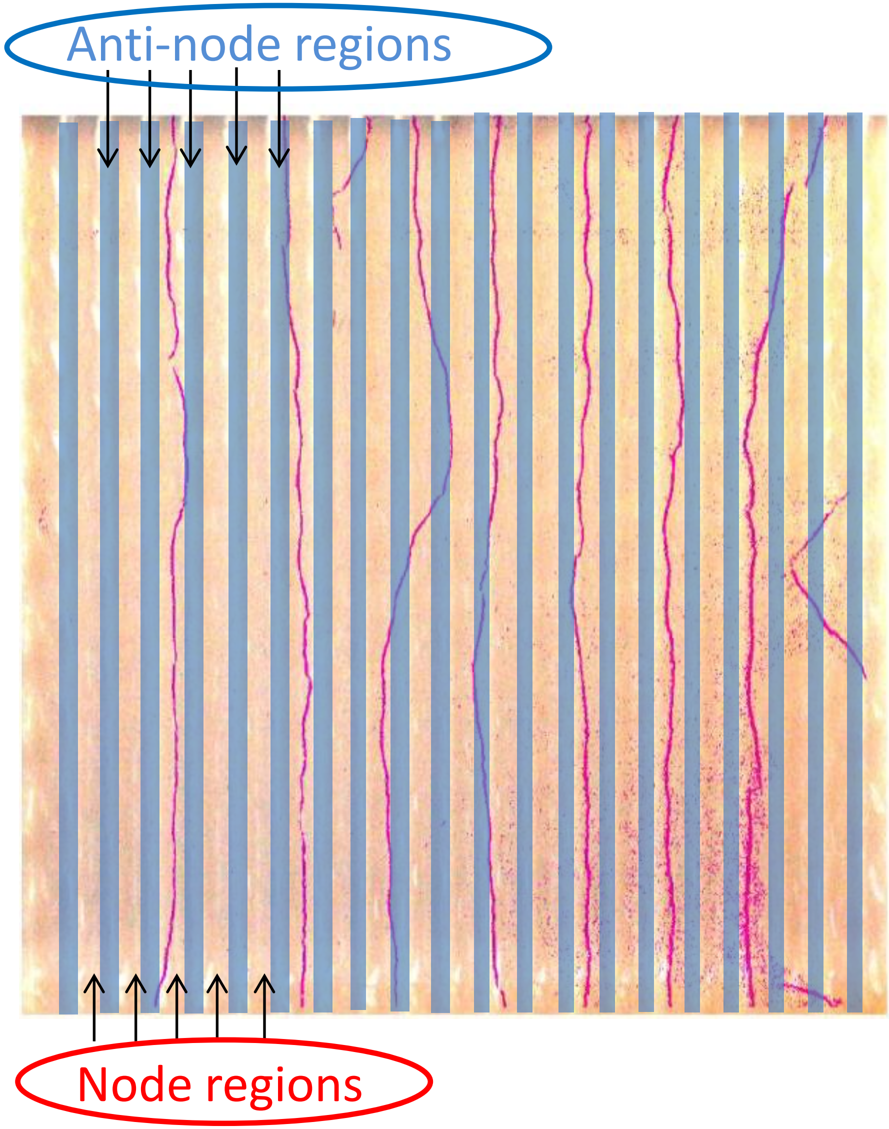
Superposition of
successive Faraday waves

desiccation cracks

Superposition of
successive Faraday waves and desiccation cracks

Statistical analysis

Ratio of observing cracks at node regions to that of anti-node regions



$$P = \log_e \frac{L_n / S_n}{L_a / S_a}$$

node regions

anti- node regions

L_n : length of cracks at node regions

L_a : length of cracks at anti-node regions

S_n : Area of node regions

S_a : Area of anti-node regions

From 10 experiments

$$P = 0.76 \pm 0.40$$

At node regions, cracks are formed

$$e^P = e^{0.76} = \boxed{2.1 \text{ times}}$$

more frequently than in anti-node regions.

Cracks emerge at node regions.

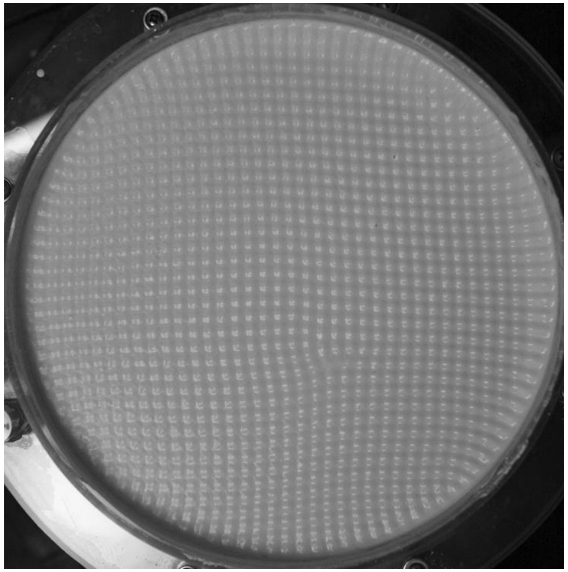
Square lattice cracks produced by vertical vibration

Vertical vibration

→ Faraday waves

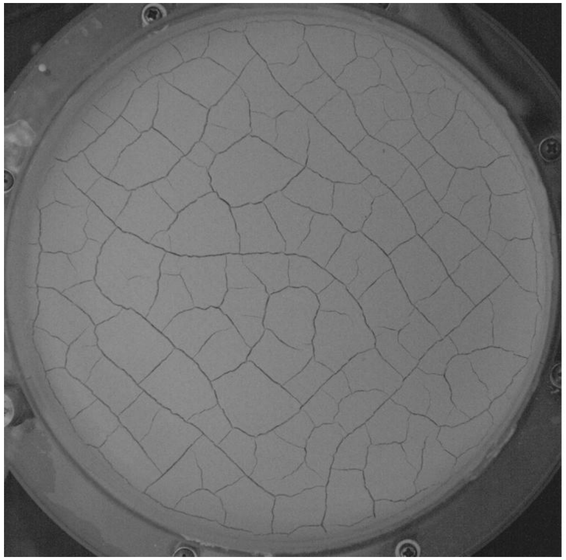
desiccation crack patterns

Square lattice

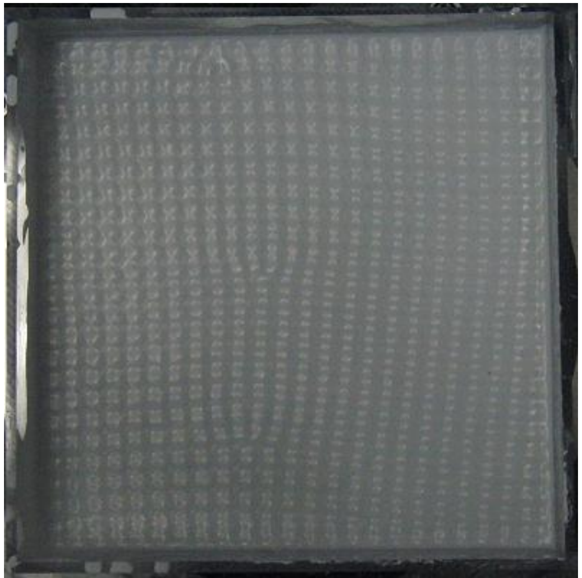


Stop & dry

Why tilt by 45^{o} ?



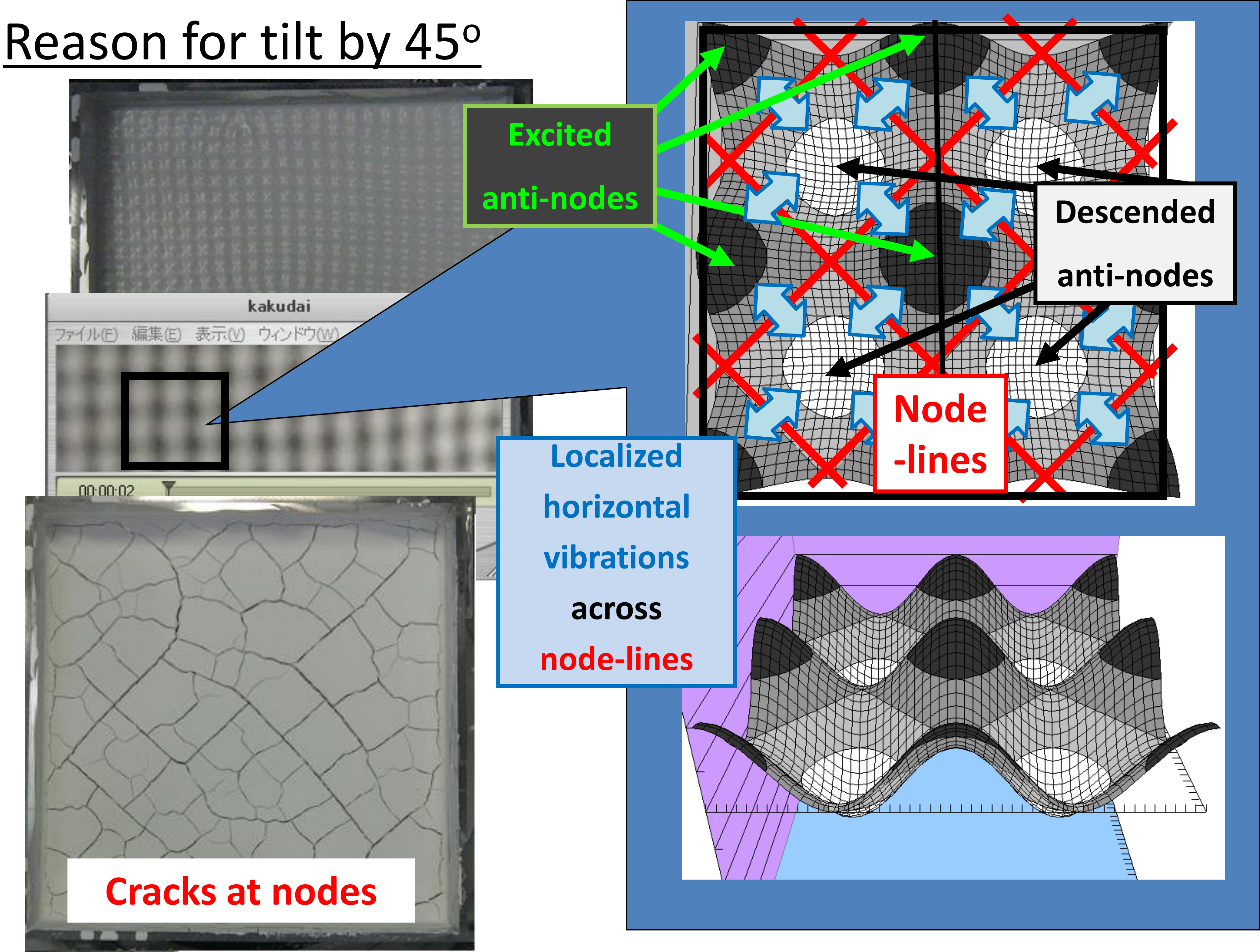
Square lattice



Stop & dry

Why tilt by 45^{o} ?

Reason for tilt by 45°



Excited anti-nodes

Descended anti-nodes

Node -lines

Localized horizontal vibrations across node-lines

Cracks at nodes

Experimental check: superposition of successive Faraday waves

Faraday waves at one snapshot

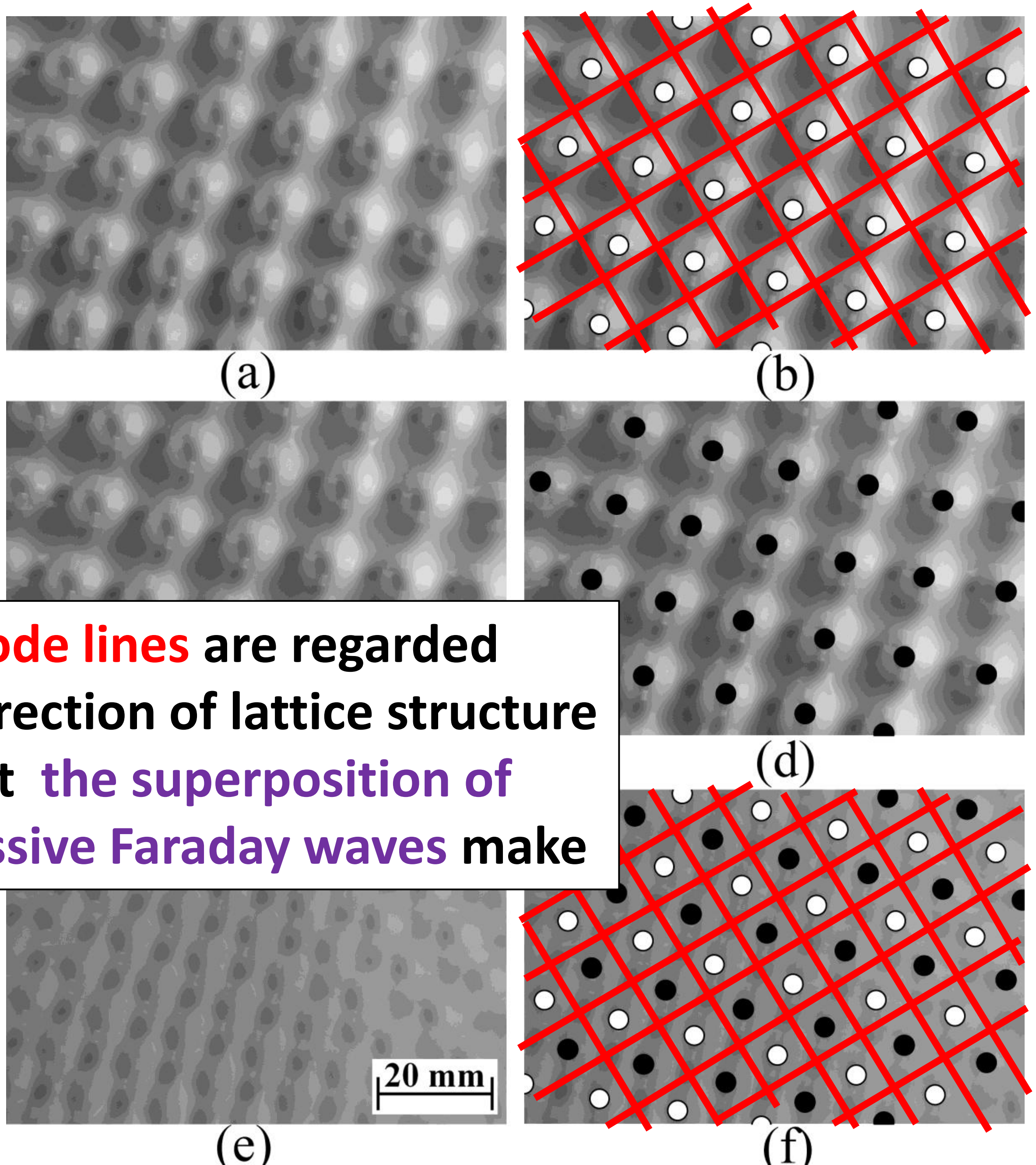
t

Faraday waves at next snapshot

$t + T_V$

Superposition of successive Faraday waves

t and $t+T_V$



Snapshot

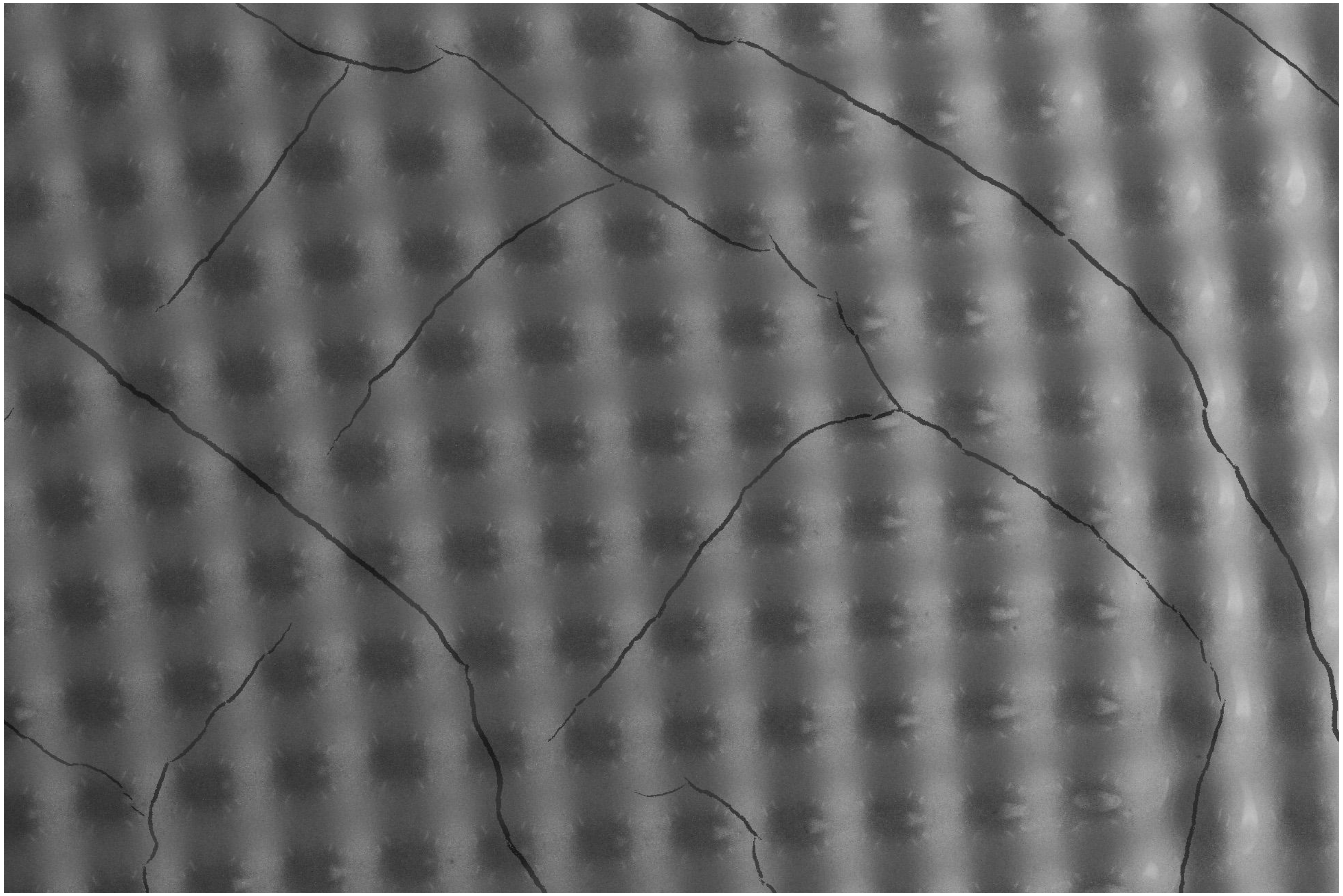
Tilt by 45° !

Top of excited anti-nodes are marked as circles

Node lines

Node lines are regarded as a direction of lattice structure that the superposition of successive Faraday waves make

Experimental check: Spatial correlation between Faraday waves at one snapshot and desiccation cracks



Structures : Tilt by 45º !

Conclusion 2

- Using vertical vibration, we can control not only the direction but also the position of cracks.
- Cracks with lattice structure can be created !

Square lattice

H. Nakayama, Y. Matsuo, Ooshida Takeshi and A. Nakahara
European Physical Journal E **36** (2013) 1.

Further applications

1. What will happen in future (Technology)

   → Control microstructure of materials,

   anisotropy in conductivity and elastic moduli,

   and macroscopic crack patterns

2. What happened before (Geoscience)

   → Know past by memories and cracks in rocks

Future plans

Flow ・・・ large deformation

- Definition and quantification of Memories in pastes

  including Memory of flow

- Theoretical model which explains Memory of flow

- Transition from Memory of vibration to Memory of flow

Future plans 2

Combine various external fields
to control crack patterns

Mechanical method (vibration, flow)

A. Nakahara & Y. Matsuo

JPSJ (2005), PRE (2006)

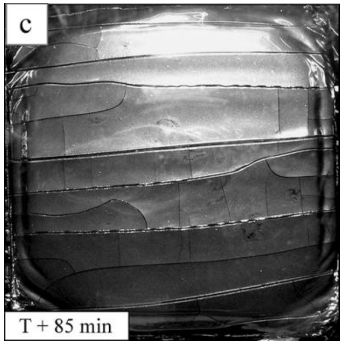
Electrical method

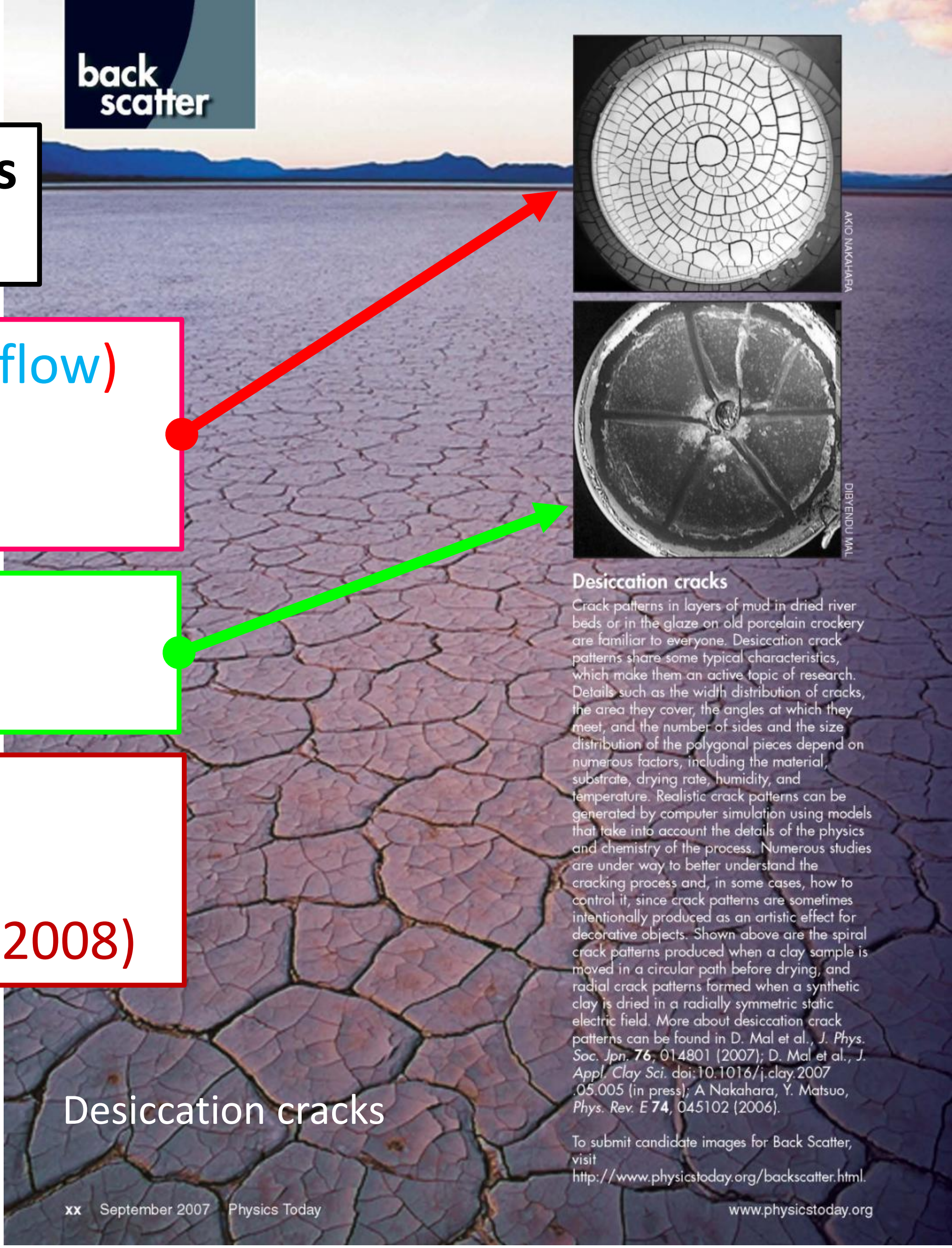
D. Mal et. al., Physica A (2007)

Magnetic method

L. Pauchard et. al., PRE, (2008)

A. T. Ngo et. el., Nano Letters (2008)




back scatter

AKIO NAKAHARA

DIBYENDU MAL

Desiccation cracks

Crack patterns in layers of mud in dried river beds or in the glaze on old porcelain crockery are familiar to everyone. Desiccation crack patterns share some typical characteristics, which make them an active topic of research. Details such as the width distribution of cracks, the area they cover, the angles at which they meet, and the number of sides and the size distribution of the polygonal pieces depend on numerous factors, including the material, substrate, drying rate, humidity, and temperature. Realistic crack patterns can be generated by computer simulation using models that take into account the details of the physics and chemistry of the process. Numerous studies are under way to better understand the cracking process and, in some cases, how to control it, since crack patterns are sometimes intentionally produced as an artistic effect for decorative objects. Shown above are the spiral crack patterns produced when a clay sample is moved in a circular path before drying, and radial crack patterns formed when a synthetic clay is dried in a radially symmetric static electric field. More about desiccation crack patterns can be found in D. Mal et al., *J. Phys. Soc. Jpn.* **76**, 014801 (2007); D. Mal et al., *J. Appl. Clay Sci.* doi:10.1016/j.clay.2007.05.005 (in press); A Nakahara, Y. Matsuo, *Phys. Rev. E* **74**, 045102 (2006).

To submit candidate images for Back Scatter, visit http://www.physicstoday.org/backscatter.html.

xx September 2007 Physics Today www.physicstoday.org

Desiccation cracks

Physics Today, September (2007) 116