6/25 Physics of Granular Flows 2013 (YITP, Kyoto Univ.)

Numerical Analysis of Granular Jet Impacts

Yukawa Institute for Theoretical Physics
Tomohiko Sano & Hisao Hayakawa

T. G. Sano and H. Hayakawa, Phys. Rev. E 86, 041308 (2012).

T. G. Sano and H. Hayakawa, Powders & Grains 2013 (in press), arXiv: 1211.3533

T. G. Sano and H. Hayakawa, arXiv:1302.6734

Outline of my talk

Introduction: “Impact Process”

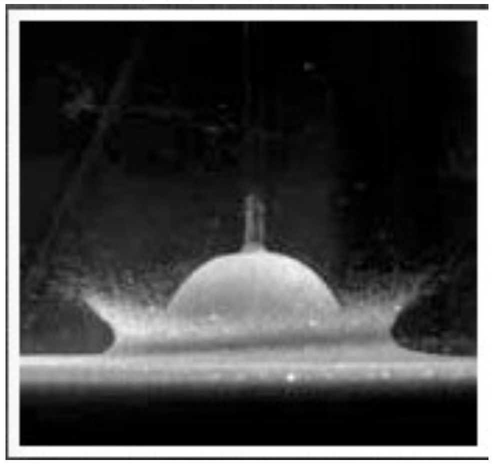
Model: Discrete Element Method (DEM)

Rheology of Granular jets in 3D
: Is granular flow “perfect fluid ?”

Rheology of Granular jets in 2D
: Jet-induced jamming

Discussion & Summary

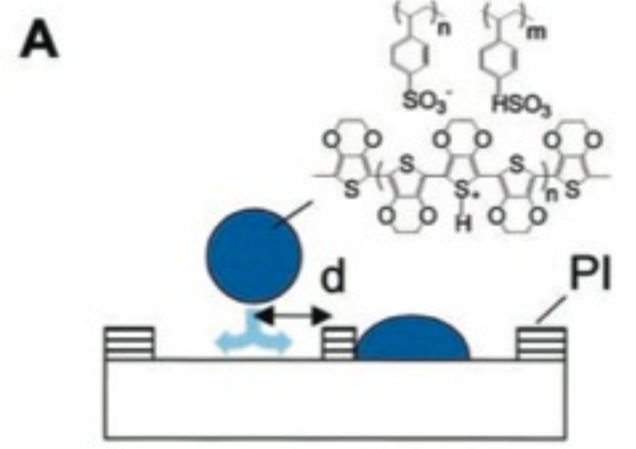
Introduction: “Impact Process”



Crater formation

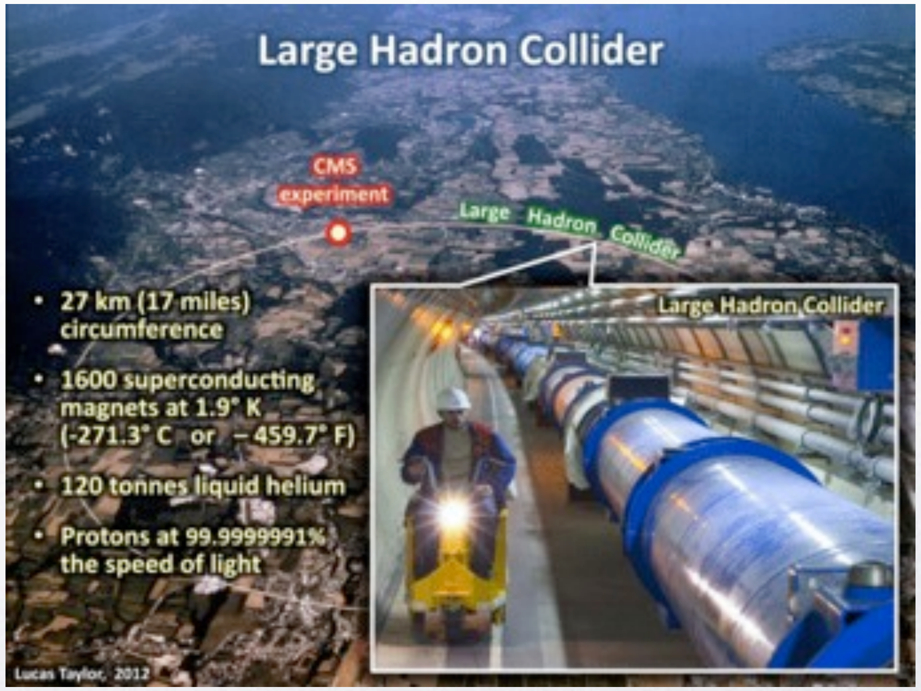
H. Katsuragi,
Phys. Rev. Lett. 104, 218001 (2010)

H. Sirringhaus, et al.
Science, 290 (5499) 2123-2126 (2000).



Inkjet

Wide Length Scale
Industrial Application
&
Natural Science



Nuclei Reaction(heavy ion)

http://lhc.web.cern.ch/lhc/

macro

Interest

“Macroscopic” Impact Process
Fluid state after the impact

Granular Jet Impact

micro

Granular Jet Impact

X. Cheng et al. Phys. Rev. Lett. **99**, 188001 (2007)

Experimental movie from Chicago group

http://nagelgroup.uchicago.edu/Nagel-Group/Granular.html

INTRODUCTION

Perfect-fluidity in Granular Jet experiment

1. From Experimental Study
2. From Numerical Study in 2D

INTRODUCTION

Perfect-fluidity in Granular Jet experiment

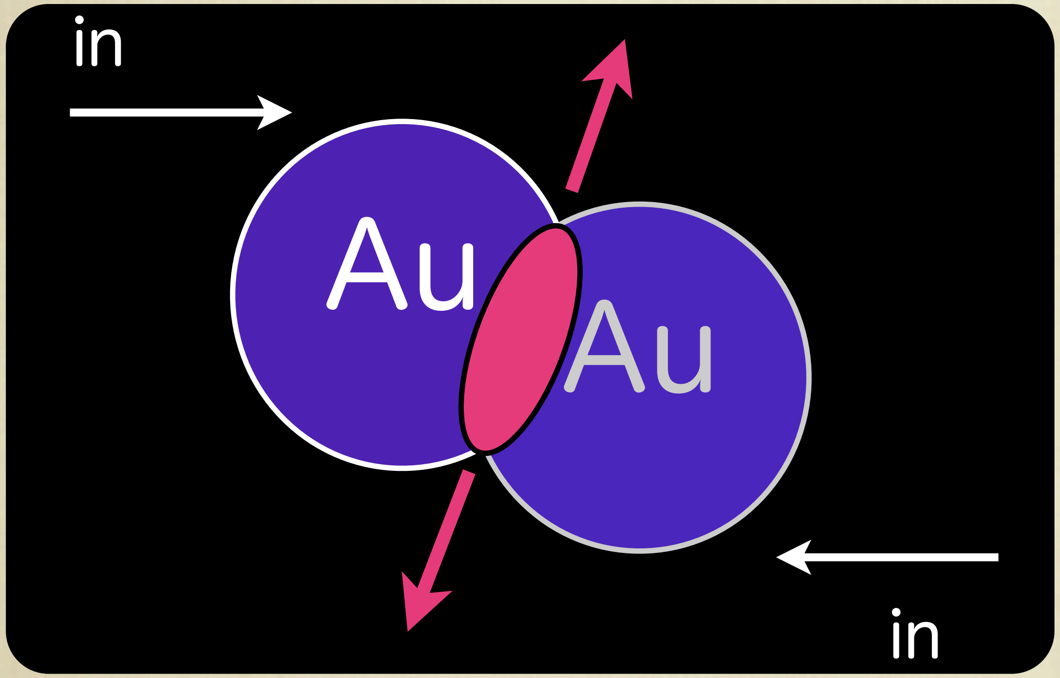
1. From experiments :

An analogy between Granular Flow & Quark Gluon Plasma(QGP)

→Perfect-Fluid like response

X. Cheng et al. Phys. Rev. Lett. 99, 188001 (2007)

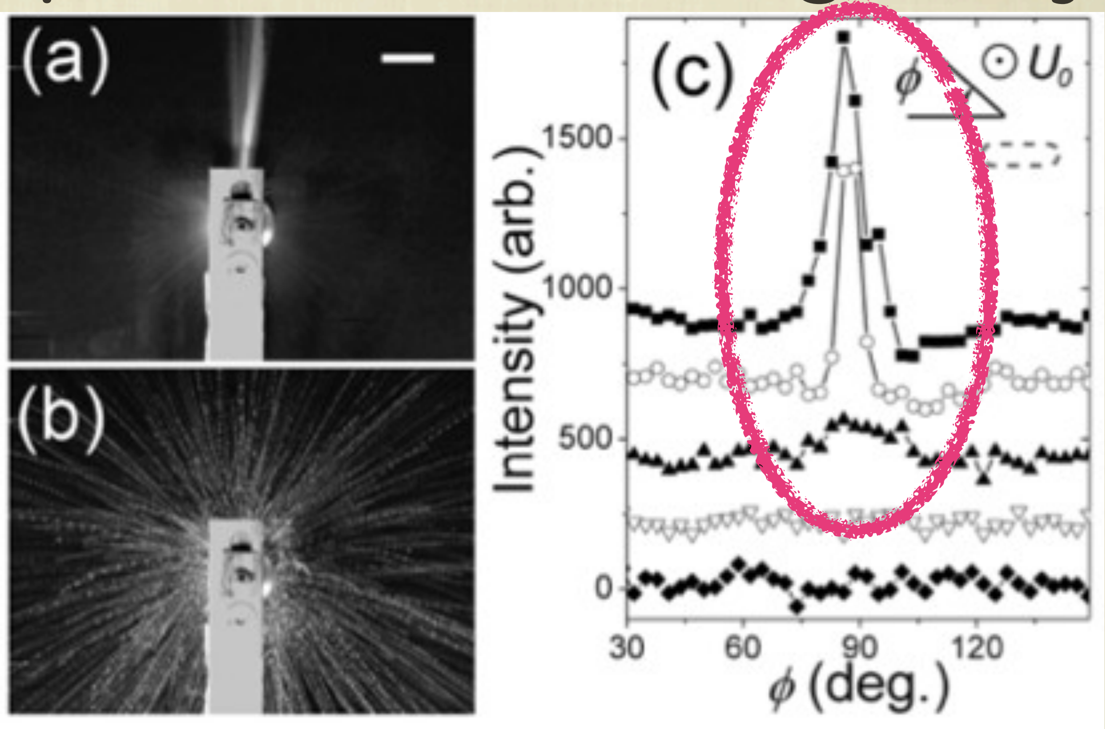
Nuclei Reaction



Elliptic Flow

QGP
→Small shear viscosity

Impact of a rectangular jet



Anisotropic flow

Perfect-Fluid like response?

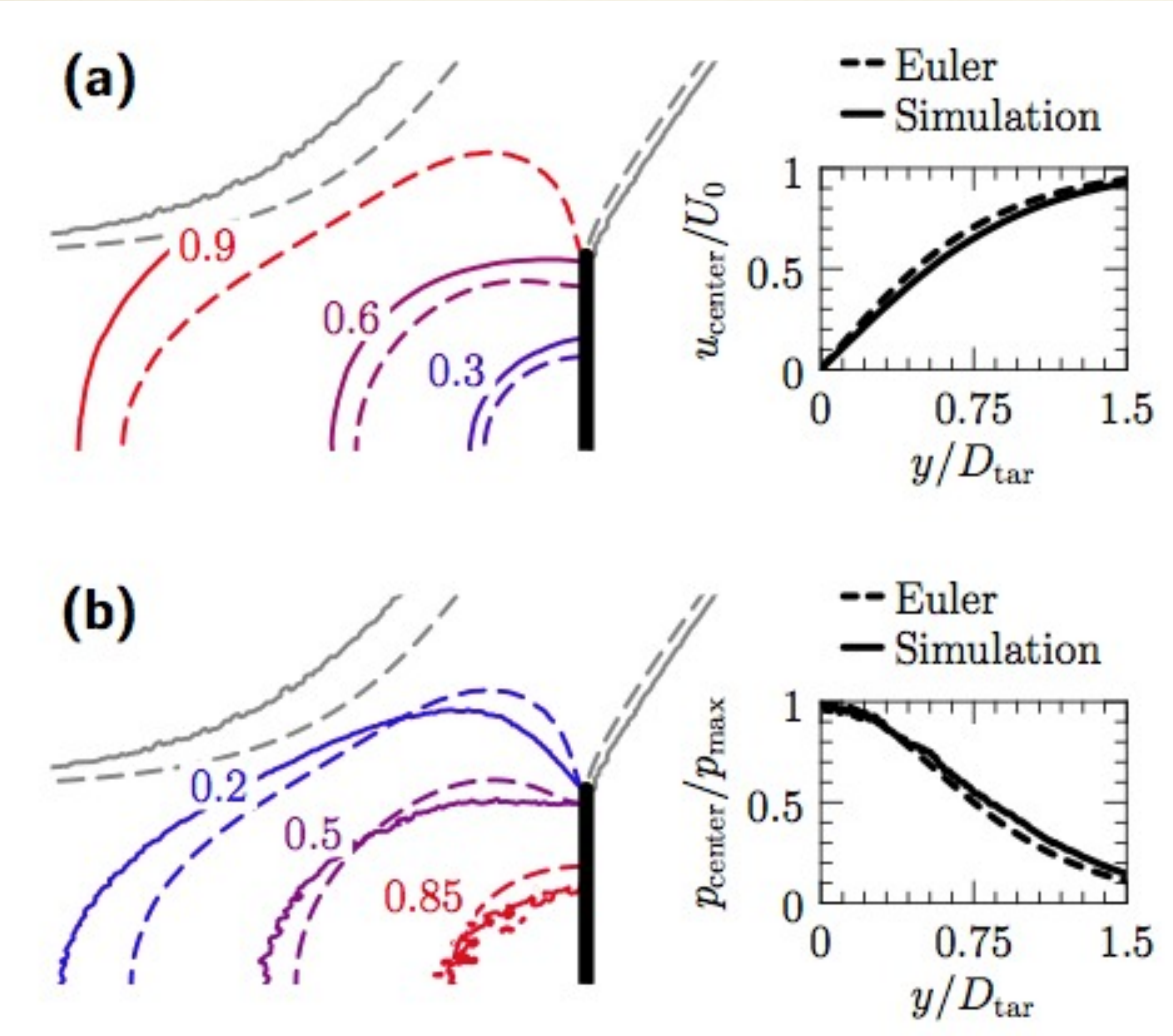
INTRODUCTION

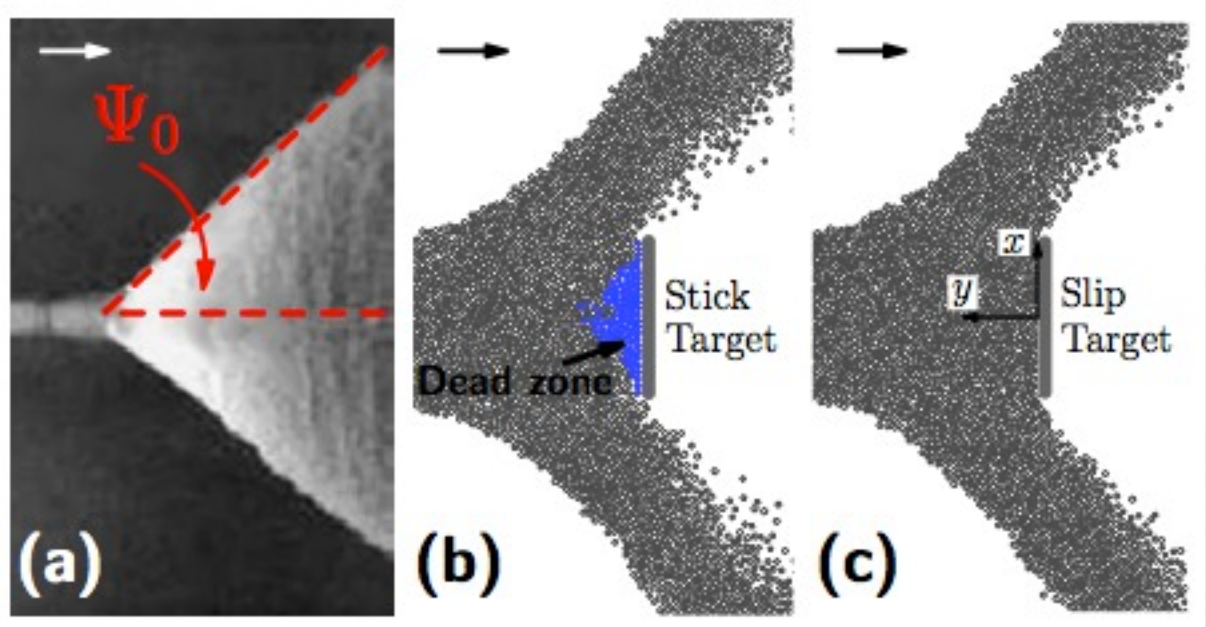
Perfect-fluidity in Granular Jet experiment

2. From Two-dimensional simulation :

A correspondence between Granular Flow & Perfect Fluid

Profile of the velocity & pressure





J. Ellowits et al. arXiv: 1201.5562

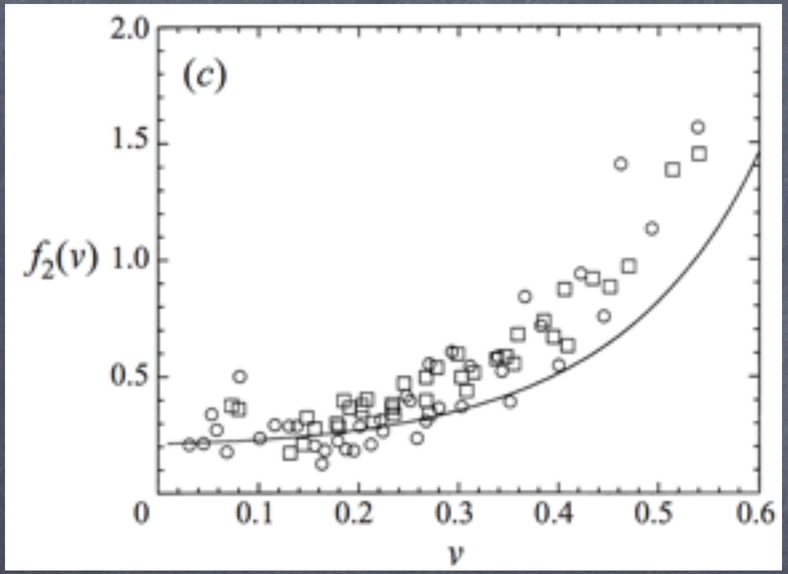
Perfect-fluidity in Granular Jet experiment

1. Experiment :Similarity between QGP and granular flow
2. Numerical study in 2D: Ellowtiz, et al. arXiv:1201.5562

Similar profile of pressure and velocity between perfect fluid and granular flow

But, granular flow cannot be a perfect fluid.

Dense granular flow



Experimental data of viscosity of granular flow

J. Fluid. Mech 400 199 (1999)

large density

→large viscosity

Perfect fluid should be

$$\sigma_{\alpha\beta} = P\delta_{\alpha\beta}$$

$$\eta = 0$$

- Why granular flow looks like a perfect fluid?
- Response to an impact in general and rheology of flows under an impact should be investigated.

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: Is granular flow “perfect fluid ?”

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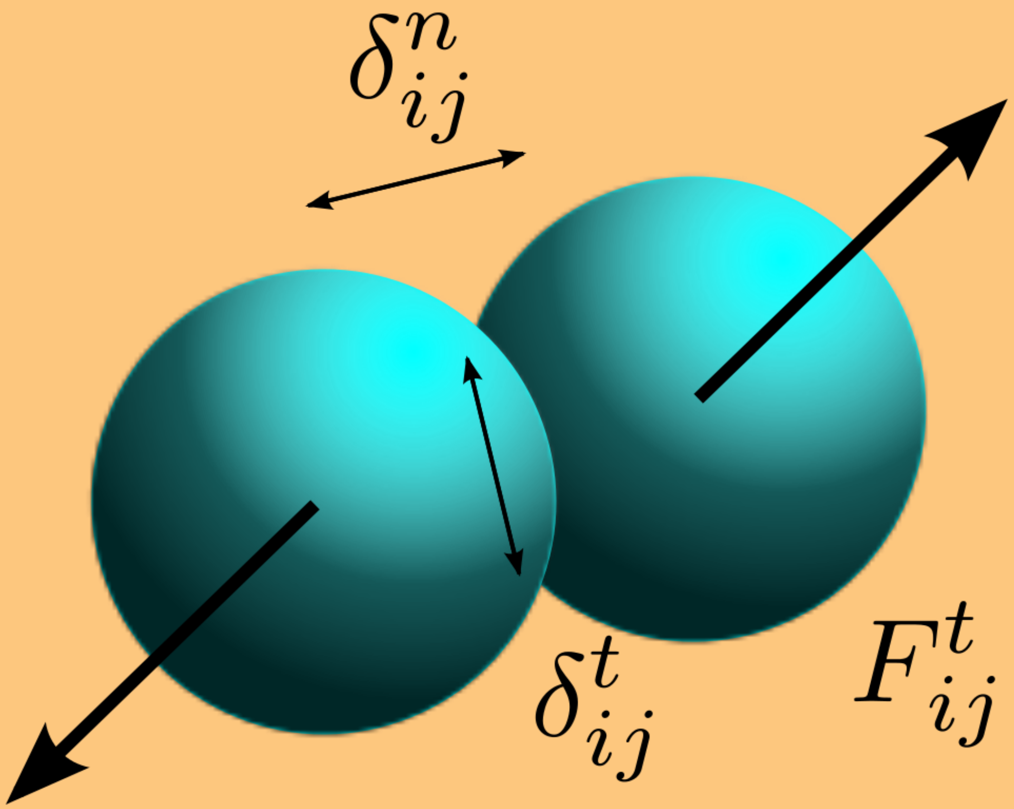
: Jet-induced jamming

Discussion & Summary

Model: Discrete Element Method (DEM)



$$\boldsymbol{F}_{ij} = F^n_{ij}\boldsymbol{n} + F^t_{ij}\boldsymbol{t}$$

$$F^n_{ij} = -k_n\delta^n_{ij} - \eta_n\dot{\delta}^n_{ij}$$

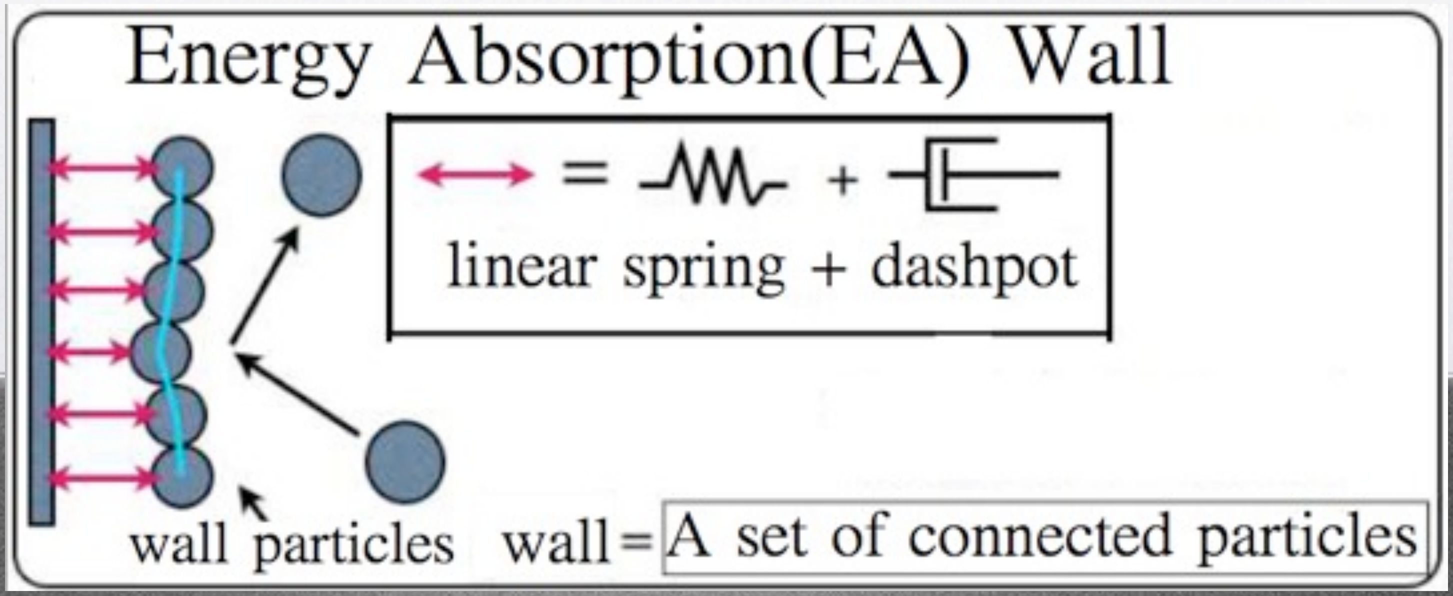
$$F^t_{ij} = \min\{|F^{t'}_{ij}|, \mu_p|F^n_{ij}|\}\mathrm{sign}(F^{t'}_{ij})$$

$$F^{t'}_{ij} = -k_t\delta^t_{ij} - \eta_n\dot{\delta}^t_{ij}$$

$e = 0.75$:Restitution Coefficient

$\mu_p = 0.2$: Coulombic const. of spheres

Wall model:



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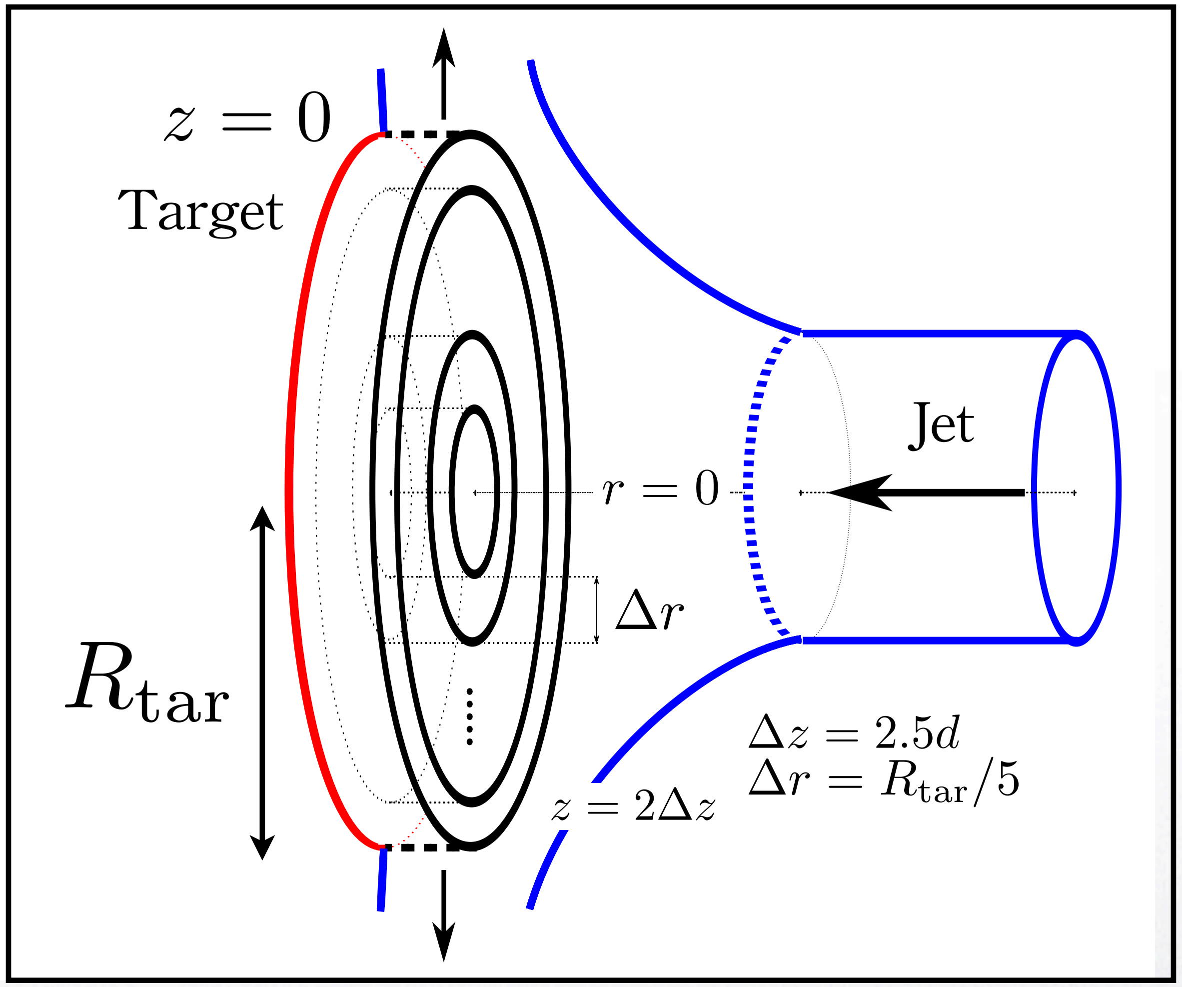
Rheology of Granular jets in 3D

Calculation Region



Simulation movie

initial value

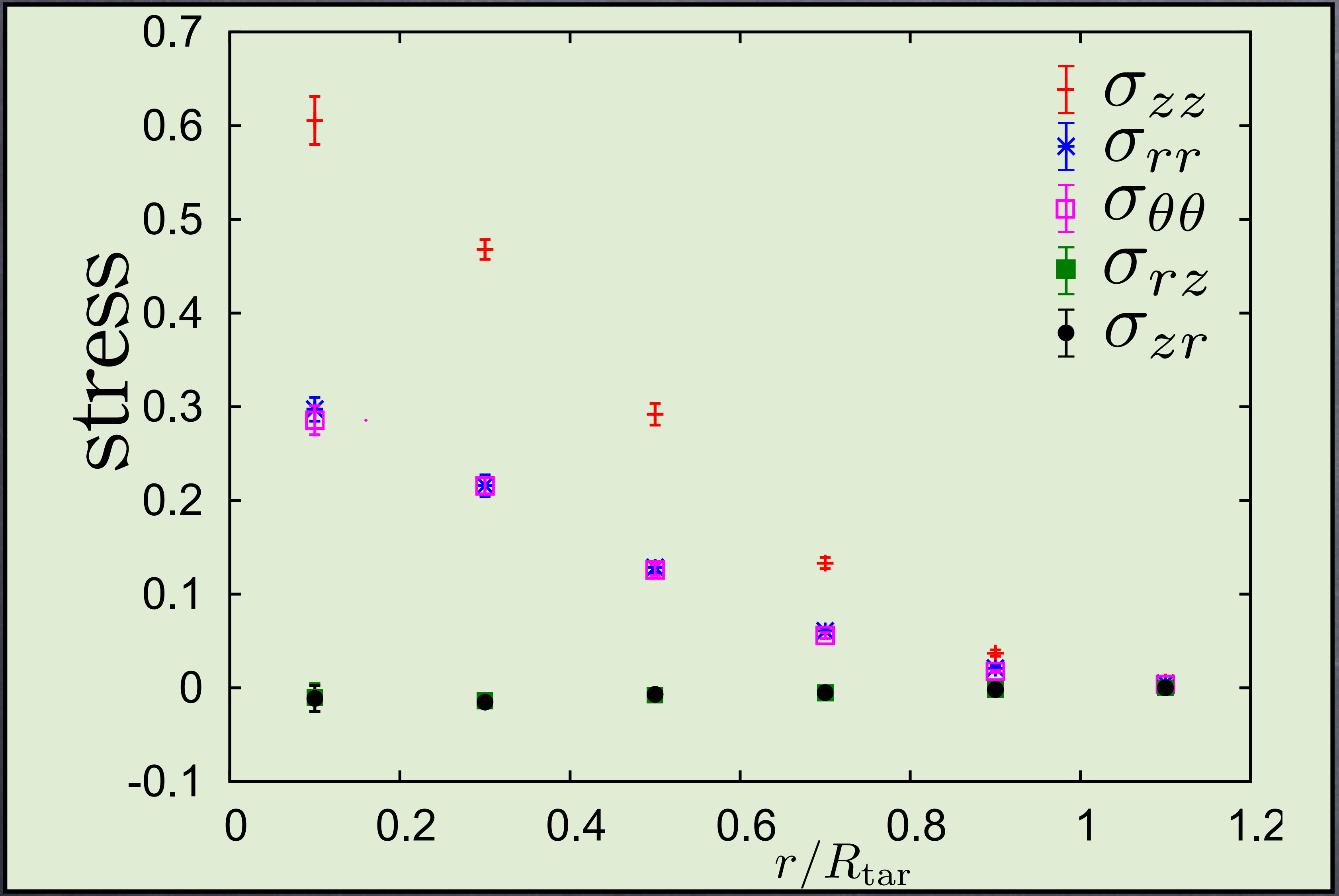
volume fraction $\phi_0/\phi_{\rm fcc} = 0.90$

granular temperature

(= fluctuation of velocity) $T_g = 0$

Rheology of Granular jets in 3D
: Is granular flow "perfect fluid ?"

Profile of the stress tensor



$$\sigma_{\alpha\beta} \equiv \sigma^k_{\alpha\beta} + \sigma^c_{\alpha\beta}$$

$$\sigma^k_{\alpha\beta} \equiv \frac{1}{V}\sum_i m u_{i\alpha} u_{i\beta}$$

$$\sigma^c_{\alpha\beta} \equiv \frac{1}{V}\sum_{i<j} F^{ij}_\alpha r^{ij}_\beta$$

$$u_{i\alpha} \equiv v_{i\alpha} - \bar{v}_\alpha$$

Large normal stress difference !! : $\sigma_{zz}, \sigma_{rr}, \sigma_{\theta\theta}$

Small off-diagonal part of stress tensor

→ Origin of Perfect-fluidity

$$\sigma_{\alpha\beta} = P_\alpha \delta_{\alpha\beta} - \eta D_{\alpha\beta} \simeq P_\alpha \delta_{\alpha\beta}$$

Is granular flow "perfect fluid ?"

How about shear viscosity ?

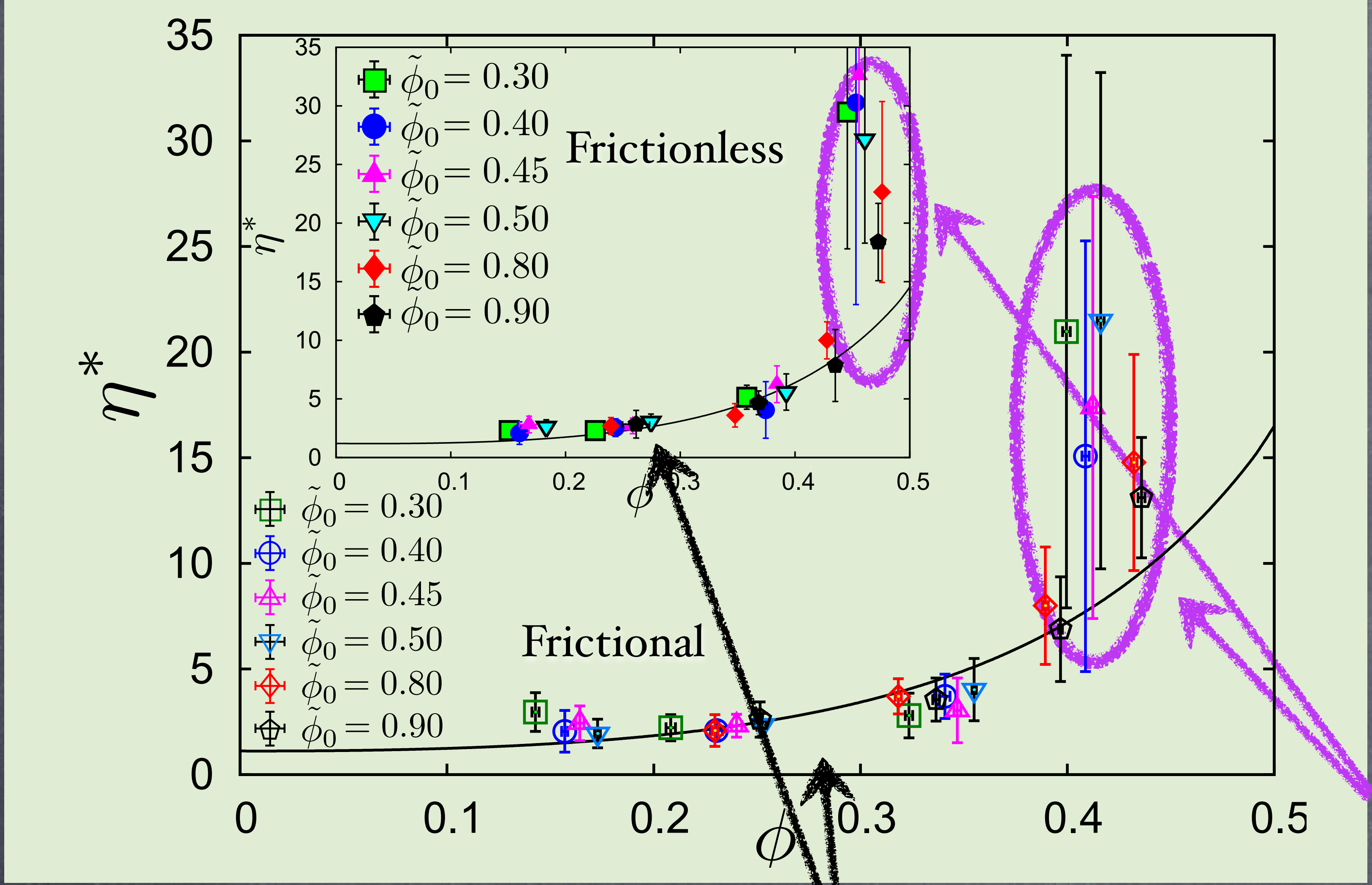
$$\sigma_{rz} = -\eta D_{rz}$$

$$\eta_0 \equiv \frac{5\sqrt{mT_g/\pi}}{16d^2}$$

$$\eta^* \equiv \eta/\eta_0$$

$$T_g = \frac{1}{N}\sum_i \frac{m\mathbf{u}_i^2}{3}$$

$$D_{\alpha\beta} \equiv \frac{1}{2}\left(\frac{\partial \bar{v}_\alpha}{\partial x_\beta} + \frac{\partial \bar{v}_\beta}{\partial x_\alpha}\right)$$



Note. In general,

$$\sigma_{rz} = \sigma_Y - \eta D_{rz}$$

However, we assume $\sigma_Y = 0$

Deviation: the effect of the source point: $r \sim 0$

The kinetic theory is not valid here.

$$\eta^* = \eta^*(\phi, e)$$

$$D^*_{rz} \equiv \frac{D_{rz}d}{\sqrt{T_g/m}} = O(0.01) \sim 0.4$$

→ Small strain rate → $\sigma_{\alpha\beta} = P_\alpha\delta_{\alpha\beta} - \eta D_{\alpha\beta} \simeq P_\alpha\delta_{\alpha\beta}$

Shear viscosity: consistent with kinetic theory

Results for Rheology of Granular jets in 3D

Granular flow cannot be a perfect fluid.

Why?

Granular flow looks like a perfect fluid.

Results for Rheology of Granular jets in 3D

Granular flow cannot be a perfect fluid.

Profile of the stress tensor

→ Shear stress looks very small in this setup.

→ $\sigma_{\alpha\beta} = P_\alpha \delta_{\alpha\beta} - \eta D_{\alpha\beta} \simeq P_\alpha \delta_{\alpha\beta}$

Granular flow looks like a perfect fluid.

Shear viscosity: consistent with kinetic theory $\eta \neq 0$

Large normal stress difference $\sigma_{\alpha\beta} \neq P\delta_{\alpha\beta}$

T. G. Sano and H. Hayakawa, Phys. Rev. E 86, 041308 (2012).

T. G. Sano and H. Hayakawa, Proceedings Powders & Grains 2013 (accepted), arXiv: 1211.3533

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Discussion & Summary

Previous Granular Jet studies

investigate 2D numerical studies to reproduce 3D experiments.

N. Guttenberg, Pys. Rev. E 85 051303 (2012).

J. Ellowitz, N. Guttenberg and W. W. Zhang, arXiv:1201.5562 (2012).

J. Ellowitz, H. Turlier, N. Guttenberg, W. W. Zhang, S. R. Nagel, arXiv:1304.4671 (2013).

However....

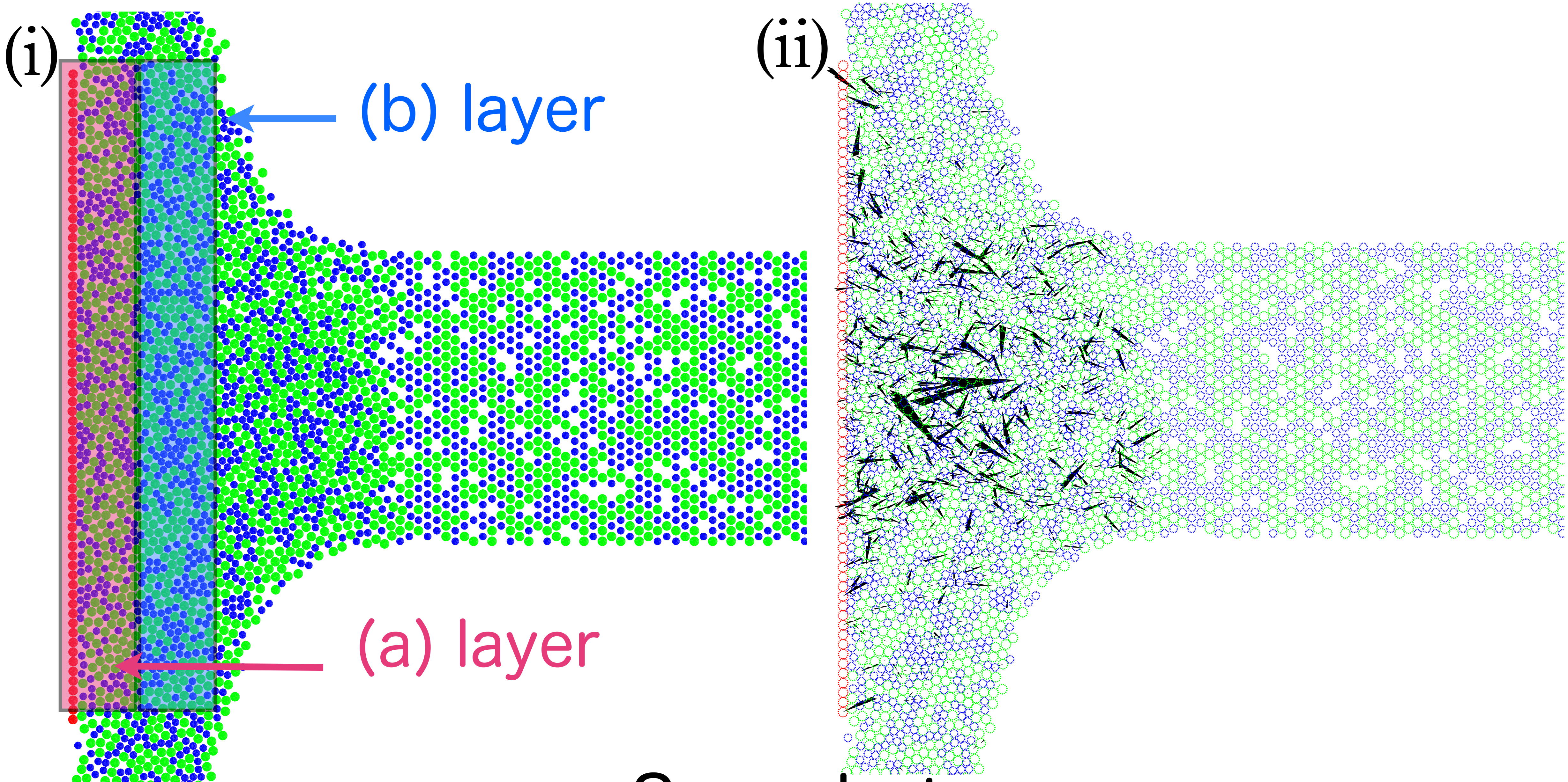
Are the rheological properties in 2D granular jets qualitatively the same as those in 3D ??

The aim of 2D rheological studies:
To clarify the qualitative difference between 2D and 3D granular jets

Rheology of Granular jets in 2D

Bi

Coordination number : $Z \simeq 0.526$ 71.5% of particles are NOT in contact.



Snapshot

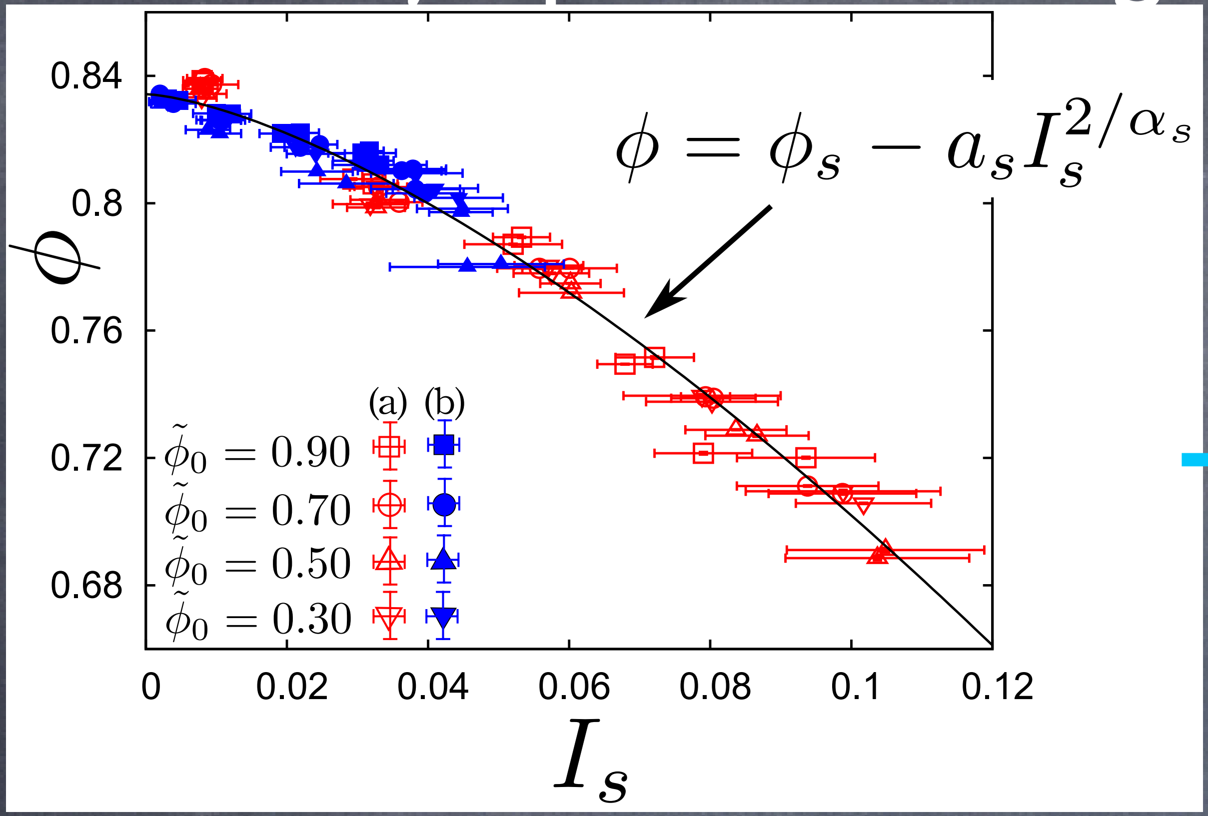
Grains are well packed

dense flow with contact-force network

Jet-induced Jammed state

Rheology of Granular jets in 2D

The asymptotic divergence of the pressure



: Frictionless case

$$I_s \equiv D_{xy}\sqrt{m/P} \quad P \equiv \frac{\sigma_{xx}+\sigma_{yy}}{2}$$

Results

$$\frac{P}{mD_{xy}^2} \sim (\phi_s - \phi)^{-\alpha_s}$$

$$\phi_s = 0.834 \pm 0.001$$

$$\alpha_s = 1.36 \pm 0.05$$

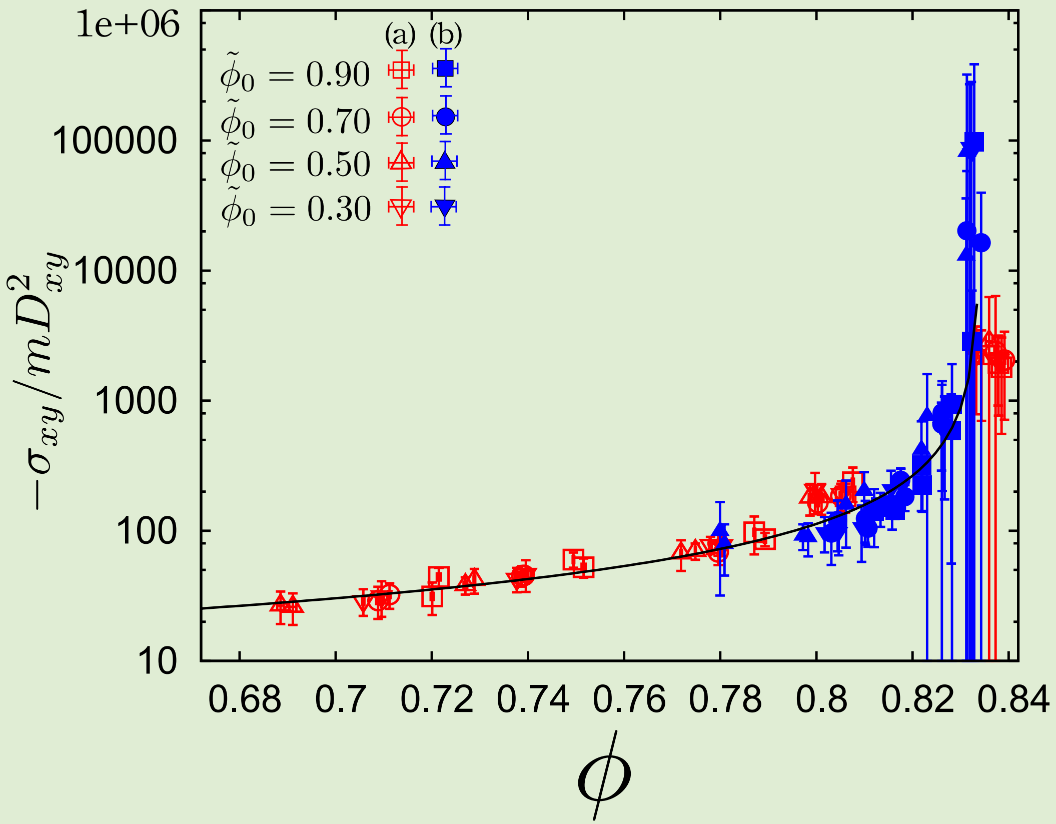
Jamming under shear

α_s
- Hatano(2008) 2.7
- Otsuki & Hayakawa(2009) 4.0 → Mean field picture of jamming
- Kinetic Theoretical regime 1.0 → $\frac{Pd^2}{T_g} \sim \frac{P}{mD_{xy}^2} \sim \phi g(\phi) \sim (\phi_c - \phi)^{-1}$
- Critical ϕ $\quad \phi_J = 0.8425 \simeq \phi_s$

Exponent is smaller than those of the sheared granular systems, and are close to the extrapolation from the kinetic theoretical regime.

The asymptotic divergence of shear stress

: Frictionless case



Shear stress: σ_{xy}

$$-\sigma_{xy} = \mu^* P \propto mD_{xy}^2(\phi_s - \phi)^{-(1-\beta/2)\alpha_s}$$

$$(1-\beta/2)\alpha_s \simeq 0.96$$

Results

$$-\sigma_{xy} \sim mD_{xy}^2(\phi_s - \phi)^{-\beta_s}$$

$$\beta_s \simeq 0.96$$

The asymptotic divergence is similar to the extrapolation from kinetic theoretical regime.

Origin of the difference between our case and systems under shear :

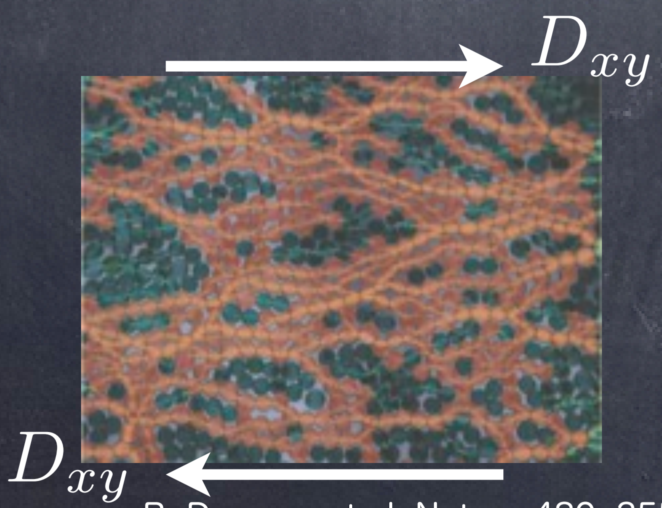
(i) Our system cannot reach the true jamming transition

→ Bagnold’s scaling regime

(ii) Uncontrollability of shear rate

→ We do nothing after the impact.

Jamming under shear	β_s
Garcia-Rojo et al. (2006) : Kinetic Theoretical	1.0
Hatano(2008)	2.6
Otsuki et al.(2010)	4.0



B. Dapeng, et al. Nature 480, 355–358 (15 December 2011)

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Discussion ~ Response to an impact ~

①Rheology of Granular jets in 3D

$$\sigma_{\alpha\beta} = P_\alpha\delta_{\alpha\beta} - \underline{\eta}\,\underline{D_{\alpha\beta}} \simeq P_\alpha\delta_{\alpha\beta}$$

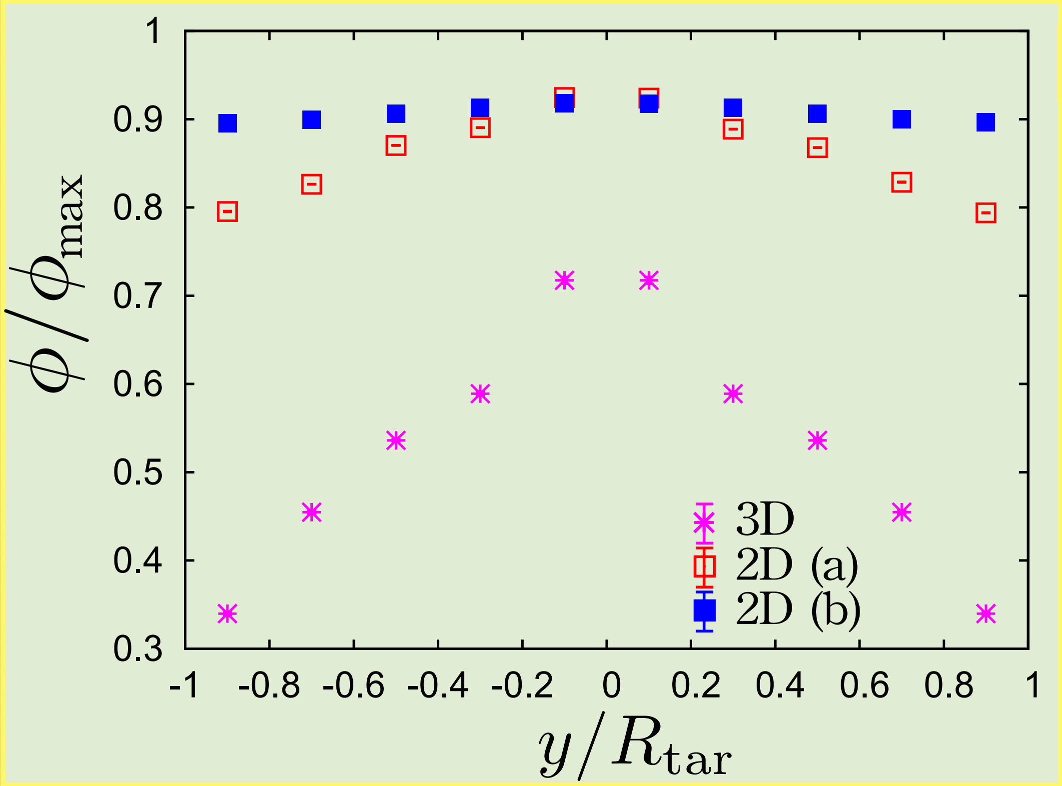
Grains: consistent with kinetic theory

Shear viscosity would be different if we use different particles.

Small: Geometrical constraint

Shear stress looks small as a whole.

②Rheology of Granular jets in 2D :Jet-induced jamming



In 2D, grains are well packed, compared with those in 3D.

Jet-induced jammed state

Note.
Critical phenomena of jamming under shear do not depend on spatial dimensions.

Summary

①Rheology of Granular jets in 3D

: Is granular flow "perfect fluid ?"

(i) $\sigma_{\alpha\beta}$: Small shear stress

Shear viscosity consistent with kinetic theory + Small strain rate

$$\longrightarrow \sigma_{\alpha\beta} = P_\alpha \delta_{\alpha\beta} - \eta D_{\alpha\beta} \simeq P_\alpha \delta_{\alpha\beta}$$

(ii) $\sigma_{\alpha\beta}$: Large normal stress difference

②Rheology of Granular jets in 2D :Jet-induced jamming

Dense flow with contact-force network

→Jet-induced "jammed" state

Asymptotic divergence of pressure and shear stress

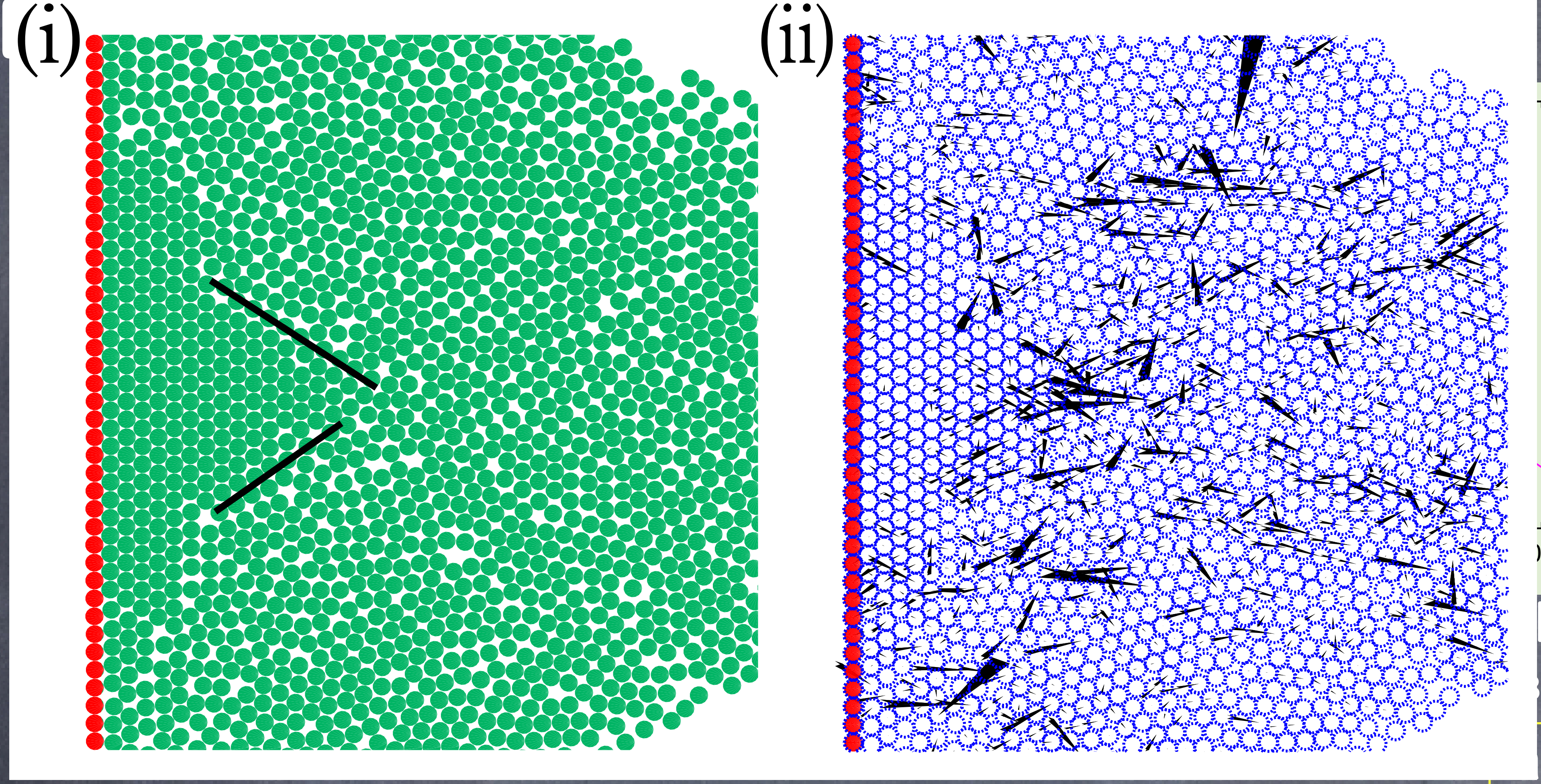
→ Extrapolation from kinetic theoretical regime

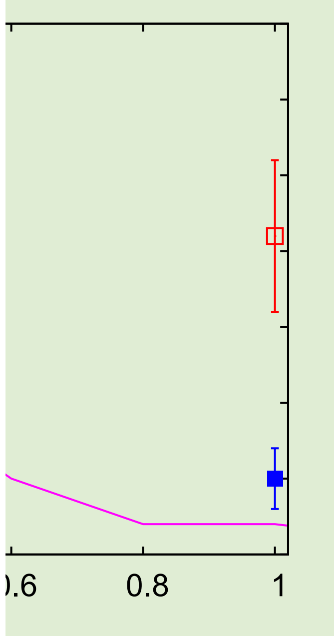
T. G. Sano and H. Hayakawa, Phys. Rev. E 86, 041308 (2012).

T. G. Sano and H. Hayakawa, Powders & Grains 2013 (in press), arXiv: 1211.3533

T. G. Sano and H. Hayakawa, arXiv:1302.6734

Thank you !





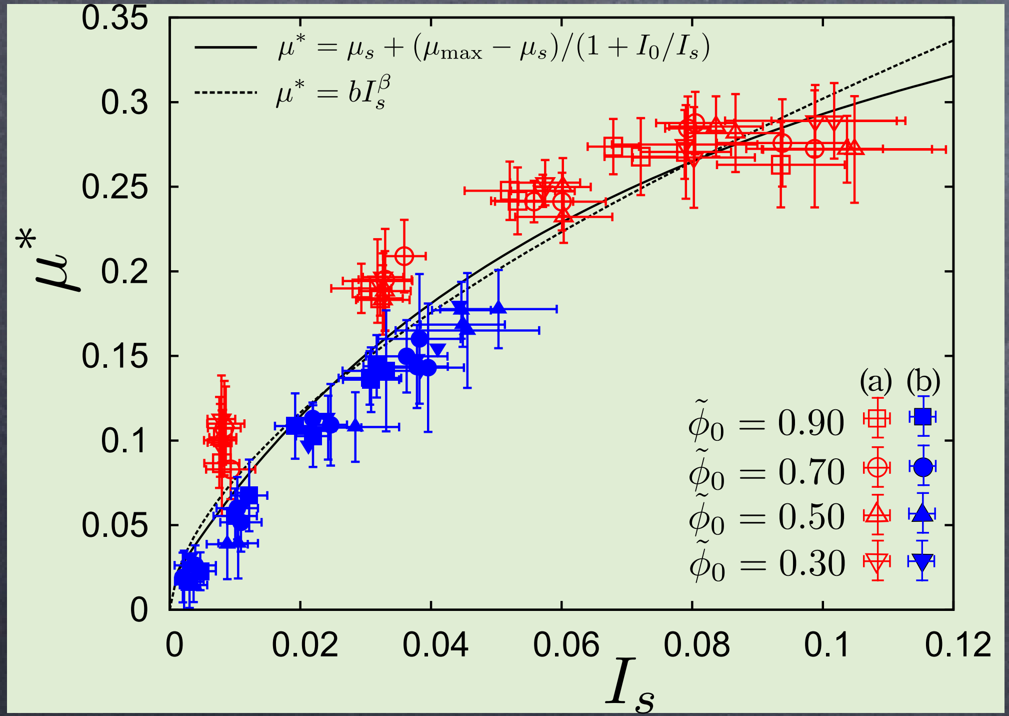
heared frictional grains

, 051301 (2011).

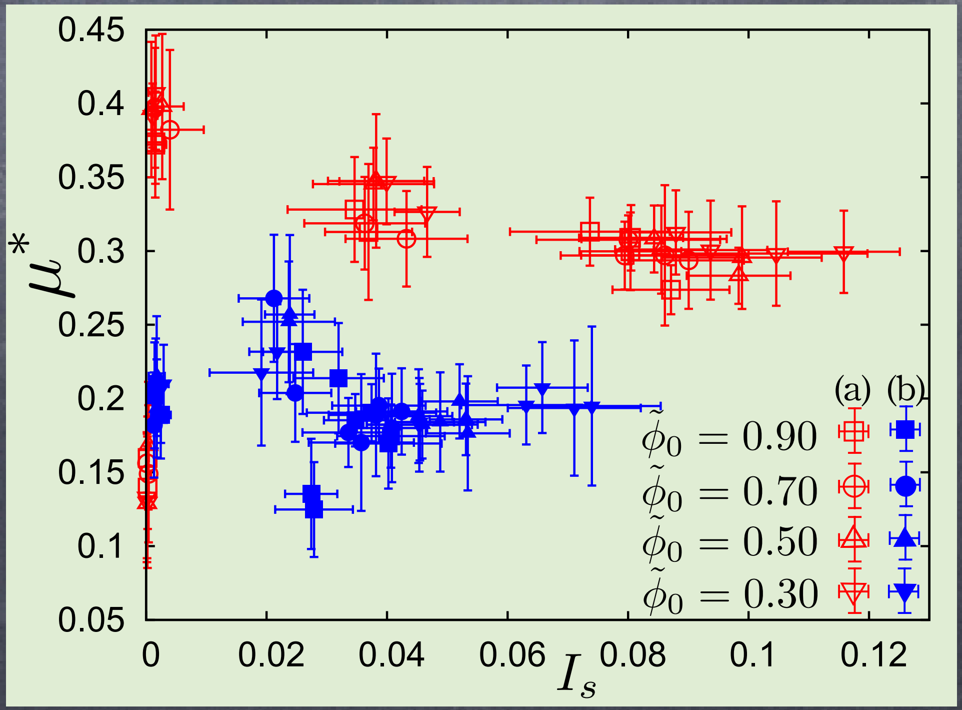
rge $\mu_p \to \phi_s$ decreases

Bi-disperse vs Mono-disperse

Effective friction const. $\mu^* \equiv -\sigma_{xy}/P$



Bi-disperse $d_1/d_2 = 0.8$

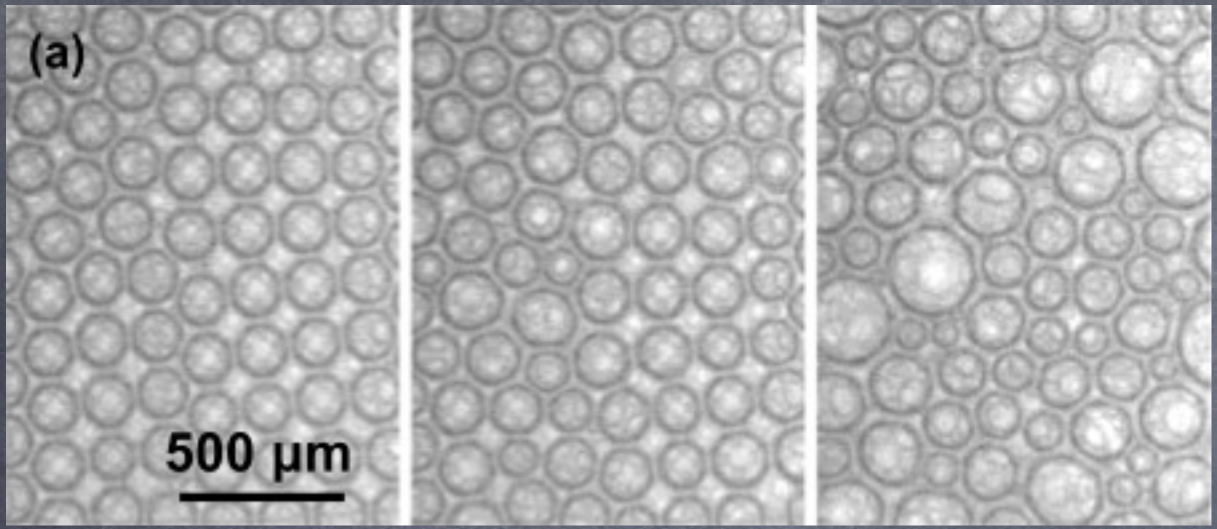


Mono-disperse $d_1/d_2 = 1.0$

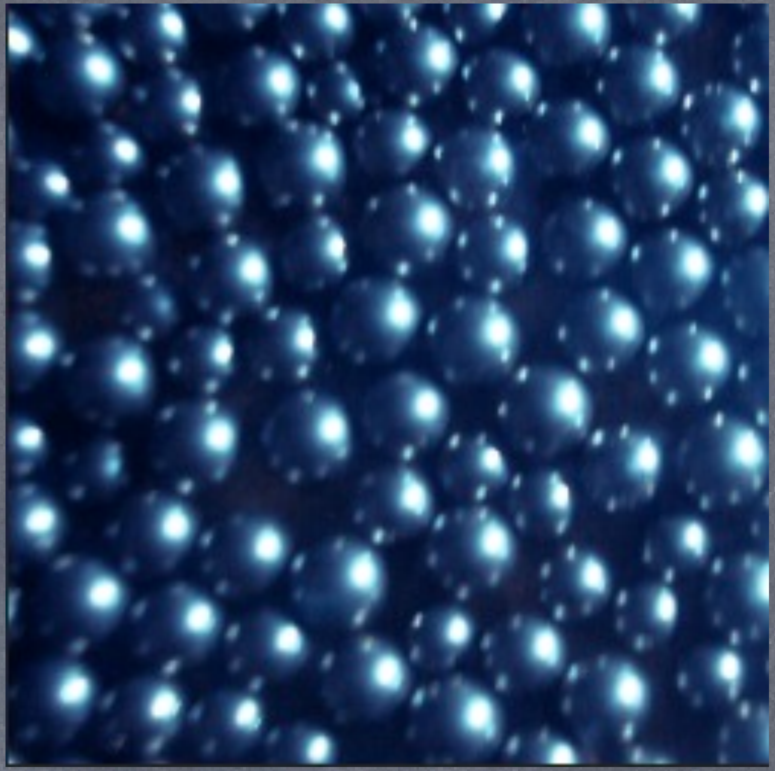
Two branches on μ^* vs I_s plane

Jamming Transition

A granular system has rigidity above a critical value of density ϕ_J, and has no rigidity below ϕ_J .



Jamming of foams
M. Le Merrer, et al. Phys. Rev. Lett. 108 188301 (2012).



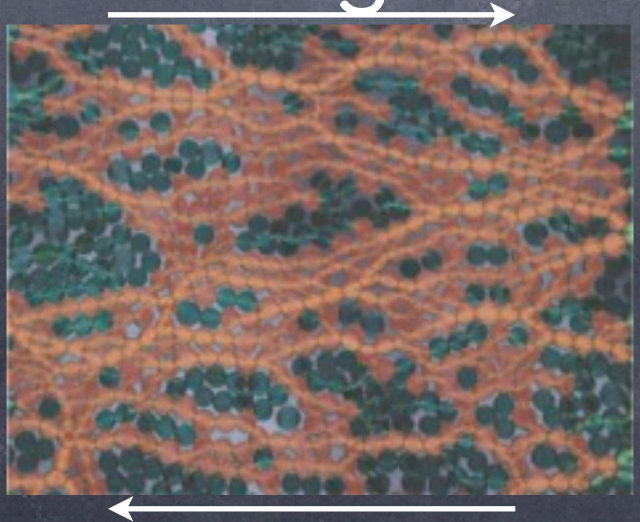
Grains Tanaka Lab., Univ. Tokyo

Jamming gripper
Univ. Chicago, Univ. Cornell,
iRobot and DARPA

Jamming of … : Grains, Foams, etc…

Characterization of Jamming

- Rigidity
- Divergence of Pressure and shear stress



Jamming under shear
B. Dapeng, et al. Nature 480, 355–358 (15 December 2011)

Phenomenology of jamming
Ex.) Kinetic theoretical divergence
Radial distribution function

$$g(\phi) \sim (\phi_c - \phi)^{-1}$$

Ex.) Mean field picture
M. Otsuki and H. Hayakawa
Prog. Theor. Phys. 121 647 (2009).