Study of K^-pp with an effective $K^{bar}N$ potential on coupled-channel Complex Scaling Method

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1. Introduction
2. Effective single-channel potential
   "Feshbach method with coupled-channel Complex Scaling Method"
3. Application to 2-body system:
   "Resonance of $K^{bar}N$-$\pi\Sigma$ = Λ^*"
4. Application to 3-body system: "K^-pp"
5. Summary and future plans

YITP workshop on "Hadron in Nucleus"
01.Nov.'13 @ YITP, Kyoto univ., Kyoto, Japan

1. Introduction

Kaonic nuclei = Nuclear system with anti-kaon "K^{bar}" (K^-, K^{0bar})

Λ(1405) ~ K^-p quasi-bound state → *Important building block*

K^-pp … *A prototype of K^{bar} nuclei*

K^-pp has been studied theoretically and experimentally

Experiment: FINUDA, DISTO

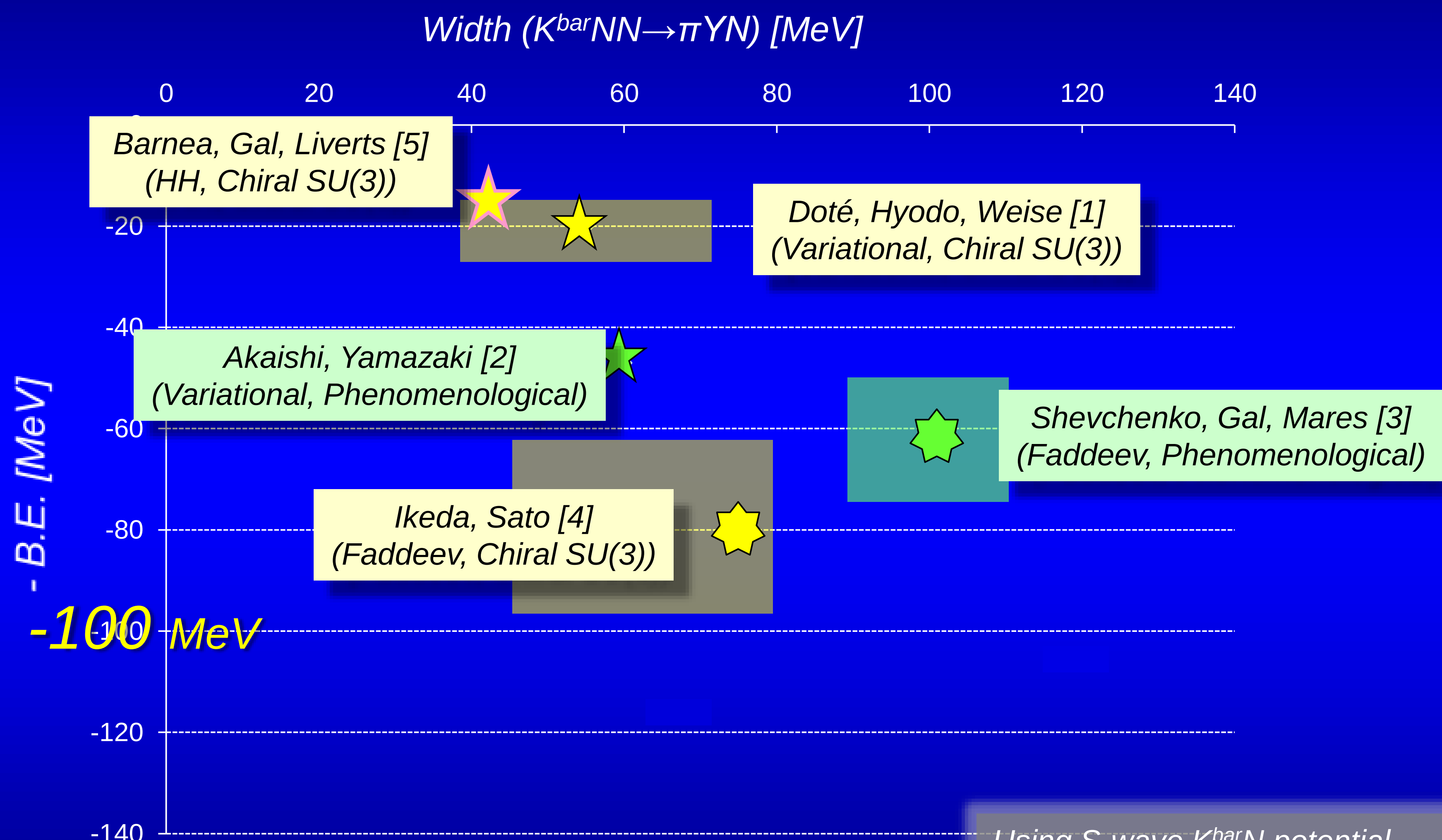
J-PARC: E27 $d(\pi^+, K^+)$, E15 ^3He(in-flight K^-, n)

LEPS: $d(\gamma, K^+)$, $d(\gamma, K^+\pi^-)$

Theory: Faddeev-AGS / Variational method (Gauss base, Hyperspherical Harmonics)

Phenomenological / Chiral SU(3)-based $K^{bar}N$ potential

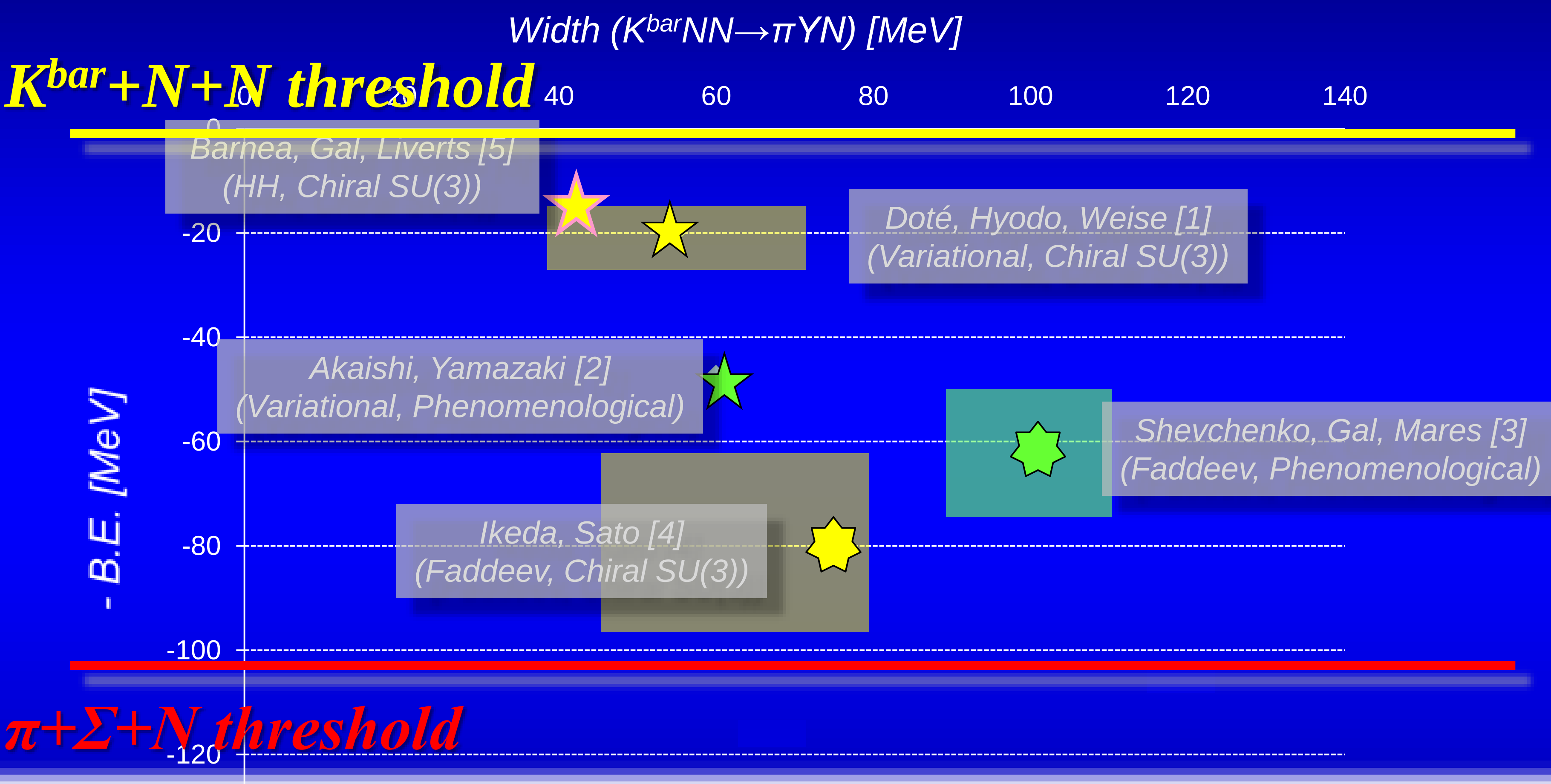
Typical results of theoretical studies of $K^{-}pp$



[1] PRC79, 014003 (2009)
[2] PRC76, 045201 (2007)
[3] PRC76, 044004 (2007)
[4] PRC76, 035203 (2007)
[5] PLB94, 712 (2012)

Using S-wave $K^{bar}N$ potential constrained by experimental data.
… $K^{bar}N$ scattering data,
Kaonic hydrogen atom data,
"Λ(1405)" etc.

Typical results of theoretical studies of K^-pp



From theoretical viewpoint,

K^-pp exists between K^{bar}-N-N and π-Σ-N thresholds!

[4] PRC76, 035203 (2007)

"Λ(1405)" etc.

Kaonic nuclei = Nuclear system with anti-kaon "K^{bar}"(K^-, K^{0bar})

Λ(1405) = Important building block

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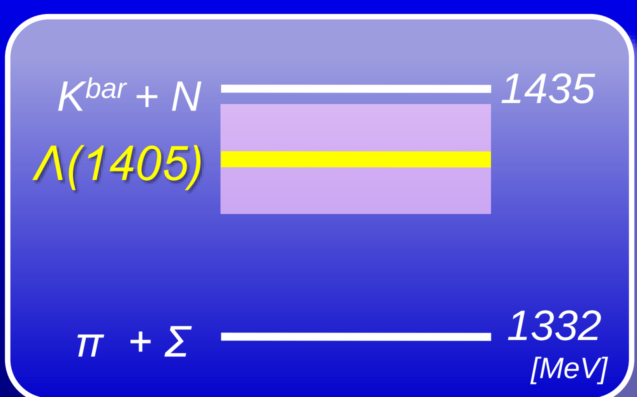
Phenomenological / Chiral SU(3)-based $K^{bar}N$ potential

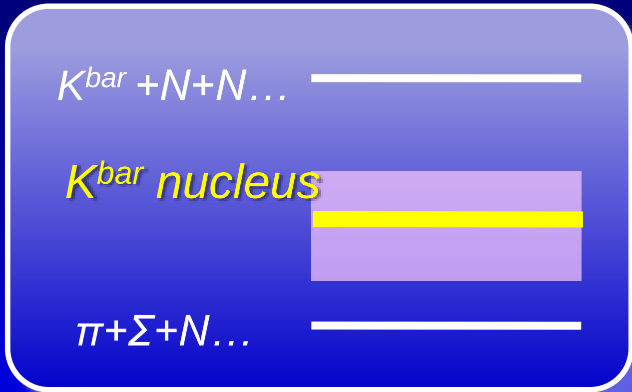


Resonant state

and

a coupled-channel system

($K^{bar}N(N) - \pi Y(N)$)



Resonant state
and
a coupled-channel system
($K^{bar}N(N) - \pi Y(N)$)

"coupled-channel Complex Scaling Method"

- ✓ *Consider a coupled-channel problem.*
- ✓ *Treat resonant states adequately.*
- ✓ *Obtain the wave function to help the analysis of the state.*
- ✓ *Applicable to many-body systems.*

coupled-channel Complex Scaling Method

A.D., T. Inoue and T. Myo, Nucl. Phys. A912, 66 (2013)

- ✓ *Applied to $K^{bar}N$-πY system*
- ✓ *Treat resonant and scattering states with Gaussian base like bound-states study*

"CSWF" method for scattering problem

A. T. Kruppa, R. Suzuki and K. Katō, PRC 75, 044602 (2007)

Chiral SU(3)-based potential (KSW-type potential)

*N. **K**aiser, P. B. **S**iegel and W. **W**eise, NPA 594, 325 (1995)*

- *Based on Chiral SU(3) theory*
  → Energy dependence
- *WT term, r-space, Gaussian form*
- *Semi-rela. / Non-rela.*

$$V_{ij}^{(I=0,1)}(r) = -\frac{C_{ij}^{(I=0,1)}}{8f_\pi^2}(\omega_i+\omega_j)\sqrt{\frac{M_iM_j}{s\,\omega_i\omega_j}}\,g_{ij}(r)$$

$$g_{ij}(r) = \frac{1}{\pi^{3/2}d_{ij}^3}\exp\left[-\left(r/d_{ij}\right)^2\right]$$

Constrained by $K^{bar}N$ scattering length

$$a_{KN(I=0)} = -1.70+i0.67fm,\quad a_{KN(I=1)} = 0.37+i0.60fm$$

A.D.Martin, NPB179, 33(1979)

Complex Scaling Method for Resonance

Complex rotation of coordinate (Complex scaling)

$$U(\theta): \quad \mathbf{r} \to \mathbf{r}\, e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}\, e^{-i\theta}$$

$$H_\theta \equiv U(\theta) H U^{-1}(\theta), \quad |\Phi_\theta\rangle \equiv U(\theta)|\Phi\rangle$$

By Complex scaling,

- Resonance wave function: divergent function ⇒ damping function

⇒ *Boundary condition is the same as that for a bound state.*

- *The pole position of resonance doesn't change.*

ABC theorem

"The energy of bound and resonant states is independent of scaling angle θ."

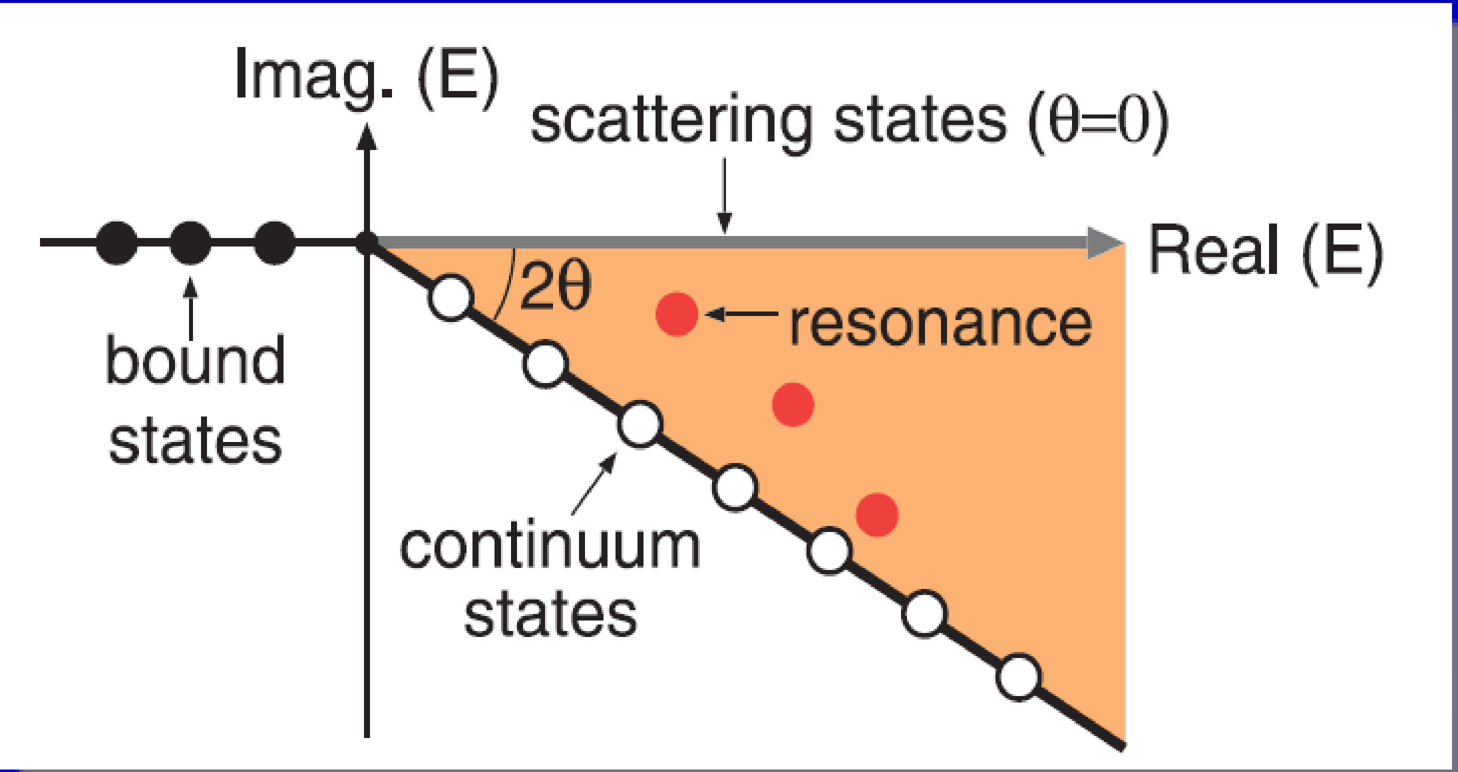
† J. Aguilar and J. M. Combes, Commun. Math. Phys. 22 (1971),269.
E. Balslev and J. M. Combes, Commun. Math. Phys. 22 (1971),280

Diagonalize H_θ with Gaussian base,

we can obtain resonant states, in the same way as bound states!

Complex Scaling Method for Resonance

Eigenvalue distribution after complex scaling



- *Continuum state appears on 2ϑ line.*
- *Resonance pole is off from 2ϑ line, and independent of ϑ.*

S. Aoyama, T. Myo, K. Kato and K. Ikeda, Prog. Theor. Phys. 116, 1 (2006)

2. Effective potential
for single-channel calculation

"Feshbach method with ccCSM"

Feshbach method

$$H\,\Phi = E\,\Phi$$: Schrödinger eq. in P and Q spaces

$$\begin{pmatrix} T_P + v_P & V_{PQ} \\ V_{QP} & T_Q + v_Q \end{pmatrix} \begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix} = E \begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix}$$

P: model space
Q: out of model space

Schrödinger eq. in P space :

$$\left(T_P + U_P^{Eff}(E) \right) \Phi_P = E\,\Phi_P$$

Effective potential

$$\begin{aligned} U_P^{Eff}(E) &= v_P + V_{PQ}\, G_Q(E) V_{QP} \\ &= v_P + V_{PQ} \frac{1}{E - H_{QQ}} V_{QP} \end{aligned}$$

Green function expressed with ECR[†]

$$G_Q(E) = \frac{1}{E - H_{QQ}}$$

† Y. Kikuchi, T. Myo, M. Takashina, K. Kato and K. Ikeda, PTP122, 499 (2009); PRC81, 044308(2010)

Complex scaling $U(\theta)$: $r \to r\, e^{i\theta}$, $k \to k\, e^{-i\theta}$

$$H^{\theta}_{QQ} = U(\theta) H_{QQ} U^{-1}(\theta)$$

$$G^{\theta}_{Q}(E) = \frac{1}{E - H^{\theta}_{QQ}}$$

$$\Rightarrow \quad G_Q(E) = U^{-1}(\theta) G^{\theta}_{Q}(E) U(\theta)$$

Extended Completeness Relation

Diagonalize H^{θ}_{QQ} with Gaussian base,

$$H^{\theta}_{QQ} \left| \chi^{\theta}_{n} \right\rangle = \varepsilon^{\theta}_{n} \left| \chi^{\theta}_{n} \right\rangle$$

$$\int_C \sum_{R+B} \left| \chi^{\theta}_{n} \right\rangle \left\langle \chi^{\theta}_{n} \right| = 1$$

$$\sum_n \left| \chi^{\theta}_{n} \right\rangle \left\langle \chi^{\theta}_{n} \right| \approx 1$$

Well approximated

T. Myo, A. Ohnishi and K. Kato, PTP99, 801 (1998)

$$G^{\theta}_{Q}(E) \approx \sum_n \left| \chi^{\theta}_{n} \right\rangle \frac{1}{E - \varepsilon^{\theta}_{n}} \left\langle \chi^{\theta}_{n} \right|$$

$$\Rightarrow \quad U^{Eff}_{P}(E) = v_P + V_P(E)$$

✓Non-local

✓Energy dependent

$$V_P(E) = V_{PQ} G_Q(E) V_{QP}$$

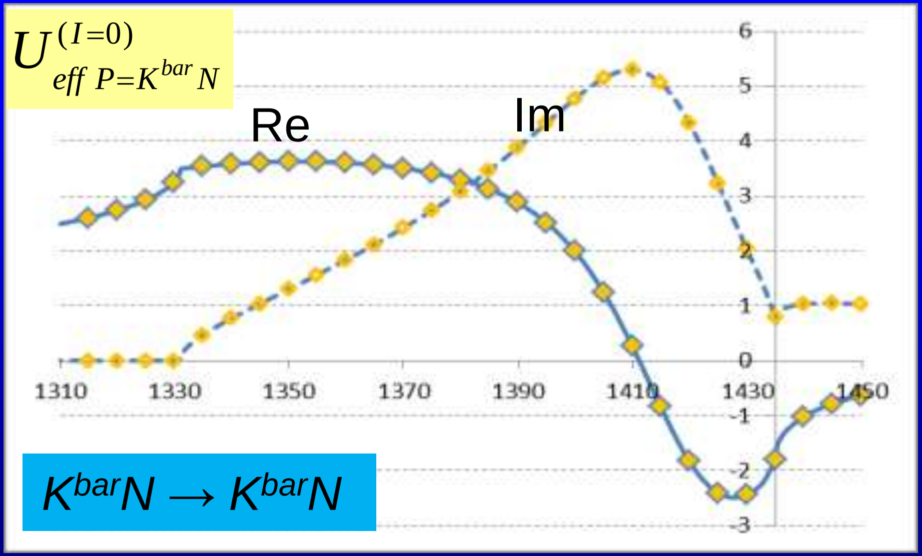
$$\approx \sum_n V_{PQ} U^{-1}(\theta) \left| \chi^{\theta}_{n} \right\rangle \frac{1}{E - \varepsilon^{\theta}_{n}} \left\langle \chi^{\theta}_{n} \right| U(\theta) V_{QP}$$

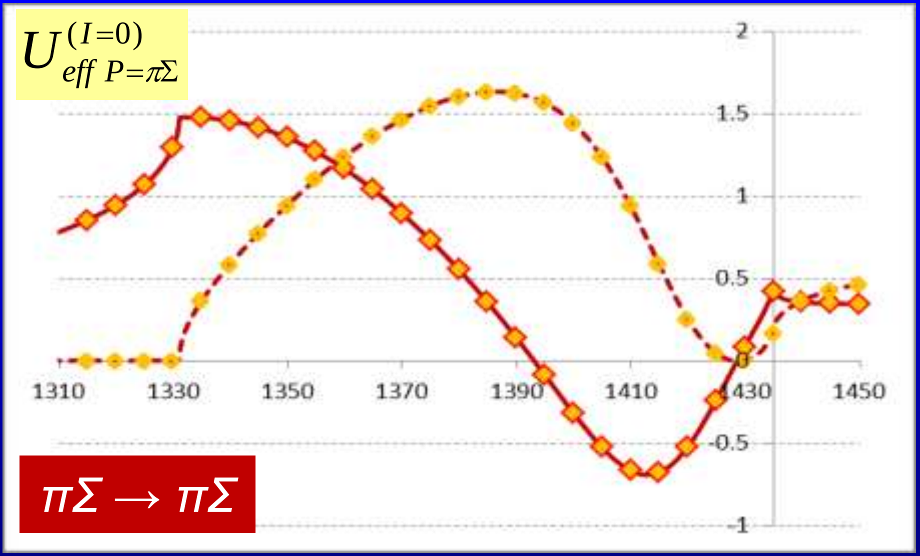
3. Application to 2-body system

"Resonance of $K^{bar}N$-$\pi\Sigma(I=0)$ = Λ*"

Confirmed that
"Feshbach with ccCSM" works well for scattering states,
But how is resonance case?

- *Scattering amplitude*





KSW - NRv2 (I=0), f_π=110 MeV

AY potential† (Non-rela. / E-indep.)

†Y. Akaishi and T. Yamazaki, PRC 52 (2002) 044005

Schrödinger eq. in P space : $\left(T_P + U_P^{Eff}(Z) \right) \Phi_P = Z \Phi_P$

Resonance → Self-consistency for complex energy "Z"

Feshbach+ccCSM

P−space	KN + π Σ	KN	π Σ
B(KN)	28.1698	28.1698	28.1698
Γ /2	20.0288	20.0288	20.0289
[MeV]			
Mean distance			
KN	1.31 − i0.35	1.25 − i0.27	*
π Σ	0.31 − i0.21	*	0.21 + i0.91
Total	1.34 − i0.39	1.25 − i0.27	0.21 + i0.91
[fm]			

KSW-type potentials (E-dep.)

$$V_{ij}^{(I=0)}(r) \sim -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j) \times g_{ij}(r)$$

- NRv2
  (Non-rela. / E-dep.)

Feshbach+ccCSM

P-space	KN+π Σ	KN	π Σ
B(KN)	17.1605	17.1606	17.1728
Γ /2	16.6176	16.6178	16.613
[MeV]			
Mean distance			
KN	1.37 − i0.37	1.28 − i0.40	*
π Σ	0.37 + i0.04	*	0.23 + i0.93
Total	1.42 − i0.34	1.28 − i0.40	0.23 + i0.93
[fm]			

- SR-A
  (Semi-rela. / E-dep.)

Feshbach+ccCSM

P-space	KN+π Σ	KN	π Σ
B(KN)	15.5336	15.5336	15.524
Γ /2	25.0158	25.015	24.9997
[MeV]			
Mean distance			
KN	1.22 − i0.47	1.07 − i0.46	*
π Σ	0.13 + i0.05	*	0.05 − i0.27
Total	1.22 − i0.47	1.07 − i0.46	0.05 − i0.27
[fm]			

$$\left(T_P + U_P^{Eff}(Z)\right)\Phi_P = Z\,\Phi_P$$

$$v_{ij}(Z)$$

Even when the original interaction has energy dependence, a self-consistent solution with complex energy can be obtained.

4. Application to 3-body system

"$K^- pp$"

… $\bar{K}NN - \pi\Sigma N - \pi\Lambda N$ ($J^\pi = 0^-$, $T = 1/2$)

4. Application to 3-body system

"$K^- pp$"

… $K^{bar}NN$ - $\pi\Sigma N$ - $\pi\Lambda N$ ($J^\pi=0^-$, $T=1/2$)

Feshbach + ccCSM

$$V\left(K^{bar}N-\pi Y;I=0,1\right)$$
$$V\left(\pi Y-\pi Y' \quad ;I=0,1\right)$$

$$\Rightarrow \quad U^{Eff}_{K^{bar}N(I=0,1)}\left(E\right)$$

Schrödinger eq. for $K^{bar}NN$ channel :

$$\left(T_{K^{bar}NN}+V_{NN}+\sum_{i=1,2}U^{Eff}_{K^{bar}N_i(I)}\left(E_{K^{bar}N}\right)\right)\Phi_{K^{bar}NN} = E\,\Phi_{K^{bar}NN}$$

How to solve …

- **Wave function**

$$\left|"K^- pp"\right\rangle = \sum_a C_a^{(KNN,1)} \left\{ G_a^{(KNN,1)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) + G_a^{(KNN,1)}\left(-\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) \right\} \left| S_{NN}=0 \right\rangle \left| \left[K \left[NN \right]_1 \right]_{T=1/2} \right\rangle$$

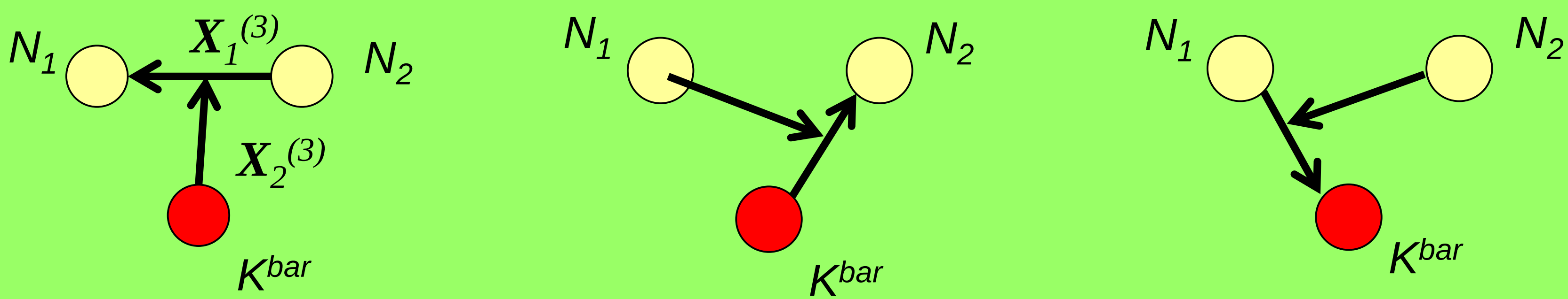
Ch. 1: $K^{bar}NN$, NN: ^1E

$$+ \sum_a C_a^{(KNN,2)} \left\{ G_a^{(KNN,2)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) - G_a^{(KNN,2)}\left(-\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) \right\} \left| S_{NN}=0 \right\rangle \left| \left[K \left[NN \right]_0 \right]_{T=1/2} \right\rangle$$

Ch. 2: $K^{bar}NN$, NN: ^1O

- **Basis function = Correlated Gaussian**

$$G_a^{(KNN,i)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) = N_a^{(KNN,i)} \exp\left[-\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) A_a^{(KNN,i)} \begin{pmatrix} \mathbf{x}_1^{(3)} \\ \mathbf{x}_2^{(3)} \end{pmatrix} \right]$$

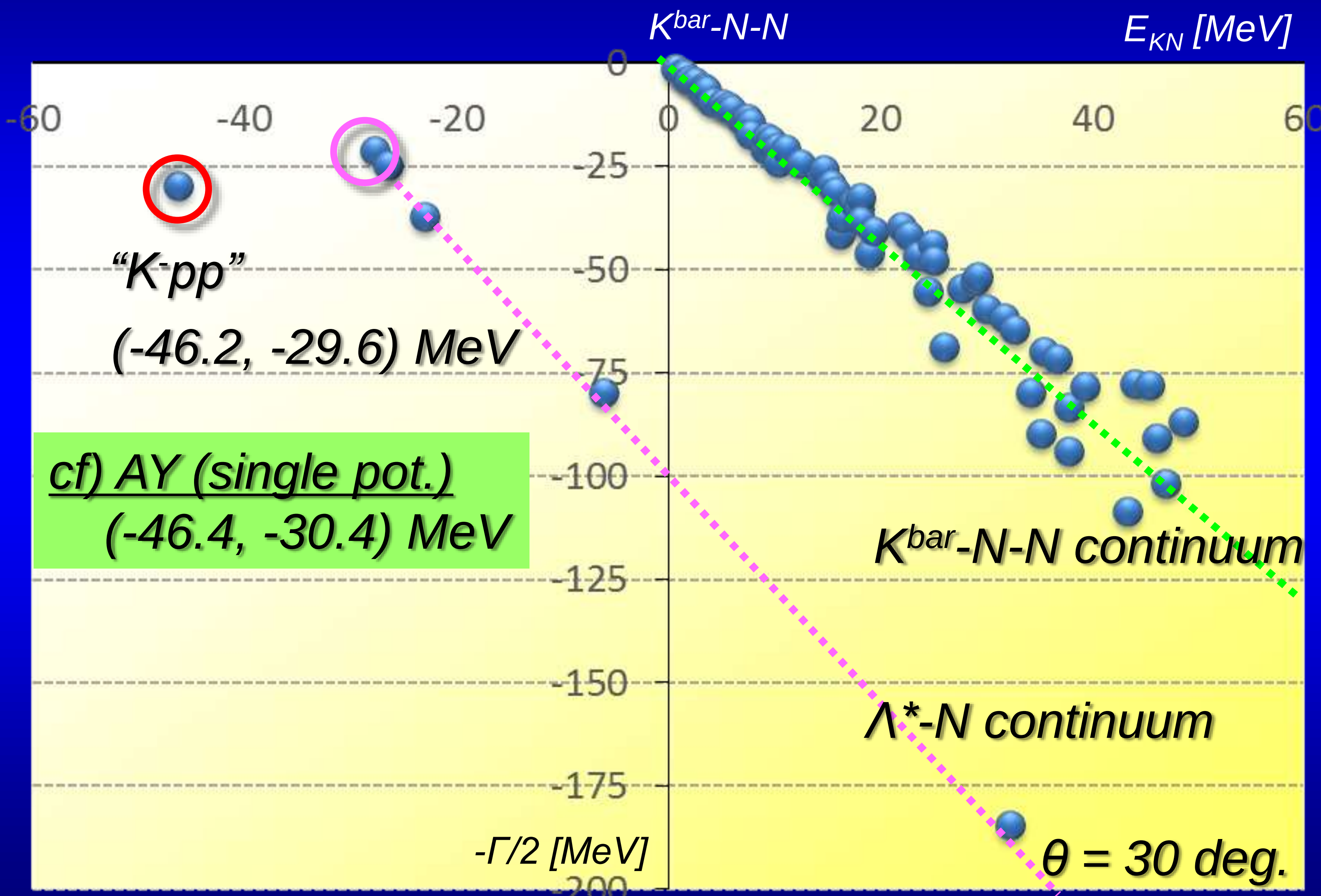


- **Diagonalize the complex Hamiltonian with base functions**

$$H_{K^{bar}NN} = T_{K^{bar}NN} + V_{NN} + \sum_{i=1,2} U^{Eff}_{K^{bar}N_i(I)}\left(E_{K^{bar}N}\right)$$

Test calculation with AY potential

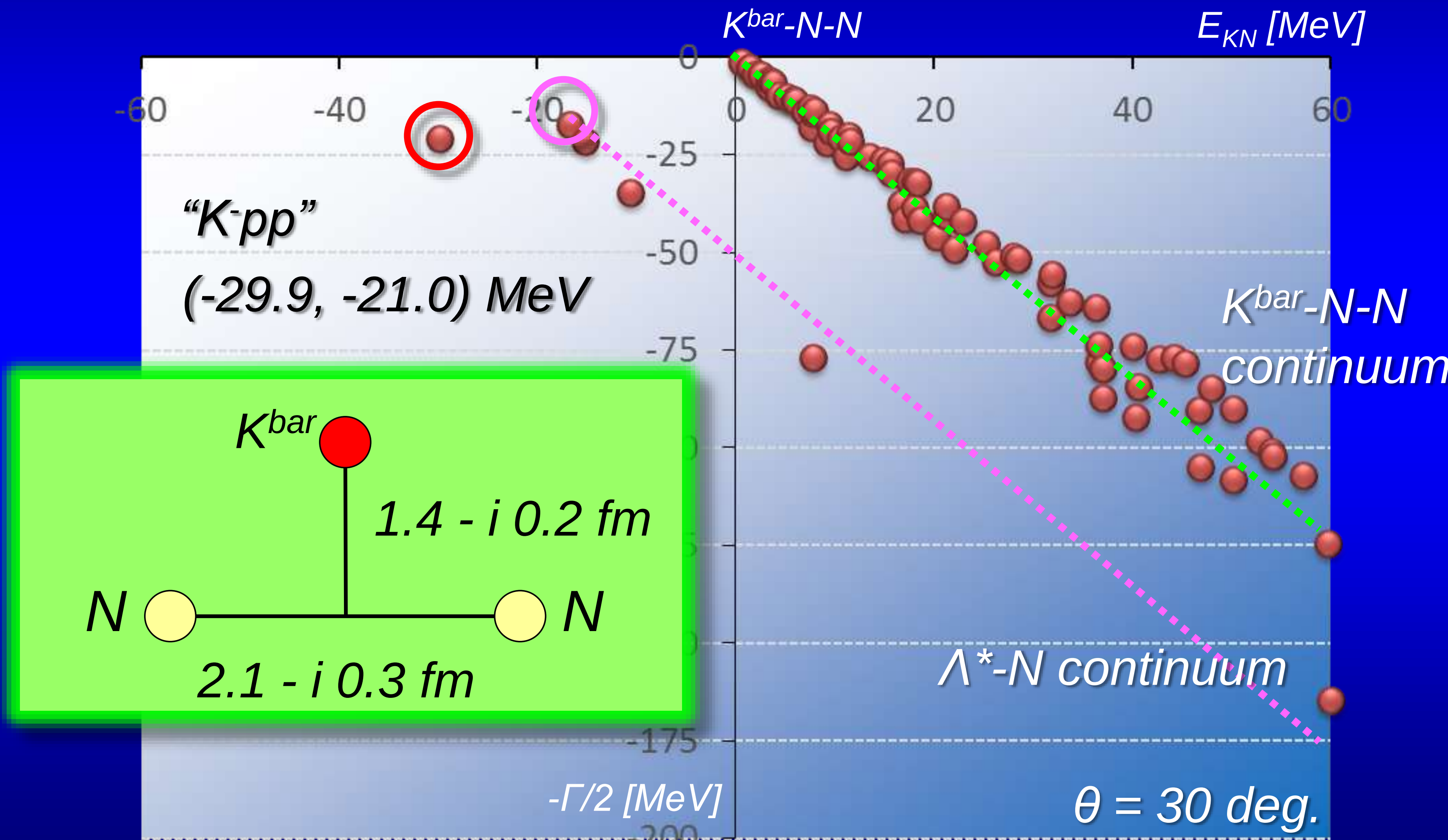
NN potential : Tamagaki potential (G3RS-case1)



Result (preliminary): KSW-NRv2 potential

NN potential : Av18 (Central + spin-spin)

$K^{bar}N$ energy is fixed at Λ*. (Not self-consistent!)



5. Summary and future plans

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The simplest K^{bar} nucleus "$K^{-}pp$" = Resonance state of K^{bar}NN-πYN coupled system

"Feshbach method + coupled-channel Complex Scaling Method"

… Represent the Green function in Q-space with the Extended Complete Set

⇒ Effective single-channel potential (for K^{bar}N , eliminate πY channels)

Two-body case "K^{bar}N-πΣ =Λ"*

Reproduce perfectly the result of the full coupled-channel calculation (AY, KSW-NRv2)

Three-body case "$K^{-}pp$"

←Use Correlated Gaussian base, diagonalize the complex Hamiltonian

・AY potential: agree with Akaishi's result

(B. E., Γ) = (46.4, 60.8) MeV with Tamagaki NN potential

・KSW-NRv2 potential:

Based on chiral SU(3) theory, Energy dependent, Non-relativistic version

Constraint by the K^{bar}N scattering length (A.D.Martin)

∗K^{bar}N energy is fixed to that of Λ   Λ*: (B. E., Γ) = (17.2, 33.2) MeV*

⇒ (B. E., Γ) ~ (30, 42) MeV with Av18 NN potential

1. *Self consistency of K^{bar}N energy in the effective potential*
2. *Full coupled-channel calculation with the coupled-channel Complex Scaling Method*