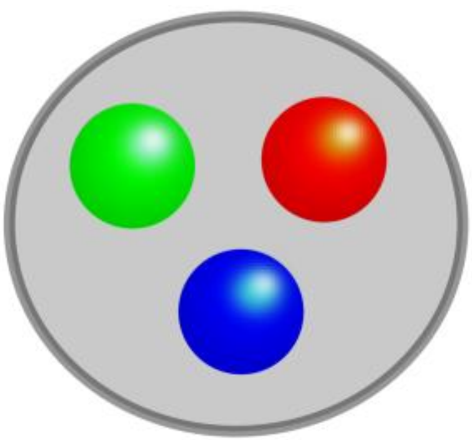
Partial Restoration of Chiral Symmetry and In-medium Pion Properties

Soichiro Goda (Kyoto Univ.)
Daisuke Jido (Tokyo Metropolitan Univ.)



Hadrons in Nuclei

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Introduction

Chiral Symmetry Breaking

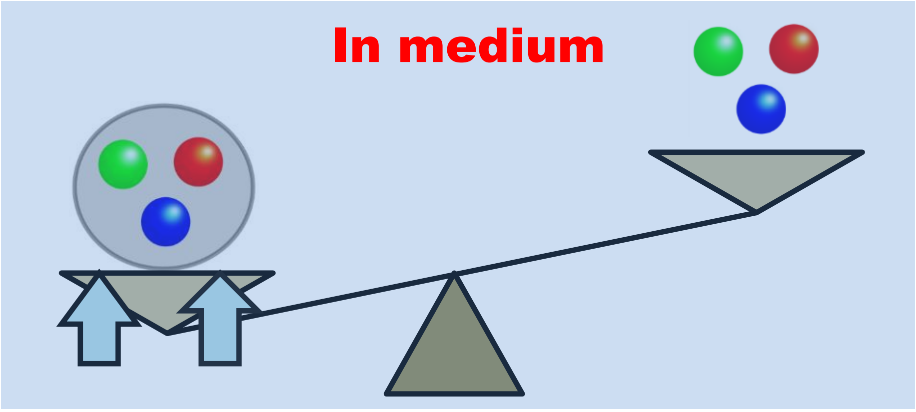
Chiral SSB characterizes Low energy QCD vacuum.

- Breaking pattern: $SU(2)_L \otimes SU(2)_R \to SU(2)_V$
- Nambu-Goldstone bosons: ***pions***
- Chiral condensate: $\langle \bar{q}q \rangle$ Characteristic scale of Hadrons

Mass generation mechanism?

How do we confirm the mechanism phenomenologically?

➢ One of the proofs is to examine partial restoration of chiral sym.



Partial restoration of chiral sym.

Partial restoration = Reduction of $|\langle\bar{q}q\rangle|$ in medium

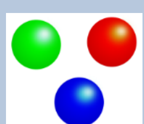
We focus on nuclear medium with finite density and 0 temperature.

$$\frac{\langle\bar{u}u+\bar{d}d\rangle^*}{\langle\bar{u}u+\bar{d}d\rangle_0} = \left(1-\frac{\rho}{m_\pi^2 f_\pi^2}\sigma_{\pi N}\right)+o(\rho)$$

⇨ In-medium hadron properties' change

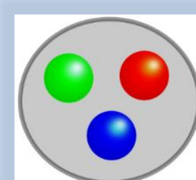
$\sigma_{\pi N} = m_q\langle N|\bar{u}u+\bar{d}d|N\rangle$ πN sigma term : πN scattering amplitude in soft limit

Complementarity between quark and hadron descriptions



Quarks

⟺



Hadrons

Baryons

Mesons

- Once we determine the density dependence of the condensate, we can **predict** in-medium hadronic quantities and vice versa.

➢ Several in-medium low energy theorems are derived model-independently using current algebras.

D. Jido, T. Hatsuda and T. Kunihiro, Phys. Lett. B **670** (2008) 109

- In-medium Glashow-Weinberg relation

$$\frac{\langle \bar{u}u + \bar{d}d \rangle^*}{\langle \bar{u}u + \bar{d}d \rangle_0} = \left(\frac{f_t}{f_\pi}\right)\left(\frac{G_\pi^*}{G_\pi}\right)$$

f_t :temporal pion decay constant

G_π^* :pseudo-scalar coupling

- In-medium Weinberg-Tomozawa relation

In-medium decay constant is related to s-wave isovector π-N sca. length.

$$T^-(\omega = m_\pi) \approx \frac{m_\pi}{2f_t^2} = -4\pi\left(1 + \frac{m_\pi}{m_N}\right) b_1^* \quad \Rightarrow \quad \frac{f_t^2}{f_\pi^2} = \frac{b_1}{b_1^*}$$

- In-medium Gell-Mann-Oakes-Renner relation

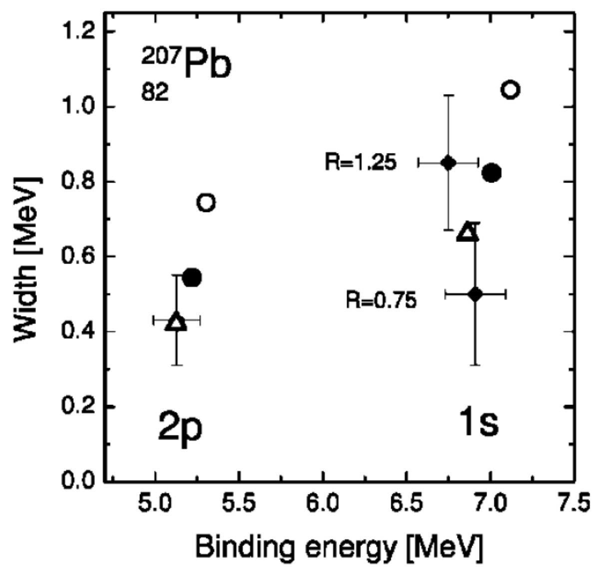
$$\frac{\langle \bar{u}u + \bar{d}d \rangle^*}{\langle \bar{u}u + \bar{d}d \rangle_0} = \left(\frac{f_t}{f_\pi}\right)^2\left(\frac{m_\pi^*}{m_\pi}\right)^2$$

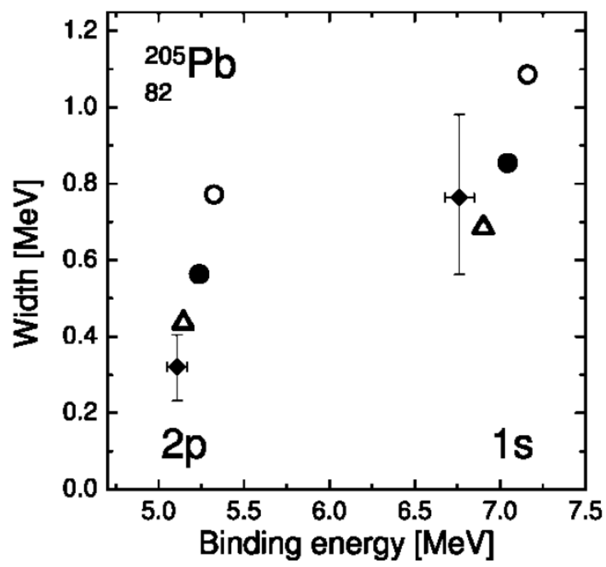
➢ These theorems suggest that in-medium pionic observables are related to in-medium chiral condensate.

➢ Deeply bound pionic atom suggests the partial restoration.

Solve KG eq. with π-Nucleus optical potential and Coulomb potential

E. Kolomeitsev, N. Kaiser and W. Weise, Phys. Rev. Lett. 90 (2003) 092501





◆ : Experiment

△ : 2-loop calculation by in-medium CHPT

○ : Energy indep. potential in Tρ approx.

● : Energy dep. Potential in Tρ approx.

- Essence: Reduction of pion decay constant

s-wave pion-nucleus optical potential (self-energy) to leading order in Tρ approx.

$$2m_\pi U_s \approx -T^-(\rho)\delta\rho = -\frac{\delta\rho}{4f_\pi^2}\left(1-\frac{\sigma_{\pi N}}{f_\pi^2 m_\pi^2}\right)^{-1} = -\frac{\delta\rho}{4f_\pi^{*2}}$$

W. Weise, Nucl. Phys. A690, 98c (2001);
W. Weise, Acta Phys. Pol. B 31, 2715 (2000).

$$\rho = \rho_p + \rho_n \qquad \delta\rho = \rho_p - \rho_n$$

Repulsive enhancement of s-wave isovector π-Nucleus sca. length can be explained by the reduction of decay constant.

Motivation

To understand partial restoration of chiral symmetry in nuclear matter beyond linear density approximation

Our work in this talk

We evaluate
- **Temporal decay constant**
- **Pion mass**
- **π0 decay width**

in nuclear matter using in-medium chiral perturbation theory(CHPT) and discuss Low energy theorems beyond linear density approximation.

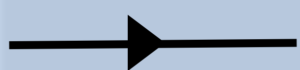
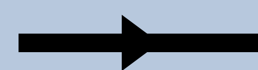
In-medium chiral perturbation theory

J. A. Oller, Phys. Rev. C **65** (2002) 025204
U. G. Meissner, J. A. Oller and A. Wirzba, Annals Phys. **297** (2002) 27

- ➢ Effective field theory for **pions in nuclear matter**
- ➢ **πN interactions: determined by chiral sym.**
- ➢ **Ground-state: Filled Fermi sea of nucleons**

Nucleon propagator is replaced into in-medium (Fermi gas) propagator.

$$iG(p) = i\frac{\not{p} + m_N}{p^2 - m_N^2 + i\epsilon} - 2\pi(\not{p} + m_N)\delta(p^2 - m_N^2)\theta(p_0)\theta(k_F - |\mathbf{p}|)$$

$=$  $+$ 

Free

Pauli blocking effect : filled up to kF

- ➢ Green fn. is characterized by **Double Expansion.**

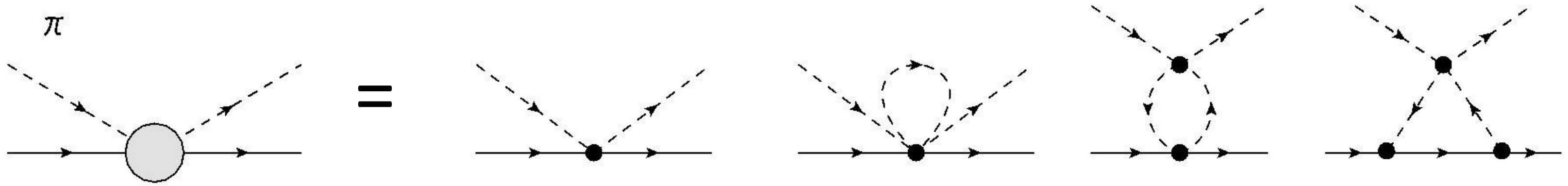
- Expansion parameters
  - ・**Pion momentum, mass** $p_\pi \sim m_\pi \sim O(p)$
  - ・**Fermi momentum of nuclear matter** $k_F \sim 2m_\pi \sim O(p)$

Classification based on density orders

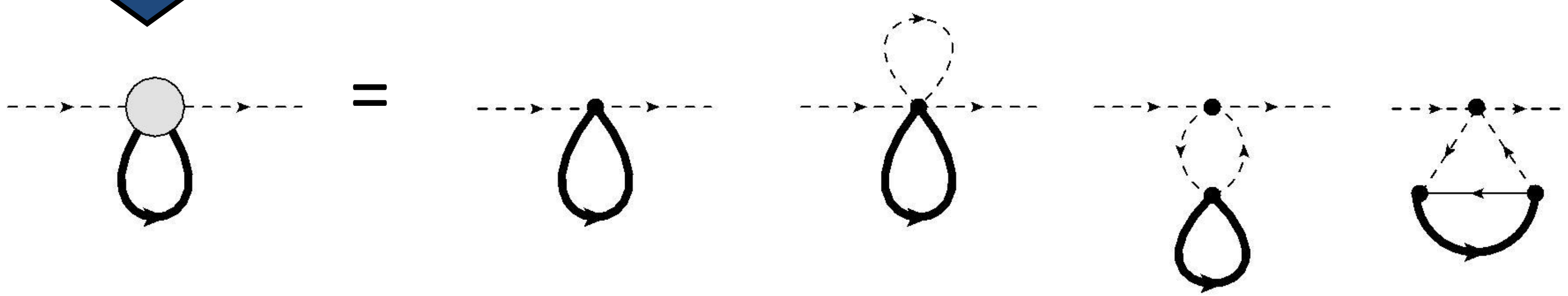
◆ Key point: Renormalization in vacuum sector

Ex. Density corrections to pion mass

Physical pi-N coupling



And then we consider density corrections



They have **different chiral orders** in chiral counting,
but the **same density orders**.

We assume that renormalizations in vacuum are already done.

We take observed value as coupling in chiral Lagrangian and focus on **density order**.

Definition of in-medium pion decay const.

$$\langle\Omega|A^a_\mu|\pi^{*b}(p)\rangle = i\hat{f}\sqrt{Z}p_\mu \quad \Rightarrow \quad f^*_\pi = \hat{f}\sqrt{Z}$$

$|\Omega\rangle$:Nuclear matter ground state A^a_μ :Axial current

$\hat{f}$:1PI pi-A vertex correction Z :Wave fn. renormalization $Z = \left(1 + \frac{\partial\Sigma}{\partial p_0^2}\right)^{-1}$

Pi op. $\langle\Omega|P^aP^b|\Omega\rangle = \delta^{ab}G^*_\pi \frac{i}{p^2 - m^{*2}_\pi + i\epsilon}G^*_\pi = \delta^{ab}\hat{G}_\pi \frac{iZ}{p^2 - m^{*2}_\pi + i\epsilon}\hat{G}_\pi \quad \Rightarrow \quad \pi^a = \frac{P^a}{\hat{G}_\pi}$

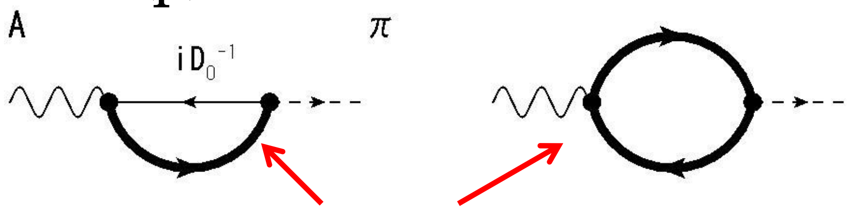
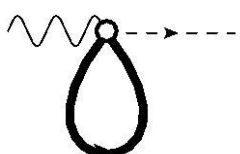
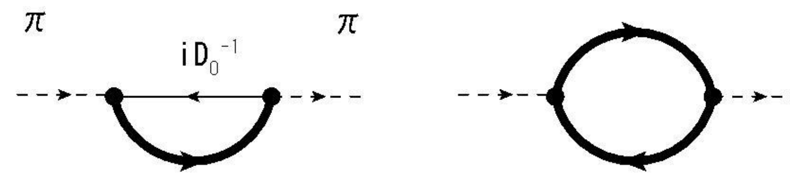
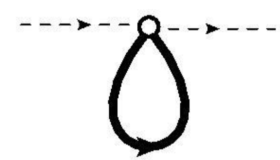
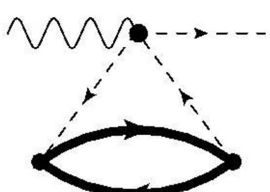
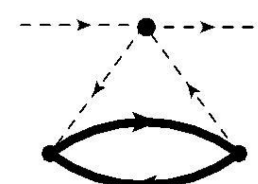
LSZ $\langle\Omega|A^a_\mu|\pi^{*b}(p)\rangle = \lim_{p^2\to m^{*2}_\pi}\left(\frac{i\sqrt{Z}}{p^2 - m^{*2}_\pi + i\epsilon}\right)^{-1}\langle\Omega|A^a_\mu\pi^b|\Omega\rangle = i\hat{f}\sqrt{Z}p_\mu$

- In-medium pion changes by a factor of √Z. $\pi^* \to \sqrt{Z}\pi$

- If we find ***the density dependence of the wave function renormalization Z and*** $\hat{f}$, we can determine the in-medium decay constant.

Density corrections of $\hat{f}$ and Σ

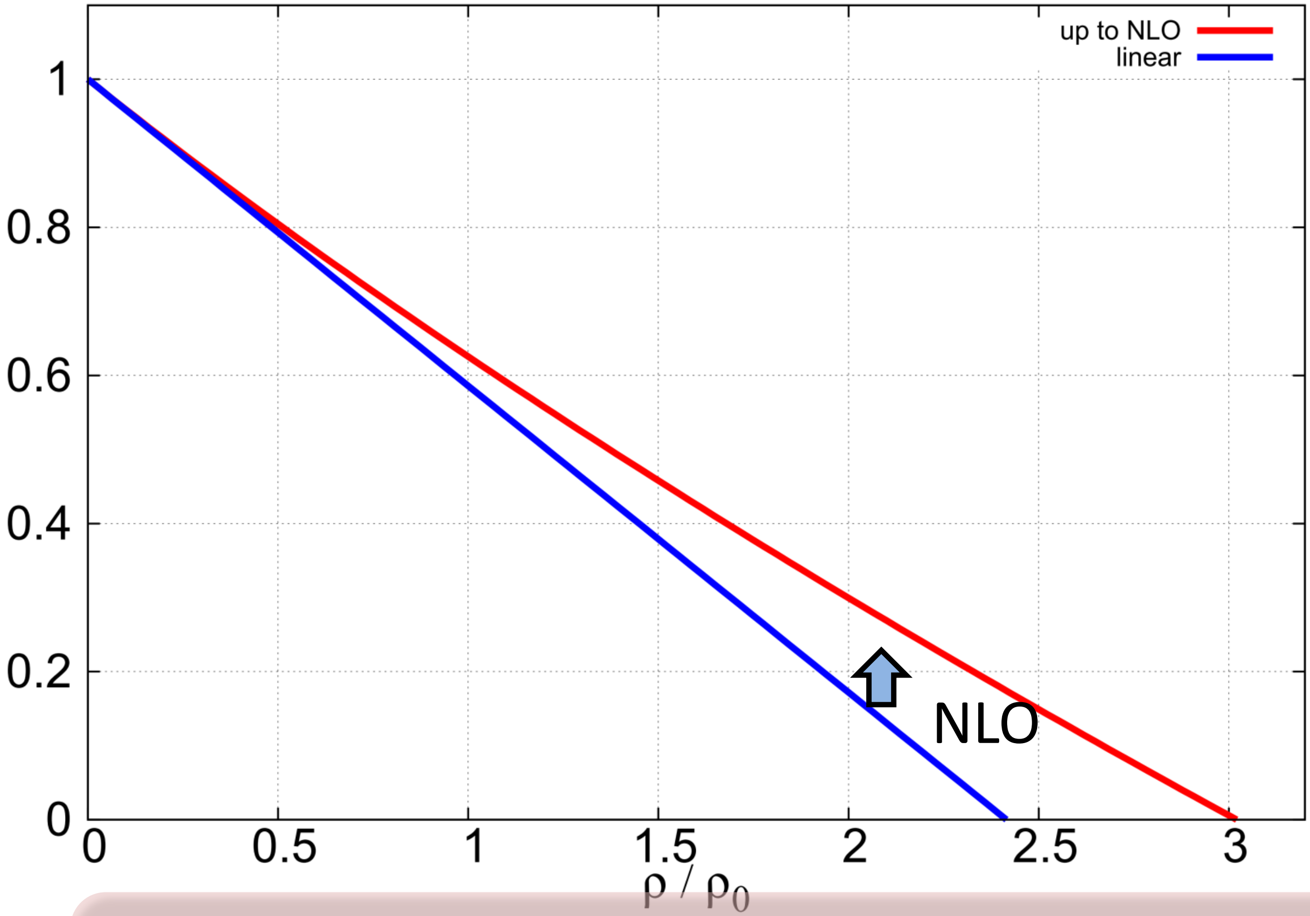
We can classify density corrections using order counting for kf.

$\hat{f}$ Aπ vertex correction	Σ Pion self energy
O(ρ)  Fermi sea effect 	 
$O(\rho^{4/3})$ 	 Z is also determined.

Density dependence of pion decay const.

$$\frac{f_\pi^{*2}}{f_\pi^2} = 1 - \frac{\rho}{f_\pi^2}\left[\frac{\sigma_{\pi N}}{m_\pi^2} + \left(1+\frac{m_\pi}{m_N}\right)\frac{4\pi f_\pi^2}{m_\pi^2}a^+\right] + \frac{g_A^2 k_F^4}{3\pi^4 f^4}F\left(\frac{k_F}{2m_\pi}\right).$$

In symmetric nuclear matter



O(ρ)

up to $O(\rho^{4/3})$

•Input

$\sigma_{\pi N} = 45\mathrm{MeV}$ $\quad g_A = 1.27$

$f = 92.4\mathrm{MeV}$

S-wave isoscalar πN sca. len.

$a^+ = 0.76(31)\cdot 10^{-2} m_\pi^{-1}$

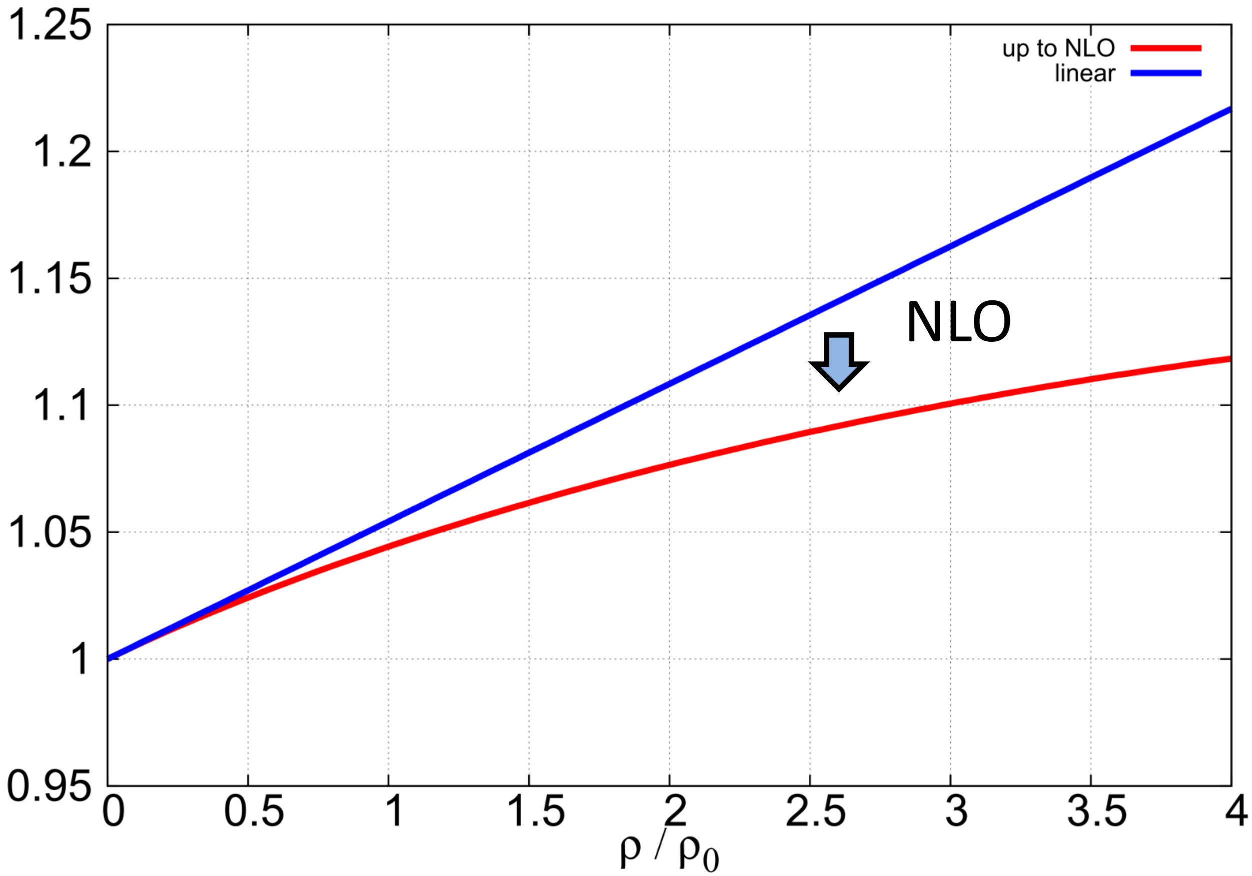
V. Baru et al, PLB (2011)

- NLO contribution is small around normal nuclear density.
- Within NLO, linear density approximation is good up to ρ_0 .

Density dependence of pion mass

$$\frac{m_\pi^{*2}}{m_\pi^2} = 1 + \rho\left(1+\frac{m_\pi}{m_N}\right)\frac{4\pi}{m_\pi^2}a^+ - \frac{g_A^2 k_F^4}{12\pi^4 f_\pi^4}F\left(\frac{k_F}{2m_\pi}\right)$$

In symmetric nuclear matter



— $O(\rho)$

— up to $O(\rho^{4/3})$

- Input

$\sigma_{\pi N} = 45\text{MeV}$ $g_A = 1.27$

$f = 92.4\text{MeV}$

S-wave isoscalar πN sca. len.

$a^+ = 0.76(31)\cdot 10^{-2} m_\pi^{-1}$

V. Baru et al, PLB (2011)

- Pion mass is almost unchanged within NLO.

In medium Low energy theorems

Within $O(\rho^{4/3})$ Low energy theorems are satisfied.

➢ Gell-Mann-Oakes-Renner relation

$$\frac{\langle \bar{u}u + \bar{d}d \rangle^*}{\langle \bar{u}u + \bar{d}d \rangle_0} = \left(\frac{f_t}{f_\pi}\right)^2 \left(\frac{m_\pi^*}{m_\pi}\right)^2$$

In-medium chiral condensate

N. Kaiser, P. de Homont and W. Weise, Phys. Rev. C77, 025204 (2008)

SG, D.Jido, arXiv: 1308.2660

➢ Glashow-Weinberg relation

$$\frac{\langle \bar{u}u + \bar{d}d \rangle^*}{\langle \bar{u}u + \bar{d}d \rangle_0} = \left(\frac{f_t}{f_\pi}\right) \left(\frac{G_\pi^*}{G_\pi}\right)$$

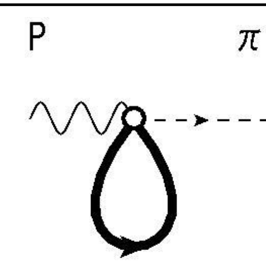
◆ In-medium pseudo-scalar coupling

$$\langle \Omega | P^a | \pi^{*b} \rangle = G_\pi^* \delta^{ab}$$

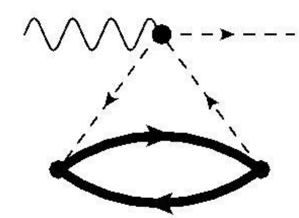
$$G_\pi^* = \hat{G}_\pi \sqrt{Z}$$

$\hat{G}_\pi$ Pseudo-scalar coupling

- $O(\rho)$



- $O(\rho^{4/3})$



In-medium pi0 decay

Pi0 decay is caused by the chiral anomaly.

$$\partial_\mu A^{\mu a} = f_\pi m_\pi^2 \pi^a - \delta^{a3} \frac{e^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} G^{\mu\nu} G^{\rho\sigma}$$

Chiral anomaly comes from Wess-Zumino-Witten term in EFT.
WZW term includes only pions.

Wave fn. Renormalization only carries medium effect.

$$\frac{\langle \pi^{0*} | \gamma\gamma \rangle}{\langle \pi^0 | \gamma\gamma \rangle} = \sqrt{Z} \quad \Rightarrow \quad \frac{\Gamma^*}{\Gamma_0} \sim Z$$

$$\begin{aligned} Z &= 1 + \frac{\sigma_{\pi N}}{f_\pi^2 m_\pi^2} \rho + \frac{4a^+}{m_\pi^2} \left(1 + \frac{m_\pi}{m_N}\right) \rho \\ &= 1 + \underline{0.4} \frac{\rho}{\rho_0} \end{aligned}$$

Summary

- We evaluated ***in-medium pion decay constant, mass and wave fn. renormalization*** using in-medium chiral perturbation theory.
- ***In-medium low energy theorems are satisfied within NLO***.
- ***In-medium pion changes by wave function renormalization.***

  Pi0 decay width increases.

Outlook

- *Pi-N scattering length, pi-pi scattering length*
- *3-flavor*

Thank you for your attention.