In-medium Hadrons -- A Theoretical Overview --

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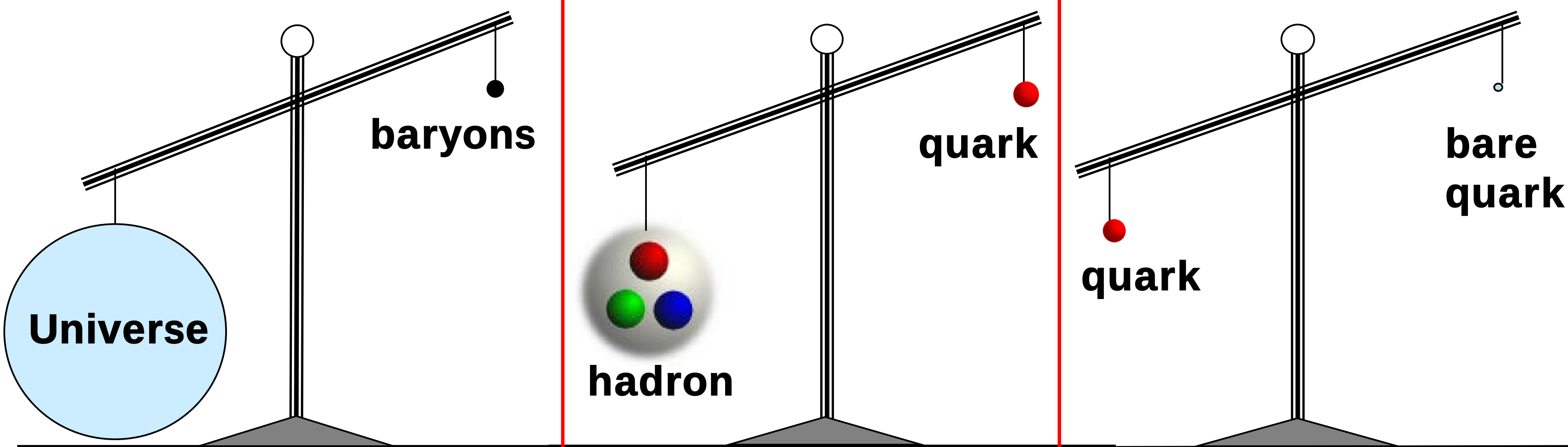


$5\ \% = \text{baryons}$
$27\ \% = \text{dark matter}$
$68\ \% = \text{dark energy}$

Cosmological constant

Einstein (1917)

$m_q \simeq 3\ \text{MeV}$
$m_N \simeq 1000\ \text{MeV}$

"Chiral" condensate

Nambu (1960)

$m_0 = 0$
$m_q \simeq 3\ \text{MeV}$

"Higgs" condensate

Englert-Brout, Higgs (1964)

Condensates ⇔ Elementary excitations

Outline

I. Status of QCD

II. Chiral order parameters

III. In-medium hadrons

IV. Summary

"Hadron Properties in the Nuclear Medium"
R. Hayano + T.H., Rev. Mod. Phys. 82 (2010) 2949

"The Phase Diagram of Dense QCD"
K. Fukushima + T.H., Rep. Prog. Phys. 74 (2011) 014001

I. Status of QCD

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

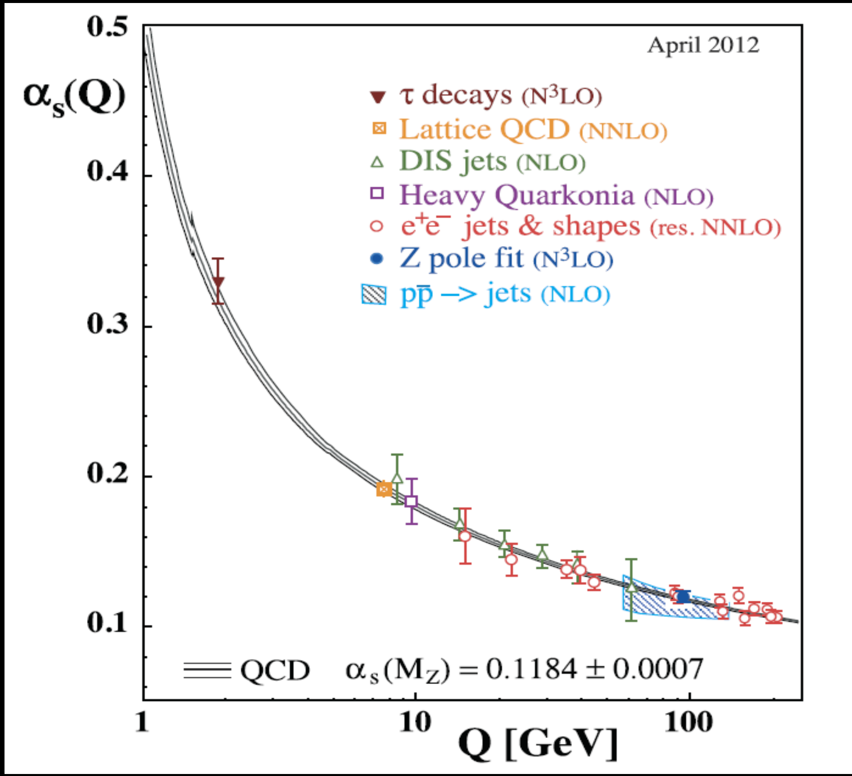
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

Running masses: $m_q(Q)$

quark masses (from lattice QCD)	**[MeV]** (MS-bar @ 2GeV)
m_u	**2.16** (9)(7)
m_d	**4.68** (14)(7)
m_s	**93.8** (2.4)

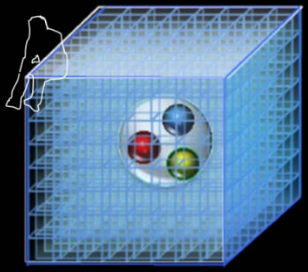
FLAG Collaboration update(July 26, 2013)
http://itpwiki.unibe.ch/flag/

Running coupling: $\alpha_s(Q)=g^2/4\pi$



PDG (2012) http://pdg.lbl.gov/

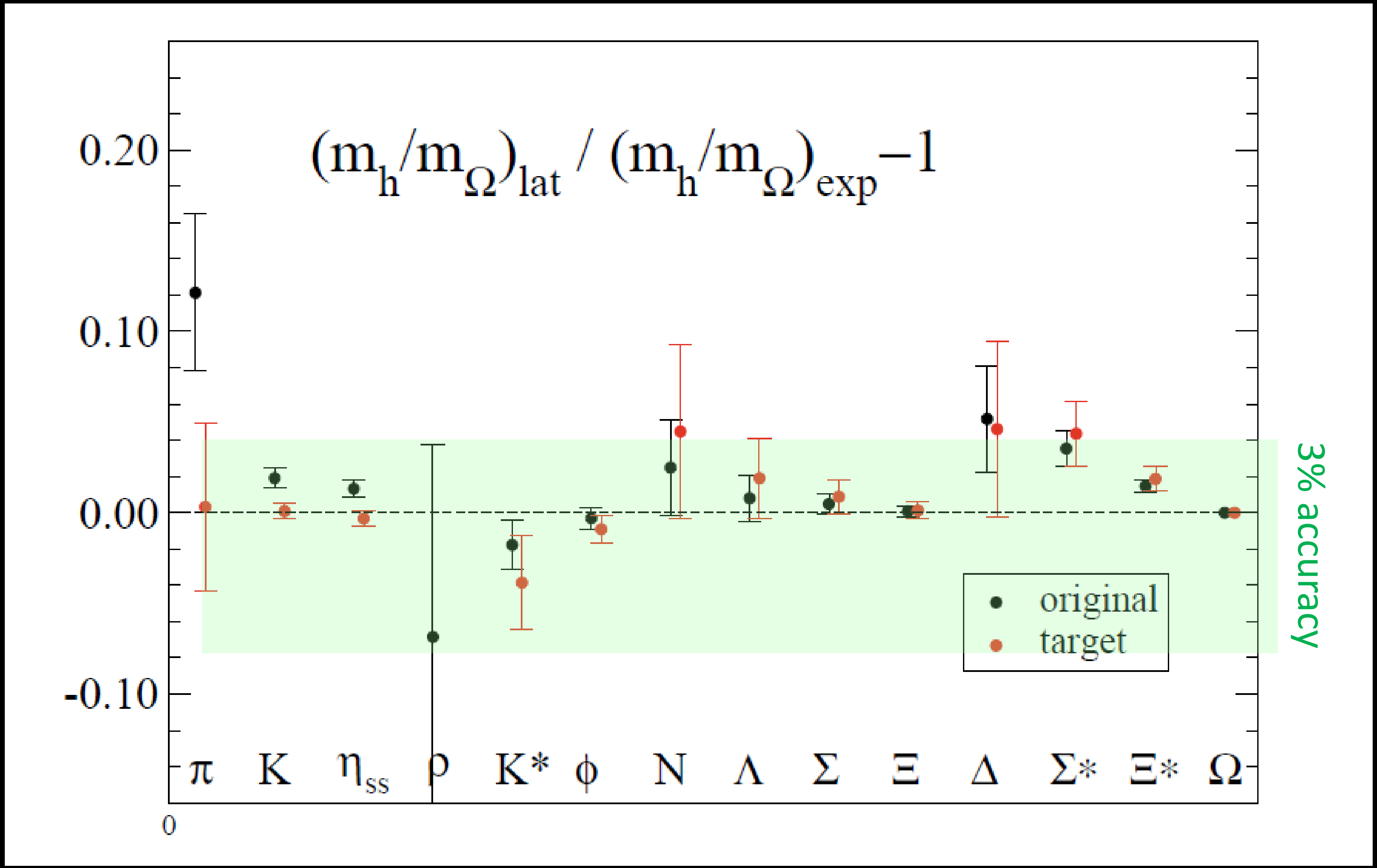
Hadron masses from Lattice QCD



Improved Wilson + Iwasaki gauge action

a = 0.09 fm, L=2.9 fm, m_π=135 MeV

PACS-CS Coll., Phys. Rev. D 81, 074503 (2010)



⇒ L~9.6 fm, m_π=135 MeV on K-computer underway

Symmetry realization in QCD vacuum

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu (i\partial_\mu - g\, t^a A^a_\mu) q - m\bar{q}q$$

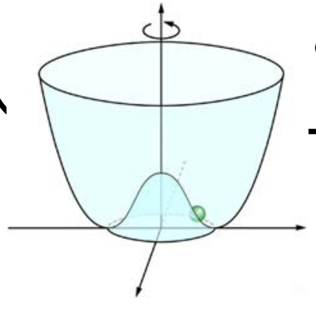
classical QCD symmetry (m=0)

$$\mathcal{G} = SU(3)_C \times [SU(N_f)_L \times SU(N_f)_R] \times U(1)_B \times U(1)_A$$

Quantum QCD vacuum (m=0)

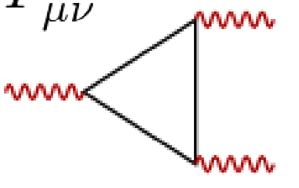
Chiral condensate : spontaneous mass generation

Axial anomaly : quantum violation of $U(1)_A$



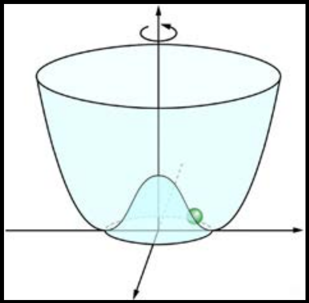
$$\langle \bar{q}_R q_L \rangle \neq 0$$

$$\partial_\mu J^\mu_A = -2N_f \frac{\alpha_s}{8\pi} F^{\mu\nu}_a \tilde{F}^a_{\mu\nu}$$



$$SU(3)_C \times SU(N_f)_{L+R} \times U(1)_B$$

Dim.3 chiral condensate in QCD



$$\langle \bar{u}u \rangle_0^{2\mathrm{GeV}} = -[(250-275)\mathrm{MeV}]^3$$

Gell-Mann-Oakes-Renner (GOR) formula (1968)

$$f_\pi^2 m_\pi^2 = -(m_u + m_d)\langle \bar{u}u \rangle_0 + O(m^2)$$

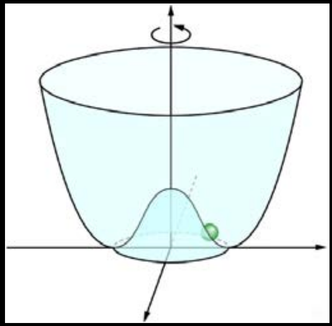
Di Vecchia-Veneziano formula (1980)

$$\chi_{\mathrm{top}} = \frac{-\langle \bar{u}u \rangle_0}{1/m_u + 1/m_d + 1/m_s} + O(m^2)$$

Banks-Casher relation (1980)

$$\langle \bar{u}u \rangle_0 = -\lim_{m\to 0} \int_0^\infty d\lambda \frac{2m}{\lambda^2 + m^2} \rho(\lambda) = -\pi\rho(0)$$

II. Chiral order parameters



Axial rotation :

$$G = e^{iQ_5}$$

Axial Charge :

$$Q_5 = \int d^3x \; A_0(\mathbf{x})$$

Order parameter : $\langle \Sigma \rangle = \langle [iQ_5, \Pi] \rangle$

= 0 (no SSB)

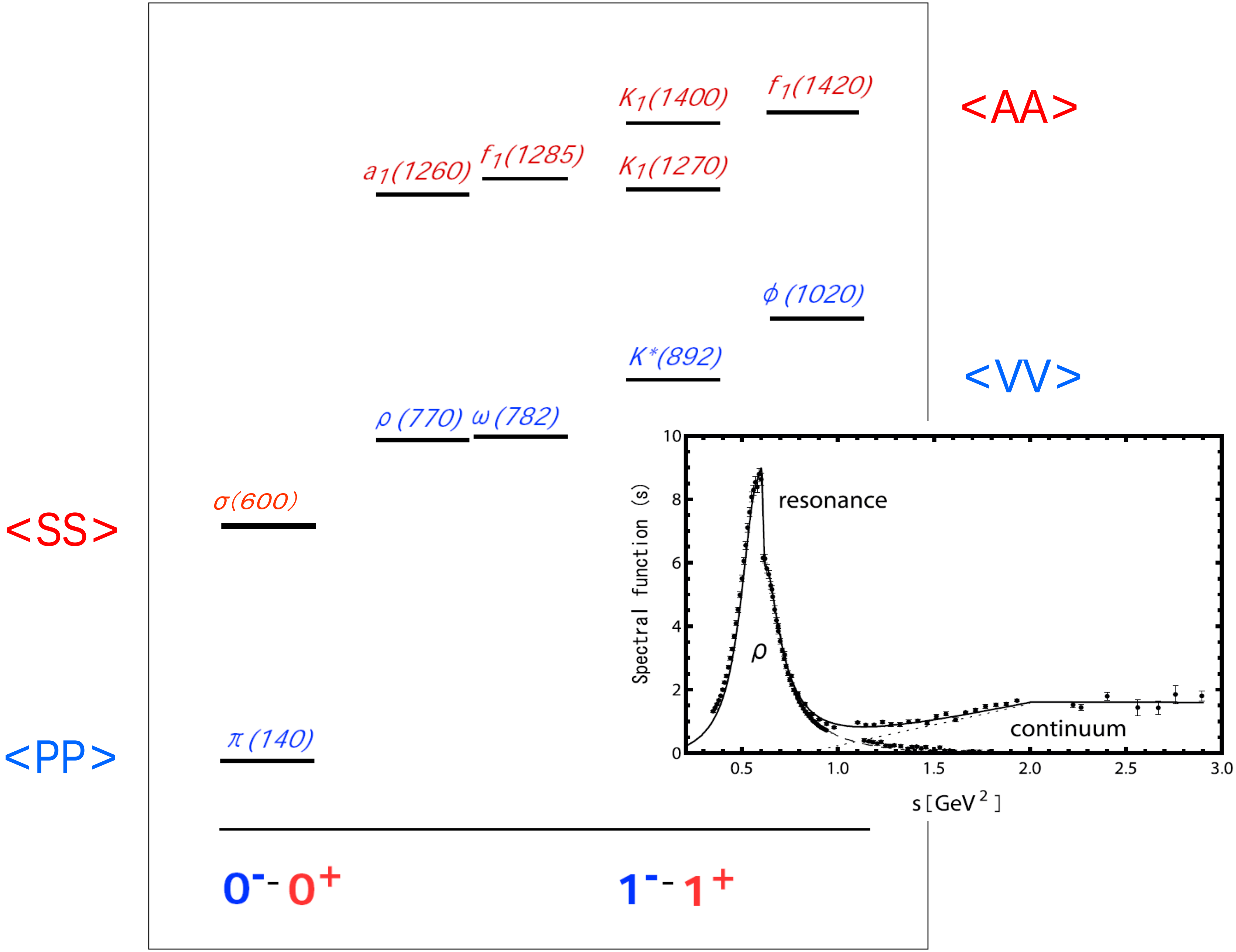
≠ 0 (SSB)

Examples

Σ	Π
$S(x) = \bar{q} t_{_F}^{0} q$	$P(x) = \bar{q} i\gamma_5 t_{_F}^{a} q$
$S(x)S(y) - P(x)P(y)$	$S(x)P(y)$
$V(x)V(y) - A(x)A(y)$	$V(x)A(y)$
$\cdots$	$\cdots$

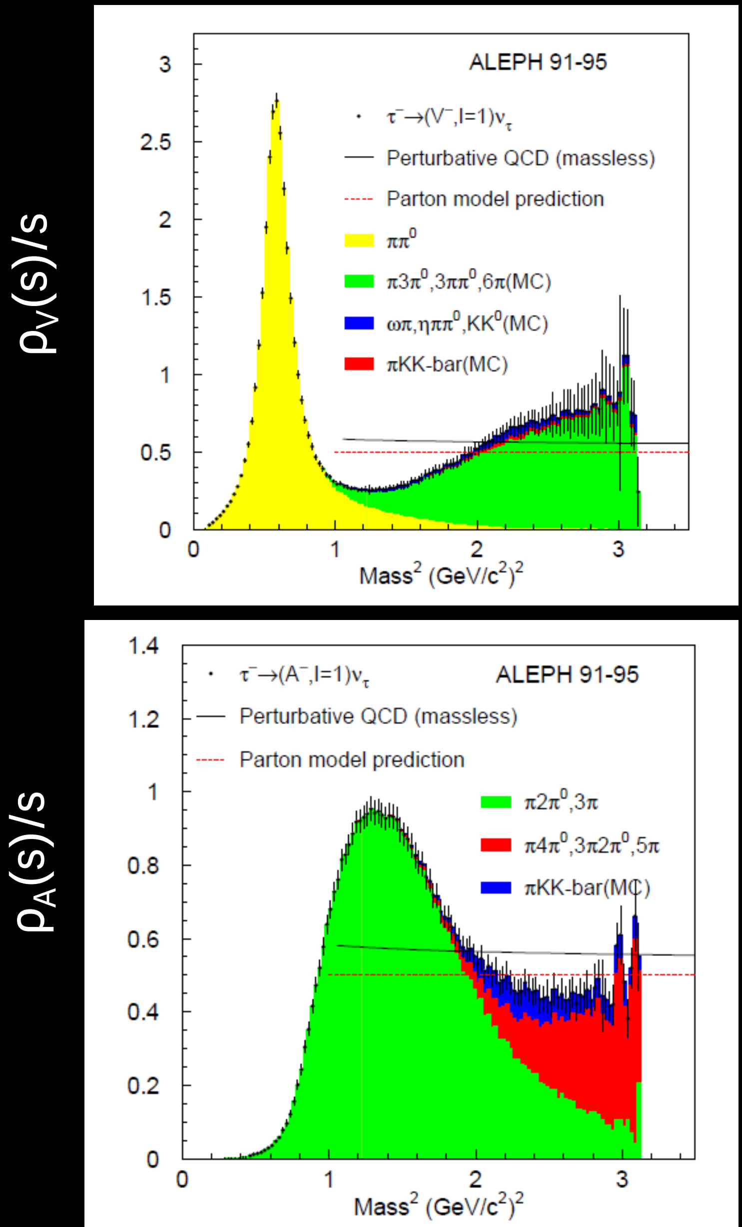
Order parameters : NOT unique !

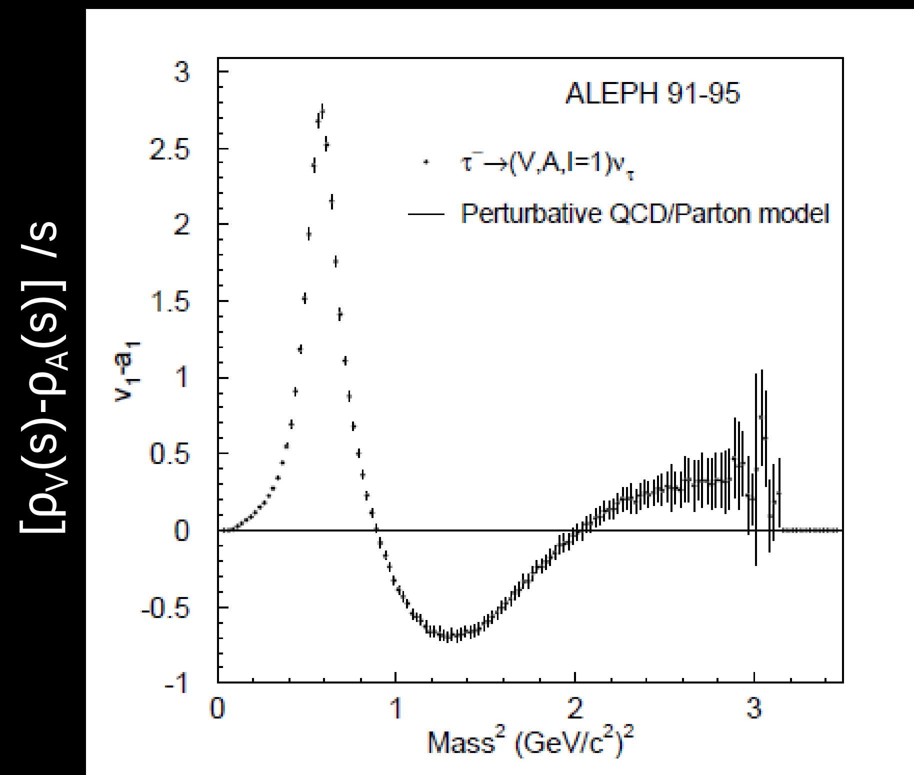
Spectral evidence of SSB in QCD



<VV> - <AA> from τ-decays at LEP-1

ALEPH Collaboration,
Phys. Rep. 421 (2005) 191





Exact sum rules in QCD

Energy weighted sum rules from OPE (m_q=0)

$$\int_0^\infty \frac{d\omega^2}{\omega^2}(\rho_V(\omega) - \rho_A(\omega)) = 0$$

$$\int_0^\infty d\omega^2(\rho_V(\omega) - \rho_A(\omega)) = 0$$

$$\int_0^\infty d\omega^2\omega^2(\rho_V(\omega) - \rho_A(\omega)) = -\frac{4\pi}{3}\alpha_s\langle\mathcal{O}_{4q}\rangle$$

Lee + T.H., PRC46 (1993) 34.
Koike, Lee + T.H., Nucl.Phys. B394 (1993) 221
Kapusta and Shuryak, PRD 49 (1994) 4694

Dim.6
chiral condensate

$$\mathcal{O}_{4q} = \mathcal{O}^\mu_\mu + 2\mathcal{O}^{00}$$

$$\mathcal{O}_{\mu\nu} = \frac{4}{3}(\bar{q}_L\gamma_\mu t^\alpha_C t^a_F q_L)(\bar{q}_R\gamma_\nu t^\alpha_C t^a_F q_R)$$

III. In-vacuum hadrons

$$\begin{aligned} M^2 &= f(m_q, \Lambda_{\rm QCD}) \\ &= m\Lambda_{\rm QCD} + \cdots \\ &= \Lambda_{\rm QCD}^2 + \cdots \end{aligned}$$

← Nambu-Goldstone bosons

← Other hadrons

Examples:

$$M_{\pi^\pm}^2 \simeq -\frac{m_u + m_d}{2f_\pi^2}\langle \bar{u}u + \bar{d}d\rangle_0$$

Gell-Mann-Oakes-Renner relation (1968)

$$M_\rho^2 \simeq \left(24\pi^3\langle(\bar{q}\gamma_\mu\gamma_5 t_C^a \tau^3 q)^2 + \frac{2}{9}(\bar{q}\gamma_\mu t_C^a q)^2\rangle_0\right)^{1/3}$$

QCDSR from OPE by SVD (1979)

$$M_\rho^2 \simeq \left(\frac{32\pi}{27}\langle\alpha_s G_{\mu\nu}^a G_a^{\mu\nu}\rangle_0\right)^{1/2}$$

QCDSR from Commutator by Hayata, PRD88 (2013)

In-medium hadrons

$$M^2 = f(\quad , \Lambda_{\rm QCD} ; T, \quad , \qquad)$$

Complex pole (even for the pion)

One-parameter example (T≠0) :

$$\frac{M^2(T)}{M^2(0)} = f\left(\frac{T}{\Lambda_{\rm QCD}}\right) = f\left(g^{-1}\left(\frac{X(T)}{X(0)}\right)\right)$$

$$\frac{X(T)}{X(0)} = g\left(\frac{T}{\Lambda_{\rm QCD}}\right)$$

* For the pion, f(x) and g(x) can be evaluated for small x.

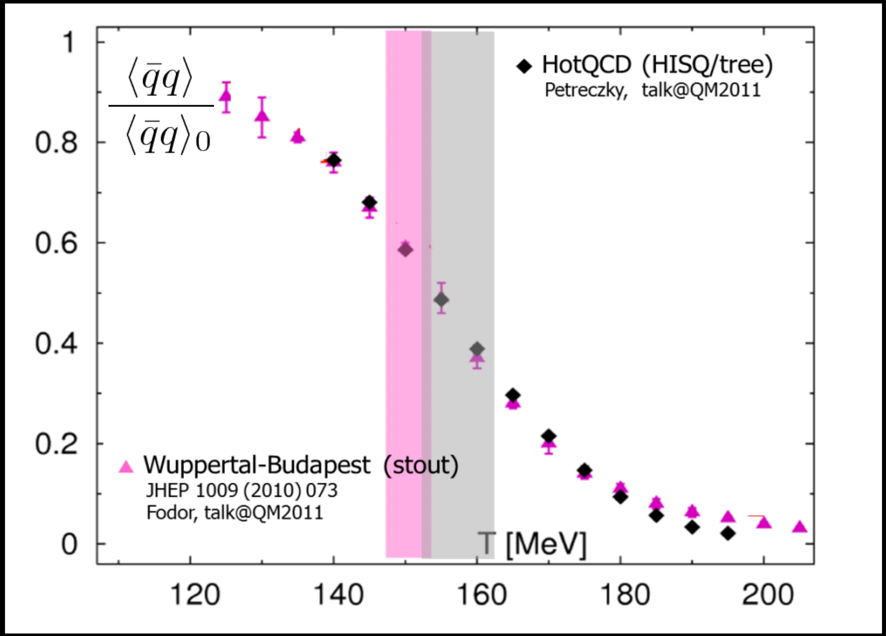
See e.g. Jido, Kunhihiro + T.H., Phys. Lett. B670 (2008)

* In general, experimental inputs are really necessary.
* Sometimes, spectral function is better to be studied.

Dim.3 Chiral condensate in the medium

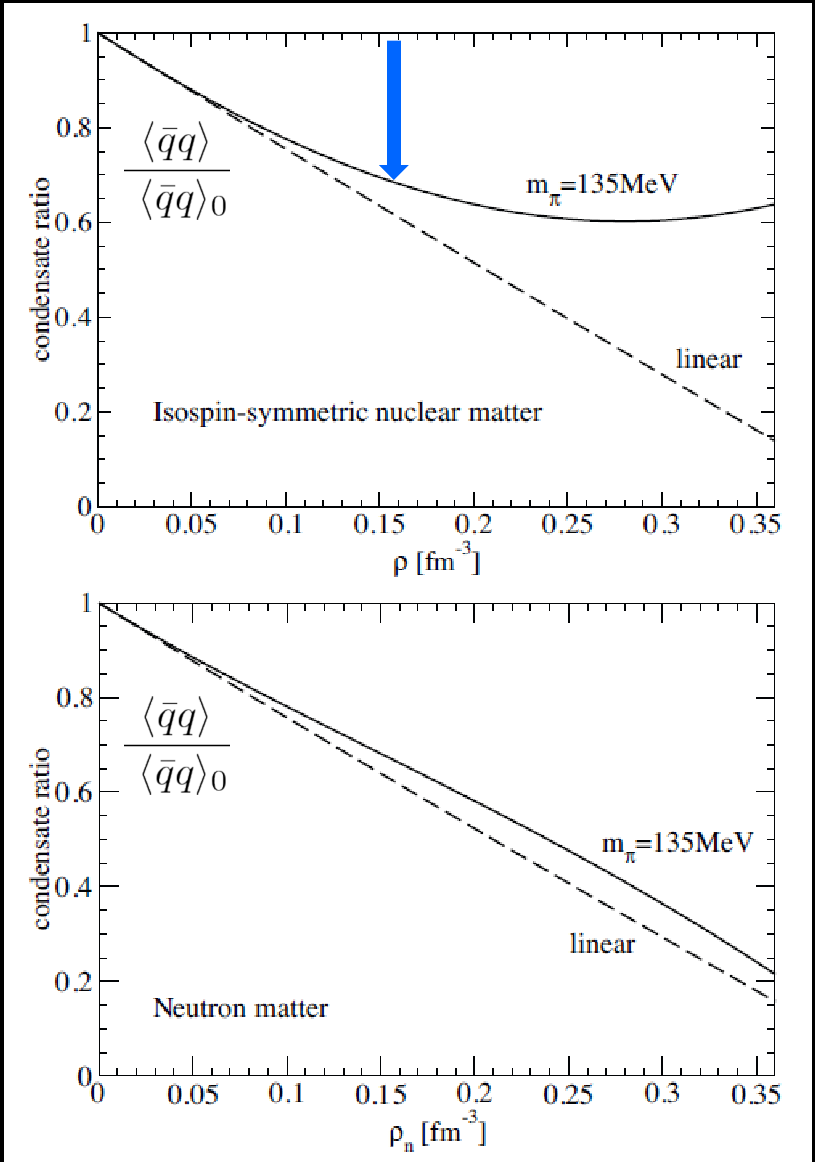
$$\langle \bar{q}q \rangle = -\frac{\partial P(T,\mu)}{\partial m_q}$$

Finite Temperature (LQCD)



Lattice QCD, (2+1)-flavor
Borsanyi et al., JHEP 1009 (2010)

Finite baryon density (χPT)



Nuclear chiral perturbation
Kaiser et al., PRC 77 (2008)

Wish list by an innocent theorist

1 Spectral difference between chiral partners

π-σ, ρ-a_1, ω-f_1, etc

Determination of D=6 chiral condensates in the vacuum?
Tau-decay in nuclei ?

2 Individual properties of NG and "Higgs" bosons

π, K, η (NG), σ (Higgs), η' (anomaly)

$\sigma \rightarrow 2\gamma$, $\eta \rightarrow 2\gamma$, $\eta' \rightarrow 2\gamma$

Mesic nuclei
Dipion

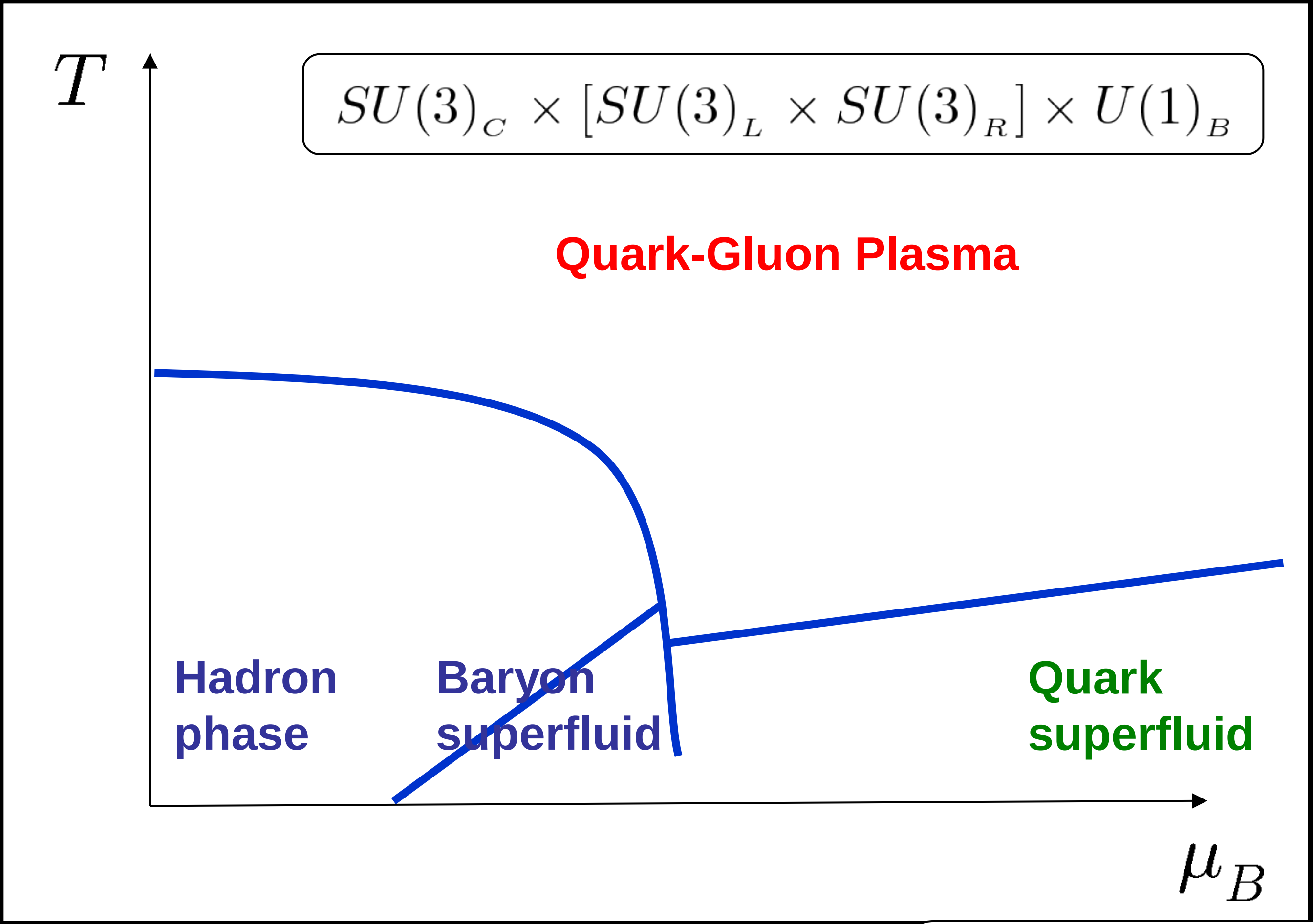
3 Individual properties of vector bosons

ρ, ω, K^* and ϕ

Dileptons
Hadronic decay

Precision/systematic studies
(dispersion relation, different targets, ...)

Hadron-Quark Continuity in dense QCD (N_c=3, N_f=3)

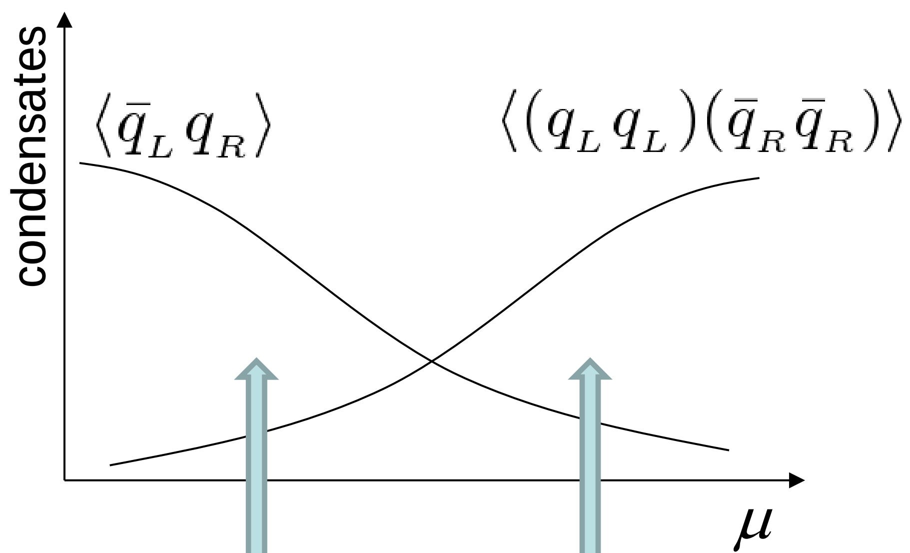


$\langle \bar{q}_L q_R \rangle \neq 0$ $SU(3)_C \times SU(3)_{L+R} \times U(1)_B$	$\langle BB \rangle \neq 0$ $SU(3)_C \times SU(3)_{L+R}$	$\langle (q_L q_L)(\bar{q}_R \bar{q}_R) \rangle \neq 0$ $SU(3)_C \times SU(3)_{L+R}$

Chiral symmetry is always broken at finite density

Possible fate of hadrons at high density (N_c=3, N_f=3)

Continuity in the ground state



excitation	Low μ	High μ
NGs	π(8) & H	π' (8) & H
Vectors	V (9)	gluons (8)
Fermions	Baryons (8)	Quarks (9)

Continuity in the excited state ?

Schafer & Wilczek, PRL 82 ('99)

Yamamoto, Tachibana, Baym + T.H., PRL97('06), PRD76 ('07)

Tachibana, Yamamoto + T.H., PRD78 ('08)

Vector Mesons = Gluons ?!
Baryons = Quarks ?!

IV. Summary

I. Status of QCD

lattice QCD : precision science with a few % accuracy
: difficult to simulate dense matter (sign problem)

II. Chiral order parameters

not unique : Dim.3 condensate, Dim.6 condensate, etc

III. In-medium hadrons

- chiral restoration can be best seen in spectral degeneracy
  i.e. <SS>-<PP>, <VV>-<AA>
- interesting possibility of hadron-quark crossover
  i.e. Vector mesons = Gluons, Baryons = Quarks