Partial restoration of chiral symmetry in the flux tube from lattice QCD

T.Iritani

KEK

Hadron in nucleus @ YITP
October 31, 2013

Ref. TI, G. Cossu, and S. Hashimoto, to appear in PoS (LATTICE 2013) 376

Quantum Chromodynamics

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}\mathrm{Tr}\; G_{\mu\nu}G^{\mu\nu} + \bar{q}\,(i\not{D} - m)\,q$$

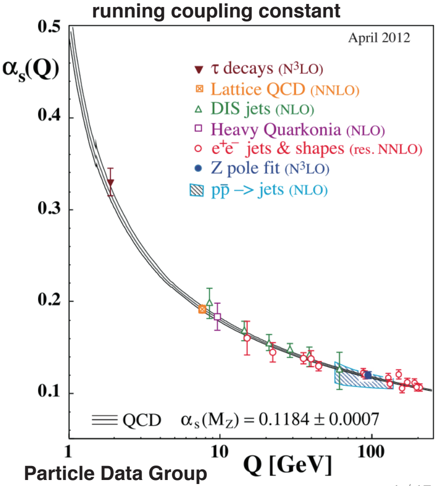
- asymptotic freedom

- chiral symmetry breaking

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$$

at finite density system

$\Rightarrow$ partially restored ?



Quantum Chromodynamics

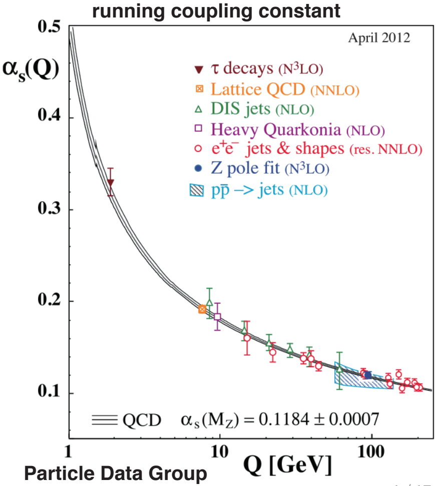
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} + \bar{q}\,(i\not{D} - m)\,q$$

- asymptotic freedom
- CONFINEMENT
  quarks are confined in hadrons
- chiral symmetry breaking

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$$

at finite density system

$\Longrightarrow$ partially restored ?

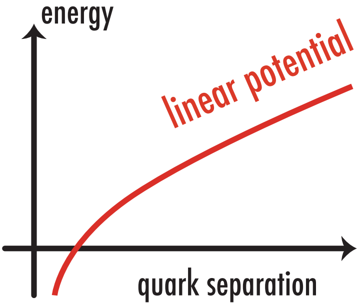


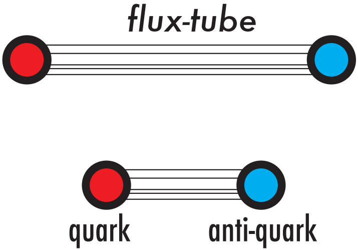
Confinement

quarks and gluons are confined in hadrons, confinement is characterized by

- linear interquark potential
  supported by Regge trajectories of hadrons, quarkonia spectra
- $\Longrightarrow$ flux-tube formation between quarks

In lattice QCD, we can show linear potential and flux-tube formation.





Lattice QCD : 1st Principle Calculation of NP-QCD Wilson '74

1. Minkowski space $\Rightarrow$ Euclidean space

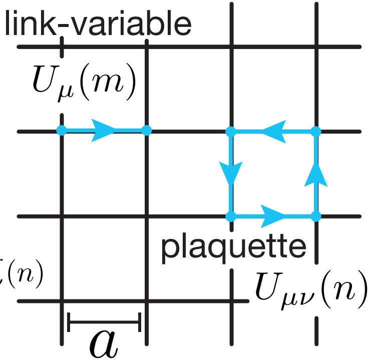
$$Z_{\rm QCD} = \int DA_\mu D\bar{q}Dq\ e^{iS_{\rm QCD}} \Rightarrow \int DA_\mu D\bar{q}Dq\ e^{-S_{\rm QCD}^{\rm Euclid.}}$$

2. discretize on lattice with spacing a $\Leftarrow$ "regularization"

$$S_{\rm QCD} = \frac{2N_c}{g^2}\sum_n \sum_{\mu>\nu}\left(1 - \frac{1}{N_c}{\rm Re\ Tr}\ U_{\mu\nu}(n)\right) + \sum_{n,m}\bar{q}(n)D(n,m)q(m)$$

- gluon fields $A_\mu(x) \in \mathfrak{su}(N_c)$
  $\Rightarrow$ link-variable
  $U_\mu(m) \equiv e^{iagA_\mu(m)} \in {\rm SU}(N_c)$
- ${\rm Tr}\ G_{\mu\nu}G_{\mu\nu}$
  $\Rightarrow$ plaquette

$$U_{\mu\nu}(n) = U_\mu(n)U_\nu(n+\hat{\mu})U_\mu^\dagger(n+\nu)U_\nu^\dagger(n)$$

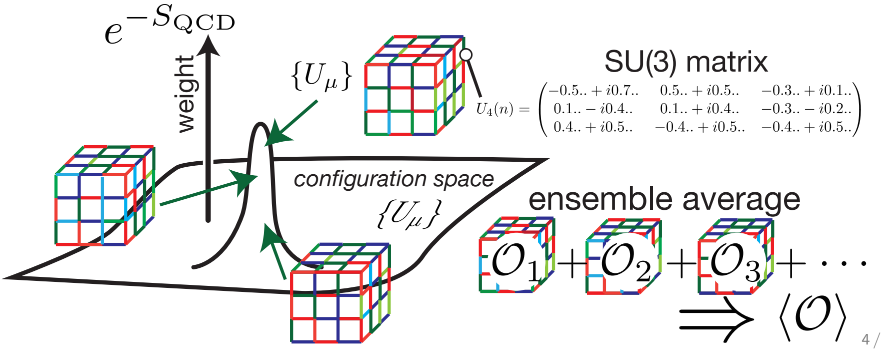


Lattice QCD : Monte Carlo Calculation — Creutz '80

expectation value of an operator $\mathcal{O}$ $\Rightarrow$ importance sampling

$$\langle \mathcal{O} \rangle = \frac{\int DU D\bar{q} Dq \; \mathcal{O}(U) \; e^{-S_{\rm QCD}}}{\int DU D\bar{q} Dq \; e^{-S_{\rm QCD}}} \simeq \frac{1}{N} \sum_i^N \mathcal{O}(U_i)$$

- "direct" calculation is impossible $\Leftarrow$ millions of multiple integrals
- use $\exp\left(-S_{\rm QCD}\right)$ as a statistical weight $\Rightarrow$ importance sampling
- *but at finite density* $\exp(-S_{\rm QCD})$ *becomes complex* $\Rightarrow$ *sign problem*



Wilson Loop and the Interquark Potential

Wilson loop

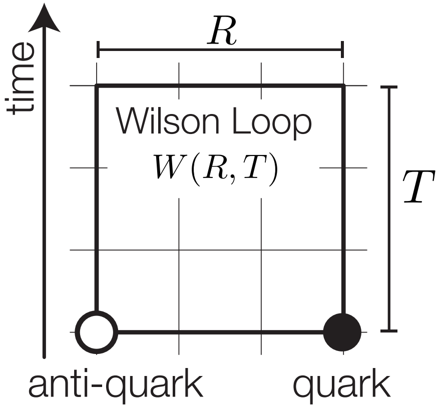
$$W(R,T) = \mathrm{Tr} \prod_{\mathrm{loop}} U_\mu$$

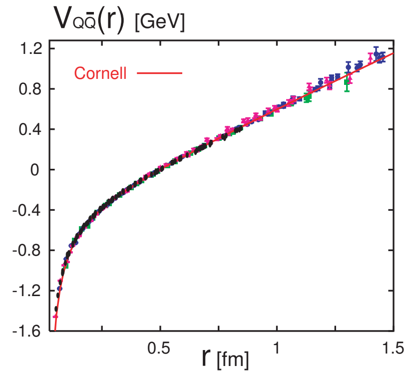
$\Rightarrow$ energy of QQ-bar system

interquark potential $V_{Q\bar{Q}}(R)$

$$\langle W(R,T)\rangle \sim \exp\left(-V_{Q\bar{Q}}(R)T\right)$$

$\Rightarrow$ Linear interquark potential

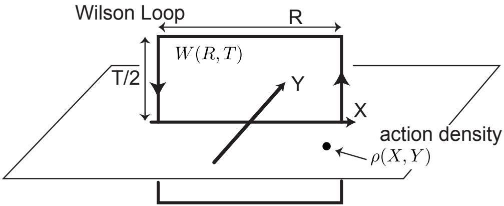




Flux-tube Measurement in Lattice QCD

action density $\rho(x)$ around Wilson loop $W(R,T)$ — QQ-bar system

$$\langle\rho(x)\rangle_W \equiv \frac{\langle\rho(x)W(R,T)\rangle}{\langle W(R,T)\rangle} - \langle\rho\rangle$$



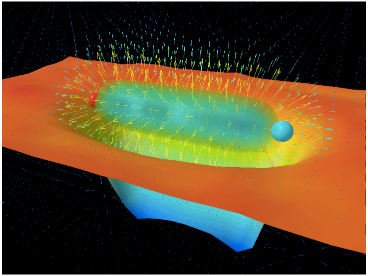
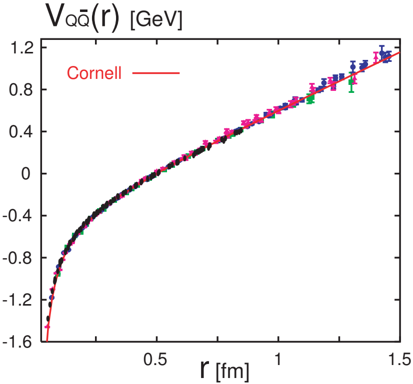
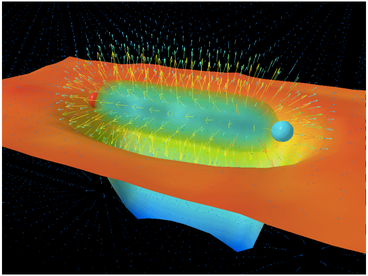


Fig. Leinweber et al. '03

Main Idea of This Work

1. Quarks are confined in hadrons.
2. Confinement is characterized by the linear potential, i.e., flux-tube formation between quarks.
3. How about "chiral symmetry breaking" in flux-tube, i.e., "hadron" ?
4. Using lattice QCD and overlap-Dirac eigenmodes, we can analyze chiral condensate inside QQ-bar and 3Q systems.





Origin of Chiral Condensate and Dirac Eigenmodes

- QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm Tr}\ G_{\mu\nu}G_{\mu\nu} + \bar{q}(\not{D} + m)q$$

- chiral condensate $\langle \bar{q}q \rangle \Longleftarrow$ *eigenvalue of Dirac operator* $\not{D}\psi_\lambda = \lambda\psi_\lambda$

$$-\langle \bar{q}q \rangle = \langle {\rm Tr} S_q \rangle = \lim_{m\to 0}\lim_{V\to\infty}\langle {\rm Tr}\frac{1}{\not{D}+m}\rangle = \lim_{m\to 0}\lim_{V\to\infty}\frac{1}{V}\sum_\lambda \frac{1}{\lambda+m}$$

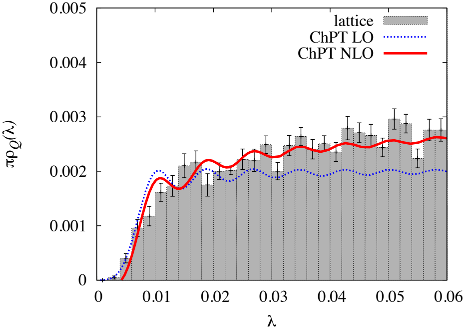
near-zero Dirac mode

$\Longrightarrow$ chiral symmetry breaking

Banks-Casher Relation

$$\langle \bar{q}q \rangle = -\pi\langle \rho(0) \rangle$$

$\rho(\lambda)$: eigenmode density



Fukaya et al. (JLQCD Coll.) '10

Local Structure of Chiral Condensate

$$\langle \bar{q}q \rangle = -\mathrm{Tr}\frac{1}{\not{D}+m} = -\frac{1}{V}\sum_{\lambda}\frac{1}{\lambda+m}$$

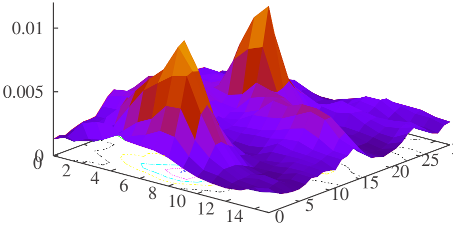
$$= -\frac{1}{V}\sum_{\lambda}\sum_{x}\frac{\psi_{\lambda}^{\dagger}(x)\psi_{\lambda}(x)}{\lambda+m} = -\frac{1}{V}\sum_{x}\bar{q}q(x)$$

with Dirac eigenfunction $\not{D}\psi_{\lambda} = \lambda\psi_{\lambda}$

"local chiral condensate" $\bar{q}q(x)$

$$\bar{q}q(x) = -\sum_{\lambda}\frac{\psi_{\lambda}^{\dagger}(x)\psi_{\lambda}(x)}{\lambda+m_q}$$

$\bar{q}q(x)$ fluctuates in QCD vacuum



a snapshot of $|\bar{q}q(x)|$

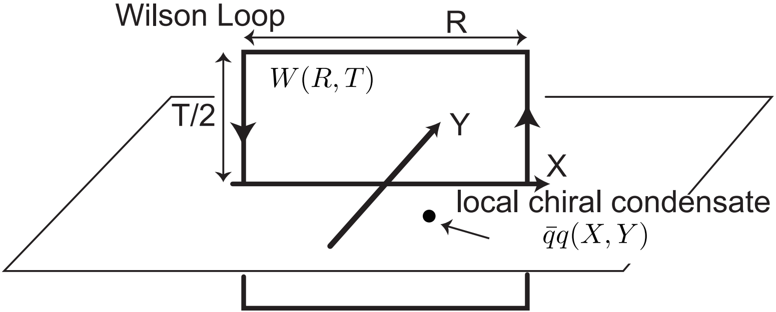
Local Chiral Condensate in Flux-tube

local chiral condensate is given by

$$\bar{q}q(x) = -\sum_{\lambda} \frac{\psi_\lambda^\dagger(x)\psi_\lambda(x)}{\lambda + m} \quad \text{with } \textit{Dirac eigenmode } \not{D}\psi_\lambda = \lambda\psi_\lambda$$

change of the chiral condensate around the Wilson loop

$$\langle \bar{q}q(x)\rangle_W \equiv \frac{\langle \bar{q}q(x) W(R,T)\rangle}{\langle W(R,T)\rangle} - \langle \bar{q}q\rangle$$



About Lattice QCD Setup

In this study, we mainly use

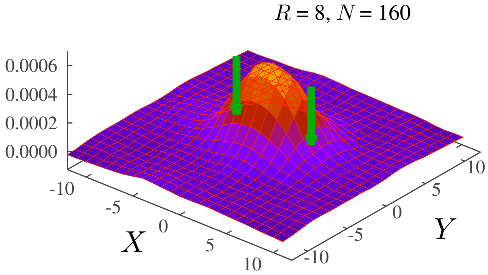
- 2+1 flavor dynamical overlap-fermion configuration by JLQCD Coll.
  $\Longrightarrow$ overlap-fermion keeps "exact chiral symmetry" on lattice
  $\Longrightarrow$ overlap-fermion Dirac eigenmodes are ideal probe
  to analyze chiral properties in lattice QCD
- pion mass $m_\pi \sim 300$ MeV, kaon mass $m_K \sim 500$ MeV
- lattice volume $24^3 \times 48$
- lattice spacing $a^{-1} = 1.759(10)$ GeV $\Longrightarrow$ $a \sim 0.112$ fm

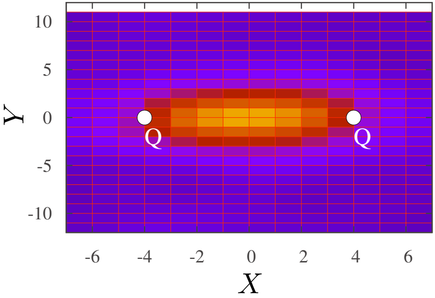
Reduction of Chiral Condensate in the Flux-tube

difference of chiral condensate around quark-antiquark

$$\langle \bar{q}q(x) \rangle_W^{(N)} \equiv \frac{\langle \bar{q}q^{(N)}(x) W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \bar{q}q \rangle^{(N)} \quad (N : \text{eigenmode number})$$

- $\langle \bar{q}q(x) \rangle_W > 0 \Longrightarrow |\langle \bar{q}q(x) \rangle|_{\text{flux}} < |\langle \bar{q}q \rangle|$
- partial restoration of chiral symmetry in the flux ("Bag-model"-like)





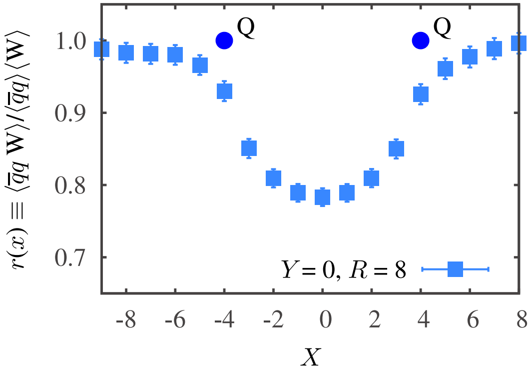
(anti-)quark at $(4, 0)$ and $(-4, 0)$. lattice unit $a \sim 0.11$ fm, pion mass ~ 300 MeV

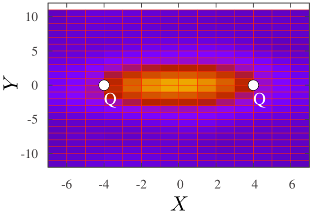
Ratio of Chiral Condensate

$$r(x) \equiv \frac{\langle \bar{q}q^{(\mathrm{subt})}(x) W(R,T)\rangle}{\langle \bar{q}q^{(\mathrm{subt})}\rangle \langle W(R,T)\rangle} < 1$$

$\Rightarrow$ about 20 % of chiral condensate is reduced at flux core

cross section of flux-tube



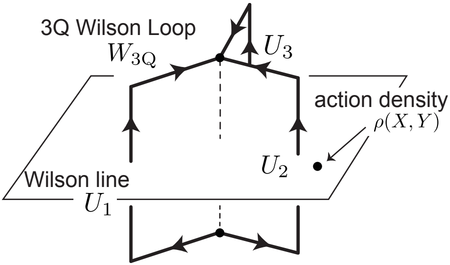


cf. subtracted condensate $\langle \bar{q}q\rangle^{(N)} = \langle \bar{q}q^{(\mathrm{subt})}\rangle + c_1^{(N)} m_q/a^2 + c_2^{(N)} m_q^3$ ref. Noaki '09

3-Quark System

3Q-Wilson loop

$$W_{3Q} \equiv \frac{1}{3!}\varepsilon_{abc}\varepsilon_{a'b'c'}U_1^{aa'}U_2^{bb'}U_3^{cc'}$$



with Wilson lines U_k
cf. Takahashi-Suganuma '01

Y-type flux formation in 3Q-system

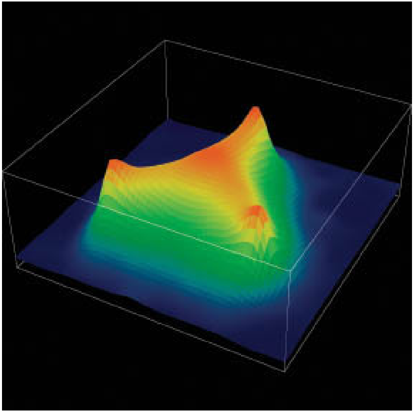
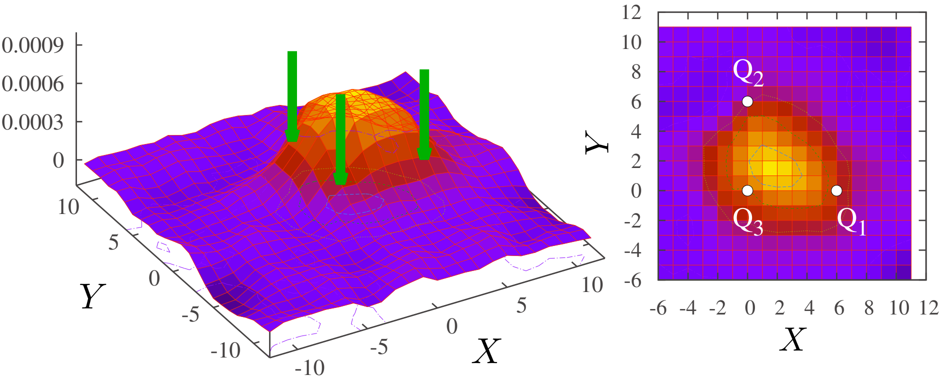


Figure : Ichie et al. '03

Partial Restoration of Chiral Symmetry in 3Q system

chiral condensate in 3Q system

$$\langle \bar{q}q(x) \rangle_{W_{3Q}} \equiv \frac{\langle \bar{q}q(x) W_{3Q} \rangle}{\langle W_{3Q} \rangle} - \langle \bar{q}q \rangle > 0$$



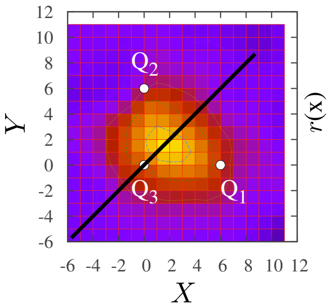
right triangle configuration : $Q_1 = (6,0), Q_2 = (0,6), Q_3 = (0,0)$

lattice unit $a \simeq 0.11$ fm

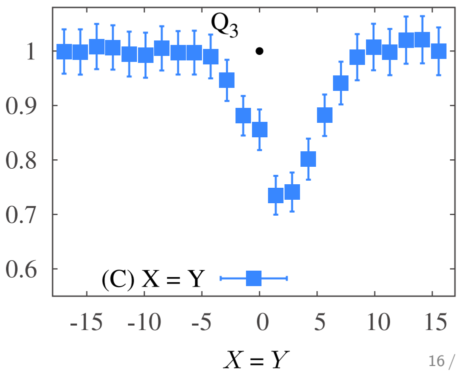
Ratio of Chiral Condensate in 3Q system

$$r_{3\mathrm{Q}}(x) \equiv \frac{\langle \bar{q}q^{(\mathrm{subt})}(x) W_{3\mathrm{Q}} \rangle}{\langle \bar{q}q^{(\mathrm{subt})} \rangle \langle W_{3\mathrm{Q}} \rangle} < 1$$

- about 20 $\sim$ 30 % of chiral condensate is reduced in 3Q-system
- "partial restoration of chiral symmetry" in 3Q-system



cross-section of 3Q-flux



Summary

In this talk, we demonstrate chiral symmetry is indeed partially restored in flux-tube from lattice QCD.

- Quarks are confined in hadrons, and confinement is characterized by linear potential, i.e., flux-tube structure between quarks.
- From lattice QCD, we measure the chiral condensate $\bar{q}q(x)$ inside flux-tube, we show partial restoration of chiral symmetry

  $$|\langle \bar{q}q \rangle|_{\text{in flux}} < |\langle \bar{q}q \rangle|_{\text{vacuum}}$$

  for both quark-antiquark ("meson") and 3-quark ("baryon") systems.
- The ratio of the chiral condensate

  $$\frac{\langle \bar{q}q \rangle_{\text{in flux}}}{\langle \bar{q}q \rangle_{\text{vacuum}}} = 0.7 \sim 0.8$$

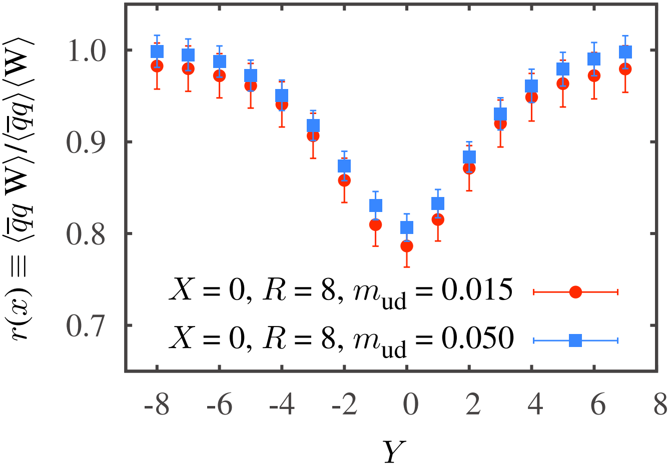
  at the core of flux.

4 Appendix

Quark Mass Dependence of Chiral Condensate Reduction

$16^3 \times 48$ lattice with low-lying 120 eigenmodes

- $m_{\rm ud} = 0.015$: $m_\pi \sim 0.30$ GeV
- $m_{\rm ud} = 0.050$: $m_\pi \sim 0.53$ GeV



Lattice QCD Configurations

- 2+1 flavor dynamical overlap-fermion configurations
- $24^3 \times 48$ with Iwasaki action $\beta = 2.3$, $a^{-1} = 1.759(10)$ GeV
- $m_\pi \sim 300$ MeV

ref. Aoki et al. (JLQCD Coll.) '09

Table : Kinds of Lattice Fermions

	chiral symmetry	flavor	computational costs
(naïve)	OK	16	cheap
Wilson fermion	explicitly broken	OK	middle
staggered fermion	OK	4	cheap
Overlap fermion	exact on lattice	OK	expensive