Nucleon spectral function in nuclear medium from QCD sum rules

Tokyo Institute of Technology   Keisuke Ohtani

Collaborators : Philipp Gubler, Makoto Oka

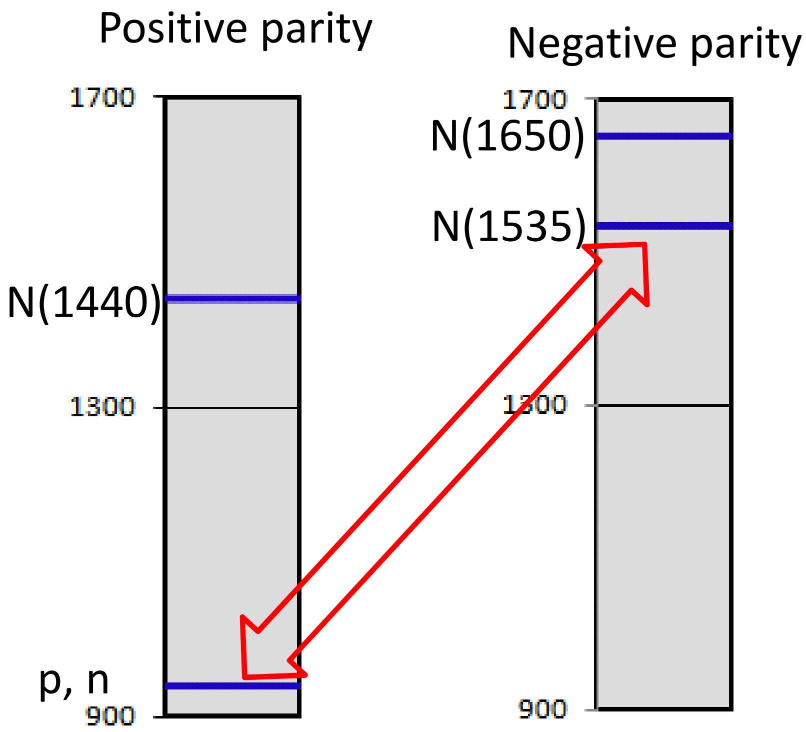
K. Ohtani, P.Gubler and M. Oka, Eur. Phys. J. A 47, 114 (2011)
K. Ohtani, P.Gubler and M. Oka, Phys. Rev. D 87, 034027 (2013)

Outline

- Introduction
- Nucleon QCD sum rules
- Nucleon QCD sum rules in nuclear medium
- Conclusion

Introduction

Mass spectrum of the nucleons



- The mass difference between nucleon ground state and N(1535) is about 600 MeV.
- It is predicted that Chiral symmetry breaking cause these difference.

When chiral symmetry is restored, the mass spectrum will change.

To investigate these properties from QCD, non perturbative method is needed.

⇨ Analysis of QCD sum rule in nuclear medium

Nucleon QCD sum rules

$$\Pi(q) \equiv i \int e^{iqx} \langle 0|T[\eta(x)\overline{\eta}(0)]|0\rangle d^4x$$

$\eta(x)$: interpolating field, which has the same quantum number as the hadron of interest.

Nucleon QCD sum rules

$$\Pi(q) \equiv i \int e^{iqx} \langle 0 | T[\eta(x)\overline{\eta}(0)] | 0 \rangle d^4x$$

$$= \int_0^\infty \frac{1}{\pi} \frac{\mathrm{Im}\Pi(t)}{t - q^2} dt = \int_0^\infty \frac{\rho(t)}{t - q^2} dt$$

is calculated by the operator product expansion (OPE)

$$\Pi(q) = \not{q}\,\Pi_1(q^2) + \Pi_2(q^2)$$

$$= \not{q} \sum_i C_i(q^2) \langle 0 | O_i | 0 \rangle + \sum_j C'_j(q^2) \langle 0 | O'_j | 0 \rangle$$

$$= \not{q} \left(C_0(q^2) + C_4(q^2) \langle \frac{\alpha_s}{\pi} G^2 \rangle + C_6(q^2) \langle \overline{q}q \rangle^2 + \cdots \right)$$

$$+ C_3(q^2) \underline{\langle \overline{q}q \rangle} + C_5(q^2) \langle \overline{q} g \sigma \cdot G q \rangle + \cdots$$

Chiral condensate

$C_i(q^2)$: Coefficient

$\langle 0 | O_i | 0 \rangle$: Condensate

$\Pi(q)$ calculated by OPE is related to the hadronic spectral function

Nucleon QCD sum rules

$$\Pi_{OPE}(q^2) = \int_0^\infty \frac{\rho(t)}{t - q^2} dt$$

" Transformation"

$$G_{OPE}(x) = \int_0^\infty K(x,\omega)\rho(\omega)d\omega$$

x: parameter

Borel sum rule: $K(M_B,\omega) = \exp(-\frac{\omega^2}{M_B^2})$ Parameter: M_B (Borel mass)

Gaussian sum rule: $K(S,\tau,\omega) = \frac{1}{\sqrt{4\pi\tau}}\exp(-\frac{(\omega^2 - s)^2}{4\tau})$ Parameter: S,τ

Parity projection

$$G^{\pm}(x) = [C_0(x) + C_4(x)\langle\frac{\alpha_s}{\pi}G^2\rangle + C_6(x)\langle\overline{q}q\rangle^2 + \cdots]$$

$$\pm[C_3(x)\langle\overline{q}q\rangle + C_5(x)\langle\overline{q}g\sigma\cdot Gq\rangle\cdots]$$

D. Jido, N. Kodama and M. Oka, Phys. Rev. D **54, 4532**

Nucleon QCD sum rules

K. Ohtani et al Phys. Rev. D 87, 034027 (2013)

$$G_{OPE}(s,\tau) = \int_0^\infty K(s,\tau,\omega)\,\rho(\omega)\,d\omega$$

$$K(s,\tau,\omega)d\omega = \frac{1}{\sqrt{4\pi\tau}}\omega e^{-\frac{(\omega^2-s)^2}{4\tau}}d\omega$$

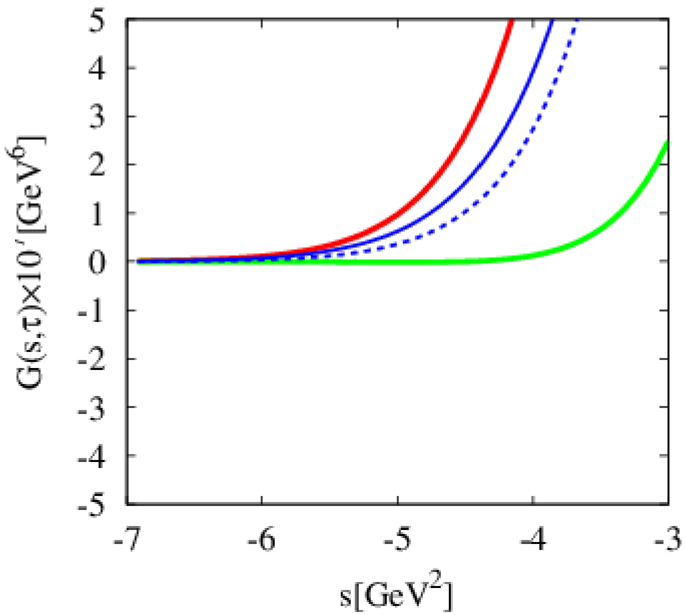
$\tau = 1.5\mathrm{GeV}^4$

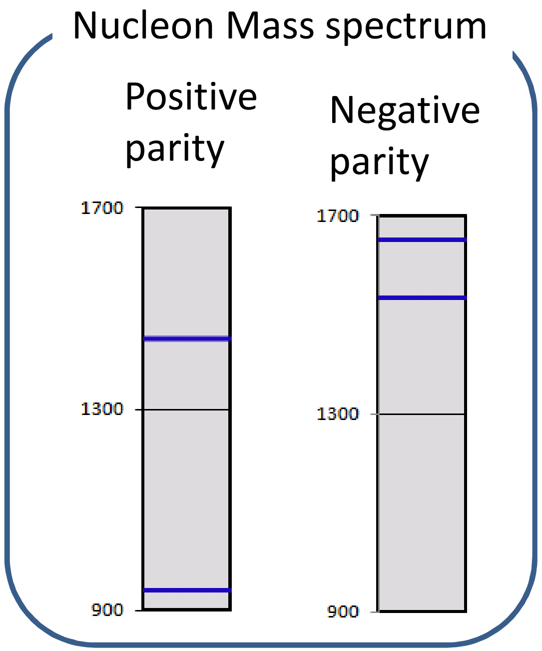
Positive parity OPE

Negative parity OPE

chiral condensate term

Perturvative term





The contribution of the chiral condensate term is dominant.

Perturbative term is also dominant.

Nucleon QCD sum rules

K. Ohtani et al Phys. Rev. D 87, 034027 (2013)

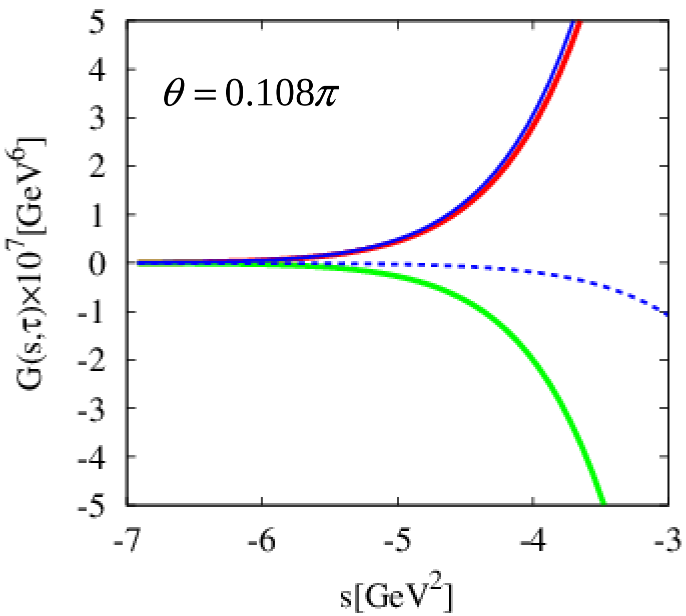
$$G_{OPE}(s,\tau) = \int_0^\infty K(s,\tau,\omega)\,\rho(\omega)\,d\omega$$

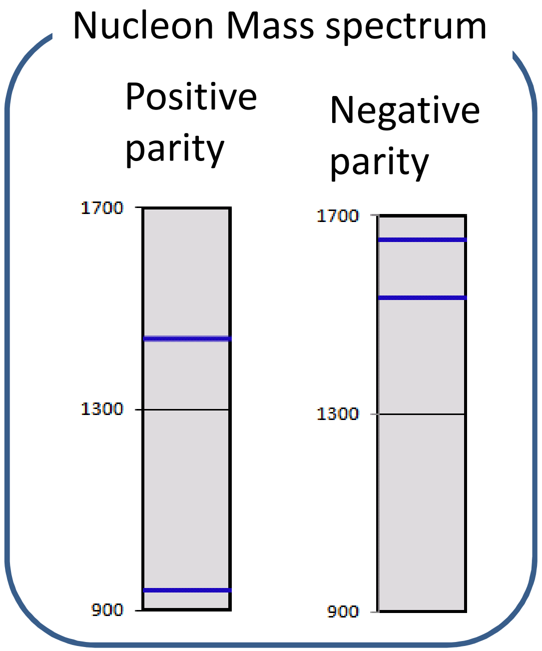
$$K(s,\tau,\omega)d\omega = \frac{1}{\sqrt{4\pi\tau}}\omega \mathrm{e}^{-\frac{(\omega^2-s)^2}{4\tau}}d\omega \quad \Rightarrow \quad \frac{1}{\sqrt{4\pi\tau}}\mathrm{Re}[\omega \mathrm{e}^{-i\theta}\mathrm{e}^{-\frac{(\omega^2\mathrm{e}^{-2i\theta}-s)^2}{4\tau}}\mathrm{e}^{-i\theta}d\omega]$$

Phase – rotated kernel

$\tau = 1.5\mathrm{GeV}^4$

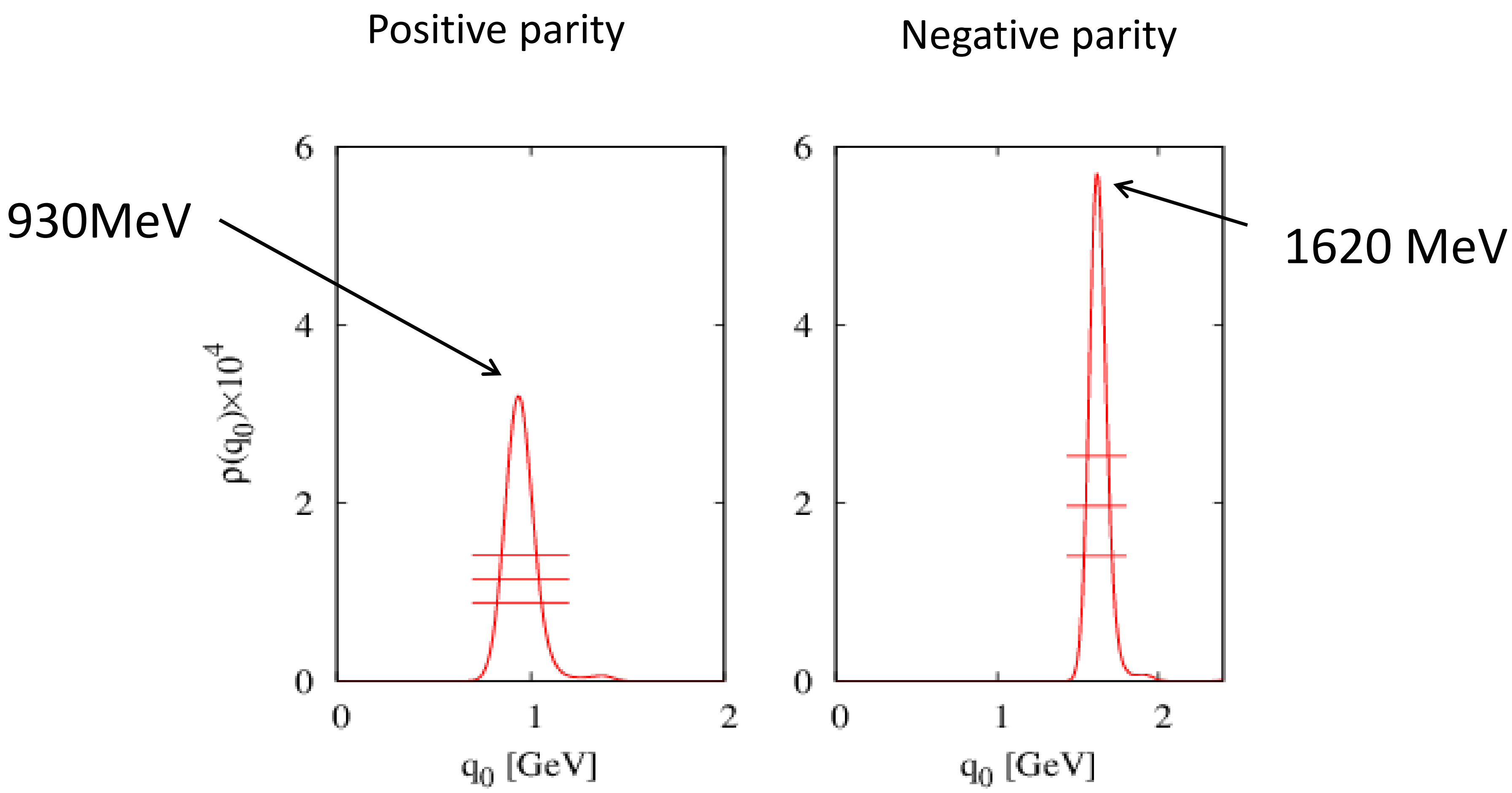
Positive parity OPE
Negative parity OPE
chiral condensate term
Perturvative term





The contribution of the chiral condensate term is dominant.

Nucleon QCD sum rules



In both positive and negative parity, the peaks are found.

In the negative parity analysis, the peak correspond to the N(1535) or (and) N(1650).

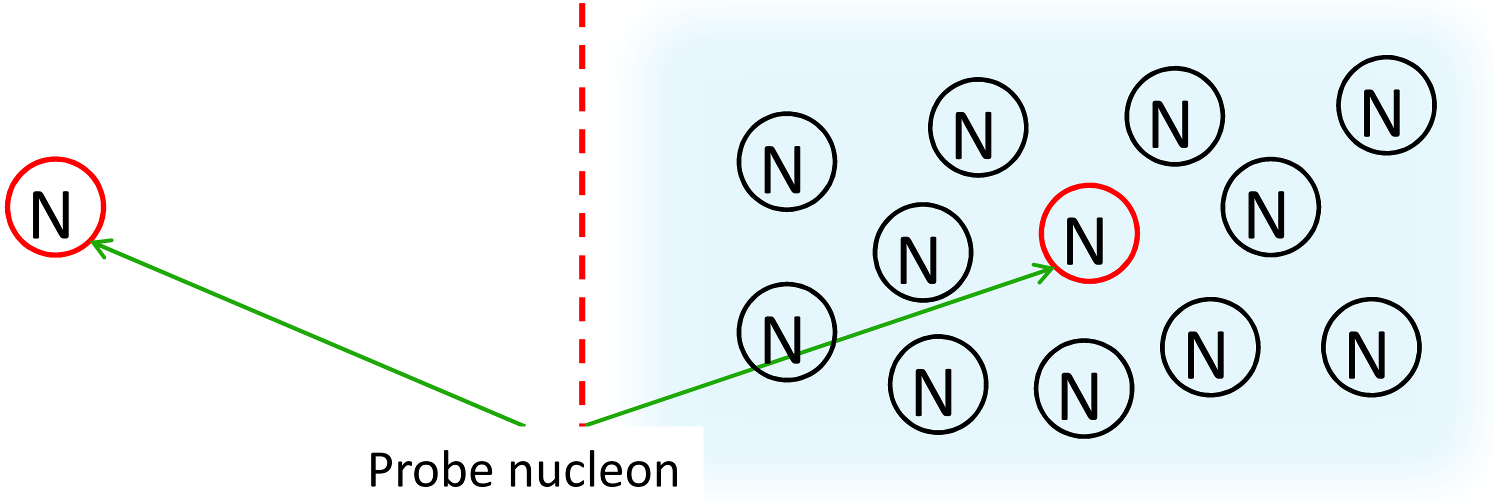
Nucleon QCD sum rules in the nuclear matter

Vacuum

$$\Pi(q) = i\int d^4x e^{iqx}\langle \underline{0}|T[\eta(x)\overline{\eta}(0)]|\underline{0}\rangle$$

Nuclear medium

$$\Pi(q) = i\int d^4x e^{iqx}\langle \underline{\Psi_0}|T[\eta(x)\overline{\eta}(0)]|\underline{\Psi_0}\rangle$$



Application of this analysis to the spectral function in nuclear matter

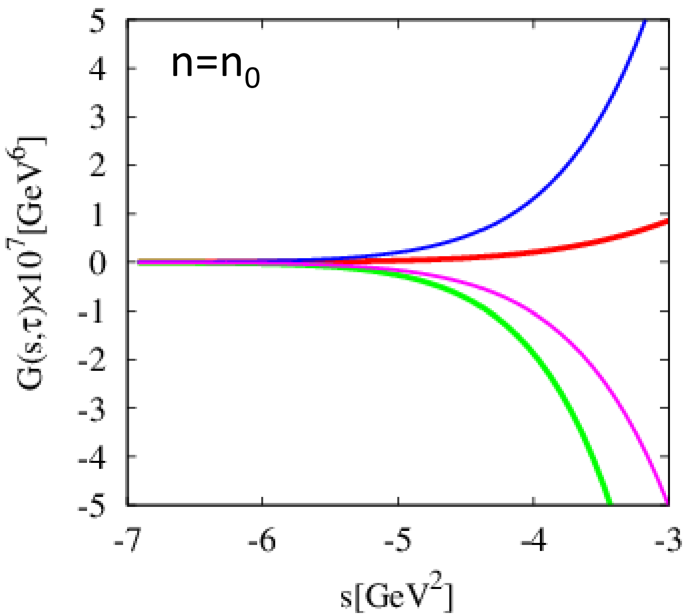
Condensate: $\langle 0|O_i|0\rangle \Rightarrow \langle \Psi_0|O_i|\Psi_0\rangle$ $\quad \Psi_0$: Nuclear matter ground state

For example,

Chiral condensate : $\langle \overline{q}q\rangle_0 \Rightarrow \langle \overline{q}q\rangle_{\rho_N} = \langle \overline{q}q\rangle_0 + \frac{\sigma_N}{2m_q}\rho_N + \cdots$

$\langle q^\dagger q\rangle_{\rho_N}$

Nucleon QCD sum rules in the nuclear matter



Positive parity OPE data

Negative parity OPE data

$\langle \overline{q}q \rangle_{\rho_N}$ term

$\langle q^\dagger q \rangle_{\rho_N}$ term

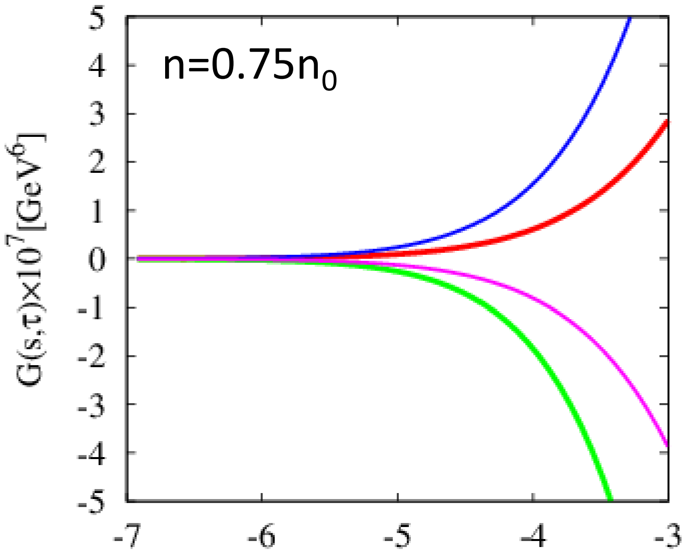
n_0: nuclear matter density

Positive parity OPE data: $+ C_1 \langle \overline{q}q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N}$

Negative parity OPE data: $- C_1 \langle \overline{q}q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N}$

Nucleon QCD sum rules in the nuclear matter

preliminary



Positive parity OPE data

Negative parity OPE data

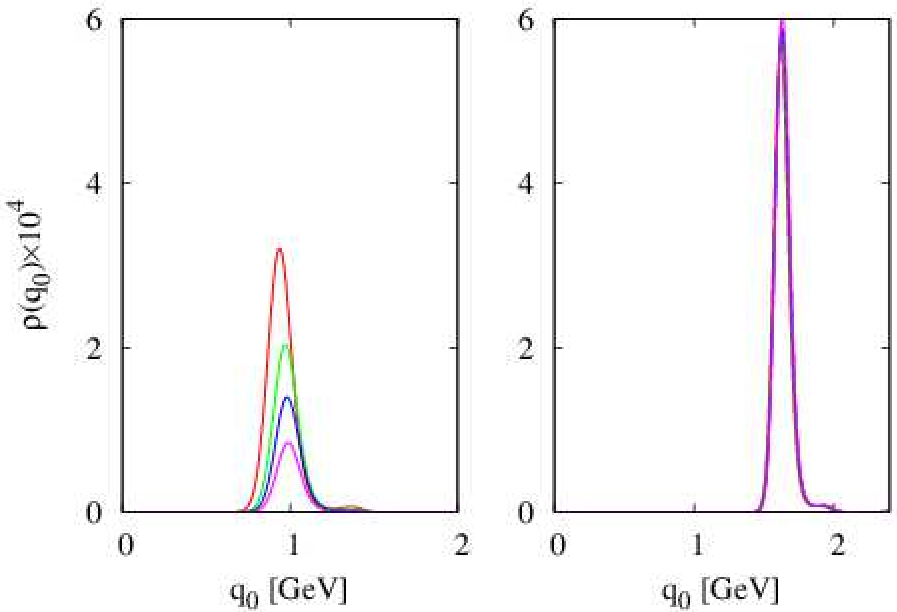
$\langle \bar{q}q \rangle_{\rho_N}$ term

$\langle q^\dagger q \rangle_{\rho_N}$ term

n$_0$: nuclear matter density

Positive parity OPE data: $+\, C_1 \langle \bar{q}q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N}$

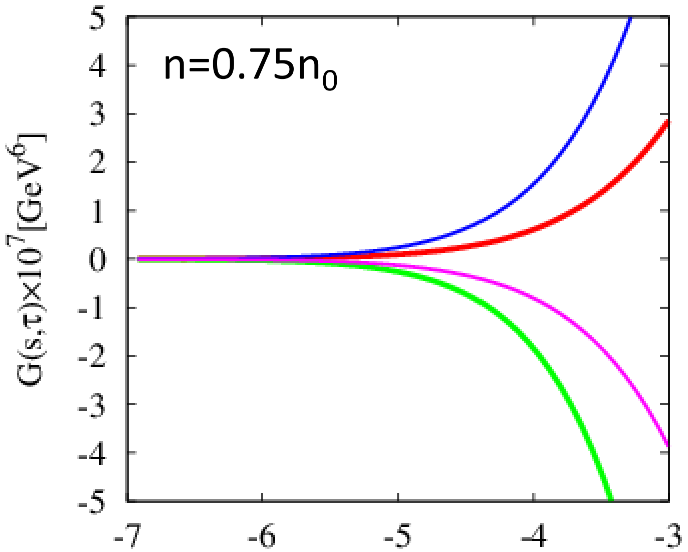
Negative parity OPE data: $-\, C_1 \langle \bar{q}q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N}$



: Vacuum

Nucleon QCD sum rules in the nuclear matter

preliminary



Positive parity OPE data

Negative parity OPE data

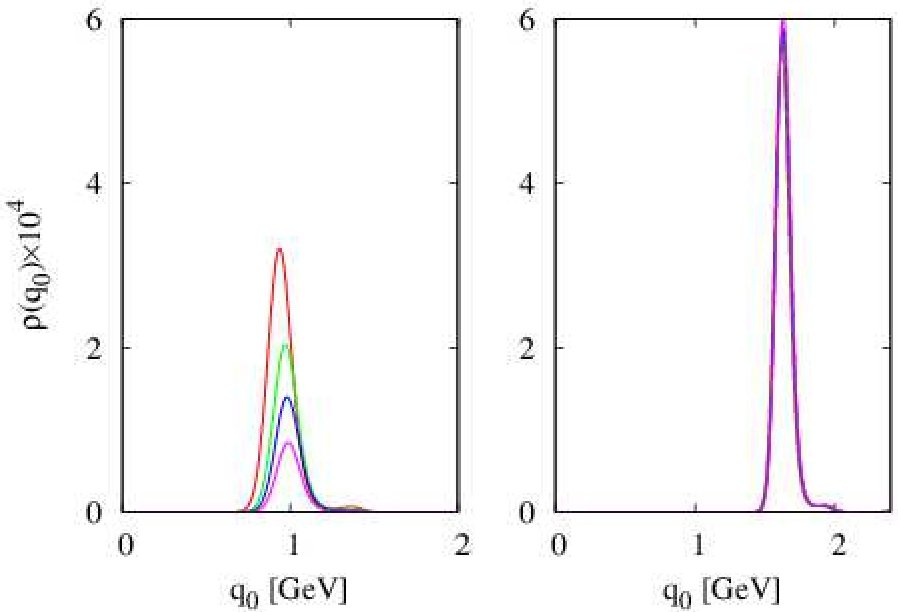
$\langle \overline{q}q \rangle_{\rho_N}$ term

$\langle q^\dagger q \rangle_{\rho_N}$ term

n_0: nuclear matter density

Positive parity OPE data: $+\, C_1 \langle \overline{q}q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N}$

Negative parity OPE data: $-\, C_1 \langle \overline{q}q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N}$



The peak position（ $E = \sqrt{q^2 + M^{*2}} + \Sigma^v$ ） is hardly sifted.

Nucleon QCD sum rules in the nuclear matter

Investigation of $M^*_{0\pm}$ and $\Sigma^v_{0\pm}$

$$\begin{aligned}\Pi(q) &= i\int d^4x e^{iqx}\langle\Psi_0|T[\eta(x)\overline{\eta}(0)]|\Psi_0\rangle \\ &= \not{q}\Pi_1(q^2, q\cdot u) + \Pi_2(q^2, q\cdot u) + \not{u}\Pi_u(q^2, q\cdot u) \\ &= \sum_n \left[\lambda^2_{n+}\frac{\not{q} + M^*_{n+} - \not{u}\Sigma^v_{n+}}{(q_0 - E_{n+} + i\epsilon)(q_0 + \overline{E}_{n+} - i\epsilon)} + \lambda^2_{n-}\frac{\not{q} - M^*_{n-} - \not{u}\Sigma^v_{n-}}{(q_0 - E_{n-} + i\epsilon)(q_0 + \overline{E}_{n-} - i\epsilon)}\right]\end{aligned}$$

$$E = \sqrt{q^2 + M^{*2}} + \Sigma^v$$

$$q_0\Pi_1 \Rightarrow \sum|\lambda_+|^2\frac{E^+}{2\sqrt{M_+^{*2} + \vec{q}^2}}\frac{1}{q_0 - E^+ + i\epsilon} + |\lambda_-|^2\frac{E^-}{2\sqrt{M_-^{*2} + \vec{q}^2}}\frac{1}{q_0 - E^- + i\epsilon},$$

$$\Pi_u \Rightarrow \sum|\lambda_+|^2\frac{-\Sigma^v_+}{2\sqrt{M_+^{*2} + \vec{q}^2}}\frac{1}{q_0 - E^+ + i\epsilon} + |\lambda_-|^2\frac{-\Sigma^v_-}{2\sqrt{M_-^{*2} + \vec{q}^2}}\frac{1}{q_0 - E^- + i\epsilon}.$$

After fitting OPE side and Phenomenological side, $M^*_{0\pm}$ and $\Sigma^v_{0\pm}$ can be obtained.

Nucleon QCD sum rules in the nuclear matter

preliminary

n_0: nuclear matter density

		Vacuum	$n=0.25n_0$	$n=0.5n_0$	$n=0.75n_0$
Positive parity	M^*_{0+}	930	850	710	470
	Σ^v_{0+}	0	120	270	500
Negative parity	M^*_{0-}	1620	1630	1650	1680
	Σ^v_{0-}	0	0	-20	-50

Conclusion

- We analyze the nucleon spectral function by using QCD sum rules with MEM
- We construct the parity projected sum rule using phase - rotated Gaussian kernel with α_s correction.
- It is found that, in this sum rule, chiral condensate term is dominant and continuum contributions is reduced.
- The information of not only the ground state but also the negative parity excited state is extracted
- We investigate the effective masses and the vector self-energies in the nuclear medium.