Quarkonia and their decay properties

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Outline

Introduction

- The number of known states for the $c\bar{c}$ and $b\bar{b}$ is constantly increasing.
- It is believed that some of these new states could be the first manifestation of the existence of exotic hadrons (tetraquarks, molecules, hybrids etc)[1, 2, 3].
- In order to explore such options, a comprehensive understanding of the heavy quarkonium spectroscopy is required.

Theory

Mass Spectrum

- We use the following Hamiltonia[4]

$$H = \sqrt{\mathbf{p}^2 + m_Q^2} + \sqrt{\mathbf{p}^2 + m_{\bar{Q}}^2} + V(\mathbf{r}) \tag{1}$$

- The inter-quark potential is of the form[5, 6, 7],

$$V(r) = -\frac{\alpha_c}{r} + Ar^\nu + V_0 \tag{2}$$

- Choose a hydrogenic trial wavefunction

$$R_{nl}(r) = \left(\frac{\mu^3 (n-l-1)!}{2n(n+l)!}\right)^{1/2} (\mu r)^l e^{-\mu r/2} L_{n-l-1}^{2l+1}(\mu r) \tag{3}$$

- The Gaussian Wavefunction has the form

$$R_{nl}(\mu, r) = \mu^{3/2} \left(\frac{2\,(n-1)!}{\Gamma\,(n+l+1/2)}\right) (\mu r)^l\, e^{-\mu^2 r^2/2} L_{n-1}^{l+1/2}(\mu^2 r^2) \tag{4}$$

Theory

Mass Spectrum

- Fix α_s using the formula[8, 9, 10]

$$\alpha_s = \frac{4\pi}{\left(11 - \frac{2}{3} n_f\right) \ln \frac{M^2 + m_B^2}{\Lambda^2}}$$

where $M = 2 m_Q m_{\bar{Q}} / \left(m_Q + m_{\bar{Q}}\right)$, $M_B = 0.95$ GeV and $\Lambda = 413$ MeV.

- By using suitable value of A, use virial theorem[11] to find the variational parameter μ.
- Solve the Schrodinger equation

$$H\psi = E\psi \tag{5}$$

to obtain the ground state spin-averaged mass of the meson.

- Match the obtained spin-avergaed mass to experimental measurement using the equation

$$M_{CW,n} = \frac{\Sigma_J 2(2J+1) M_{nJ}}{\Sigma_J 2(2J+1)} \tag{6}$$

by fixing the value of parameter V_0

Theory

Mass Spectrum

- Use the following equation to get excited state spectrum[12, 13, 14],

$$
\begin{aligned}
V_{SD}(\mathbf{r}) &= \left(\frac{\mathbf{L}\cdot\mathbf{S_Q}}{2m_Q^2}+\frac{\mathbf{L}\cdot\mathbf{s_{\bar{Q}}}}{2m_{\bar{Q}}^2}\right)\left(-\frac{dV(r)}{dr}+\frac{8}{3}\alpha_S\frac{1}{r^3}\right)+ \\
&\quad \frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{Q}}}\frac{\mathbf{L}\cdot\mathbf{S}}{r^3}+\frac{4}{3}\alpha_S\frac{2}{3m_Q m_{\bar{Q}}}\mathbf{S_Q}\cdot\mathbf{s_{\bar{Q}}}4\pi\delta(\mathbf{r})+ \\
&\quad \frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{Q}}}(3(\mathbf{S_Q}\cdot\mathbf{n})-(\mathbf{s_{\bar{Q}}}\cdot\mathbf{n})-\mathbf{S_Q}\cdot\mathbf{s_{\bar{Q}}})\frac{1}{r^3}, \\
&\quad \text{where } \mathbf{n}=\frac{\mathbf{r}}{r} \qquad (7)
\end{aligned}
$$

Theory

Decay Constants

- The decay constants are evaluated using the relation[15],

$$f_{P/V}^2 = \frac{12\left|\psi_{P/V}(0)\right|^2}{M_{P/V}}\bar{C}^2(\alpha_S) \tag{8}$$

Where $\bar{C}(\alpha_S)$ is the QCD correction factor given by[16]

$$\bar{C}^2(\alpha_S) = 1 - \frac{\alpha_S}{\pi}\left[2 - \frac{m_Q - m_{\bar{Q}}}{m_Q + m_{\bar{Q}}}\ln\frac{m_Q}{m_{\bar{Q}}}\right] \tag{9}$$

Theory

Electric Dipole Transitions

- The E1 radiative transition rate is given by[17, 18]

$$\Gamma(i \to f + \gamma) = \frac{4\alpha \langle e_Q \rangle^2}{27} k^3 (2J_f + 1) \left| \langle f \left| r \right| i \rangle \right|^2 S_{if} \tag{10}$$

where S_{if} is the statistical factor with $S_{if} = 1$ for the transitions between spin-triplet states and $S_{if} = 3$ for the transition between spin-triplet states, $\langle e_Q \rangle$ is an effective quark charge given by

$$\langle e_Q \rangle = \frac{m_{\bar{Q}} e_Q - m_Q e_{\bar{Q}}}{m_Q + m_{\bar{Q}}} \tag{11}$$

Theory

Magnetic dipole transitions

- The M1 rate for transitions between S-wave levels is given by[17, 19, 18]

$$\Gamma_{M1}(i \to f + \gamma) = \frac{16\alpha}{3}\mu^2 k^3 (2J_f + 1) \left| \langle f | j_0(kr/2) | i \rangle \right|^2$$

where the magnetic dipole moment is

$$\mu = \frac{m_{\bar{Q}} e_Q - m_Q e_{\bar{Q}}}{4 m_{\bar{Q}} m_Q}$$

and k is the photon energy.

Theory

Two photon decay widths

In the non-relativistic limit, the two-photon decay widths of $c\bar{c}$; $^1S_0$, $^3P_0$, and $^3P_2$ can be written as [20]

$$\Gamma^{NR}(^1S_0 \to \gamma\gamma) = \frac{3\alpha^2 e_c^4 \left|R_{nS}(0)\right|^2}{m_c^2}, \tag{12}$$

$$\Gamma^{NR}(^3P_0 \to \gamma\gamma) = \frac{27\alpha^2 e_c^4 \left|R'_{nP}(0)\right|^2}{m_c^4}, \tag{13}$$

$$\Gamma^{NR}(^3P_2 \to \gamma\gamma) = \frac{36\alpha^2 e_c^4 \left|R'_{nP}(0)\right|^2}{5m_c^4}. \tag{14}$$

The first-order QCD radiative corrections to the two-photon decay rates can be accounted for as[20]

$$\begin{aligned}\Gamma(^1S_0 \to \gamma\gamma) &= \Gamma^{NR}(^1S_0 \to \gamma\gamma) \quad \times \\ &\quad \left[1 + \frac{\alpha_S}{\pi}\left(\frac{\pi^2}{3} - \frac{20}{3}\right)\right],\end{aligned} \tag{15}$$

Theory

Two photon decay widths

$$\Gamma(^3P_0 \to \gamma\gamma) = \Gamma^{NR}(^3P_0 \to \gamma\gamma) \times \left[1 + \frac{\alpha_S}{\pi}\left(\frac{\pi^2}{3} - \frac{28}{9}\right)\right], \quad (16)$$

$$\Gamma(^3P_2 \to \gamma\gamma) = \Gamma^{NR}(^3P_2 \to \gamma\gamma) \times \left[1 - \frac{16\alpha_S}{3\pi}\right]. \quad (17)$$

In calculating these decay widths we have made use of the spectroscopic parameters obtained from the present model using the gaussian wavefunction only.

Results

Fitted and Used Parameters

Table : Value of V_0(in GeV)

ν	$c\bar{c}$		$b\bar{b}$	
	Gauss.	Hydro.	Gauss.	Hydro.
1.0	-0.287	-0.213	-0.370	-0.336

$A = 0.175$ for $c\bar{c}$ while $A = 0.24$ for $b\bar{b}$, $m_b = 4.88\ GeV$, $m_c = 1.55\ GeV$

Results

Spin Averaged Masses

Table : S wave Spin Averaged Masses of $c\bar{c}$

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $\lvert R(0)\rvert$ $GeV^{3/2}$	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $\lvert R(0)\rvert$ $GeV^{3/2}$	Gaussian $E(\mu)$ (GeV)	Expt.[1] (GeV)	Theory (GeV)
1S	0.5	1.362	1.123	3.068	0.526	0.573	3.068	3.068	3.068[2]
	1.0	1.692	1.556	3.068	0.655	0.797	3.068		3.068[21]
	1.5	2.014	2.020	3.068	0.771	1.016	3.068		3.068[22]
	2.0	2.317	2.493	3.068	0.872	1.222	3.068		
2S	0.5	1.097	0.406	3.392	0.294	0.195	3.367	3.674[1]	3.672[2]
	1.0	1.705	0.787	3.668	0.460	0.382	3.685		3.661[21]
	1.5	2.290	1.225	3.972	0.624	0.604	4.075		3.666[22]
	2.0	2.811	1.667	4.298	0.778	0.842	4.483		
3S	0.5	1.072	0.262	3.545	0.233	0.123	3.522		4.026[2]
	1.0	1.847	0.592	4.063	0.405	0.283	4.122		4.064[21]
	1.5	2.619	0.999	4.691	0.584	0.490	4.922		

nL	ν	Hydrogenic			Gaussian			Theory
		μ (GeV)	$\|R(0)\|$ $GeV^{3/2}$	$E(\mu)$ (GeV)	μ (GeV)	$\|R(0)\|$ $GeV^{3/2}$	$E(\mu)$ (GeV)	(GeV)
4S	0.5	1.084	0.199	3.652	0.203	0.093	3.634	4.420[2]
	1.0	1.985	0.495	4.390	0.376	0.234	4.492	
	1.5	2.909	0.877	5.354	0.564	0.430	5.704	
	2.0	3.682	1.249	6.702	0.755	0.667	7.115	
5S	0.5	1.103	0.164	3.738	0.184	0.076	3.724	4.830[2]
	1.0	2.112	0.434	4.680	0.357	0.204	4.822	
	1.5	3.162	0.795	5.993	0.551	0.391	6.445	
	2.0	4.009	1.135	8.026	0.752	0.625	8.403	
6S	0.5	1.125	0.141	3.811	0.171	0.064	3.801	5.164[2]
	1.0	2.226	0.392	4.946	0.343	0.183	5.126	
	1.5	3.384	0.734	6.622	0.541	0.363	7.159	
	2.0	4.285	1.045	9.402	0.751	0.594	9.724	

Results

Spin averaged masses

Table : S wave Spin Averaged Masses of $b\bar{b}$

nL	ν	Hydrogenic			Gaussian			Expt.[1]	Theory
		μ (GeV)	$\lvert R(0)\rvert$ $GeV^{3/2}$	$E(\mu)$ (GeV)	μ (GeV)	$\lvert R(0)\rvert$ $GeV^{3/2}$	$E(\mu)$ (GeV)	(GeV)	(GeV)
1S	0.5	2.647	3.046	9.453	1.019	1.544	9.453	9.453	9.443[2]
	1.0	3.007	3.686	9.453	1.160	1.876	9.453		9.445[21]
	1.5	3.299	4.237	9.453	1.261	2.128	9.453		9.442[22]
	2.0	3.555	4.741	9.453	1.339	2.328	9.453		
2S	0.5	2.043	1.033	9.826	0.548	0.498	9.792		10.015[2]
	1.0	2.852	1.703	10.024	0.771	0.831	10.021		10.015[21]
	1.5	3.533	2.348	10.202	0.965	1.162	10.250		9.996[22]
	2.0	4.110	2.946	10.355	1.131	1.476	10.463		
3S	0.5	1.968	0.651	9.992	0.429	0.308	9.958		10.348[2]
	1.0	3.044	1.252	10.377	0.670	0.602	10.403		10.348[21]
	1.5	3.996	1.882	10.757	0.893	0.926	10.896		10.329[22]

nL	ν	Hydrogenic			Gaussian			Theory
		μ (GeV)	$\lvert R(0)\rvert$ $GeV^{3/2}$	$E(\mu)$ (GeV)	μ (GeV)	$\lvert R(0)\rvert$ $GeV^{3/2}$	$E(\mu)$ (GeV)	(GeV)
4S	0.5	1.975	0.491	10.107	0.371	0.229	10.077	10.583[2]
	1.0	3.252	1.037	10.663	0.618	0.494	10.723	
	1.5	4.429	1.648	11.252	0.857	0.806	11.487	
	2.0	5.480	2.267	11.827	1.080	1.140	12.286	
5S	0.5	2.003	0.401	10.197	0.335	0.186	10.172	10.864[2]
	1.0	3.448	0.905	10.914	0.585	0.429	11.006	
	1.5	4.823	1.498	11.717	0.834	0.730	12.046	
	2.0	6.068	2.114	12.539	1.073	1.064	13.175	
6S	0.5	2.038	0.343	10.273	0.310	0.158	10.252	11.076[2]
	1.0	3.628	0.815	11.143	0.561	0.384	11.266	
	1.5	5.182	1.390	12.161	0.818	0.675	12.583	
	2.0	6.603	1.999	13.252	1.069	1.009	14.061	

Results

Spin averaged masses

Table : P and D wave Spin Averaged Masses of $c\bar{c}$.

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $E(\mu)$ (GeV)	Expt.[1] (GeV)	Theory (GeV)
1P	0.5	1.066	3.358	0.331	3.331	3.525[1]	3.525[2]
	1.0	1.607	3.569	0.498	3.531		3.526[21]
	1.5	2.117	3.798	0.649	3.740		3.492[22]
	2.0	2.590	4.022	0.761	5.255		
2P	0.5	1.057	3.526	0.247	3.498		3.926[2]
	1.0	1.796	4.004	0.420	3.990		3.945[21]
	1.5	2.534	4.576	0.594	4.602		
	2.0	3.226	5.184	0.754	6.550		

nL	ν	Hydrogenic		Gaussian		Theory
		μ (GeV)	$E(\mu)$ (GeV)	μ (GeV)	$E(\mu)$ (GeV)	(GeV)
3P	0.5	1.074	3.640	0.210	3.614	4.337[2]
	1.0	1.953	4.351	0.385	4.370	
	1.5	2.866	5.256	0.570	5.392	
	2.0	3.714	6.325	NS	NS	
1D	0.5	1.028	3.488	0.272	3.457	3.802[2]
	1.0	1.697	3.874	0.448	3.831	3.811[21]
	1.5	2.342	4.317	0.614	4.246	
	2.0	2.949	4.763	0.751	7.228	
2D	0.5	1.056	3.615	0.223	3.584	
	1.0	1.893	4.263	0.399	4.230	
	1.5	2.752	5.078	0.580	5.051	
	2.0	3.577	5.970	NS		

Results

Spin averaged masses

Table : P and D wave Spin Averaged Masses of $b\bar{b}$.

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $E(\mu)$ (GeV)	Expt.[1] (GeV)	Theory (GeV)
1P	0.5	1.990	9.792	0.617	9.757	9.899	9.900[2]
	1.0	2.703	9.941	0.836	9.899		9.901[21]
	1.5	3.286	10.073	0.907	10.673		9.873[22]
	2.0	3.780	10.188	1.167	11.035		
2P	0.5	1.941	9.973	0.453	9.936	10.260	10.260[2]
	1.0	2.968	10.326	0.695	10.300		10.261[21]
	1.5	3.865	10.675	0.865	11.268		10.231[22]
	2.0	4.647	10.995	1.078	11.920		

nL	ν	Hydrogenic		Gaussian		Theory (GeV)
		μ (GeV)	$E(\mu)$ (GeV)	μ (GeV)	$E(\mu)$ (GeV)	
3P	0.5	1.958	10.094	0.384	10.059	10.544[2]
	1.0	3.204	10.628	0.632	10.627	
	1.5	4.344	11.195	0.840	11.831	
	2.0	5.367	11.740	1.071	12.802	
1D	0.5	1.890	9.922	0.500	9.895	10.163[2]
	1.0	2.812	10.217	0.742	10.168	10.158[21]
	1.5	3.587	10.481	0.849	11.591	10.127[22]
	2.0	4.251	10.719	1.071	12.396	
2D	0.5	1.927	10.055	0.407	10.029	
	1.0	3.110	10.554	0.656	10.512	
	1.5	4.178	11.061	NS		
	2.0	5.129	11.546	NS		

Results

Mass spectrum

Table : Mass spectrum of charmonium(in GeV).

State	This Work		Expt.[1]	Ref[2]	Ref.[21]	Ref[22]
	Gaussian	Hydrogenic				
1^1S_0	2.972	2.800	2.980	2.981	2.979	2.980
1^3S_1	3.100	3.157	3.097	3.096	3.096	3.097
1^3P_0	3.455	3.405	3.415	3.413	3.424	3.436
1^3P_1	3.519	3.535	3.511	3.511	3.510	3.486
1^1P_1	3.531	3.569	3.525	3.525	3.526	3.493
1^3P_2	3.554	3.622	3.556	3.555	3.556	3.507
2^1S_0	3.665	3.585	3.637	3.635	3.588	3.608
2^3S_1	3.692	3.695	3.686	3.685	3.686	3.686
1^3D_1	3.823	3.848	3.773	3.783	3.798	
1^3D_2	3.831	3.873		3.795	3.813	
1^1D_2	3.833	3.874		3.807	3.811	
1^3D_3	3.833	3.886		3.813	3.815	

State	This Work		Expt.[1]	Ref[2]	Ref.[21]
	Gaussian	Hydrogenic			
2^3P_0	3.930	3.756		3.870	3.854
2^3P_1	3.980	3.948		3.906	3.929
2^1P_1	3.990	4.004		3.926	3.945
2^3P_2	4.008	4.088	3.927	3.949	3.972
3^1S_0	4.111	4.015		3.989	3.991
3^3S_1	4.126	4.079	4.039	4.039	4.088
2^3D_1	4.221	4.210	4.153	4.150	
2^3D_2	4.230	4.256		4.190	
2^1D_2	4.231	4.263	4.156	4.196	
2^3D_3	4.232	4.291		4.220	
3^3P_0	4.314	4.021		4.301	
3^3P_1	4.361	4.273		4.319	
3^1P_1	4.370	4.351		4.337	
3^3P_2	4.388	4.463	4.351	4.354	
4^1S_0	4.484	4.356		4.401	
4^3S_1	4.494	4.401	4.421	4.427	

Results

Mass Spectrum

Table : Mass spectrum of bottomonium(in GeV).

State	This Work		Expt.[1]	Ref[2]	Ref.[21]	Ref[22]
	Gaussian	Hydrogenic				
1^1S_0	9.421	9.331	9.391	9.398	9.400	9.377
1^3S_1	9.464	9.494	9.460	9.460	9.460	9.464
1^3P_0	9.879	9.887	9.859	9.859	9.863	9.834
1^3P_1	9.892	9.930	9.893	9.892	9.892	9.864
1^1P_1	9.899	9.941	9.898	9.900	9.901	9.873
1^3P_2	9.908	9.959	9.912	9.912	9.913	9.886
2^1S_0	10.014	9.998		9.990	9.993	9.963
2^3S_1	10.023	10.033	10.023	10.023	10.023	10.007
1^3D_1	10.165	10.207		10.154	10.153	10.120
1^3D_2	10.168	10.216	10.164	10.161	10.158	10.126
1^1D_2	10.168	10.217		10.163	10.158	10.127
1^3D_3	10.171	10.222		10.166	10.162	10.130

State	This Work		Expt.[1]	Ref.[2]	Ref.[21]	Ref[22]
	Gaussian	Hydrogenic				
2^3P_0	10.282	10.251	10.232	10.233	10.234	10.199
2^3P_1	10.294	10.308	10.255	10.255	10.255	10.224
2^1P_1	10.300	10.325	10.259	10.260	10.261	10.231
2^3P_2	10.307	10.352	10.269	10.268	10.268	10.242
3^1S_0	10.400	10.362		10.329	10.328	10.298
3^3S_1	10.404	10.381	10.355	10.355	10.355	10.339
2^3D_1	10.508	10.536		10.435		10.573
2^3D_2	10.511	10.551		10.443		10.602
2^1D_2	10.512	10.554		10.445		
2^3D_3	10.514	10.563		10.449		
3^3P_0	10.613	NS		10.521		
3^3P_1	10.622	NS		10.541		
3^1P_1	10.627	NS		10.544		
3^3P_2	10.633	NS		10.550		
4^1S_0	10.720	10.653		10.573		
4^3S_1	10.723	10.666	10.579	10586		
5^1S_0	11.005	10.906		10.851		

Results

Decay Constant

Table : Pseudoscalar Decay Constants of $c\bar{c}$ (in GeV)

State		$c\bar{c}$		
		Gauss.	Hydro.	Others
$1S$	Uncor.	0.468	0.809	0.335 ± 0.075[23]
	Cor.	0.316	0.549	
$2S$	Uncor.	0.194	0.397	
	Cor.	0.132	0.269	
$3S$	Uncor.	0.136	0.285	
	Cor.	0.092	0.194	
$4S$	Uncor.	0.108	0.230	
	Cor.	0.073	0.156	

Results

Decay Constants

Table : Pseudoscalar Decay Constants of $b\bar{b}$ (in GeV)

State		$b\bar{b}$		
		Gauss.	Hydro.	Others
$1S$	Uncor.	0.595	1.164	0.711[24]
	Cor.	0.471	0.920	
$2S$	Uncor.	0.256	0.525	
	Cor.	0.203	0.415	
$3S$	Uncor.	0.182	0.380	
	Cor.	0.144	0.300	
$4S$	Uncor.	0.147	0.310	
	Cor.	0.116	0.245	

Results

Decay Constants

Table : Vector Decay Constants of $c\bar{c}$ (in GeV)

State		$c\bar{c}$		
		Gauss.	Hydro.	Others[25]
$1S$	Uncor.	0.479	0.887	0.459 ± 0.028
	Cor.	0.324	0.603	
$2S$	Uncor.	0.195	0.403	
	Cor.	0.132	0.274	
$3S$	Uncor.	0.136	0.287	
	Cor.	0.092	0.195	
$4S$	Uncor.	0.108	0.231	
	Cor.	0.073	0.157	

Results

Decay constants

Table : Vector decay constants of $b\bar{b}$ and (in GeV)

State		$b\bar{b}$		
		Gauss.	Hydro.	Others[26]
$1S$	Uncor.	0.597	1.174	0.708 ± 0.008
	Cor.	0.472	0.928	
$2S$	Uncor.	0.257	0.526	
	Cor.	0.203	0.416	
$3S$	Uncor.	0.182	0.380	
	Cor.	0.144	0.300	
$4S$	Uncor.	0.147	0.310	
	Cor.	0.117	0.245	

Results

E1 Transitions Rates

Table : $E1$ Transition Rates of $b\bar{b}$ Meson.

Transition	k (GeV)		Γ(keV)		Expt. [1]	[21]
	Hydr.	Gauss.	Hydr.	Gauss.		
$1^3P_2 \to 1^3S_1\gamma$	0.454	0.406	44.117	20.703		40.2
$1^3P_1 \to 1^3S_1\gamma$	0.426	0.419	36.527	22.640		36.6
$1^3P_0 \to 1^3S_1\gamma$	0.385	0.406	26.926	20.680		29.9
$1^1P_1 \to 1^1S_0\gamma$	0.591	0.466	97.289	31.289		52.6
$2^3S_1 \to 1^3P_2\gamma$	0.074	0.114	0.869	0.946	2.287	2.46
$2^3S_1 \to 1^3P_1\gamma$	0.102	0.130	1.400	0.837	2.207	2.45
$2^3S_1 \to 1^3P_0\gamma$	0.145	0.143	1.320	0.370	1.251	1.62
$2^1S_0 \to 1^1P_1\gamma$	0.057	0.114	0.717	1.703		3.09

Results

E1 Transition rates

Table : E1 transition rates of the $c\bar{c}$ meson.

Transition	k (GeV)		Γ(keV)		Expt.[1]	[21]	[27]
	Hydr.	Gauss.	Hydr.	Gauss.			
$1^3P_2 \to 1^3S_1\gamma$	0.435	0.428	492.114	326.643	386.1	327	309
$1^3P_1 \to 1^3S_1\gamma$	0.358	0.398	274.242	261.210	295.8	265	244
$1^3P_0 \to 1^3S_1\gamma$	0.239	0.340	81.961	164.171	176.8	121	117
$1^1P_1 \to 1^1S_0\gamma$	0.686	0.505	1900.210	531.564	< 510	560	323
$2^3S_1 \to 1^3P_2\gamma$	0.072	0.135	9.235	17.840	26.5	18.2	34
$2^3S_1 \to 1^3P_1\gamma$	0.157	0.169	56.242	20.776	27.9	22.9	36
$2^3S_1 \to 1^3P_0\gamma$	0.279	0.229	105.487	17.318	29.4	26.3	25
$2^1S_0 \to 1^1P_1\gamma$	0.016	0.132	0.179	29.437		41	104

Results

M1 transition rates

Table : M1 transitions rates of the $b\bar{b}$ meson.

Transition	k (GeV)		Γ(eV)		[21]
	Hydr.	Gauss.	Hydr.	Gauss.	
$1^3S_1 \to 1^1S_0\gamma$	0.162	0.043	191	4	5.8
$2^3S_1 \to 2^1S_0\gamma$	0.035	0.009	02	0	1.4
$3^3S_1 \to 3^1S_0\gamma$	0.019	0.004	0	0	0.8
$4^3S_1 \to 4^1S_0\gamma$	0.013	0.003	0	0	

Results

M1 transition rates

Table : M1 transition rates of the $c\bar{c}$ meson.

Transition	k (GeV)		Γ(keV)		Expt.[1]	[21]
	Hydr.	Gauss.	Hydr.	Gauss.		
$1^3S_1 \to 1^1S_0\gamma$	0.337	0.110	68.266	2.381	1.58	1.05
$2^3S_1 \to 2^1S_0\gamma$	0.108	0.027	2.286	0.035		0.043
$3^3S_1 \to 3^1S_0\gamma$	0.081	0.015	0.962	0.006		
$4^3S_1 \to 4^1S_0\gamma$	0.045	0.010	0.161	0.002		
$5^3S_1 \to 5^1S_0\gamma$	0.032	0.008	0.058	0.001		

Results

Two photon decay widths

Table : Two photon decay widths in charmonia(Gaussian wavefunction).

Transition	Mass (GeV)	μ (GeV)	$R_{nS}(0)$ (GeV$^{3/2}$)	$R'_{nP}(0)$ (GeV$^{5/2}$)	Width (keV) This work Γ	Γ_{cor}	Expt.[1]	Ref[27]	Ref.[28]
$1^1S_0 \to \gamma\gamma$	2.984	0.655	0.797		8.34	3.86	5.29	8.5	3.5
$2^1S_0 \to \gamma\gamma$	3.665	0.460	0.382		1.92	0.89	< 5	2.4	1.38
$3^1S_0 \to \gamma\gamma$	4.111	0.405	0.283		1.05	0.49		0.88	0.94
$1^3P_0 \to \gamma\gamma$	3.455	0.498		0.215	2.27	2.33	2.32	2.5	1.39
$2^3P_0 \to \gamma\gamma$	3.930	0.420		0.140	0.97	0.99		1.7	1.11
$3^3P_0 \to \gamma\gamma$	4.314	0.385		0.113	0.63	0.64		1.2	0.91
$1^3P_2 \to \gamma\gamma$	3.554	0.498		0.215	0.60	0.09	0.51	0.31	0.44
$2^3P_2 \to \gamma\gamma$	4.008	0.420		0.140	0.26	0.04		0.23	0.48
$3^3P_2 \to \gamma\gamma$	4.388	0.385		0.113	0.17	0.02		0.17	0.014

Conclusion

- The spin averaged masses of these mesons are in excellent agreement with the experimental results in Ref.[1] and also with the results of Ref.[2].
- The results obtained using the Gaussian wavefunction are in good agreement with experiemental measurements and also with Ref.[2].
- Spectra produced due to the hydrogenic wavefunction is overestimated.
- E1 transition widths in the case of $c\bar{c}$ obtained with Gaussian wave function are in good agreement with experimental results.
- Two photon decay widths in the case of $c\bar{c}$ are in satisfactory agreement with experimental measurements.

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Thanks