In-medium η' mass and η'N interaction in vacuum based on a chiral effective theory

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S.S., D.Jido, arXiv:1309.4845.

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  - The contribution from the $U_A(1)$ anomaly
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  - The in-medium η' mass
  - η'N 2body interaction
  - Possible bound state of η'N

analyzed with the linear sigma model

- Summary and future prospects

The η' mass and $U_A(1)$ anomaly

(considering 3flavor case)

QCD:micro.theory of hadrons

The global symmetry of QCD lagrangian

$\sim U(3)_L \times U(3)_R$ (in chiral limit)

No parity doubling

→spontaneous break down of chiral symmetry

$(U(3)_L \times U(3)_R \rightarrow U(3)_V)$

9 Nambu-Goldstone(NG) bosons?

The pseudoscalar meson mass

- π:135MeV
- K:497MeV
- η:548MeV

Relatively large difference

- η':958MeV

NG boson of $U(3)_L \times U(3)_R \rightarrow U(3)_V$?

$U_A(1)$ anomaly : Non-zero divergence of the $U_A(1)$ current: $\partial_\mu J_5^\mu = -\frac{g^2}{4\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$

No necessity of the $U_A(1)$ NG boson (The relatively large mass of η' is permitted.)

(e.g. K.Fujikawa,H.Suzuki,Path Integrals and Quantum Anomalies (2004))

$U_A(1)$ anomaly:One of the origin of the η' mass

The effect of the $U_A(1)$ anomaly to the meson mass spectrum may change in medium.

(The reduction of the non-perturbative instanton effect in medium)

E.Shuryak,NPB203(1982)93,140.

The η' mass is discussed in vacuum and medium from the $U_A(1)$ anomaly.

E.Witten:NPB156(1979)269.,G.Veneziano:NPB159(1979)213.,R.D.Pisarski,F.Wilczec:PRD29(1984)338.,
H.Kiuch,et al.PLB200(1988)543, J.Kapsta,et al.,PRD53(1996)5028,....

The effect of chiral symmetry breaking to the η' mass

The pseudoscalar singlet(η') and octet mesons(η) degenerate in chiral SU(3) symmeric phase(@high T or ρ).

T.D. Cohen, Phys. Rev.D54 (1996) 1867., S.H. Lee, T. Hatsuda, Phys.Rev. D54(1996)54., D. Jido, H. Nagahiro, S. Hirenzaki, Phys.Rev. C85(2012)032201(R).

η and η' are contained in the same multiplet of $SU(3)_L \times SU(3)_R$.

Singlet-pseudoscalar meson

$$\eta' \sim \bar{q} i \gamma_5 q \xrightarrow{\text{Axial trans.}} [Q_5^a, \eta'] = -\bar{q} i \frac{\lambda^a}{\sqrt{6}} q$$

$$\xrightarrow{\text{Axial trans.}} [Q_5^a, [Q_5^b, \eta']] = d^{abc} \bar{q} i \gamma_5 \frac{\lambda^c}{\sqrt{6}} q \sim d^{abc} \boxed{\eta^c}$$

Octet-pseudoscalar mesons

(Q_5^a:axial charge (generator of axial trans. of $SU(3)_L \times SU(3)_R$))

⇨ η and η' degenerate when chiral symmetry is restored.

※1. We used only the axial transformation of $SU(3)_L \times SU(3)_R$.
※2. We cannot transform the singlet to the octet ps-meson in the 2-flavor case using the $SU(2)_L \times SU(2)_R$ transformation.

⇨ ***Chiral symmetry breaking is also responsible for the generation of the η' mass.***

Partial restoration of chiral symmetry (PRCS)

<u>**Quark condensate @low density**</u>

$$\langle \bar{q}q \rangle^{*} = \left(1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho\right)\langle \bar{q}q \rangle + \mathcal{O}(\rho^{n>1})$$

E.G.Durkarev, E.M.Levin, Nucl.Phys. A511, 679(1990).

ρ:nuclear density[fm^{-3}] $\langle \bar{q}q \rangle$:quark condensate in vacuum

$\sigma_{\pi N} = 2m_q \langle N|\bar{q}q|N\rangle$ $\langle \bar{q}q \rangle^{*}$:quark condensate in nuclear matter

The possibility of the reduction of 〈q^{bar}q〉 in the nuclear matter

<u>The reduction of 〈q^{bar}q〉 can affect to the hadron properties.</u>

Gell-Mann-Oakes-Renner relation

$$f_\pi^2 m_\pi^2 = -m_q \langle \bar{q}q \rangle$$

Change of the hadron properties (decay const. or mass)

※Experimental investigation of PRCS @normal nuclear density with nucleus target

The 35% reduction of 〈q^{bar}q〉@normal nuclear density is suggested.

- π atom:K.Suzuki, et al., PRL92,72302(2004).
- π-nucleus elastic scattering:E.Friedman, et al., PRL93,122302(2004).

The degeneracy of η and η' when chiral SU(3) symmetry is restored
& partial restoration of chiral symmetry

<u>The possibility of the η' mass reduction in the nuclear matter</u>
<u>through the partial restoration of chiral symmetry</u>

- The studies related to the η' mass in finite T & ρ with chiral model exist.

  V.Bernard et al., PRD38(1988)1551.,T.Hatsuda, T.Kunihiro, Phys. Rep247(1994)221., P.Costa,et al.,Phys.Lett.B569(2003)171.,J.T.Renaghan, et al. PRD62,085008(2000). H.Nagahiro,et al.PRC74(2006)045203.,…

- The possibility of the η' mass reduction about a few 100 MeV from the analysis of the relativistic heavy ion collision data.

  T.Csoro,PRL105(2010)182301.

- With some simple assumptions,

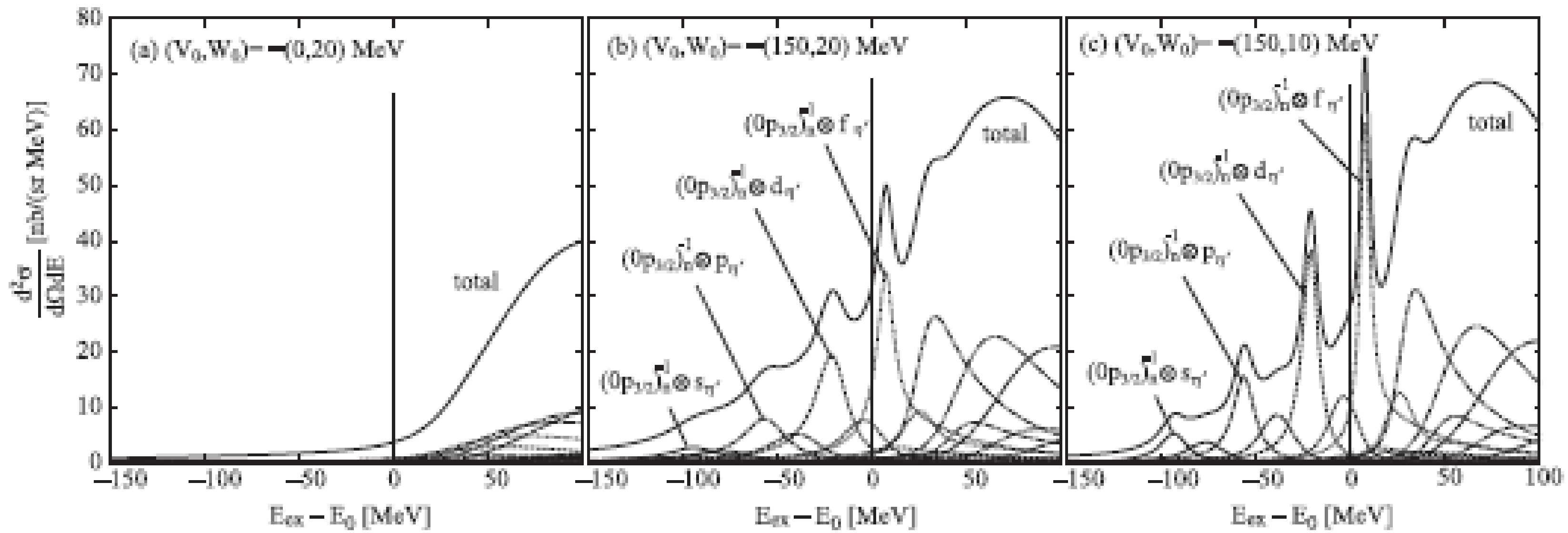
$$\Delta m_{\eta'} = \frac{2}{3}\frac{m_{\eta'}^2 - m_\eta^2}{2m_{\eta'}}\frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho$$

→80-100MeV mass reduction of η'
@normal nuclear density

η' mesic nuclei

K.Saito,et al.,Prog.Part.Nucl.Phys.58(2007)1.
Itahasihi et al,PTP,128(2012)601.

The observation of the η' mesic nuclei is discussed recently.



The analysis through $^{12}C(p,d)^{11}C \otimes \eta'$ reaction with 2.50GeV proton beam

Theoretical calculation suggests the characteristic structure
related to the η' optical potential

investigation the η' optical potential

In-medium *η'* mass and *η'*N 2body interaction in vacuum

In-medium mass (in-medium self energy) :the self-energy contains the nuclear matter effect.

$$m^2_{\eta'}(\rho) = m^2_{\eta'} + \boxed{\Sigma_{\eta'}(\rho)}$$

mass modification from medium effect

Non-relativistic limit etc.

Optical potential

$$\Sigma_{\eta'}(\rho) \sim 2m_{\eta'}V_{\eta'}(\rho)$$

Linear density approximation

$$V_{\eta'}(\rho) \sim \underline{V_{\eta' N}}\rho$$

*η'*N 2body interaction

- Not known well
- The important quantity as the foundation of in-medium η' property

In-medium mass reduction of *η'* means the *η'*N attraction in vacuum.

The purpose

- Calculation of *η'* mass in the nuclear matter

(Only the symmetry cannot say the reduction of *η'*
→the analysis with a particular model)

- Evaluation of the *η'*N 2body interaction

(Expectation of the attractive *η'*N interaction
accompanied with the η' mass reduction)

with a chiral effective model.

The important effect:

- The chiral SU(3) symmetry
- $U_A(1)$ anomaly
- Nucleon degree of freedom
- Introduction of symmetric nuclear matter
  with a consistent way with the partial restoration of chiral symmetry

➡ **Analysis with the linear sigma model**

Lagrangian of linear sigma model

J.T.Renaghan, et al. PRD62,085008(2000).
J.Schechter,Y.Ueda,Phys.Rev.D3,168(1971).

$$\mathcal{L} = \frac{1}{2}\mathrm{tr}(\partial_\mu M \partial^\mu M^\dagger) - \frac{\mu^2}{2}\mathrm{tr}(MM^\dagger) - \frac{\lambda}{4}\mathrm{tr}\left[(MM^\dagger)^2\right]$$

$$-\frac{\lambda'}{4}\left[\mathrm{tr}(MM^\dagger)\right]^2 + A\mathrm{tr}\chi M^\dagger + \sqrt{3}B\det M + \mathrm{h.c.}$$

The effect from the current quark mass

$U_A(1)$ anomaly effect

$$+\bar{N}i\partial\!\!\!/ N - g\bar{N}\left(\frac{1}{\sqrt{3}}\sigma_0 + \frac{1}{\sqrt{6}}\sigma_8 + i\gamma_5\frac{\vec{\tau}\cdot\vec{\pi}}{\sqrt{2}} + i\gamma_5\frac{1}{\sqrt{3}}\eta_0 + i\gamma_5\frac{1}{\sqrt{6}}\eta_8\right)N$$

Contribution from nucleon

$$M = \sum_{a=0}^{8}\frac{\sigma_a\lambda_a}{\sqrt{2}} + i\sum_{a=0}^{8}\frac{\pi_a\lambda_a}{\sqrt{2}} \quad N = \begin{pmatrix} p \\ n \end{pmatrix}$$

(λ_a:Gell-Mann matrix, τ_i:Pauli matrix)

※6 free parameters are fixed to reproduce
in-vacuum meson properties and 35% reduction of quark condensate @normal nuclear density.

The calculation of the in-medium η' mass with the linear sigma model

Some points of the calculation

✓The introduction of the nuclear matter

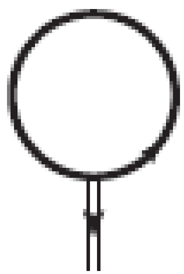
- the nucleon mean field: $\bar{N}N \to \rho$
- the symmetric nuclear matter: $\rho_p = \rho_n$

✓The determination of ⟨σ⟩ (⟨σ⟩:order parameter of χSSB)

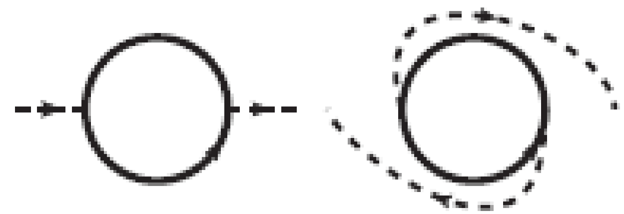
- minimum point of the effective potential: $\dfrac{\partial V}{\partial \sigma}(\langle \sigma \rangle ; \rho) = 0$

In-medium meson mass(1)

- The meson masses contain
  the contribution from the 3diagrams.



The contribution
from the nucleon mean field



The contribution
from the particle-hole excitation

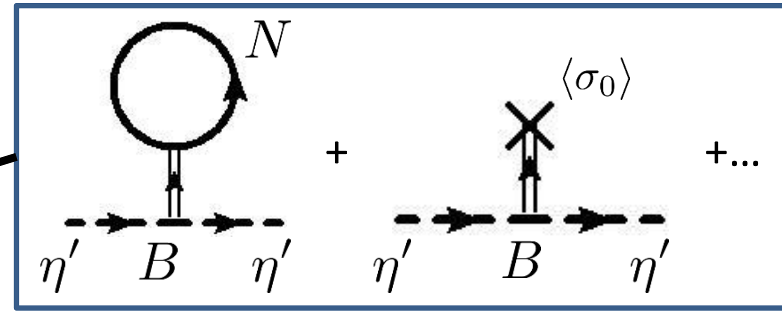
The nuclear matter affect through the nucleon loop.

$$\underbrace{\frac{i}{p^2 - m^2 + i\epsilon}}_{\text{Free propagation}} \underbrace{- 2\pi\theta(p_0)\delta(p^2 - m^2)\theta(k_f - |\vec{p}|)}_{\text{Pauli Blocking}}$$

In-medium meson mass(2)

- <u>*η’* mass in chiral limit</u>

$$m_{\eta'}^{*2} = 6\boxed{B}\boxed{\langle\sigma_0\rangle^*}$$



The contribution
from the $U_A(1)$ anomaly

The contribution
from the chiral symmetry breaking

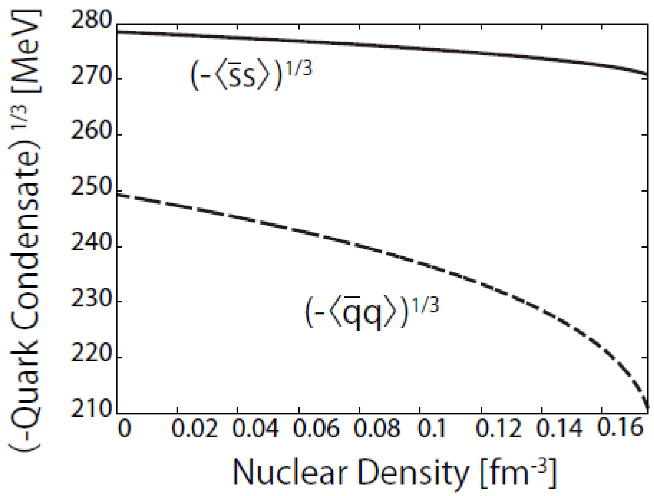
The necessity of both the $U_A(1)$ anomaly and chiral symmetry breaking
for the generation of the *η’* mass

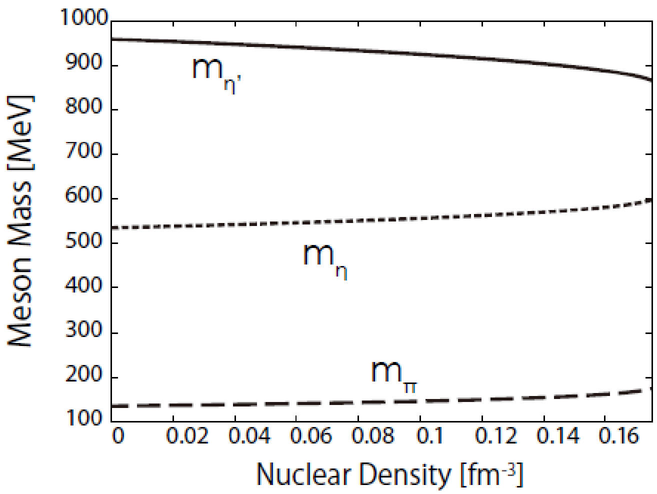
※The π mass vanishes in chiral limit

$$m_\pi^{*2} = \frac{6Am_q}{\langle\sigma_0\rangle^* + \frac{\langle\sigma_8\rangle^*}{\sqrt{2}}} \to 0 \quad (m_q \to 0)$$

In-medium η' mass with linear sigma model

S.S,D.Jido,arXiv:1309.4845





35% reduction of $\langle q^{bar}q \rangle$ @$\rho=\rho_0$ is input.

π atom:K.Suzuki, et al., PRL92,72302(2004).
π-nucleus elastic scattering:E.Friedman, et al., PRL93,122302(2004).

About 80MeV reduction of η' mass @ $\rho=\rho_0$
About 50MeV enhancement of η mass @ $\rho=\rho_0$

Mass difference between η and η' reduces about 130MeV.

(The partial restoration of chiral symmetry leads to the degeneracy of η and η')

From the η' mass reduction, the η'N 2body interaction is expected to be attractive.

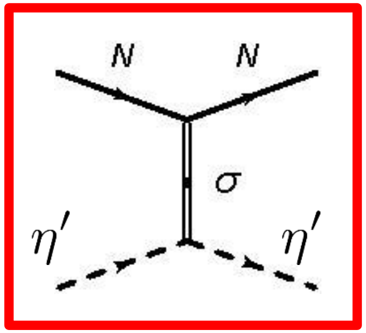
The evaluation of the η'N 2body interaction with the linear sigma model

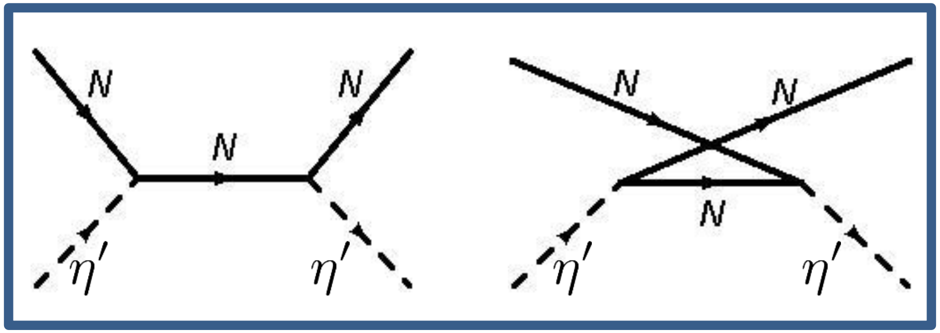
η'N 2body interaction in vacuum

- η'N interaction in the linear sigma model

@tree level

Tree level diagrams





Scalar meson exchange

Born term (containing nucleon intermediate state)

$$\frac{g}{\sqrt{3}}\frac{2\lambda\langle\sigma_0\rangle/3+2\lambda'\langle\sigma_0\rangle+4B}{q^2-m^2_{\sigma_0}}\delta_{ab}+\frac{ig}{\sqrt{3}}\gamma_5\frac{1}{\not{p}+\not{k}-m_N}\frac{ig}{\sqrt{3}}\gamma_5+\frac{ig}{\sqrt{3}}\gamma_5\frac{1}{\not{p}-\not{k}'-m_N}\frac{ig}{\sqrt{3}}\gamma_5$$

η'N 2body interaction in vacuum

S.S,D.Jido,arXiv:1309.4845

The low-energy η'N interaction in chiral limit

$$V_{\eta_0 N} = -\frac{6gB}{\sqrt{3}m_{\sigma_0}^2}$$

- The contribution from the scalar meson exchange term
- momentum independent interaction
- $V_{\eta 0N}$ is proportional to B.

The effect of the $U_A(1)$ anomaly.

✓different from the ordinary NG bosons
(scalar meson exchange terms are cancelled out .)

Substituting the fixed parameters to $V_{\eta 0 N}$,

<u>the η'N interaction is strongly attractive comparable to the $K^{bar}N$.</u>

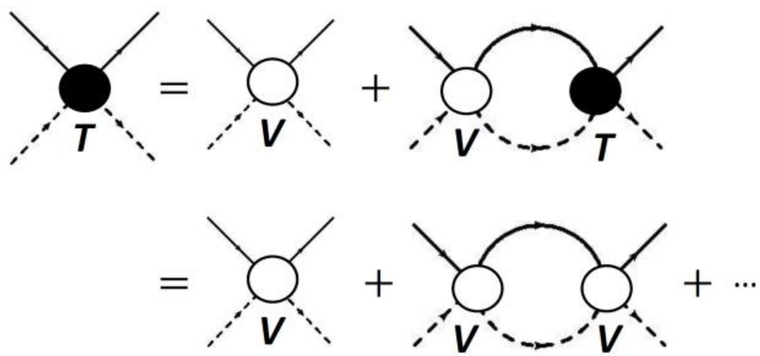
※There is a bound state in the $K^{bar}N$ system:Λ(1405)

<u>The possibility of the existence of the η'N bound state.</u>

The bound state appears as a pole of the T matrix below the threshold.

Single channel Lippmann-Schwinger equation

$$T = V + VGT$$



T:T matrix of η'N
V:interaction kernel
G:2body Green function

- The inteaction kernel=η'N interaction with LσM: $V_{\eta_0 N} = -\frac{6gB}{\sqrt{3}m_{\sigma_0}^2}$

Solving the scattering equation

Single channel Lippmann-Schwinger equation

$$T = V + VGT$$

T:T matrix of η'N
V:interaction kernel
G:2body Green function

– The interaction kernel:momentum independent: $V_{\eta_0 N} = -\frac{6gB}{\sqrt{3}m_{\sigma_0}^2}$

→LS eq. can be solved in algebraic way : $T = \frac{1}{V^{-1} - G}$

✓ Contain a divergence in the loop integral G.

→ some regularization and fixing the subtraction constant is needed.

Dimensional regularization and natural renormalizaion scheme are used.

T.Hyodo,D.Jido,A.Hosaka,PRC78025203(2008).

Excluding other dynamics than η' and N.

Possible bound state in η'N system

In the obtained T-matrix,

<u>a bound state (the sub-threshold pole of T-matrix) is found.</u>

Obtained values from T matrix of η'N system:

Binding energy	Scattering length	Effective range
6.2MeV	-2.7fm	0.25fm

※1.The existence of the bound state→scattering length is order 1 : $a_{\eta' N} = \frac{1}{\sqrt{2\mu E_B}}$

※2.Comparison with experimentally suggested values: $|\mathrm{Re}a_{\eta' p}| < 0.8\mathrm{fm}$ P.Moskal,et al.,PLB482,356(2000).

$|a_{\eta' p}| \sim 0.1\mathrm{fm}$ P.Moskal,et al.,PLB474,416(2000).

Summary

Using LσM,

- the η' mass in symmetric nuclear matter.
  - the mass reduction of η' in nuclear matter
- η'N 2body interaction in vacuum
  - the strong attraction comparable to $K^{bar}N$
- Possible η'N bound state
  - binding energy:6.2MeV
  - scattering length:-2.7fm
  - effective range:0.25fm

Future prospects

- Transition to the other mesons
  - Imaginary part of the optical potential

- Application to the photoproduction through final state interaction

Transformation ($\eta_0 \rightarrow \eta_8$)

Vector trans.: $q \longrightarrow e^{i\theta_V^a \lambda^a/2} q$

Axial trans.: $q \longrightarrow e^{i\theta_A^a \lambda^a \gamma_5/2} q$

$$\delta_A^a q = [Q_5^a, q] = -\frac{1}{2}\lambda^a \gamma_5 q$$

Q_5^a:axial charge (generator of axial trans.)

$$\eta' \sim \bar{q} i \gamma_5 q \xrightarrow{\text{Axial trans.}} [Q_5^a, \eta'] = -\bar{q} i \frac{\lambda^a}{\sqrt{6}} q$$

$$\xrightarrow{\text{Axial trans.}} [Q_5^a, [Q_5^b, \eta']] = d^{abc} \boxed{\bar{q} i \gamma_5 \frac{\lambda^c}{\sqrt{6}} q}$$

Octet-pseudoscalar meson

※1. used only axial trans.

※2. cannot transform $\eta_0 \rightarrow \eta_8$ in SU(2)

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = i f^{abc} \frac{\lambda^c}{2} \quad \left\{\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right\} = d^{abc} \frac{\lambda^c}{2}$$

η' properties

- Experimental efforts
  - pp→ppη' process P.Moskal,et al.,PLB482,356(2000). $|\mathrm{Re}\, a_{\eta' p}| < 0.8\mathrm{fm}$ P.Moskal,et al.,PLB474,416(2000). $|a_{\eta' p}| \sim 0.1\mathrm{fm}$
  - γp→η'p process M.Nanova, et al.,PLB710(2012)600.
- Theoretical efforts
  - SU(3) chiral effective model A.Ramos,E.Oset,et al.,PLB704(2011)334.

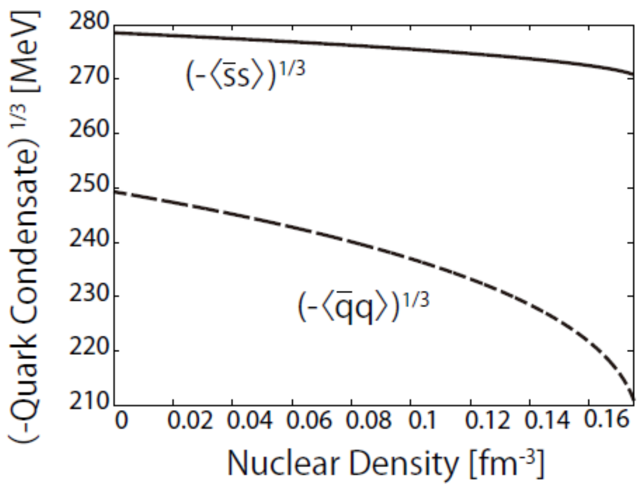
<u>The η'N interaction is not known well.</u>

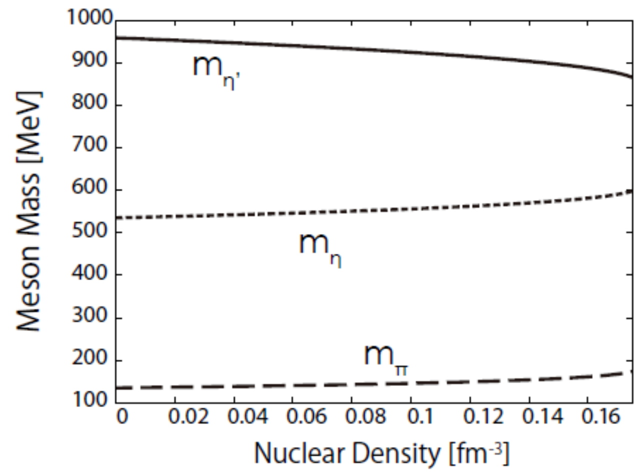
Important as the foundation of the in-medium η' properties

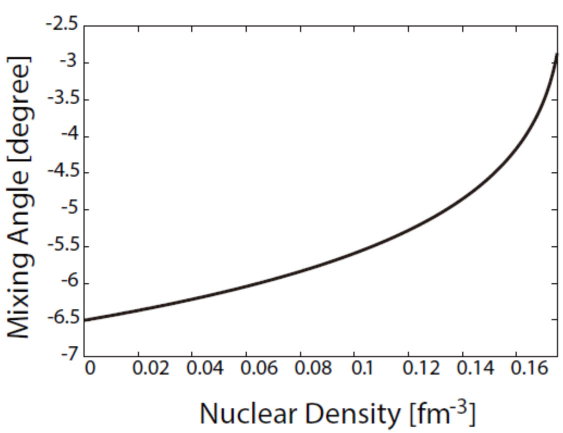
⇨ **Analyze the η'N 2body interaction with a consistent way with the chiral restoration in the nuclear matter.**

The in-medium quantities

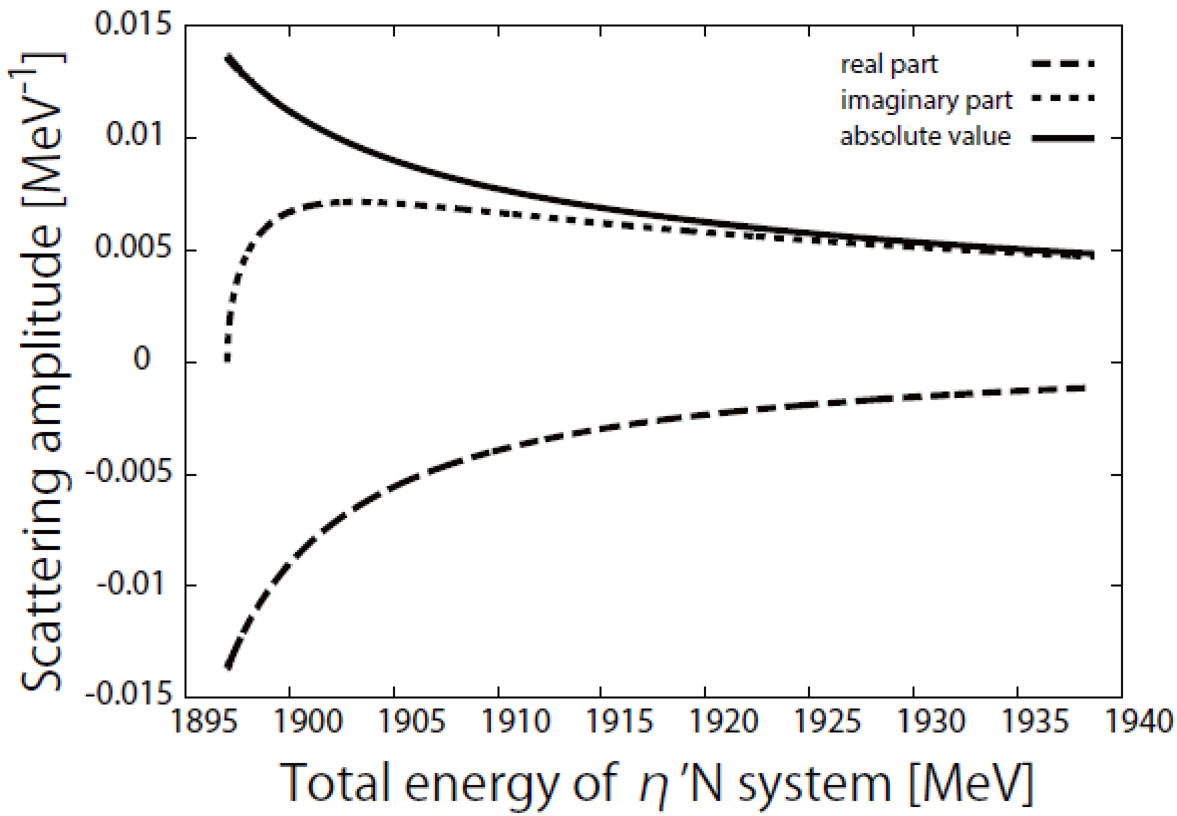
-quark condensate, meson mass, η_0-η_8 mixing angle







The energy dependence of T matrix



Assuming that the mass difference of singlet and octet meson is proportional to the quark condensate,

(The mass difference vanishes when chiral symmetry is restored.)

$$m_{\eta'}^2 - m_\eta^2 = C\left(2\langle\bar{q}q\rangle + \langle\bar{s}s\rangle\right) \quad \text{(in vacuum)}$$

$$m_{\eta'}^{*2} - m_\eta^{*2} = C\left(2\langle\bar{q}q\rangle^* + \langle\bar{s}s\rangle^*\right) \quad \text{(in medium)}$$

$$C = \frac{m_{\eta'}^2 - m_\eta^2}{2\langle\bar{q}q\rangle + \langle\bar{s}s\rangle} \sim \frac{m_{\eta'}^2 - m_\eta^2}{3\langle\bar{q}q\rangle}$$

The ⟨s^bar s⟩ and η assumed not to change in the nuclear matter.

$$m_\eta^* = m_\eta \quad \langle\bar{s}s\rangle^* = \langle\bar{s}s\rangle$$

(η:NG boson, symmetric nuclear matter)

$$m_{\eta'}^2 - m_{\eta'}^{*2} = 2C(\langle\bar{q}q\rangle - \langle\bar{q}q\rangle^*)$$

$$\boxed{\langle\bar{q}q\rangle^* = \left(1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho\right)\langle\bar{q}q\rangle + \mathcal{O}(\rho^{n>1})}$$

(The relation obtained with the linear denisty approx.)

$$m_{\eta'}^2 - m_{\eta'}^{*2} = 2C\frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\langle\bar{q}q\rangle\rho$$

$$\Rightarrow \quad \Delta m_{\eta'} = \frac{2}{3}\frac{m_{\eta'}^2 - m_\eta^2}{2m_{\eta'}}\frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho$$

The equations to determine the vacuum

$$\frac{\partial V_\sigma}{\partial \sigma_0} = \mu^2\sigma_0 + \frac{\lambda}{6}(2\sigma_0^3 + 6\sigma_0\sigma_8^2 - \sqrt{2}\sigma_8^3) + \lambda'\sigma_0(\sigma_0^2 + \sigma_8^2)$$
$$-2Am_0 - 2B(\sigma_0^2 - \frac{\sigma_8^2}{2}) + \frac{g\rho}{\sqrt{3}} = 0,$$
$$\frac{\partial V_\sigma}{\partial \sigma_8} = \mu^2\sigma_8 + \lambda\sigma_8(\sigma_0^2 - \frac{\sigma_0\sigma_8}{\sqrt{2}} + \frac{\sigma_8^2}{2}) + \lambda'\sigma_8(\sigma_0^2 + \sigma_8^2)$$
$$-2Am_8 + 2B\sigma_8(\sigma_0 + \frac{\sigma_8}{\sqrt{2}}) + \frac{g\rho}{\sqrt{6}} = 0.$$

$m_0=2m_q+m_s$
m8=$\sqrt{2}(m_q-m_s)$

$\langle\sigma_0\rangle$, $\langle\sigma_8\rangle$ is determined to fulfill these equations.

The effect of the change of B

H.Nagahiro,M.Takizawa,S.Hirenzaki,PRC74,045203(2006)

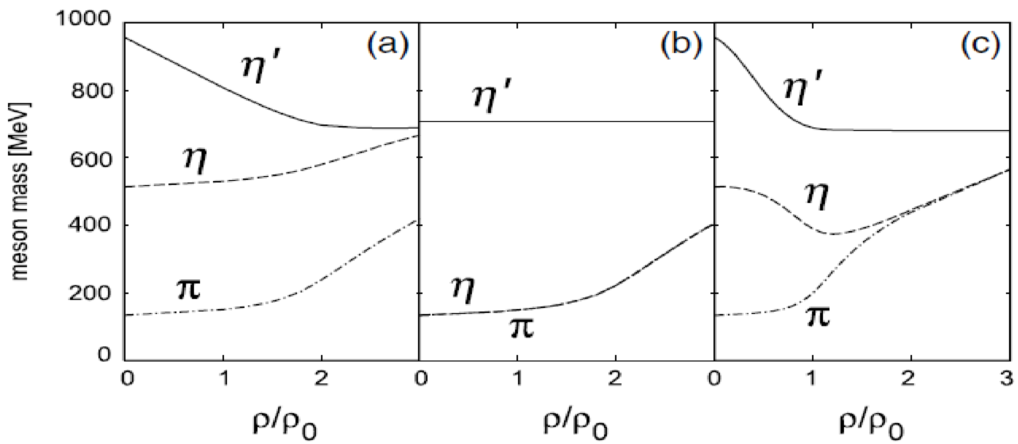
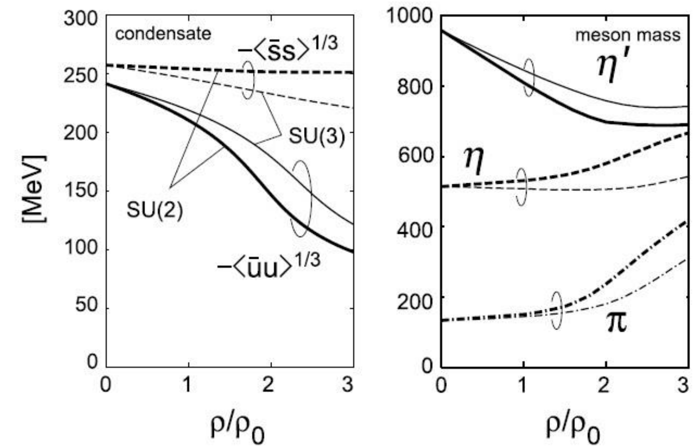


FIG. 2. Density dependence of the meson mass spectra. Three panels corresponds to the cases (a), (b), and (c) defined in Eq. (19), respectively. The nucleon density ρ is defined in Eq. (7) and ρ_0 is the normal nuclear density $\rho_0 = 0.17\ \mathrm{fm}^{-3}$.



(a) $g_D(\rho)$=const.   (b) $g_D(\rho)$=0   (c) $g_D(\rho)=\exp(-\rho^2/\rho_0^2)$

(g_D represents the strength of the effect of UA(1) anomaly.)

Brief proof of η and η’ degeneracy

T.D. Cohen, Phys. Rev.D54 (1996) 1867.,
S.H. Lee, T. Hatsuda, Phys.Rev. D54(1996)54
N.Evans, et al.PLB375(1996)262.

- 2 pt. correlation function and meson mass

$$\Pi_\Gamma(x,y) = \langle J_\Gamma(x), J_\Gamma(y)\rangle \propto e^{-m|x-y|} \ (|x-y| \sim \infty)$$

$$(J_\Gamma(x) = \bar{q}\Gamma q(x))$$

The minimum mass of meson which has same quantum number as the current

(in the Euclidian QCD)

Coincidence of the 2pt. Correlator

=degeneracy of the meson mass of the lowest excitation.

The correlation function of pseudoscalar current

$$\begin{aligned}\Pi_\pi(x,y) &= \frac{1}{Z}\int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-S_{YM}-\int d^4x\bar{\psi}(\not{D}-m_q)\psi(x)}\left[\bar{q}i\gamma_5\frac{\lambda^a}{2}q(x)\bar{q}i\gamma_5\frac{\lambda^a}{2}q(y)\right] \\ &= \frac{1}{Z}\sum_\nu\int [DA]_\nu\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-S_{YM}-\int d^4x\bar{\psi}(\not{D}-m_q)\psi(x)}\left[\bar{q}i\gamma_5\frac{\lambda^a}{2}q(x)\bar{q}i\gamma_5\frac{\lambda^a}{2}q(y)\right]\end{aligned}$$

Expanding the complete set of the Dirac zero mode,

$$\not{D} u_k(x) = i\lambda_k u_k(x)$$

(Euclidian action and anti-hermicity of Dirac op. and $\{\gamma_5, \not{D}\} = 0$)

$$\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\exp\left(-\int d^4x\bar{\psi}(\not{D}-m_q)\psi(x)\right) = \mathrm{Det}(\not{D}-m_q) = \Pi_k m_q^{|\nu|N_f}(i\lambda_k - m_q) = \mathcal{O}(m_q^{|\nu|N_f})$$

$$S_A(x,y) = \sum_k \frac{u_k^\dagger(x)u_k(y)}{i\lambda_k - m_q}$$

$$\Pi_\pi(x,y) = \frac{1}{Z}\sum_\nu \int [DA]_\nu \mathcal{D}\psi\mathcal{D}\bar{\psi}\, \boxed{e^{-S_{YM}-\int d^4x\bar{\psi}(\not{D}-m_q)\psi(x)}} \left[\boxed{\bar{q}i\gamma_5\frac{\lambda^a}{2}q(x)\bar{q}i\gamma_5\frac{\lambda^a}{2}q(y)}\right]$$

$\mathcal{O}(m_q^{|\nu|N_f})$

from fermion det.

$\mathcal{O}(m_q^{-2})$ (disconnected)

$\mathcal{O}(m_q^{-1})$ (connected)

from denom. of propagator
of the Dirac zero mode

(The perturbative QCD will be good @high energy.)

➡ The **contribution from the disconnected diagram** is $\mathcal{O}(m_q^{|\nu|N_f-2})$

It gives the difference of the flavor-singlet and octet meson mass.

(The difference of the flavor trace)

※ν=net number of Dirac zero mode from Atiyah-Singer index theorem.

➡ The contribution from the ν>0 sector vanishes in the chiral limit if N_f>2.

※One can take the naïve chiral limit only in the Wigner phase due to the V→∞(removing the regulator).

The contribution form the ν=0 sector

$$\langle \bar{q}q \rangle = \int \mathcal{D}A e^{-S_{YM}} \mathrm{Det}(\not{D} - m_q) \mathrm{tr} S_A(x) = \mathcal{O}(m_q)$$

in the chiral symmetry restored phase.

$\mathrm{tr} S_A(x) = \mathcal{O}(m_q)$ in the chiral symmetry restored phase and ν=0 sector.

$$|\mathrm{tr} S_A(x)\gamma_5| \leq |\mathrm{tr} S_A(x)|$$

from $0 \leq |u_{kL}(x)|^2 = |(1+\gamma_5)u_k(x)|^2 = u_k^\dagger (1+\gamma_5)^2 u_k(x)$

Disconnected part : the difference of the singlet and octet mesons

$$\frac{1}{Z}\int [DA]_{\nu=0} e^{-S_{YM}} Det(\not{D} + m_q) \boxed{[\mathrm{tr} S_A(x,x)\gamma_5 \mathrm{tr} S_A(y,y)\gamma_5]}$$

$$\leq \frac{1}{Z}\int [DA]_{\nu=0} e^{-S_{YM}} Det(\not{D} + m_q) [\mathrm{tr} S_A(x,x) \mathrm{tr} S_A(y,y)] = \mathcal{O}(m_q^2)$$

QCD vacuum and θ term

Instanton : contribution from pure gauge which changes the winding number.

$$U_\mu = g^{-1}\partial_\mu g$$

$|\nu\rangle$ → $|\nu + n\rangle$

The vacuum with winding number ν → The vacuum with winding number ν+n

The gauge invariance of QCD partition function is assumed.

→ The QCD vacuum is superposition of the states with different winding number: $|\theta\rangle = \sum_\nu |\nu\rangle$

$$\begin{aligned} Z_{QCD} &= \langle\theta|\, e^{iH_{QCD}t}\, |\theta\rangle \\ &= \sum_{m,n} e^{i(m-n)\theta} \langle m|\, e^{iH_{QCD}t}\, |n\rangle \\ &\equiv \sum_\nu \int [DA]_\nu e^{i\int(\mathcal{L}+\nu\theta)} \end{aligned}$$

θ : free parameter of QCD ←constrained from neutron magnetic dipole moment

$$\theta \lesssim 10^{-10}$$

The $U_A(1)$ anomaly avoid the Nambu-Goldstone theorem.

K.Fujikawa,H.Suzuki,Path Integrals and Quantum Anomalies (2004)

The effect of $U_A(1)$ anomaly : $\partial_\mu J_5^\mu = -\frac{g^2}{4\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$

(non-zero divergence of the $U_A(1)$ current from the quark loop)

∵) Ordinary chiral Ward id.,

$$-ip_\mu \int d^4x e^{ip\cdot x} \left\langle 0 \left| T\bar{\psi}(x)\gamma^\mu\gamma_5\psi(x)\bar{\psi}(x)\gamma_5\psi(0)\right| 0\right\rangle = -2i \left\langle 0 \left| \bar{\psi}(0)\psi(0)\right| 0\right\rangle$$

Take account of the $U_A(1)$ anomaly,

$\underline{-i\frac{g^2}{16\pi^2}\int d^4x \mathrm{tr} \left\langle 0 \left| TF_{\mu\nu}\tilde{F}^{\mu\nu}\bar{\psi}(0)\psi(0)\right| 0\right\rangle}$ should be added to the rhs of the equation.

Topologiacal charge:momentum independent constant

⇒ Lhs of the equation do not have to possess the massless pole
due to the anomaly even if the rhs is non-zero in the NG phase for any p_μ.

※Accidental massless pole cannot be excluded only from the symmetry.

Partial restoration of chiral symmetry in the π atom system

Measurement of the double differential cross section of Sn(d,^{3}He)Sn'

K.Suzuki, et.al. Phys.Rev.Lett.92(2004)072302.

$$U_s(r) = -\frac{2\pi}{m_\pi}\left[\epsilon_1\left\{b_0\rho(r) + \boxed{b_1}\left[\rho_n(r) - \rho_p(r)\right]\right\} + \epsilon_2 B_0 \rho^2(r)\right]$$

Related to $1/f_\pi^2$

$$\frac{b_1^{\rm free}}{b_1^*(\rho)} \sim \frac{f_\pi^*(\rho)^2}{f_\pi^2}$$

Larger asymmetry enhances the PRCS

Sn(d,^{3}He)Sn'反応

Parameters b_0,b_1,B_0 to fit the experimental data

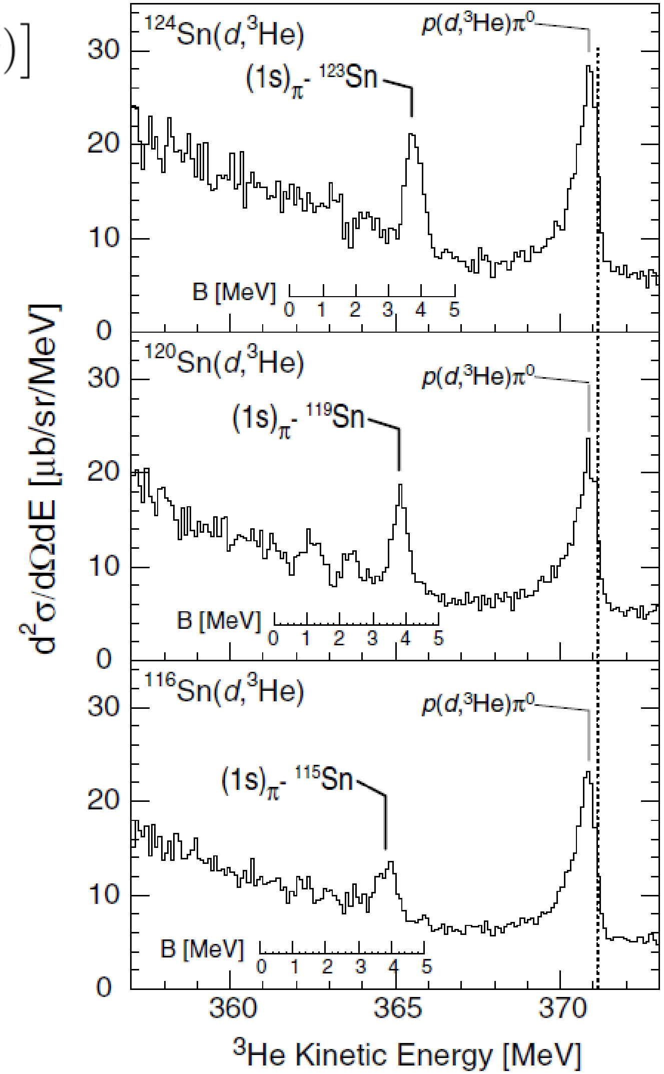
b_1^{free}(the value in free space) is determined in π-Hydrogen.

$$\frac{b_1^{\rm free}}{b_1^*(\rho_{\rm e})} = 0.78 \pm 0.05 \quad (\rho_e = 0.6\rho_0)$$

Gell-Mann-Oakes-Renner relation

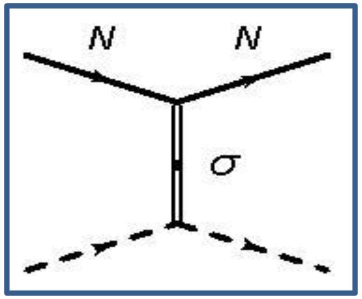
in medium and the π mass in medium

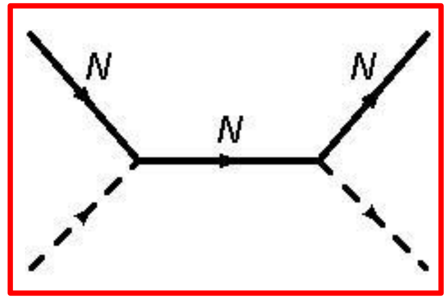
→about 30% reduction of chiral condensate.

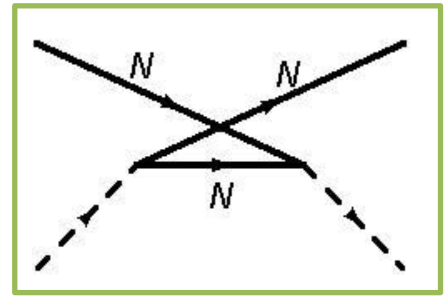


Low energy interaction of NG boson in linear sigma model

In chiral limit and low energy







<u>Ex) πN interaction</u>

$$\frac{g}{\sqrt{3}}\frac{2\lambda\langle\sigma_0\rangle/3+2\lambda'\langle\sigma_0\rangle-2B}{q^2-m_{\sigma_0}^2}\delta_{ab}+\frac{g}{\sqrt{6}}\frac{\sqrt{2}\lambda\langle\sigma_0\rangle/3-2\sqrt{2}B}{q^2-m_{\sigma_8}^2}\delta_{ab}$$

$$+\frac{ig}{\sqrt{2}}\gamma_5\lambda_a\frac{1}{\not{p}+\not{k}-m_N}\frac{ig}{\sqrt{2}}\gamma_5\lambda_b+\frac{ig}{\sqrt{2}}\gamma_5\lambda_b\frac{1}{\not{p}-\not{k}'-m_N}\frac{ig}{\sqrt{2}}\gamma_5\lambda_a$$

$$\sim-\frac{g^2m_\pi}{8m_N^2}[\lambda_a,\lambda_b]+\frac{g^2}{4m_N}(\{\lambda_a,\lambda_b\}-2\delta_{ab})$$ (q^2~0, p~m_N, k, k'~$m_{\eta 0}$)

No contribution to the self-energy of NG boson @LO.

π mass in medium

$$m_\pi^2 = \mu^2 + \frac{\lambda}{3}(\langle\sigma_0\rangle^2 + \sqrt{2}\langle\sigma_0\rangle\langle\sigma_8\rangle + \frac{\langle\sigma_8\rangle^2}{2}) + \lambda'(\langle\sigma_0\rangle^2 + \langle\sigma_8\rangle^2) - 2B(\langle\sigma_0\rangle - \sqrt{2}\langle\sigma_8\rangle) + \frac{\sqrt{3}g\rho}{2(\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}})}$$

The contribution from $\Sigma_{ph}(\rho)$

$$= \frac{6Am}{\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}} - \frac{\sqrt{3}g\rho}{2(\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}})} + \frac{\sqrt{3}g\rho}{2(\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}})} = \frac{6Am}{\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}}$$

The contribution from quark mass

The contribution from $V_{MF}(\rho)$

The contribution from $\Sigma_{ph}(\rho)$

The equation to determine ⟨σ⟩

$$m_N = \frac{g}{\sqrt{3}}\left(\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}\right)$$

η’ mass and chiral symmetry

- η’ should degenerate with η @Wigner phase (high ρ, T)

T.D.Cohen,PRD54(1996)1867.
S.H.Lee,T.Hatsuda,PRD(1996)1871.
D.Jido,H.Nagahiro,S.Hirenzaki,PRC84(2012)032201.

$$
\begin{tikzcd}
\eta_0 = \bar{q}i\gamma_5 q \arrow[r, leftrightarrow, "U_A(1)"] \arrow[dr, leftrightarrow, "SU_A(3)"] & \sigma_0 = \bar{q}q \arrow[dl, leftrightarrow, "SU_A(3)"'] \\
\eta_8 = \bar{q}i\gamma_5\lambda_8 q \arrow[r, leftrightarrow, "SU_A(3)"'] & \sigma_8 = \bar{q}\lambda_8 q
\end{tikzcd}
$$

Similar figure:T.D.Cohen,NPB195(2009)59.

$U(3)_L \times U(3)_R$

(ignoring the $U_A(1)$ anomaly)

$SU(3)_L \times SU(3)_R \times U(1)_V$

$$[Q_5^a, \eta_0] = -\frac{i}{\sqrt{3}}\sigma_a, \ [Q_5^a, [Q_5^b, \eta_0]] = \frac{d^{abc}}{\sqrt{3}}\pi^c$$

(Even if no $U_A(1)$ symmetry)

Breaking of chiral symmetry

The mixing of η_0 and η_8 prevented.

The notion of singlet and octet
∼the decomposition in terms of $SU(3)_V$

Natural renormalization scheme

T.Hyodo,D.Jido,A.Hosaka,PRC78025203(2008).

This scheme removes the contribution from the CDD poles.

CDD pole:the contribution from <u>the other dynamics than considering hadrons.</u>

Ex)more microscopic degree of freedom (quark and gluon)

The condition to determine the subtraction constant: $G(M) = 0$

$$G(W) \quad = \quad i\int \frac{d^4q}{(2\pi)^4}\frac{2M}{(W-q)^2 - M^2 + i\epsilon}\frac{1}{q^2 - m^2 + i\epsilon}$$

M:baryon mass
m:meson mass

(W:the energy in c.m. system)

G(W):meson-baryon 2-body Green function.

$$G(W) \sim \sum_n \frac{\rho(W)}{W - E_n + i\epsilon}$$ with spectral representation.

ρ(W)≧0 and no state below threshold(no CDD pole)

⇒ G(W)<0 when W<M+m

and <u>The condition to dominate the s-wave contribution:W>M</u>

η’ in finite T

Ex)NJL model

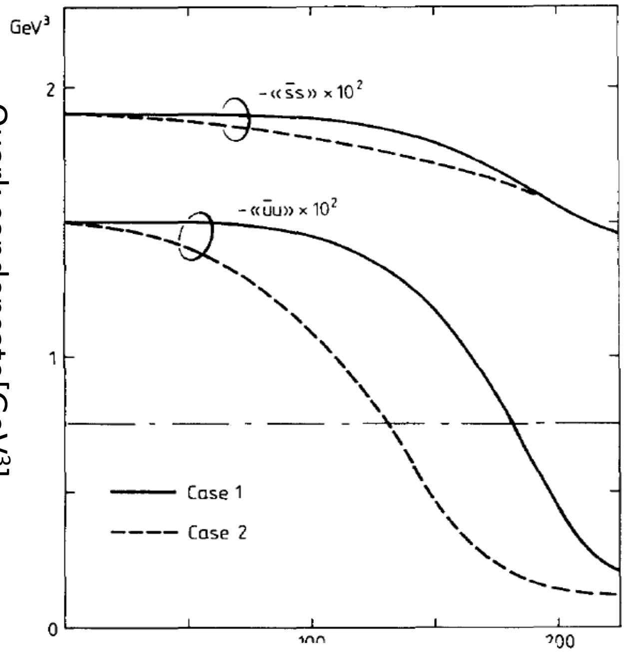
T.Hatsuda, T.Kunihiro, Phys.Rep.247(1994)221.

$$\mathcal{L} = \bar{q}(i\gamma\cdot\partial - m)q + \frac{1}{2}g_s\sum_{a=0}^{8}[(\bar{q}\lambda_a q)^2 + (\bar{q}i\lambda_a\gamma_5 q)^2] + \boxed{g_D[\det\bar{q}_i(1-\gamma_5)q_j + \text{h.c.}]}$$

(λ_i (i=1~8):Gell-Mann matrices, λ_0=**1**/√3)

KMT term (anomaly effect)

Quark condensate[GeV³]

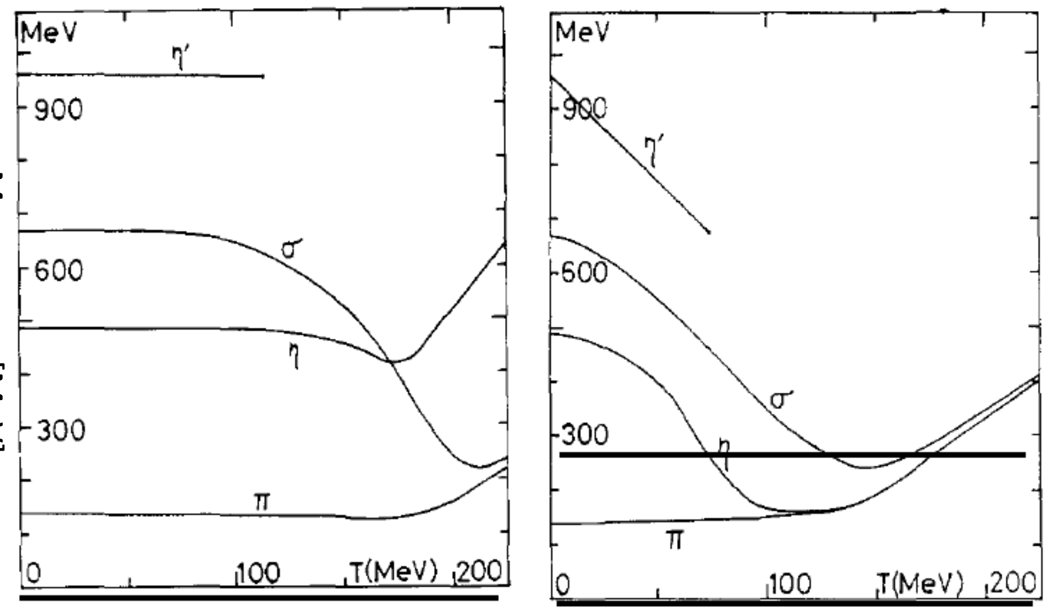


Temperature[MeV]

Case1

Case2

Meson mass[MeV]



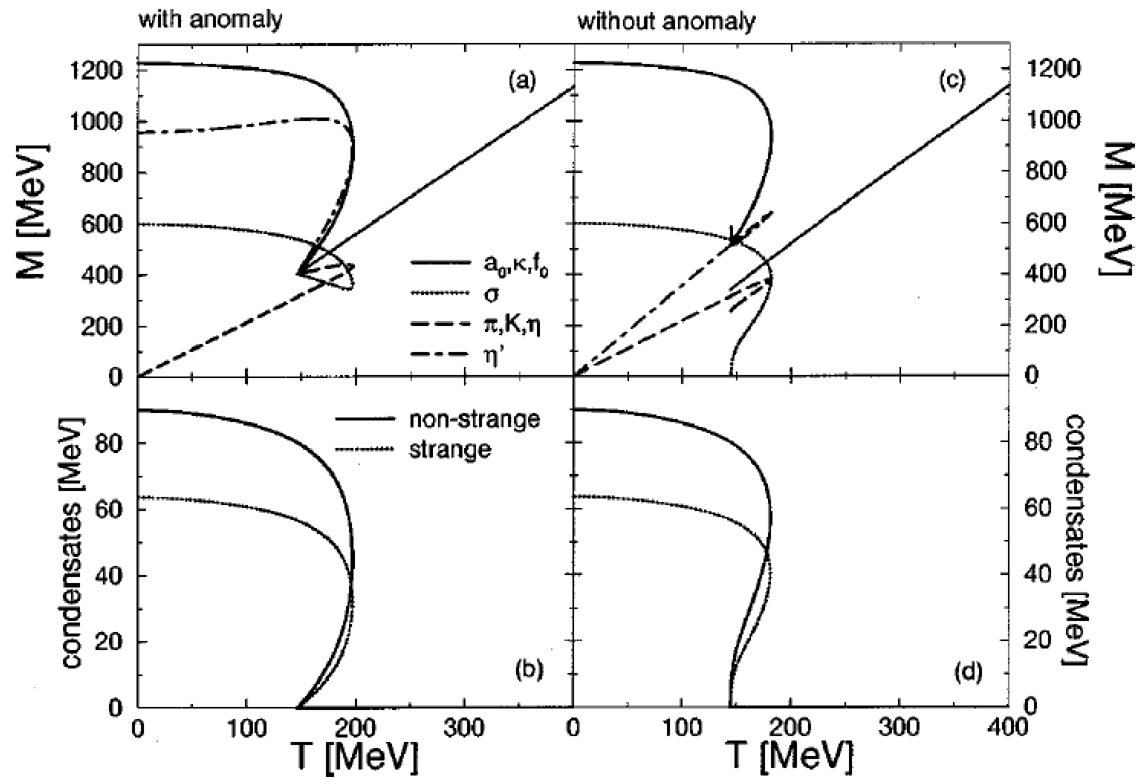
Temperature[MeV]

Case1: $g_D(T) = \text{const.}$

Case2: $g_D(T) = \text{const.}\times\exp\left[-(T/T_0)^2\right]$ (T_0=100MeV)

η' in finite T

J.T.Lenaghan, et.al. Phys.Rev.D62(2000)085008.



The analysis of meson mass in finite T with LσM

η’ in finite T

Yin Jiang, Pengfei Zhuang, arXiv:1209.0507

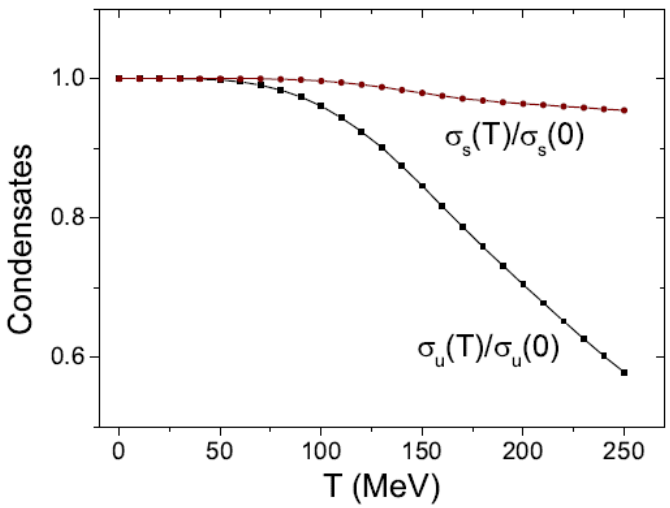


FIG. 2: (Color online) The temperature dependence of the light and strange quark condensates $\bar{\sigma}_{u0}$, $\bar{\sigma}_{s0}$, scaled by their vacuum values at $T = 0$.

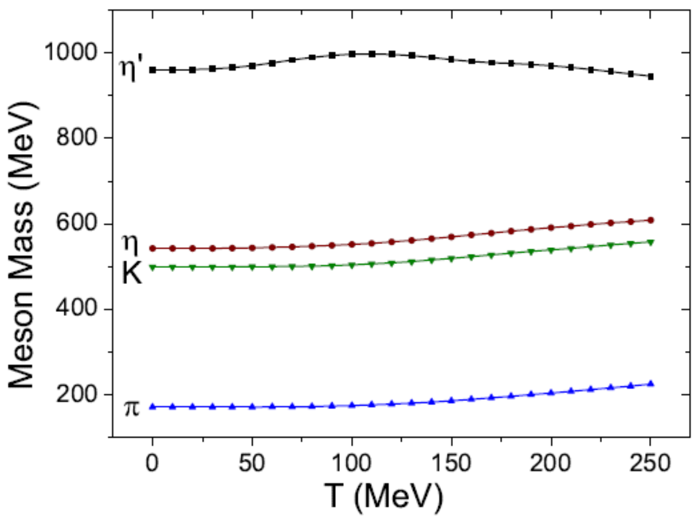


FIG. 7: (Color online) The temperature dependence of the pseudoscalar meson masses.

Analysis with Functional Renormalization Group of the LσM of finite T.

η’ mass does not change so much.

η’ mass in high T (@RHIC)

T.Csörgo,et al.Phys.Rev.Lett.105(2010)182301.

Mass reduction of η’→the number of η’ generated in the matter inclearse.

η’ : long life time→not decay in the fireball

Out side of the fireball : ordinary hadron phase
→the η’ mass is recover to the in-vacuum value.
(The momentum reduces due to the energy cons.)

From the $\eta' \to \eta\pi^+\pi$ process, the number of the low-momentum π enhances.

The analysis of the # of π with enhancement factor which comes from the η’ mass reduction
→**more than 200MeV reduction of η’ mass**

Absorption of η’ in the nuclear matter

M.Nanova,et al.Phys.lett.B710(2012)600.

Analysis through the η’ photoproduction with nucleus target : A(γ,η’)A’

Transparency ratio : $T_A = \dfrac{\sigma_{\sigma A \to \eta' A'}}{A\sigma_{\gamma N \to \eta' N}}$

T_A is much smaller than 1→large absorption to the nuclear matter

due to the many body effect

self-energy in medium

Width : $\Gamma_{\eta'}(\rho) = -\dfrac{\mathrm{Im}\Pi_{\eta'}(\rho)}{E_{\eta'}}$ $\Pi_{\eta'}(\rho) \sim t_{\eta' \to \eta' N}\rho \sim t_{\eta' N \to \eta' N}\rho(\vec{r})$

linear density approx. local density approx.

$\sigma_{\gamma A \to \eta' A'}$: pγ→η’p＋absorption of η’ into the matter（integration with the distance in the nuclear matter）

$$\sigma_{\gamma A \to \eta' A'} = C\int d^3r\rho(r)\int d\phi^{\eta'}_{\mathrm{c.m.}}\int d\cos\theta^{\eta'}_{\mathrm{c.m.}}\frac{d\sigma}{d\Omega}(\gamma p \to \eta' p)P_s(\vec{k}_{\eta'},\vec{r})$$

$$P_s(\vec{k}_{\eta'},\vec{r}) = \exp\left[\int_0^\infty dl\frac{\mathrm{Im}\Pi(\rho(\vec{r}'))}{|\vec{k}_{\eta'}|}\right] \quad \vec{r}' = \vec{r} + l\frac{\vec{k}_{\eta'}}{|\vec{\eta'}|}$$

Γ=15-25MeV @$\rho=\rho_0$ (~200keV @ρ=0)

The μ_q dependence of B

E.V.Shuryak,Nucl.Phys.B203(1982)140.
H.Nagahiro,M.Takizawa,S.Hirenzaki,Phys.Rev.C74(2006)045203

Instanton density in vacuum($T,\mu_q=0$)

asuming

$$B \propto n_{\rm inst}(T,\mu_q) \quad = \left(\frac{8\pi^2}{g^2}\right)^{2N_c} e^{-8\pi^2/g(\rho_i)}\rho_i^{-5}$$
$$\times \exp\left[-\pi^2\rho_i^2 T^2\left(\frac{2N_c}{3}+\frac{N_f}{3}\right) - N_f\rho_i^2\mu_q^2\right]$$

The T dependence of the instanton density

The density dependence

μ_q:quark chemical potential
ρ_i:instanton radius (typical value:0.3fm)
g:gauge coupling

※$n_{\rm inst}(\mu_q=300\text{MeV})/n_{\rm inst}(\mu_q=0)\sim 0.5$

⇨ The effect of the $U_A(1)$anomaly is suppressed in the nuclear matter.

$$m^2_{\eta_0} - m^2_{\eta_8} = 6B\langle\sigma_0\rangle$$

Such an attraction leads to the mass reduction in nuclear matter at nucleon 1loop.