It’s Halloween, today. My talk is:

In-medium Tomozawa-Weinberg Relation with nuclear correlation effects

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What pions do

π –N in free space:

Tomozawa-Weinberg

$$b_0 = 0$$

$$b_1 = -\frac{1}{4\pi\epsilon_1}\frac{m_\pi}{2f_\pi^2} = -0.079 m_\pi^{-1}$$

Experiment

$$b_0 \equiv (a_n + a_p)/2 = -0.0069 \pm 0.0031 m_\pi^{-1}$$

$$b_1 \equiv (a_n - a_p)/2 = -0.0864 \pm 0.0012 m_\pi^{-1}$$

Seemingly nice, here

In nuclei:

A recent pionic-atom measurement [K. Suzuki *et al.*, Phys. Rev. Lett. **92**, 072302 (2004)]

$$(b_0)_{atom} = -0.0233 \pm 0.0038 m_\pi^{-1}$$

$$(b_1)_{atom} = -0.1149 \pm 0.0074 m_\pi^{-1} .$$

More repulsive

But

b_0 and b_i are more repulsive by about the same amount!

In free space, too!

$$-0.0233 \approx -0.00 \quad - 0.02$$

$$-0.1149 \approx -0.08 \quad - 0.03$$

Wicked, isn't it?

Comments on

Precision Spectroscopy of Pionic 1s States of Sn Nuclei and Evidence for Partial Restoration of Chiral Symmetry in the Nuclear Medium

K. Suzuki et al., Phys. Rev. Lett. 92, 072302 (2004) . [cited :66 so far]

Pionic atom spectra
generated by
pions in real nuclei

↓

b_0 and b_1

in

$$V_{opt}(r) = -\frac{4\pi}{2\mu}\epsilon_1[b_0(\rho_n(r)+\rho_p(r)) + b_1(\rho_n(r)-\rho_p(r))] + \cdots$$

$(\mathbf{p}^2/{m_\pi}^2)$ expansion $\sim$ eff. theory

↔

?

Partial restoration
of chiral symmetry breaking
in nuclear medium :

$$f_\pi \quad \rightarrow \quad f_\pi^{\,*}$$

in

$$(b_0)_{TW} = 0$$

$$(b_1)_{TW} = -\frac{1}{4\pi\epsilon_1}\frac{m_\pi}{2f_\pi^2}$$

π – N scattering amplitude
in nuclear medium

Nuclear density
N-N correlation

$\sim \; P_F \sim m_\pi$ << $2\pi f_\pi, \; \Lambda$

Lorentz-Lorenz (Clausius-Mossotti) Effect

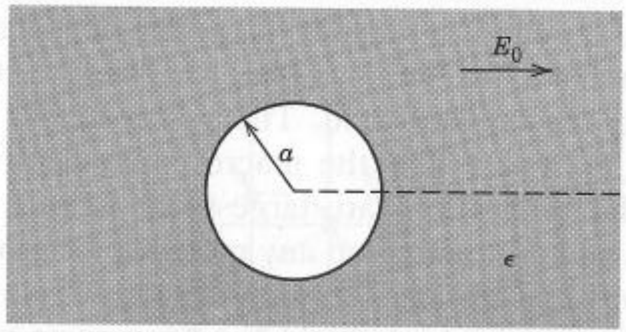


Figure 4.8 Spherical cavity in a dielectric with a uniform field applied.

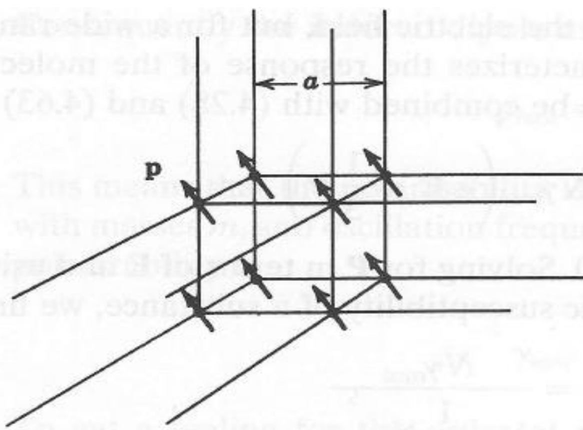


Figure 4.9 Calculation of the internal field: contribution from nearby molecules in a simple cubic lattice.

Pictures are taken from J. D. Jackson, Classical Electrodynamics 3rd ed. (John Wiley 1998)

1) M. Ericson & T. E. O. Ericson, Ann. Phys. 36, 323 (1966).
2) Theory of Meson Interactions with Nuclei, J. M. Eisenberg & D. S. Koltun (John Wiley, 1980) ←

$$V_{opt} = \sum_{i=1}^{A} < 0|[\tau_i + \sum_{j(\neq i)} \tau_i G^{ij}(1-P_0)\tau_j + \cdots]|0 >$$
$$= V_{\mathrm{I}} + V_{\mathrm{II}} + \cdots \ .$$

τ_i is t-matrix of π-nucleon scattering **IN NUCLEAR MEDIUM**.

For pionic atoms,

$$\tau_i \to \frac{4\pi}{2\mu} a_i \cdot \delta(\mathbf{r}_i - \mathbf{r})$$

$$V_{\mathrm{I}} = -\frac{2\mu}{4\pi}\epsilon_1 \sum_{i=1}^{A} a_i < 0|\delta(\mathbf{r}_i - \mathbf{r})|0 >= -\frac{2\mu}{4\pi}\epsilon_1 [a_p \rho_p(\mathbf{r}) + a_n \rho_n(\mathbf{r})] \ ,$$

$$
\begin{aligned}
V_{opt} &= \sum_{i=1}^{A} < 0|[\tau_i + \sum_{j(\neq i)} \tau_i G^{ij}(1-P_0)\tau_j + \cdots]|0 > \\
&= V_{\mathrm{I}} + V_{\mathrm{II}} + \cdots \ .
\end{aligned}
$$

$$
\tau_i \to \frac{4\pi}{2\mu} a_i \qquad G^{ij} \to -\frac{2\mu}{4\pi} \frac{1}{|\mathbf{r}_i - \mathbf{r_j}|} \ ,
$$

$$
1 - P_0 = 1 - |0 >< 0| \ .
$$

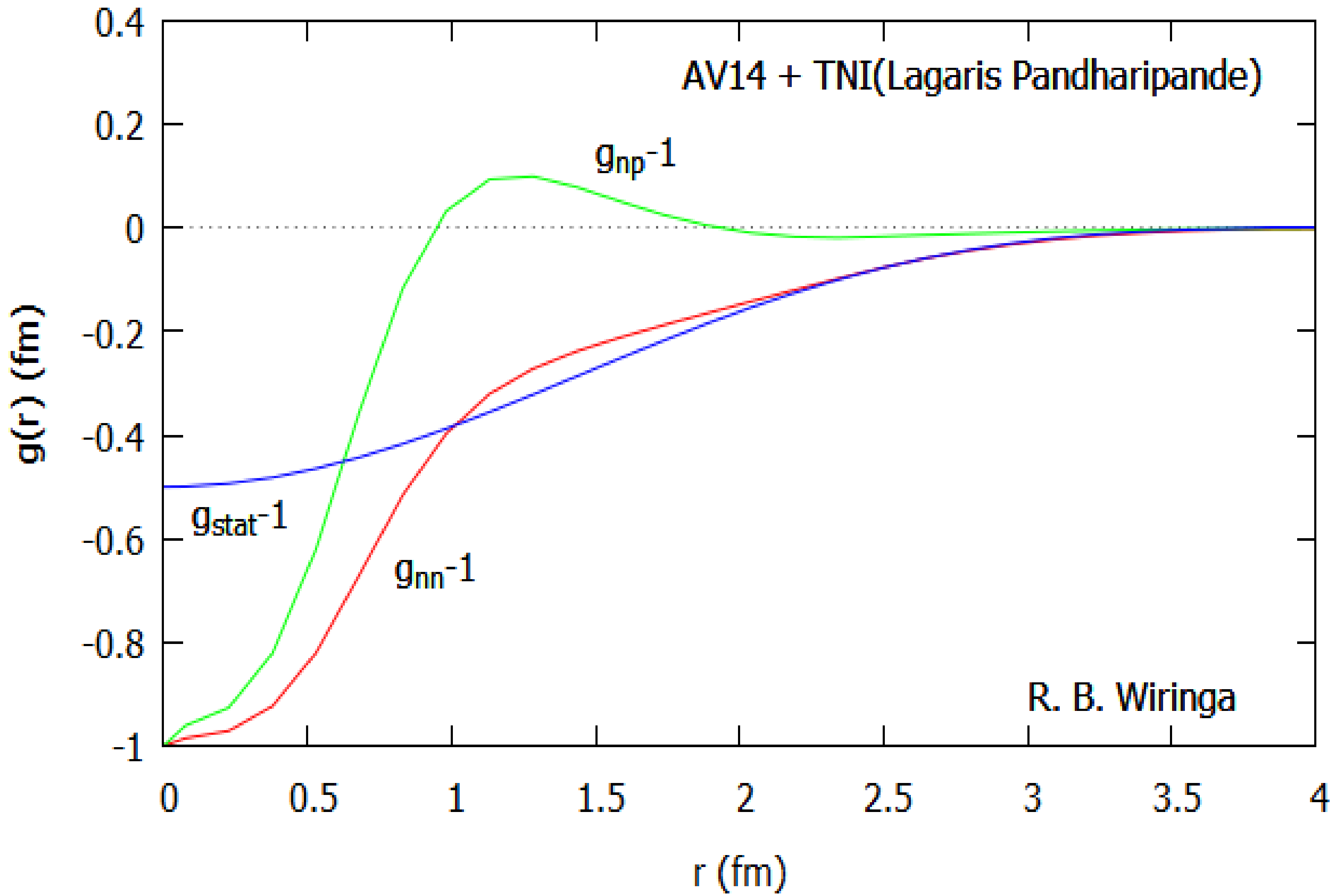
$$
\begin{aligned}
< 0|\tau_i[G(1-P_0)]^{ij}\tau_j|0 > &= < 0|\delta(\mathbf{r}_i - \mathbf{r})\delta(\mathbf{r}_j - \mathbf{r}')|0 > - < 0|\delta(\mathbf{r}_i - \mathbf{r})|0 >< 0|\delta(\mathbf{r}_j - \mathbf{r}')|0 > \\
&= \rho_{2,ij}(\boldsymbol{r}, \boldsymbol{r}') - \rho_{1,i}(\boldsymbol{r})\rho_{1,j}(\boldsymbol{r}') \equiv \rho_{1,i}(\boldsymbol{r})\rho_{1,j}(\boldsymbol{r}')g_{ij}(\boldsymbol{r}, \boldsymbol{r}') \ .
\end{aligned}
$$

$$
V_{\mathrm{II}} = -\frac{4\pi}{2\mu}\epsilon_1^2 \sum_{i=1}^{A} a_i \rho_1(\boldsymbol{r}_i) \sum_{j(\neq i)} C_{ij}(\boldsymbol{r}_i) a_j \ ,
$$

$$
C_{ij}(\boldsymbol{r}_i) \equiv \int d^3 r_j \rho_1(\boldsymbol{r}_j) \frac{1}{|\boldsymbol{r}_i - \boldsymbol{r}_j|} g_{ij}(\boldsymbol{r}_i, \boldsymbol{r}_j) \approx \rho_j \int d^3 r_{ij} \frac{1}{|\boldsymbol{r}_{ij}|} g_{ij}(\boldsymbol{r}_{ij}) \equiv < \frac{1}{r} >_{ij} \ .
$$

Local density approx.: nonlocal $\rightarrow$ local potential

Wiringa AV14 and Fermi Gas Pair Distribution Functions



$$V_{\rm II} = -\frac{4\pi}{2\mu}\epsilon_1^2[\Delta a_n \rho_n(\boldsymbol{r}) + \Delta a_p \rho_p(\boldsymbol{r})] \ ,$$

$$\Delta a_n = a_n C_{nn} a_n + a_n C_{np} a_p$$

$$\Delta a_p = (a_p c_{pp} a_p + a_p C_{pn} a_n) + a_{pn} C_{nn} a_{np} \ .$$

V_I and V_{II} together, finally we obtain: (SNM version)

$$(b_0)_{corr} = b_0 - \frac{1}{2}(\Delta a_n + \Delta a_p) = b_0 - \epsilon_1 \left[(b_0^2 + 2b_1^2) C_{nn} + (b_0^2 - b_1^2) C_{np} \right]$$

$$(b_1)_{corr} = b_1 - \frac{1}{2}(\Delta a_n - \Delta a_p) = b_1 - \epsilon_1 (2b_0 b_1 - b_1^2) C_{nn} \ .$$

*C'*s using the Wiringa V14 and Fermi Gas pair distribution functions

	$C_{nn}, C_{pp}(\mathrm{fm}^{-1})$	$C_{np}, C_{pn}(\mathrm{fm}^{-1})$
Wiringa	0.555	0.093
Fermi gas	0.537	0.

(preliminary)

$\rho_n = \rho_p = \rho_0/2 = 0.048\mathrm{fm}^{-3}$

b_0 and b_1 for symmetric nuclear matter

	$b_0(m_\pi{}^{-1})$	$b_1(m_\pi{}^{-1})$	$b_0(m_\pi{}^{-1})$	$b_1(m_\pi{}^{-1})$	
$\pi - N$	-0.007	-0.086	-0.007	-0.086/0.78	*
Fermi gas	-0.020	-0.081	-0.028	-0.101	
Wiringa	-0.019	-0.081	-0.027	-0.101	

* Possibly mimicking $f_\pi \to f_\pi^*$

K. Suzuki *et al.*, Phys. Rev. Lett. **92**, 072302 (2004):

$$(b_0)_{atom} = -0.0233 \pm 0.0038 m_\pi^{-1}$$

$$(b_1)_{atom} = -0.1149 \pm 0.0074 m_\pi^{-1} \ .$$

Summary

A simple picture

$$\text{Tomozawa-Weinberg: } b_0 = 0. \qquad (b_0)_{corr} \approx 0. \quad - 2\epsilon_1 b_1^2 C_{nn}$$

$$\text{No } n-p \text{ correlation: } C_{np} = C_{pn} = 0. \quad \rightarrow \quad (b_1)_{corr} \approx b_1 \quad + \epsilon_1 b_1^2 C_{nn}$$

V_I V_{II}

Findings:

1) b_0 is largely determined by V_{II}, and b_1 by V_I.

2) b_0 and b_1, both seem to have "Missing Repulsion Problems".

3) Both problems appear to be largely cured by enhancing the b_1 strength.

Issues:

1) Need more care on the C calculation. The local density approximation is really OK for b_1 ?

2) More important issues:

a) The form of the optical potential. $b_1(\rho)$? Nonlocal?

b) The treatment of π - nucleon t-matrix in nuclear medium, τ_i.