YITP workshop Hadron in Nucleus

October 31 – November 2, 2013 Kyoto, Japan

STRUCTURE CHANGES OF THE NUCLEON IN NUCLEAR MATTER

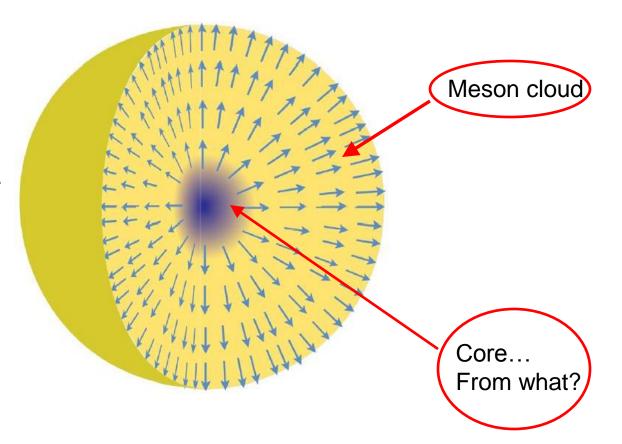
Ulugbek Yakhshiev Inha University

Content

- Topological models and soliton
- Medium modifications
 - "Outer shell" modifications
 - "Inner core" modifications
- Nuclear matter
 - □ Symmetric matter
 - □ Asymmetric matter
- Summary
- Outlook

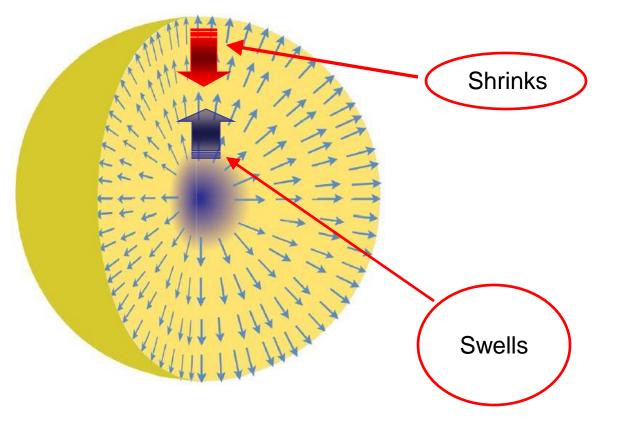
Structure

- What is a nucleon and, in particular, its core?
- At large number of colors it still has the mesonic content



Stabilization

- Soliton has finite size and finite energy
- One needs at least two contrterms in the effective Lagrangian



Skyrme model

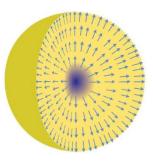
[T.H.R. Skyrme, Pros.Roy.Soc.Lond. A260(1961)]

Nonlinear chiral effective meson (pionic) theory)

$$\mathcal{L} = \frac{F_{\pi}^{2}}{16} \operatorname{Tr}(\partial_{\alpha}U)(\partial^{\alpha}U^{+}) + \frac{1}{32e^{2}} \operatorname{Tr}[U^{+}\partial_{\alpha}U, U^{+}\partial_{\beta}U]^{2}$$
Shrinks Swells

Hedgehog soliton (nontrivial mapping)

$$U = \exp\left\{\frac{i\bar{\tau}\bar{\pi}}{2F_{\pi}}\right\} = \exp\left\{i\bar{\tau}\bar{n}F(r)\right\}$$



Original Lagrangian in use

[G.S. Adkins et al. Nucl. Phys. B228 (1983)]

$$\mathcal{L}_{\text{free}} = \frac{F_{\pi}^2}{16} \operatorname{Tr}\left(\partial^{\alpha}U\right) \left(\partial_{\alpha}U^+\right) + \frac{1}{32e^2} \operatorname{Tr}\left[U^+\partial_{\alpha}U, U^+\partial_{\beta}U\right]^2 + \frac{F_{\pi}^2 m_{\pi}^2}{16} \operatorname{Tr}\left(U + U^+ - 2\right)$$

- Nontrivial mapping
- It has topologically nontrivial solitonic solutions (in separated different topological sectors) with the corresponding conserved topological number A
- Nucleon is quantized state of the classical solitonskyrmion

$$U = \exp\{i\bar{\tau} \,\bar{\pi}/2F_{\pi}\} = \exp\{i\bar{\tau} \,\bar{n}F(r)\}$$

$$B^{\mu} = \frac{1}{24\pi^2} \varepsilon^{\mu\nu\alpha\beta} Tr(L_{\nu}L_{\alpha}L_{\beta}) \qquad L_{\alpha} = U^+\partial_{\alpha}U$$

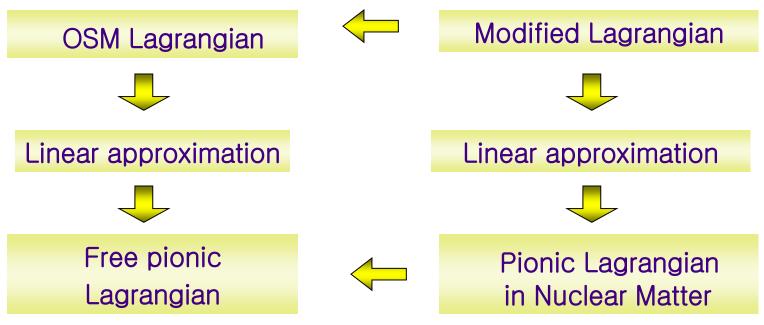
$$A = \int d^3 r B^0$$

$$H = M_{cl} + \frac{S^2}{2I} = M_{cl} + \frac{T^2}{2I},$$

$$|S = T, s, t > = (-1)^{t+T} \sqrt{2T + 1} D_{-t,s}^{S=T}(A)$$

- What happens in a medium?
- One should be able to describe the possible phenomena
 - Deformations
 - Mass change
 - Swelling or shrinking
 - Effective NN interactions
 - Etc.

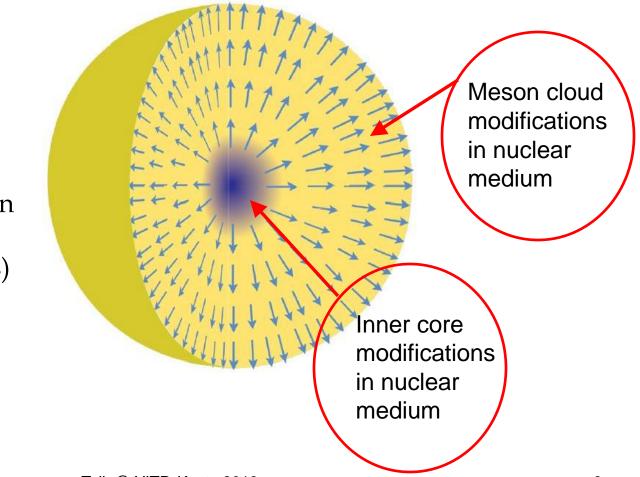
Modification in the mesonic sector modifies the baryonic sector



• How to modify the mesonic sector?

Soliton in Nuclear Medium (structure changes)

- Outer shell modifications plus
- Inner core modifications (in particular, at higher densities)



"Outer shell" modifications

- Three types of pions can be treated separately
- In nuclear matter, one considers three types of polarization operators
- There will be some parameters which correspond to the isospin breaking effects in the surrounding environment

$$\left(\partial^{\mu}\partial_{\mu}+m_{\pi^{(\pm,0)}}^{2}\right)\vec{\pi}^{(\pm,0)}=0$$

$$\left(\partial^{\mu} \partial_{\mu} + m_{\pi^{(\pm,0)}}^2 + \hat{\Pi}^{(\pm,0)} \right) \vec{\pi}^{(\pm,0)} = 0$$

	$\pi\text{-}\mathrm{atom}$	$T_{\pi} = 50 \text{ MeV}$
$b_0 [m_\pi^{-1}]$	- 0.03	- 0.04
$b_1 [m_{\pi}^{-1}]$	- 0.09	- 0.09
$c_0 \left[m_{\pi}^{-3} \right]$	0.23	0.25
$c_1 [m_{\pi}^{-3}]$	0.15	0.16
g'	0.47	0.47

"Outer shell" modifications [U.Meissner et al., EPJ A36 (2008)]

$$\mathcal{L}_{2}^{*} = \frac{F_{\pi}^{2}}{16} \left\{ \alpha_{s}^{02} \operatorname{Tr}(\partial_{0}U\partial_{0}U^{\dagger}) - \alpha_{p}^{0} \operatorname{Tr}(\partial_{i}U\partial_{i}U^{\dagger}) \right\}$$
$$\mathcal{L}_{\chi SB}^{*} = \frac{F_{\pi}^{2}m_{\pi}^{2}}{8} \alpha_{s}^{00} \operatorname{Tr}(U-1),$$

- Due to the nonlocality of optic potential the kinetic term is also modified
- Due to energy and momentum dependence of the optic potential parameters following parts of the kinetic term is modified in different form:
 - □ Temporal part
 - □ Space part

$$\hat{\Pi} = 2\omega U_{opt} = \chi_s + \vec{\nabla} \cdot \chi_p \vec{\nabla}$$

	$\pi\text{-}\mathrm{atom}$	$T_{\pi} = 50 \text{ MeV}$
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$c_1 \left[m_{\pi}^{-3} \right]$	0.15	0.16
g'	0.47	0.47

"Inner core" modifications

[UY & H.Ch. Kim, PRC83 (2011); UY, JKPS62 (2013); UY, PRC88 (2013)]

□ May be related to

- vector meson properties in nuclear matter
- nuclear matter properties

$$\mathcal{L}_{4}^{*} = -\frac{1}{16e_{\tau}^{*2}} \operatorname{Tr}[L_{0}, L_{i}]^{2} + \frac{1}{32e_{s}^{*2}} \operatorname{Tr}[L_{i}, L_{j}]^{2}$$

$$e \rightarrow e^* = e \gamma^{1/2}(\rho)$$

Final Lagrangian

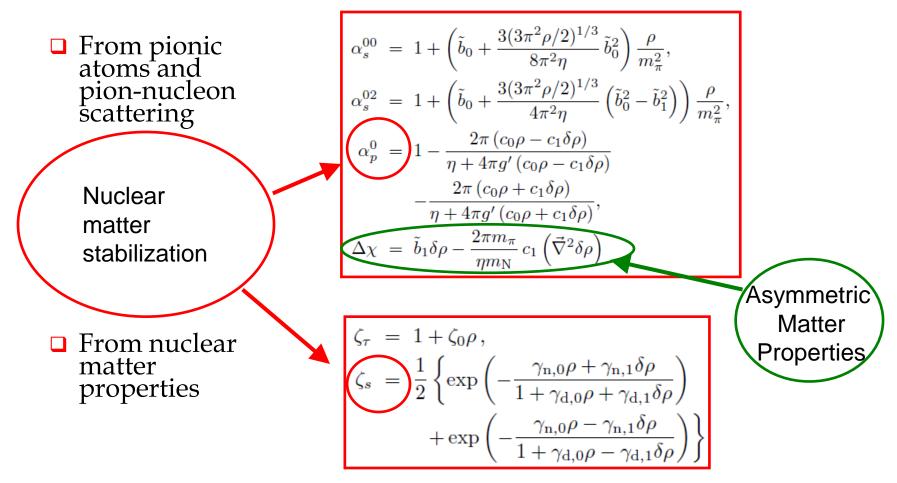
[U.Meissner et al., EPJ A36 (2008); UY, JKPS62 (2013), UY, PRC88 (2013)]

Separated into two parts $\mathcal{L}_{2}^{*} = \frac{F_{\pi}^{2}}{16} \Big\{ \alpha_{s}^{02} \operatorname{Tr} \left(\partial_{0} U \partial_{0} U^{\dagger} \right) \Big\}$ $\mathcal{L}^* = \mathcal{L}^*_{\text{sym}} + \mathcal{L}^*_{\text{asym}}$ $-\alpha_p^0 \operatorname{Tr}(\vec{\nabla} U \cdot \vec{\nabla} U^{\dagger}) \bigg\},$ $\mathcal{L}_4^* = -\frac{1}{16e^2 \zeta_\tau} \operatorname{Tr}[U^{\dagger} \partial_0 U, U^{\dagger} \partial_i U]^2$ Isoscalar part $\mathcal{L}^*_{\text{sym}} = \mathcal{L}^*_2 + \mathcal{L}^*_4 + \mathcal{L}^*_{\text{ySB}}$ $+\frac{1}{32e^{2\zeta_{z}}}\operatorname{Tr}\left[U^{\dagger}\partial_{i}U,U^{\dagger}\partial_{j}U\right]^{2},$ Isovector part $\mathcal{L}_{\chi SB}^{*} = \frac{F_{\pi}^{2} m_{\pi}^{2}}{2} \alpha_{s}^{00} \operatorname{Tr} (U-1),$ $\mathcal{L}_{asym}^* = \Delta \mathcal{L}_{mes} + \Delta \mathcal{L}_{env}^*$ $\Delta \mathcal{L}_{\text{mes}} = -\frac{F_{\pi}^2}{32} \sum_{a=1}^2 (m_{\pi^{\pm}}^2 - m_{\pi}^2) \text{Tr}(\tau_a U) \text{Tr}(\tau_a U^{\dagger}),$ $\Delta \mathcal{L}_{\text{env}}^* = -\frac{F_{\pi}^2}{32} \sum_{a,b=1}^2 \varepsilon_{ab3} \frac{\Delta \chi}{m_{\pi}} \text{Tr}(\tau_a U) \text{Tr}(\tau_b \partial_0 U^{\dagger}).$

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Medium functionals and their parameters

[U.Meissner et al., EPJ A36 (2008); UY, JKPS62 (2013), UY, PRC88 (2013)]



Nucleon in nuclear matter

Isoscalar mass

$$m_{\rm N}^{\rm S*} = M_{\rm NP}^* + \frac{3}{8\Lambda^*} + \frac{\Lambda^*}{2} \left(a^{*2} + \frac{\Lambda_{\rm env}^{*2}}{\Lambda^{*2}} \right)$$

Isovector mass

$$\Delta m_{\rm np}^* = a^* + \frac{\Lambda_{\rm env}^*}{\Lambda^*}$$

□ Mass of the nucleon

$$m_{\mathrm{n,p}}^* = m_{\mathrm{N}}^{\mathrm{S}*} - \Delta m_{\mathrm{np}}^* T_3,$$

Nuclear matter

Origin of the binding-energy-formula terms in present model

$$\varepsilon(A,Z) = -a_V + a_S \frac{(N-Z)^2}{A^2} + \dots,$$

- Volume term
 - Infinite and asymmetric nuclear matter
- Asymmetry term
 - Isospin asymmetric environment
- Surface and Coulomb terms
 - Nucleons in a finite volume
- Finite nuclei properties
 - Local density approximation

Nuclear matter

Volume term and Symmetry energy

- At infinite nuclear matter approximation the binding energy formula takes form $\varepsilon(\lambda, \delta) = -a_V(\lambda) + \varepsilon_S(\lambda)\delta^2 + O(\delta^4)$ $\equiv \varepsilon_V(\lambda) + \varepsilon_A(\lambda, \delta),$
 - λ is normalized nuclear matter density
 - β is asymmetry parameter
 - *a_s* is symmetry energy
- In our model
 - Symmetric matter $\varepsilon_V(\lambda) = \frac{1}{2} [m_p^*(\lambda, 0) + m_n^*(\lambda, 0)] m_N^S$ $= m_N^{S*}(\lambda, 0) - m_N^S,$
 - Asymmetric matter $\varepsilon(\lambda, \delta) = \frac{Zm_{p}^{*}(\lambda, \delta) + Nm_{n}^{*}(\lambda, \delta)}{A} - \frac{Zm_{p} + Nm_{n}}{A}$

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Volume term [UY, JKPS62 (2013)]

- Volume term (binding energy per nucleon) in the binding energy formula can be defined as
 - Model I solid curve
 - Model II dashed curve
 - Model III dotted curve

Model	γ_0	$\gamma_{ m n,0}$	$\gamma_{ m d,0}$
	$[m_{\pi}^{-3}]$	$[m_{\pi}^{-3}]$	$[m_{\pi}^{-3}]$
Ι	0.0	1.901	0.070
II	0.5	1.867	0.049
III	1.0	1.840	0.031

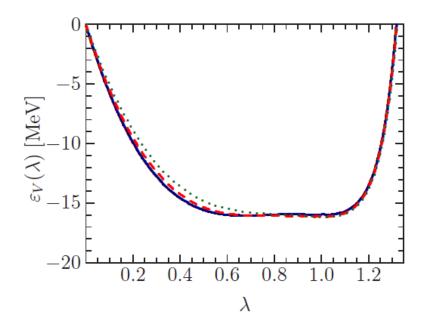


TABLE I: The volume term coefficient $a_V(1)$ at the normal nuclear matter density $\lambda = 1$ and the compression modulus K_0 of symmetric nuclear matter. Their values are given for the three different sets of parameters. The variational parameters $\gamma_{n,0}$ and $\gamma_{d,0}$ are chosen in such a way that at saturation point $\rho = \rho_0 = 0.15$ fm⁻³ the value of volume energy per nucleon is close to its experimental value, $\varepsilon_V^{exp} \simeq -16$ MeV.

γ_0	$\gamma_{ m n,0}$	$\gamma_{ m d,0}$	$a_V(1)$	K_0
$[m_{\pi}^{-3}]$	$[m_{\pi}^{-3}]$	$[m_{\pi}^{-3}]$	[MeV]	[MeV]
0.0	1.901	0.070	15.94	202
0.5	1.867	0.049	16.11	218
1.0	1.840	0.031	16.12	366
	$[m_{\pi}^{-3}]$ 0.0 0.5	$ \begin{array}{c c} [m_{\pi}^{-3}] & [m_{\pi}^{-3}] \\ \hline 0.0 & 1.901 \\ 0.5 & 1.867 \end{array} $	$\begin{bmatrix} m_{\pi}^{-3} \end{bmatrix} \begin{bmatrix} m_{\pi}^{-3} \end{bmatrix} \begin{bmatrix} m_{\pi}^{-3} \end{bmatrix}$ 0.0 1.901 0.070 0.5 1.867 0.049	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

• Asymmetry energy

$$\begin{split} \varepsilon_A(\lambda,\delta) &= \varepsilon(\lambda,\delta) - \varepsilon_V(\lambda) \\ &= m_{\rm N}^{\rm S*}(\lambda,\delta) - m_{\rm N}^{\rm S*}(\lambda,0) + [\Delta m_{\rm np}^*(\lambda,\delta) - \Delta m_{\rm np}] \frac{\delta}{2}. \end{split}$$

Symmetry energy

$$\varepsilon_{S}(\lambda) = \frac{1}{2} \frac{\partial^{2} \varepsilon_{A}(\lambda, \delta)}{\partial \delta^{2}} \Big|_{\delta=0}$$

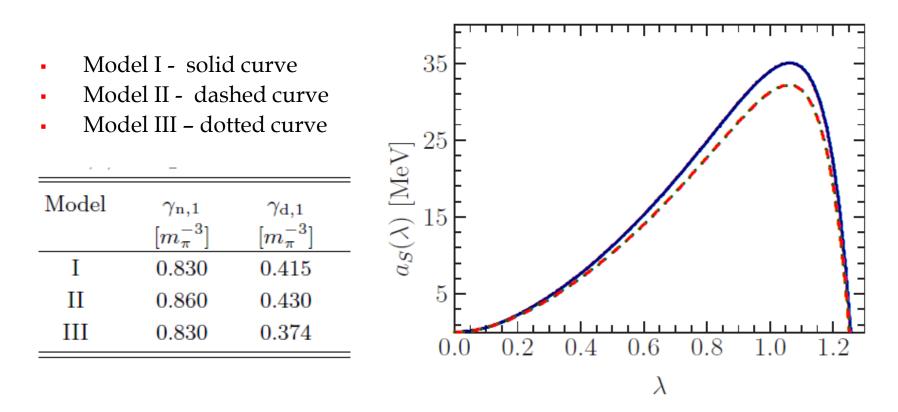
$$= \frac{1}{2} \frac{\partial^{2}}{\partial \delta^{2}} \left(m_{N}^{S*}(\lambda, \delta) + \Delta m_{np}^{*}(\lambda, \delta) \frac{\delta}{2} \right)_{\delta=0}$$

• Symmetry energy coefficients

$$\varepsilon_{\mathcal{S}}(\lambda) = \varepsilon_{\mathcal{S}}(1) + \frac{L_{\mathcal{S}}}{3}(\lambda - 1) + \frac{K_{\mathcal{S}}}{18}(\lambda - 1)^2 + \cdots$$

Symmetry energy [UY, JKPS62 (2013)]

• Symmetry energy as function of normalized density

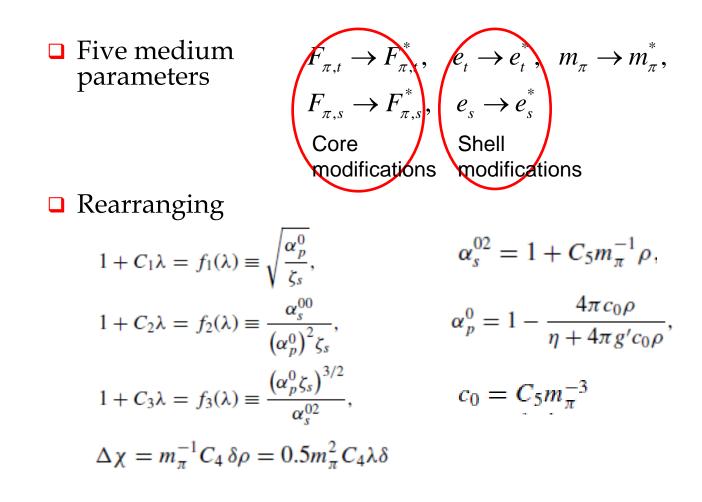


Symmetry energy [UY, JKPS62 (2013)]

TABLE II: The slope L_S and the curvature K_S of symmetry energy. The variational parameters $\gamma_{n,1}$ and $\gamma_{d,1}$ are chosen in such a way that at normal nuclear matter density $\rho = \rho_0 =$ 0.15 fm^{-3} the value of symmetry energy $a_S(1)$ is close to its experimental value, $a_S^{\exp} \approx 32$ MeV. Other parameters $\gamma_{n,0}$ and $\gamma_{d,0}$ are given in Table I.

Model	$\gamma_{ m n,1}$	$\gamma_{ m d,1}$	$a_S(1)$	L_S	K_S
	$[m_{\pi}^{-3}]$	$[m_{\pi}^{-3}]$	[MeV]	[MeV]	[MeV]
Ι	0.830	0.415	33.99	91.75	-3428
II	0.860	0.430	31.21	85.66	-2761
III	0.830	0.374	31.21	76.41	-2800

Reparametrization [UY, PRC88 (2013)]



After reparametrization [UY, PRC88 (2013)]

□ Volume energy

TABLE I. The variational parameters and the coefficients of the volume term at the saturation density ρ_0 . Here the isospin-breaking effect in the mesonic sector is ignored, i.e., $\mathcal{M}_{-} = 0$.

Set	C_1	<i>C</i> ₂	<i>C</i> ₃	$\varepsilon_V(\rho_0)$ (MeV)	K ₀ (MeV)	Q (MeV)
I	-0.279	0.737	1.782	-16	240	-410
Π	-0.273	0.643	1.858	-16	250	-279
Ш	-0.277	0.486	2.124	-16	260	-178

$$K_0 = 9\rho^2 \frac{\partial^2 \varepsilon_V}{\partial \rho^2} \bigg|_{\rho = \rho_0}$$

$$Q = 27\lambda^3 \frac{\partial^3 \varepsilon_V(\lambda)}{\partial \lambda^3} \Big|_{\lambda=0}$$

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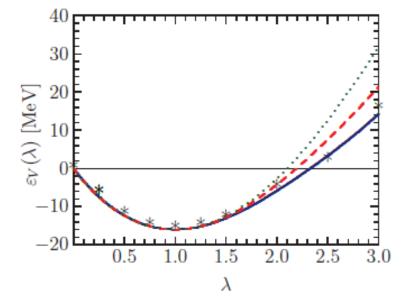


FIG. 1. (Color online) The volume energy ε_V as a function of normalized density $\lambda = \rho / \rho_0$. The parameters of medium functionals are given in Table I: the solid curve corresponds to Set I, the dashed one to Set II, and the dotted curve to Set III. Akmal-Pandharipande-Ravenhall predictions [68] are marked by stars.

[68] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C 58, 1804 (1998).

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After reparametrization [UY, PRC88 (2013)]

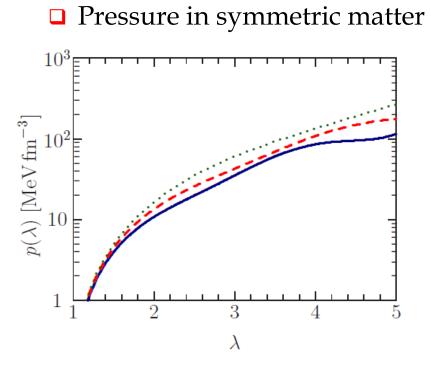
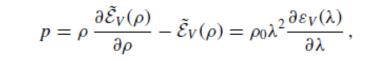
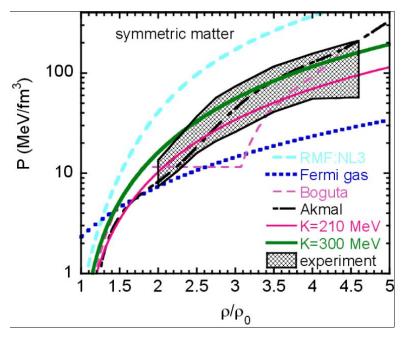


FIG. 2: (Color online) The pressure p in symmetric matter as a function of normalized density $\lambda = \rho/\rho_0$. The parameters of medium functionals are given in Table I: the solid curve corresponds to the Set I, the dashed one to the Set II and the dotted curve belongs to the Set III.





Right figure from:

P. Danielewicz, R. Lacey, and W. G. Lynch, Science 298, 1592 (2002).

After reparametrization [UY, PRC88 (2013)]

□ Symmetry energy

 $c_0 = C_5 m_{\pi}^{-3}$

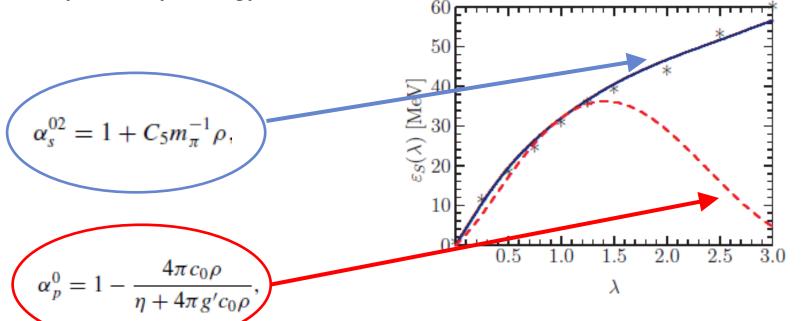
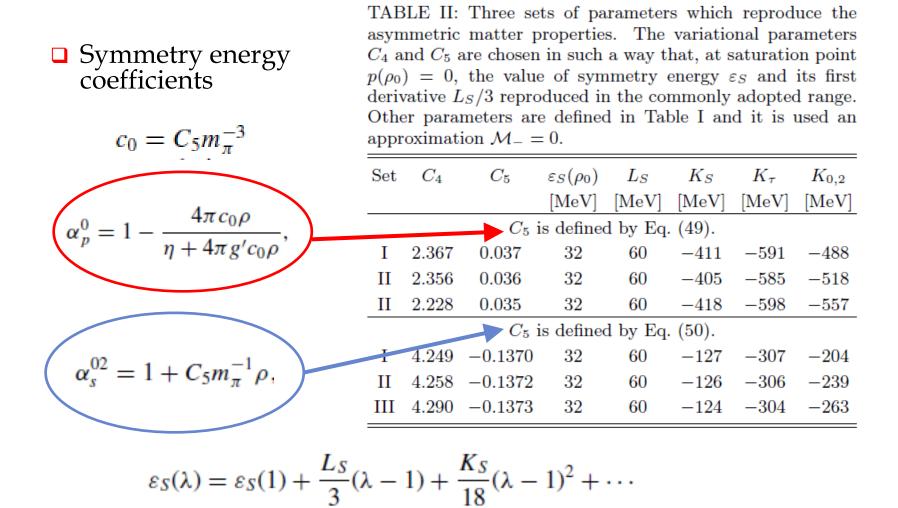


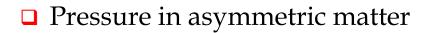
FIG. 3. (Color online) The symmetry energy as a function of normalized density $\lambda = \rho/\rho_0$. The solid curve corresponds to Eq. (49) and to the parameters defined by Set II, while the dashed one corresponds to Eq. (50) and to the parameters defined by Set II (see Tables I and II). APR predictions [68] are marked by stars.

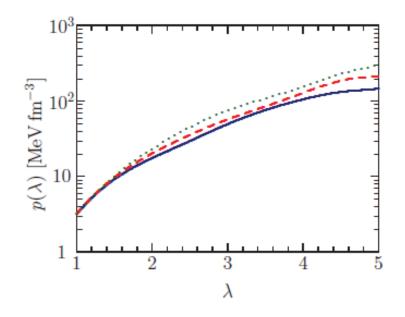
After reparametrization [UY, PRC88 (2013)]



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After reparametrization [UY, PRC88 (2013)]





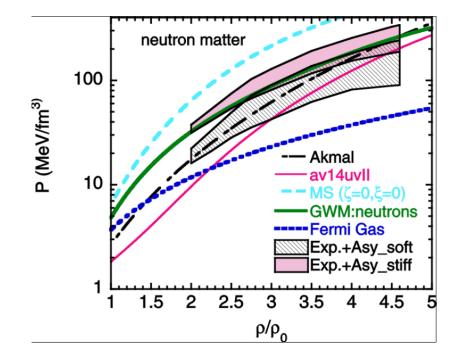


FIG. 4. (Color online) The pressure p in asymmetric matter as a function of normalized density $\lambda = \rho/\rho_0$. The parameters of medium functionals are given in Table II [C_5 is defined by Eq. (50)]: the solid curve corresponds to Set I, the dashed one to Set II, and the dotted curve to Set III.

Right figure from:

P. Danielewicz, R. Lacey, and W. G. Lynch, Science 298, 1592 (2002).

After reparametrization

Low density behavior of symmetry energy and correlations between coefficients	TABLE III: Correlations of symmetry energy coeffic The variational parameters C_4 and C_5 are chosen in s way that, at saturation point $p(\rho_0) = 0$, the value of sy try energy ε_S and its first derivative $L_S/3$ are reprodu- the commonly adopted range. Other parameters are d in Table I (see Set II).						n in such a of symme- produced in	
coefficients	C_4	C_5	$\varepsilon_S(ho_0)$					$\varepsilon_S(0.1 {\rm fm}^{-3})$
			[MeV]	[MeV]	[MeV]	[MeV]	[MeV]	[MeV]
	1.294	0.132	32	40	-181	-301	-257	25.15
	1.168	0.022	32	50	-160	-310	-254	24.15
	1.064	-0.069	32	60	-126	-306	-239	23.22
	0.978	-0.144	32	70	-80	-290	-211	22.37
From analysis of	0.904	-0.209	32	80	-21	-261	-172	21.57
GDR (208Pb)	0.841	-0.264	32	90	50	-220	-119	20.82

32

100

134 - 166

-55

 $23.3 < \varepsilon_S(\rho = 0.1 \text{ fm}^{-3}) < 24.9 \text{ MeV}$

L. Trippa, G. Colo, and E. Vigezzi, Phys. Rev. C 77, 061304 (2008).

0.786 - 0.313

20.13

Nuclear matter

After reparametrization [UY, PRC88 (2013)]

The energy per nucleon in symmetric matter and in neutron matter

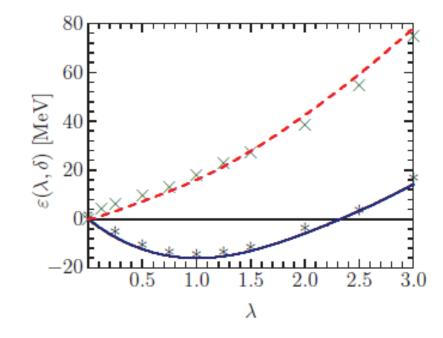


FIG. 5. (Color online) The energy per nucleon $\varepsilon(\lambda, \delta)$ as a function of normalized nuclear matter density $\lambda = \rho/\rho_0$. The parameters are taken from Model II (see the lower part of Table II). The solid curve represents symmetric nuclear matter ($\delta = 0$) while the dashed curve represents neutron matter ($\delta = 1$). For comparison, APR predictions [68] are marked by crosses and stars.

Summary

Within the applicability range, the model describes

- the single hadrons properties
 - in separate state
 - in the community of their partners
- as well as the properties of that whole community at same footing

Outlook

Extensions and applicability of the approach

- Nucleon tomography in nuclear matter [H.Ch. Kim, UY, PLB726 (2013), arXiv:1304.5926]
- NN interactions in nuclear matter
- Neutron stars
- □ Finite nuclei properties
 - Mirror nuclei
 - Exotic nuclei
 - Halo nuclei
- Nucleon-knock out reactions
- Vector mesons in nuclear matter [J.H.Jung, UY, H.Ch.Kim, PLB 723 (2013), arXiv:1212.4616]

Thank you for your attention!