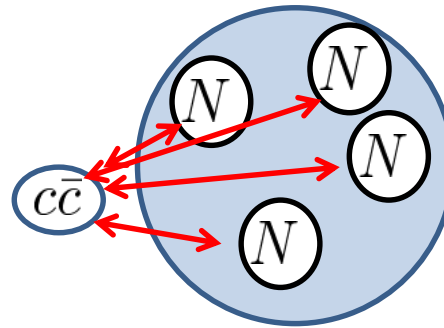


Possible existence of charmonium-nucleus bound states

A. Y., E. Hiyama and M. Oka, [arXiv:1308.6102](https://arxiv.org/abs/1308.6102), accepted by PTEP



Akira Yokota
Tokyo Institute of Technology

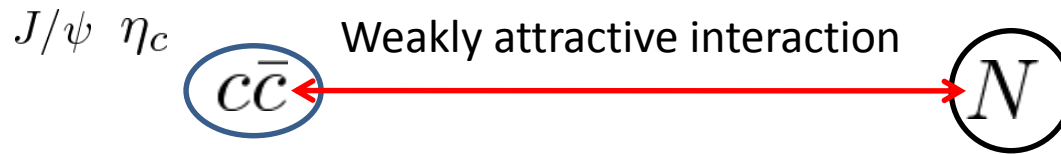
Collaborating with
Emiko Hiyama^a and Makoto Oka^b
RIKEN Nishina Center^a
Tokyo Institute of Technology^b

Contents

- **Introduction**
 - Charmonium-**nucleon** interaction (multiple-**gluon** exchange)
 - Charmonium-**nucleus** bound states
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- **Results**
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 - Charmonium- ^4He system ($J/\psi - \alpha$)
 - Charmonium- ^8Be system ($J/\psi - \alpha - \alpha$)
- **The decay and mixing of J/ψ and η_c in nucleus**
- **Summary and conclusion**

Interaction between $c\bar{c} - N$

Dominated by multiple gluon exchange (weakly attractive)



$c\bar{c}$ and $N(uud, udd)$

- They have **no valence quarks in common** :
 ➔ Meson exchange is suppressed by the OZI rule
- They are **color singlet** : ➔ Single gluon exchange is forbidden

Therefore

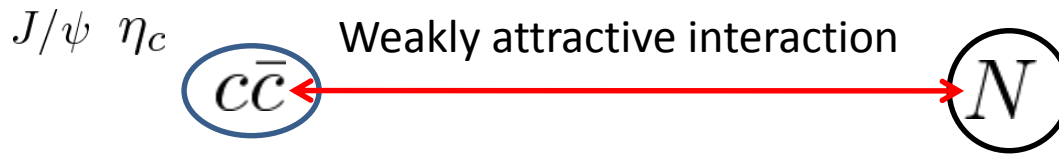
- It is dominated by multiple gluon exchange.
- No repulsive core coming from the Pauli blocking of common quarks.
- It is a short range force due to the color confinement.
- cc-N interaction is weakly attractive force. (Kawanai & Sasaki, PRD.82,091501)
(M. Luke, et al. PLB 288, 355 (1992), D.Kharzeev, H.Satz, PLB 334, 155(1994),
S. J. Brodsky, et al., PRL 64 (1990) 1011, S. J. Brodsky, G. A. Miller PLB 412 (1997) 125)

Study of $c\bar{c} - N$ interaction is suitable for understanding

- the role of gluon and QCD in low energy hadronic interaction
- hadronic interactions in short range region which could not be described only by one meson exchange

Interaction between $c\bar{c} - N$

Dominated by multiple gluon exchange (weakly attractive)



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S. J. Brodsky, (1997) 125)

But the details of the interaction are not yet known.

Study of $c\bar{c} - N$ inte

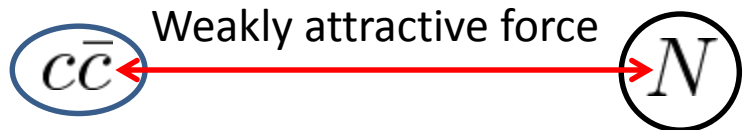
- the role of gluon and
- hadronic interactions in short range region which could not be described only by one meson exchange

Why studying $c\bar{c}$ – nucleus bound state?

- Low energy $c\bar{c} - N$ scattering experiment is not feasible.
- We have to study without direct information about the $c\bar{c} - N$ interaction.



Precise study of the **binding energy** and the **structure** of the $c\bar{c}$ – nucleus bound states from both **accurate theoretical calculations** and **experiments** is **the only way** to determine the properties of the $c\bar{c} - N$ interaction. (cf. The study of Hyperon-Nucleon interaction from the spectroscopy of hypernuclei.)

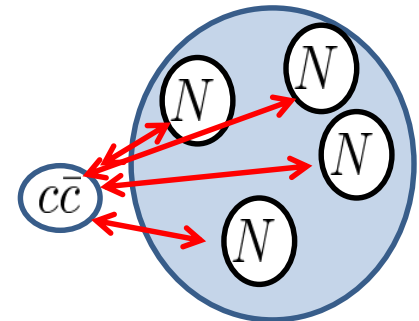


It should make a bound state with nucleus of large A (A : the nucleon number)

S. J. Brodsky et al., PRL 64 (1990) 1011

D. A. Wasson, PRL 67 (1991) 2237

V. B. Belyaev et al., NPA 780, (2006) 100



Also, it is a new type of hadronic state in which particles with **no common valence quarks** are bound mainly by **(multiple-)gluon exchange interaction**.

Such bound states have not yet been found by experiment.
Therefore, we give an estimation of the binding energy of cc^{bar} -nucleus bound states.

Effective potential between $c\bar{c} - N$

- We only consider **S wave** ($L=0$). (We only want to see the ground state.)
- Since the attraction is relatively **weak** and **short ranged**, the interaction could be expressed well by **scattering length**.
- We assume **Gaussian type** potential.

$$\eta_c \quad (J^\pi = 0^-)$$

$$v_{\eta_c - N}(r) = v_{\text{eff}} e^{-\mu r^2}$$

$$\mu = (1.0 \text{ fm})^{-2}$$

(taken from color confinement scale)

$$J/\psi \quad (J^\pi = 1^-)$$

$$v_{J/\psi - N}(r) = (v_0 + v_s (\mathbf{S}_{J/\psi} \cdot \mathbf{S}_N)) e^{-\mu r^2}$$

$$\equiv v_{\text{eff}} (S_{J/\psi - N}) e^{-\mu r^2}$$

$$v_{\text{eff}}(S_{J/\psi - N}) = \begin{cases} v_0 - v_s & (S_{J/\psi - N} = 1/2) \\ v_0 + \frac{1}{2}v_s & (S_{J/\psi - N} = 3/2) \end{cases}$$

Our strategy:

- 1, Solve the Schrödinger equation for $c\bar{c} - N$ 2-body system and obtain the relation between the potential depth v_{eff} and the scattering length a .
- 2, Solve the Schrödinger equation for $c\bar{c}$ -nucleus system (by GEM) and obtain the relation between v_{eff} and the binding energy B .
- 3, By combining these results, we obtain the relation between a and B .

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Can be calculated by lattice QCD

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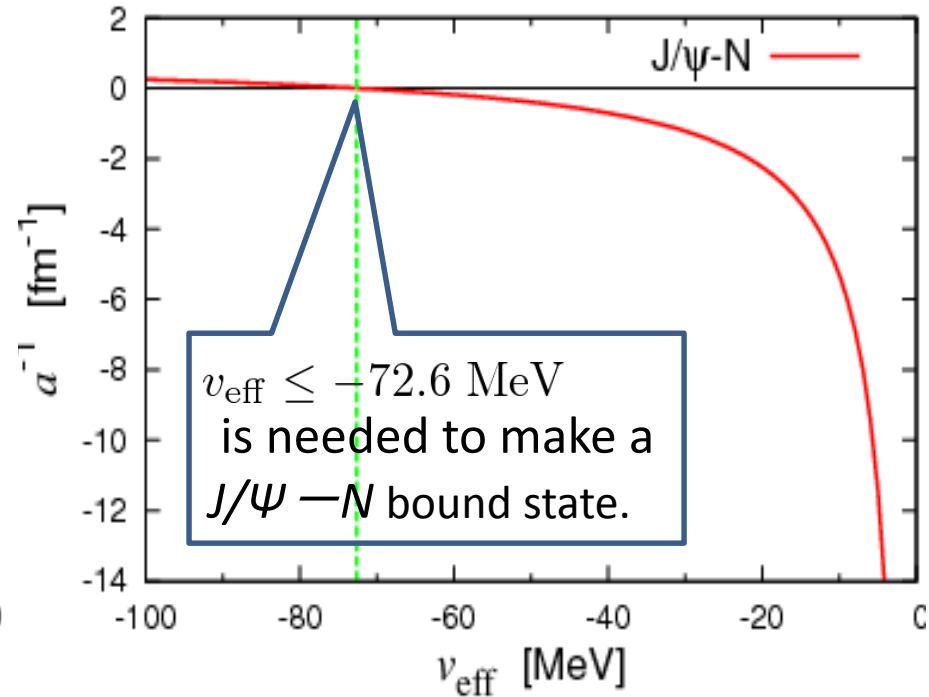
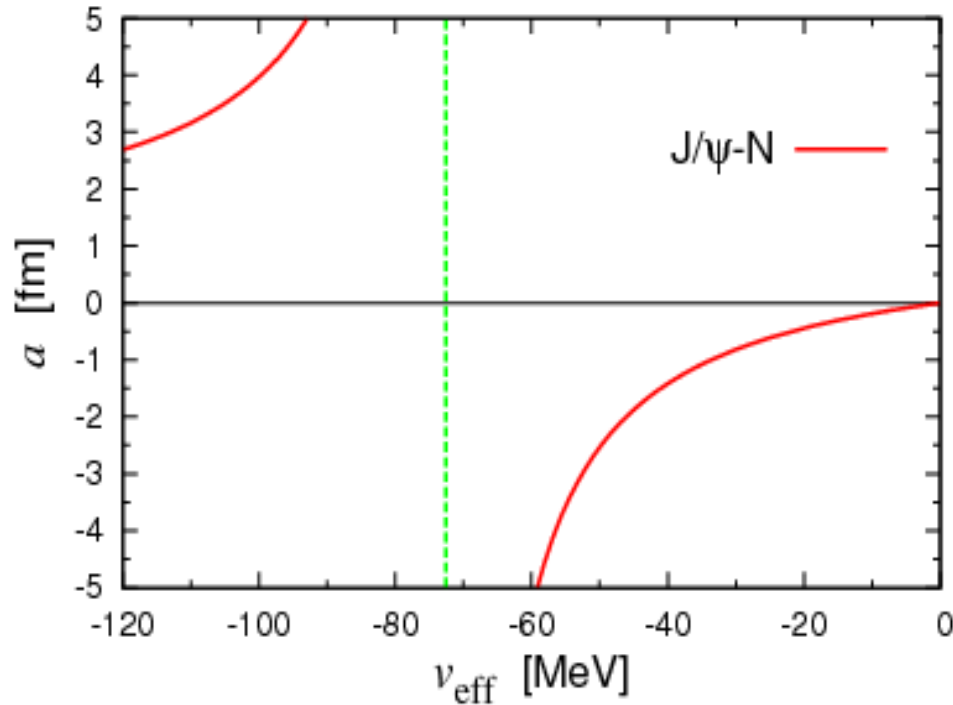
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- 3, By combining these results, we obtain the relation between a and B .

Combining lattice QCD data, we estimate the binding energy of cc^{bar} -nucleus bound states.

The relation between potential strength and the scattering length



$$v_{J/\psi-N}(r) = v_{\text{eff}}(S_{J/\psi-N})e^{-\mu r^2}$$

$$\mu = (1.0 \text{ fm})^{-2}$$

By the results, we can convert the value of v_{eff} into $a_{J/\psi-N}$.

A $J/\psi - N$ bound state is formed when $v_{\text{eff}} \leq -72.6 \text{ MeV}$.

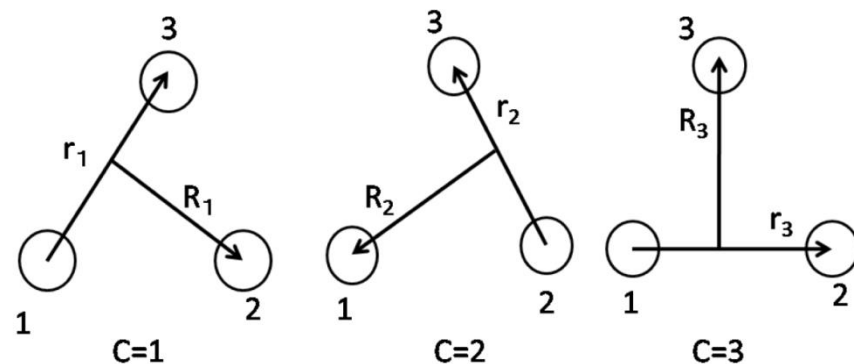
Calculation of J/ψ -nucleus bound states

GEM 3-body calculation

(variation method)

It is known empirically that setting range parameters in geometric progression as shown below produce accurate eigenvalues and eigenfunctions with a relatively few basis functions.

E. Hiyama et al. Prog. Part. Nucl. Phys. 51, 223 (2003)



$$\Psi_{JM} = \sum_{c=1}^3 \sum_{n=1}^{n_{max}} \sum_{N=1}^{N_{max}} \sum_I C_{nNI}^c \phi_{nlm}^c(\mathbf{r}_c) \psi_{NLM}^c(\mathbf{R}_c) [[\chi_s(1)\chi_s(2)]_I \chi_s(3)]_{JM}$$

$$\phi_{nlm}^c(\mathbf{r}) = \phi_{nl}^c(r) Y_l^m(\hat{\mathbf{r}})$$

$$\phi_{nl}^c(r) = r^l e^{-\nu_n r^2}$$

$$\nu_n = \frac{1}{r_n^2} \quad r_n : \text{geometric progression}$$

$$r_n = r_1 a^{n-1} \quad (n = 1, \dots, n_{max})$$

$$\psi_{NLM}^c(\mathbf{R}) = \psi_{NL}^c(R) Y_L^M(\hat{\mathbf{R}})$$

$$\psi_{NL}^c(R) = R^L e^{-\lambda_N R^2}$$

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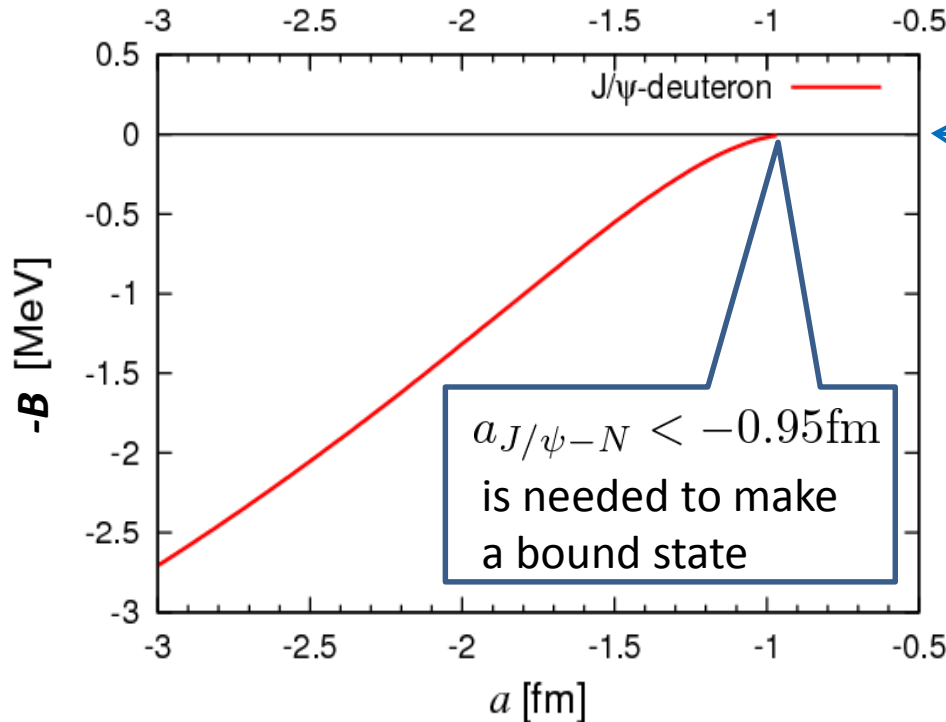
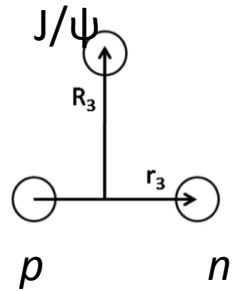
$$\langle \phi_{n00}^c \psi_{N00}^c | [-\frac{\hbar^2}{2\mu_{13}} \nabla_{\mathbf{r}_1}^2 - \frac{\hbar^2}{2\mu_{123}} \nabla_{\mathbf{R}_1}^2 + V(\mathbf{r}_1, \mathbf{R}_1) - E] | \Psi_{JM} \rangle$$

$$= \sum_{c'.n'.N'.I} [(T_{nN,n'N'}^{c,c'} + V_{nN,n'N'}^{c,c'}) - EN_{nN,n'N'}^{c,c'}] C_{n'N'}^{c'} = 0$$

Generalized eigenvalue problem of symmetric matrix.

$J/\psi - NN$ 3-body bound state

A. Y., E. Hiyama and M. Oka, [arXiv:1308.6102](https://arxiv.org/abs/1308.6102), accepted by PTEP



lattice QCD $a \sim -0.35 \text{ fm}$
T. Kawanai, S. Sasaki,
Pos (Lattice 2010)156

Relation between $a_{J/\psi-N}$ of $J/\psi - N$ and binding energy B of $J/\psi - NN$ (Isospin $T=0$).

The binding energy is measured from $J/\psi + \text{deuteron}$ breakup threshold -2.2 MeV .

$N-N$ potential: Minnesota potential

I. Reichstein, Y. C. Tang, Nucl. Phys. A, 158, 529 (1970)

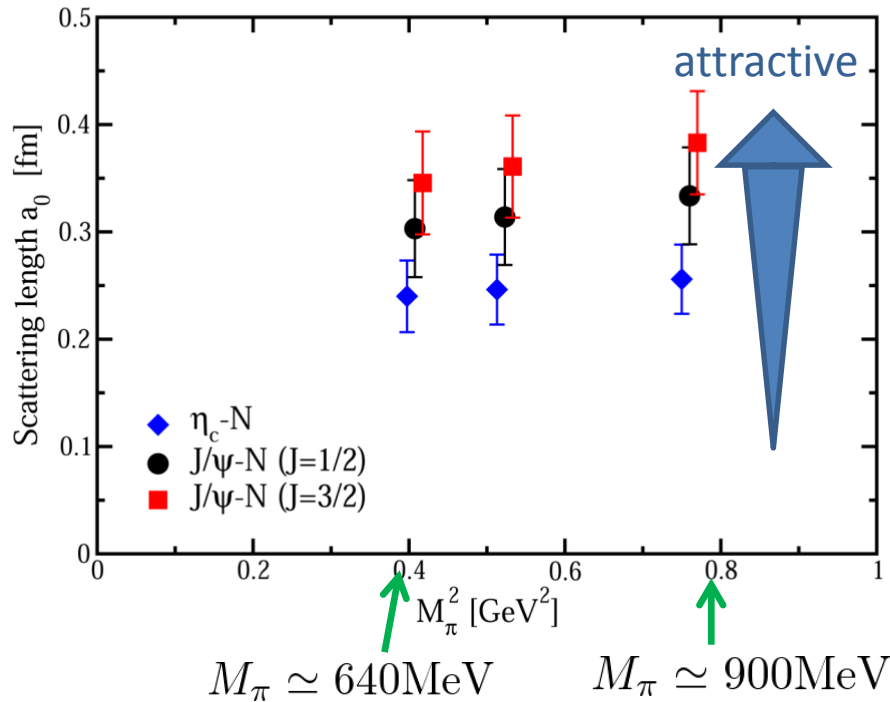
D. R. Thompson et al., Nucl. Phys. A, 286, 53, (1977)

A bound state is formed when
 $a_{J/\psi-N} < -0.95 \text{ fm}$

Charmonium-nucleon scattering length from recent lattice QCD

Scattering lengths as functions of the square mass of π derived by quenched lattice QCD using Luscher's formula.
 (The notation of the sign of scattering length is opposite.)

(T. Kawanai, S. Sasaki, Pos (Lattice 2010)156) ▪ Tendency of $a_{J=3/2}^{J/\psi-N} \geq a_{J=1/2}^{J/\psi-N} \geq a^{\eta_c-N}$ can be seen for the central values although there are overlaps of error-bars. (The size of the error-bars are about 0.1 fm.)



- The small spin dependence may exist.

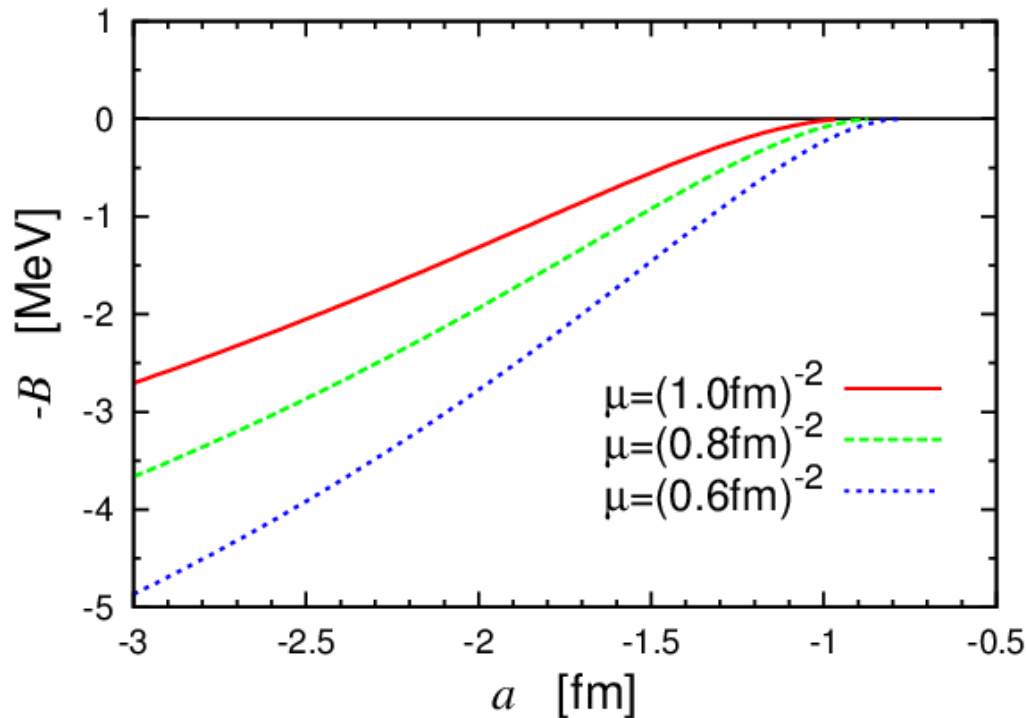
$$a_{\text{SAV}}^{J/\psi-N} \simeq -0.35 \text{ fm}$$

(our notation of the sign)

$$a_{\text{SAV}}^{J/\psi-N} \equiv \frac{1}{3} (a_{J=1/2}^{J/\psi-N} + 2a_{J=3/2}^{J/\psi-N})$$

Range dependence of the binding energy

J/ ψ -NN binding energy



$$V(r) = V_{\text{eff}} e^{-\mu r^2}$$

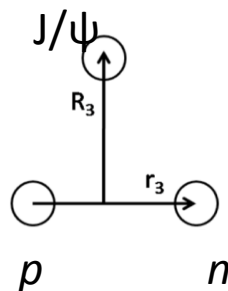
- So far we have assumed that there are one to one correspondence between a and B .
- But additionally there is range dependence.
- A potential with smaller range gives deeper binding for the same value of the scattering length.
- But the difference becomes small when attraction become weak.
- Our results do not change qualitatively by the difference of the potential range.

Difference between using

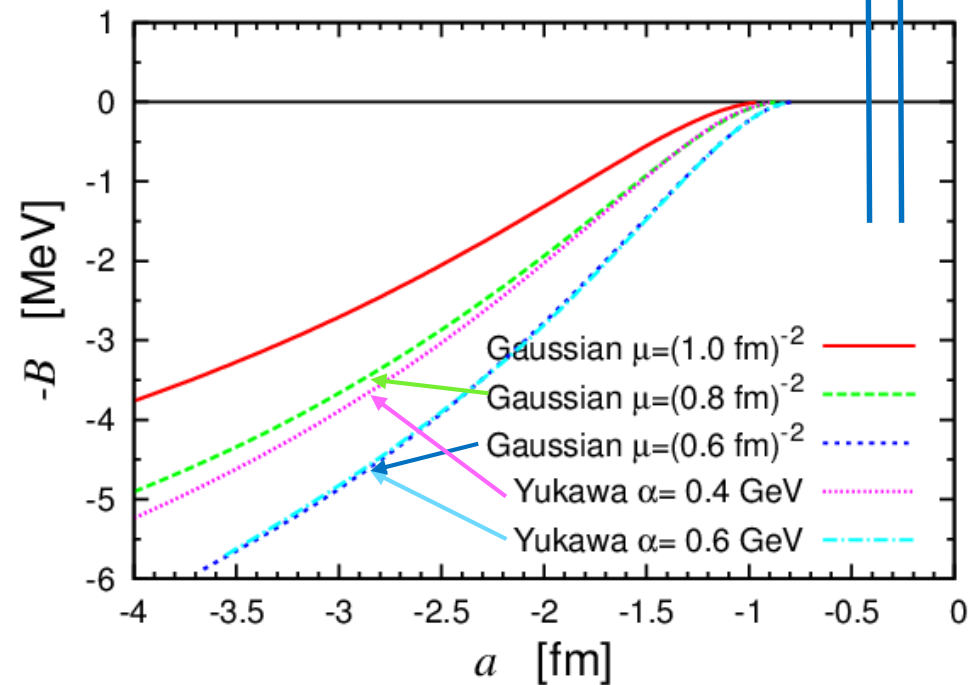
Gaussian-type Potential $V(r) = V_{\text{eff}} e^{-\mu r^2}$

and

Yukawa-type Potential $V(r) = -\frac{\gamma}{r} e^{-\alpha r}$

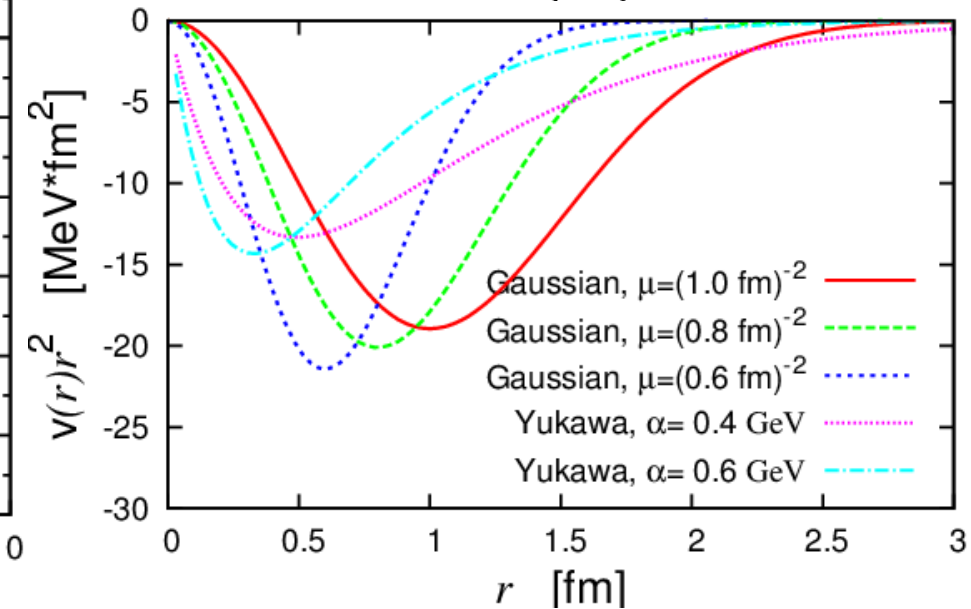
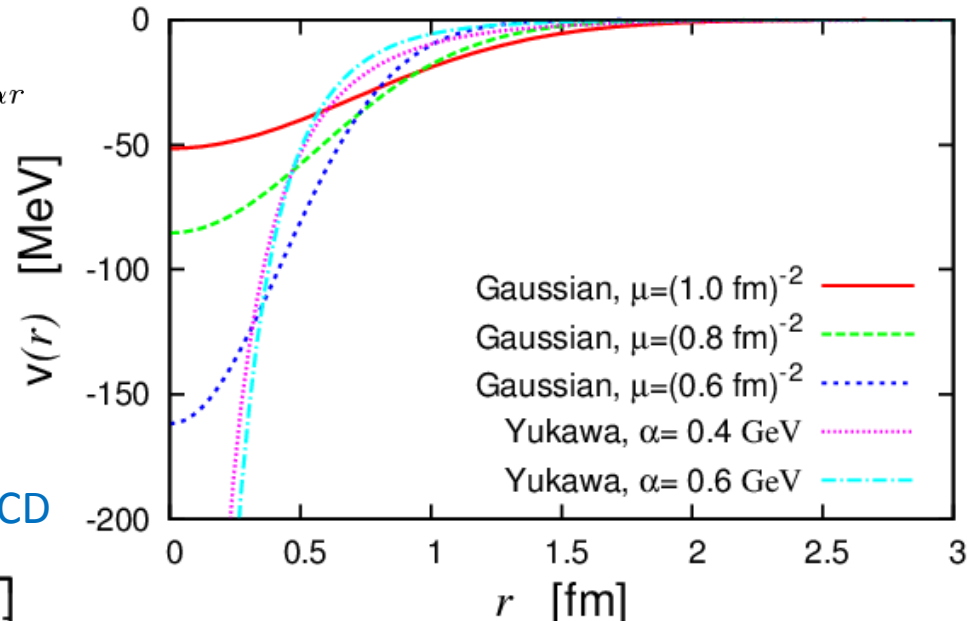


J/ ψ -NN binding energy Lattice QCD



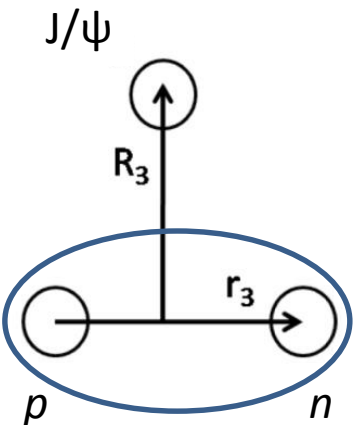
Potentials

$a = -2.8 \text{ fm}$

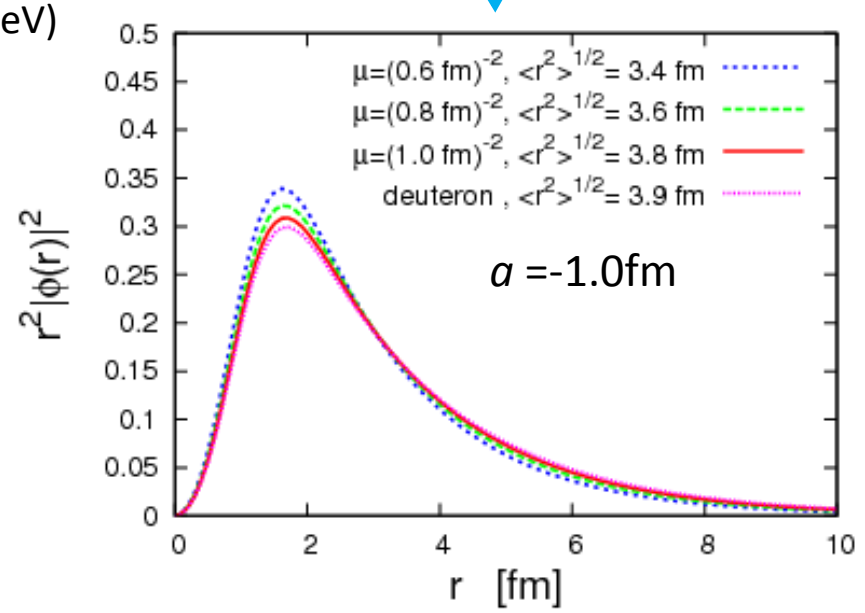
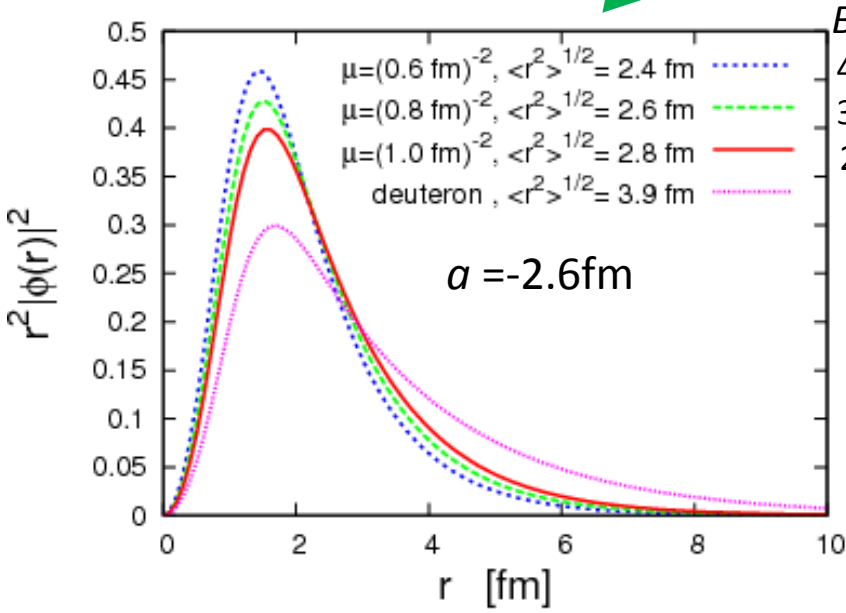
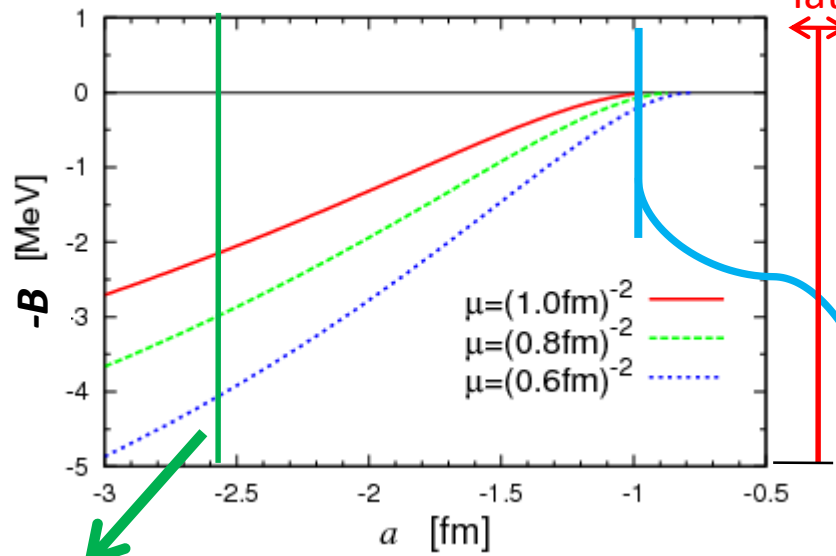


“Glue-like role” of J/ψ (J/ψ -deuteron system)

lattice QCD $a \sim -0.35$ fm



Shrinking of p - n
density distribution in
the deuteron by the
emergence of $c\bar{c}$



Glue like effect is suppressed for weak attraction

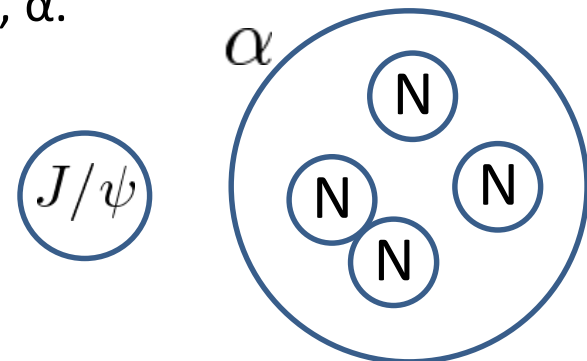
J/ ψ - ^4He potential

Since J/ ψ -N interaction is weak and ^4He is a deeply bound state ($B.E. = 28\text{MeV}$), nucleon density distribution in ^4He may not be disturbed by J/ ψ .

Therefore, it is reasonable to treat ^4He as one stable particle, α .

For J/ ψ - α potential, we use folding potential given by

$$V_{J/\psi-\alpha}(\mathbf{r}) = \sum_{i=1}^4 \int \rho_{N_i}(\mathbf{r}') V_{J/\psi-N}(\mathbf{r} - \mathbf{r}') d^3\mathbf{r}'$$



$$\left\{ \begin{array}{l} \rho_{N_i}(r') = \frac{4}{b^3\sqrt{\pi}} \exp\left(-\frac{1}{b^2}r'^2\right) \quad (\text{nucleon density distribution in } ^4\text{He}) \\ b = 1.358\text{fm} \end{array} \right.$$

$$\int_0^\infty \rho_{N_i}(r)r^2 dr = 1$$

Ref: R. Hofstadter, Annu. Rev. Nucl. Sci. 7, 231 (1957)

R.F. Frosch et al., Phys. Rev. 160, 4 (1967)

J.S. McCarthy et al., PRC15, 1396 (1977)

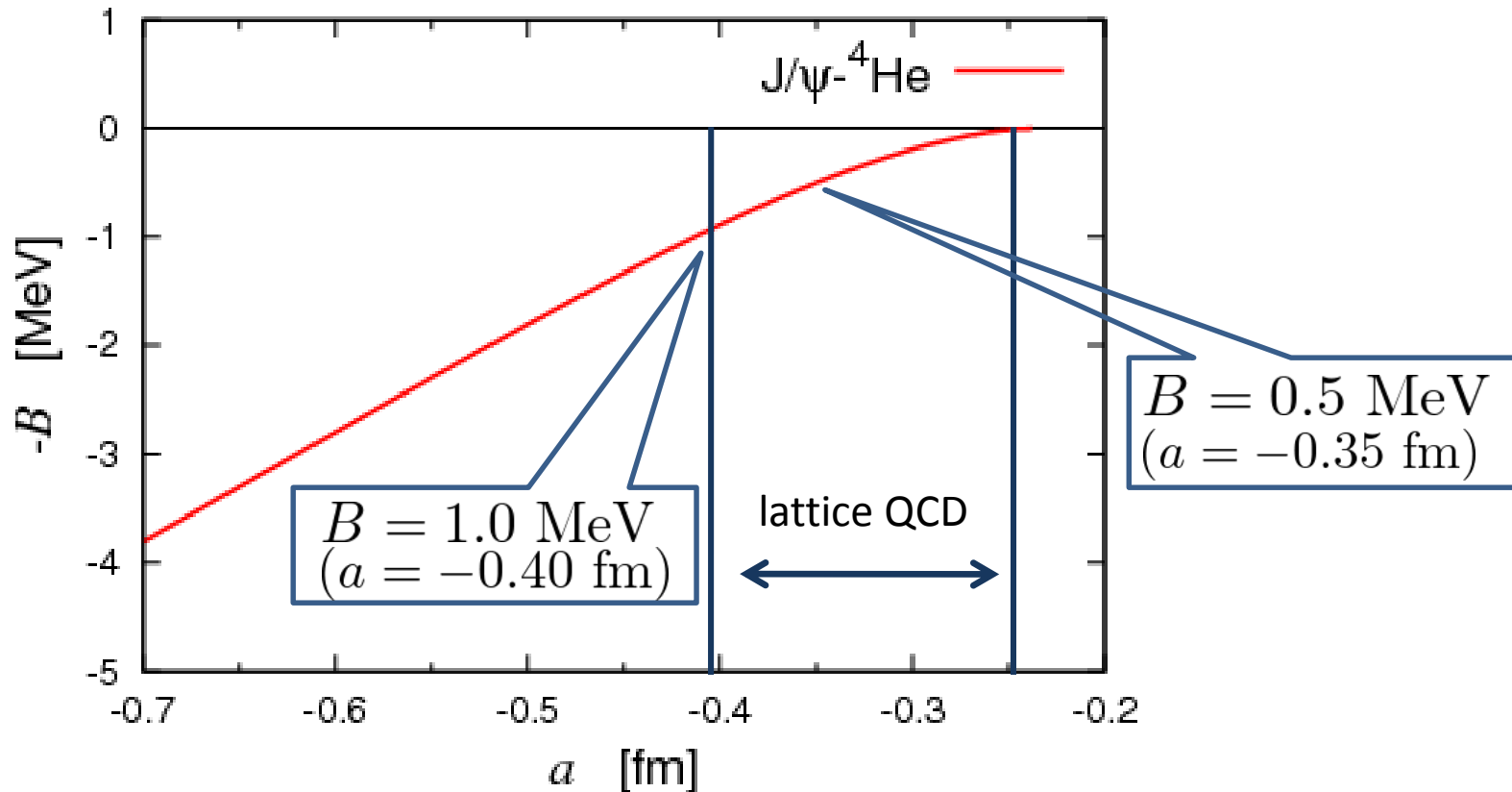
Also, we implement **the Center of Mass Correction** to the folding potential.

$$V_{\text{fold}}(r) = 4 \left(\frac{4}{4 + 3\mu b^2} \right)^{3/2} v_0 e^{-4\mu r^2 / (4 + 3\mu b^2)}$$

r : the relative distance between J/ ψ and the center of mass of ^4He .

$J/\psi - {}^4\text{He}$ Binding Energy

A. Y., E. Hiyama and M. Oka, [arXiv:1308.6102](https://arxiv.org/abs/1308.6102), accepted by PTEP



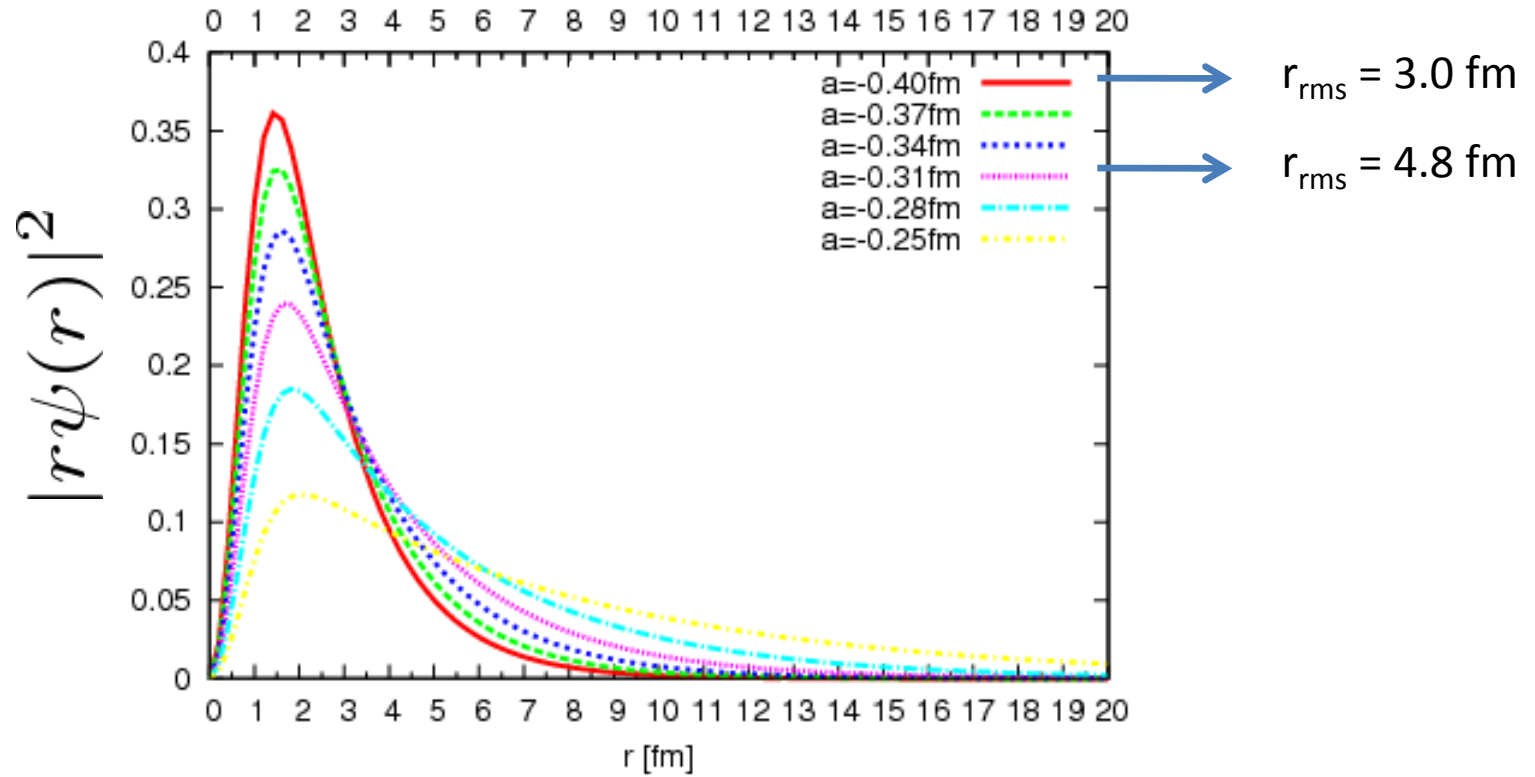
$J/\psi - {}^4\text{He}$ bound state is formed when $a_{J/\psi-N} \leq -0.24$ fm

$J/\psi - {}^4\text{He}$ bound state may exist!

Also, since J/ψ -N interaction is attractive, bound states may exist for nuclei with $A \geq 4$.

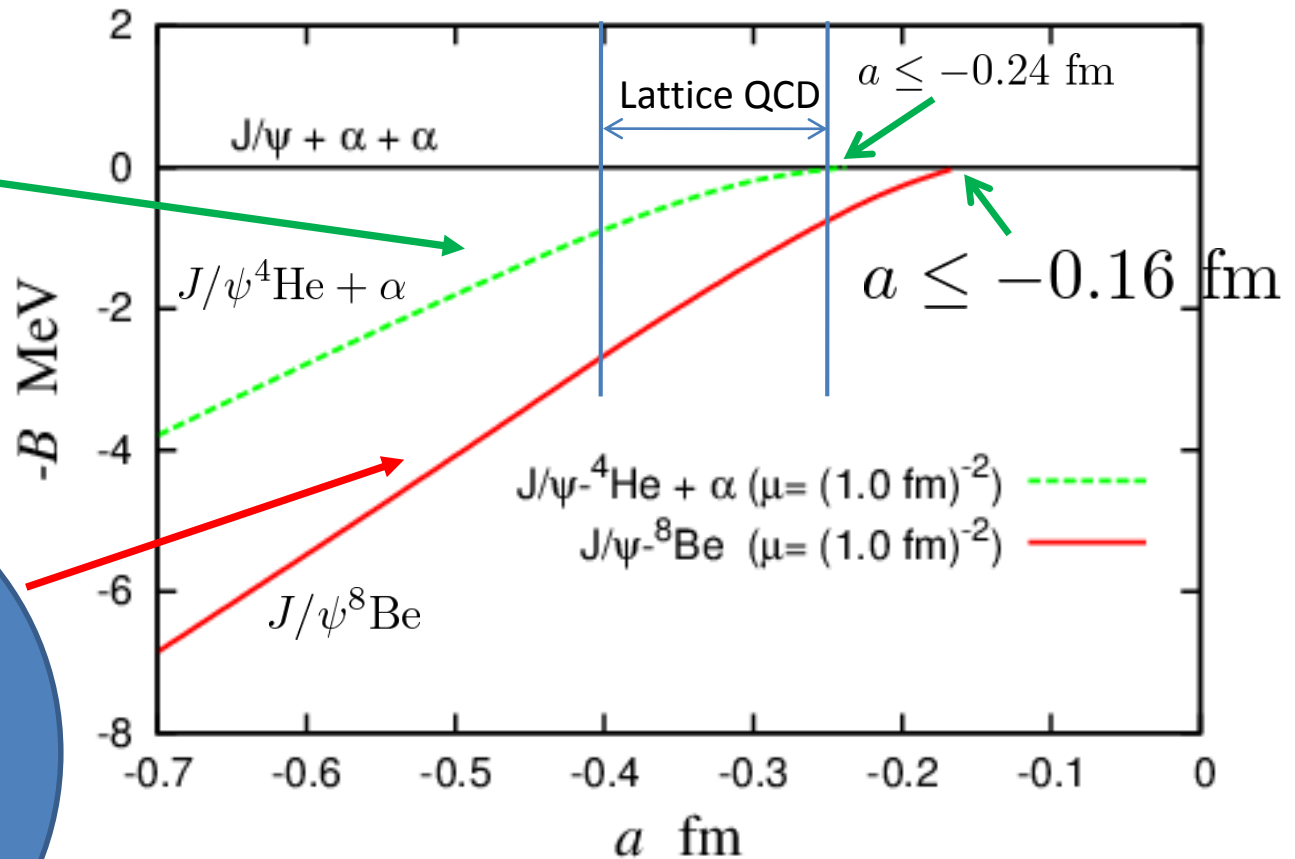
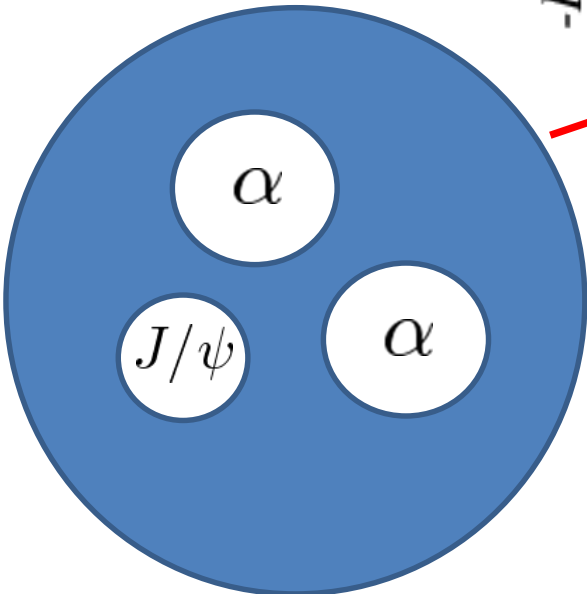
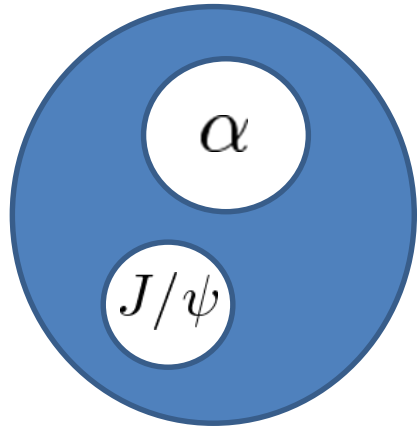
Density distribution between J/ψ - ^4He

A. Y., E. Hiyama and M. Oka, [arXiv:1308.6102](https://arxiv.org/abs/1308.6102), accepted by PTEP



$J/\psi - \alpha - \alpha$ 3-body system

Relations between scattering length a of $J/\psi - N$ and binding energy B of $J/\psi - {}^4\text{He}$ and $J/\psi - \alpha - \alpha$



α - α interaction : folding Hasegawa-Nagata potential with OCM

A. Hasegawa, S. Nagata, Prog. Theor. Phys. 45, 1786 (1971)

${}^8\text{Be}$ is a resonance state, 0.09 MeV above the $\alpha + \alpha$ break-up threshold with narrow width $\Gamma = 6 \text{ eV}$.

Summary and Conclusion

- We calculate the binding energies of $J/\psi - {}^4\text{He}$ and $J/\psi - \alpha - \alpha$ by using Gaussian Expansion Method and give **the relations** between ***the $J/\psi - N$ scattering length*** and ***the $J/\psi - \text{nucleus binding energies}$*** .
- By comparing these results with the recent **lattice QCD data**, we see that **a shallow bound state of $J/\psi - {}^4\text{He}$ and $J/\psi - \alpha - \alpha$ ($J/\psi - {}^8\text{Be}$) may exist.**
- Since $J/\psi - N$ interaction is *attractive*,
 $J/\psi - \text{nucleus bound states}$ may be formed with **$A > 4$ nucleus**.
- The decay mechanisms which could make the width of J/ψ -nucleus larger are considered to be small.
- Therefore, we conclude that if $J/\psi - \text{nucleus bound states exist}$, it would be narrow states comparable to J/ψ in the vacuum.

Back up slides

The decay and the mixing of J/ψ and η_c in nuclei

- $\Gamma_{J/\psi} = 93 \text{ keV}$, $\Gamma_{\eta_c} = 30 \text{ MeV}$ in vacuum.
- We expect narrow J/ψ states in nuclei.
- **Two possible mechanisms**, which make the decay width of $cc^{\text{bar}}(J/\psi)$ -nucleus states larger are
 - (1) The **final state interaction (FSI)** of cc^{bar} decay products with nucleons in nucleus
 - (2) The **mixing of cc^{bar} -nucleon state with other hadronic states** which retain c and c^{bar} quarks.

(1) The final state interaction(FSI) of $c\bar{c}$ decay products with nucleons in nucleus

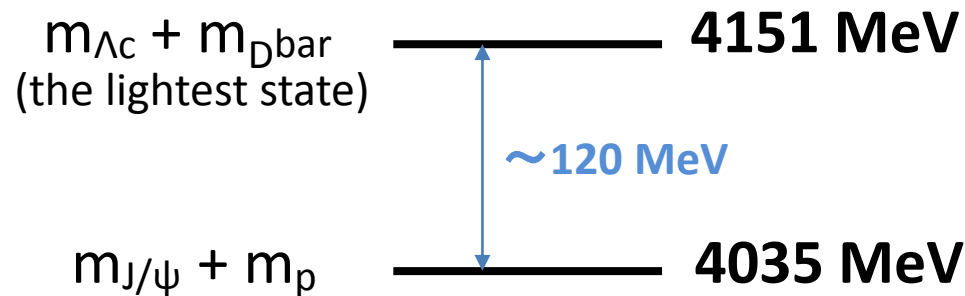
- For J/ψ , FSI in nucleus (e.g. π absorption by N) may enhance the decay width of J/ψ in nucleus several times larger than in vacuum.
- But $\Gamma_{J/\psi}$ in vacuum is so small (= 93 keV) that even if it is enhanced for several times in nucleus, it still would be small (< 1 MeV).

(2) The mixing of $c\bar{c}$ -nucleon state with other hadronic states which retain c and \bar{c} quarks.

- **The decay of J/ψ going through the mixing**



is **prohibited** since all such hadronic states have larger masses.



- Then, **the only possible decay process** of J/ψ in this case goes through **the mixing of J/ψ -nucleus and ηc -nucleus** channels which have the same conserving quantum numbers.
- This mixing process can be divided into two groups:
 - (a) **coherent mixing** (retains the nucleus to its ground state)
 - (b) **incoherent mixing** (nucleus is excited or broken)

(e.g. $J/\psi\text{-}^4\text{He} \rightarrow \eta c\text{-}^3\text{H} + p$)

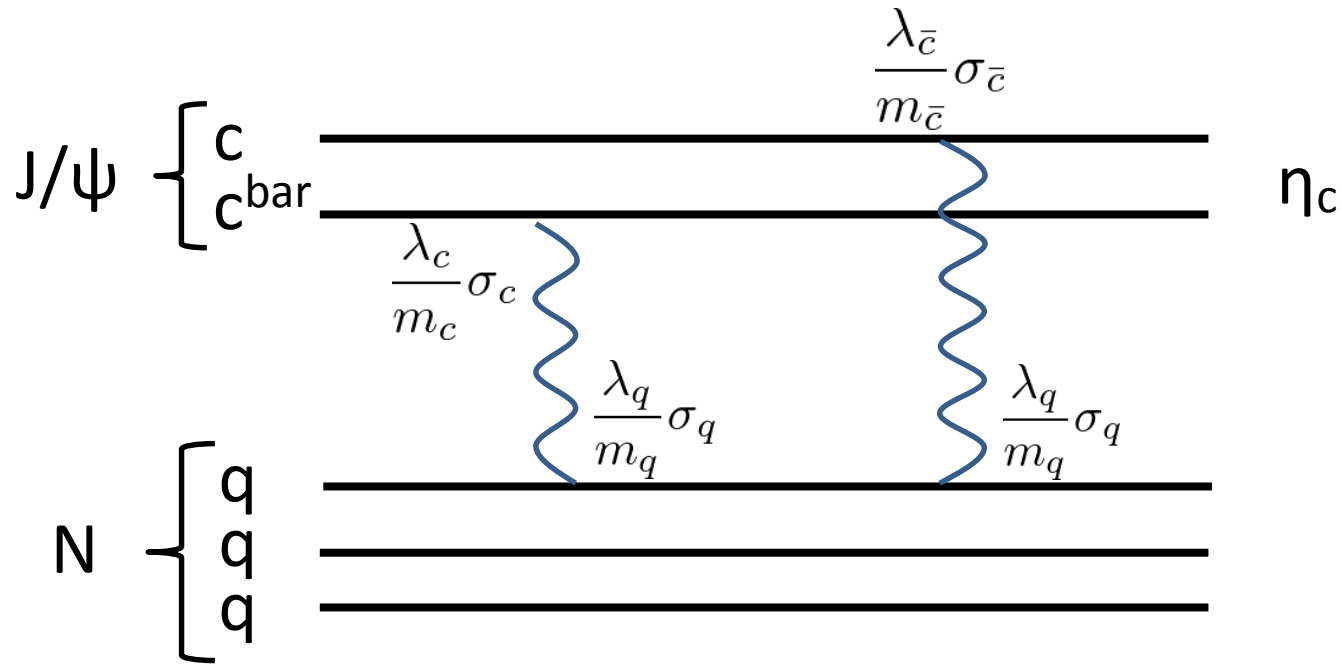
(a) Coherent mixing channels

	J			L
$J/\psi - N$	$1/2^-$	\rightarrow	$\eta_c - N$	0
	$3/2^-$	\rightarrow	$\eta_c - N$	2
$J/\psi - \text{deuteron}$	0^-	\rightarrow	\times	
	1^-	\rightarrow	$\eta_c - \text{deuteron}$	0, 2
	2^-	\rightarrow	$\eta_c - \text{deuteron}$	2
$J/\psi - {}^4\text{He}$	1^-	\rightarrow	\times	
$J/\psi - {}^6\text{Li}$	0^-	\rightarrow	\times	
	1^-	\rightarrow	$\eta_c - {}^6\text{Li}$	0, 2
	2^-	\rightarrow	$\eta_c - {}^6\text{Li}$	2
$J/\psi - {}^7\text{Li}$	$1/2^-$	\rightarrow	$\eta_c - {}^7\text{Li}$	0
	$3/2^-$	\rightarrow	$\eta_c - {}^7\text{Li}$	2
$J/\psi - {}^8\text{Be}$	1^-	\rightarrow	\times	

	J^π
${}^2\text{H}$ (d)	1^+
t	$1/2^+$
${}^3\text{He}$	$1/2^+$
${}^4\text{He}$ (α)	0^+
${}^6\text{Li}$	1^+
${}^7\text{Li}$	$1/2^+$
${}^8\text{Be}$	0^+

Table 1: Coherent mixing channels of J/ψ -nucleus and η_c -nucleus systems for several light nuclei. L denotes the orbital angular momentum between charmonium and nucleus.

The mixing of J/ψ-nucleus and η_c-nucleus channels



$$\propto \frac{1}{m_c m_q} (\vec{\lambda}_c \cdot \vec{\lambda}_q) (\vec{\sigma}_c \cdot \vec{\sigma}_q) \delta(r) \quad \text{Color magnetic interaction}$$

- The spin flip process is suppressed by $1/m_c^2$.

Spin averaged J/ψ - N potential in J/ψ - NN system

Possible states for J/ψ - NN system

T (isospin)	J	S_{NN}	$S_{J/\psi-N}$
0	0	1	1/2
0	1	1	1/2, 3/2
0	2	1	3/2

J/ψ - N potential

$$\begin{aligned}
 \underline{v_{J/\psi-N}(r)} &= (v_0 + v_s(\mathbf{S}_{J/\psi} \cdot \mathbf{S}_N))e^{-\mu r^2} \\
 &= \begin{cases} \underline{(v_0 - v_s)} e^{-\mu r^2} & (S_{J/\psi-N} = 1/2) \\ \underline{(v_0 + \frac{1}{2}v_s)} e^{-\mu r^2} & (S_{J/\psi-N} = 3/2) \end{cases} \\
 &\equiv v_{\text{eff}}(S_{J/\psi-N}) e^{-\mu r^2}
 \end{aligned}$$

Spin averaged J/ψ - N potential in J/ψ - NN system

$$V_{\text{eff}}^{(J,T)} e^{-\mu r^2} \equiv \left\langle (NN)_{S_{NN}}, J/\psi; J \left| \underline{v_{J/\psi-N}(r)} \right| (NN)_{S_{NN}}, J/\psi; J \right\rangle$$

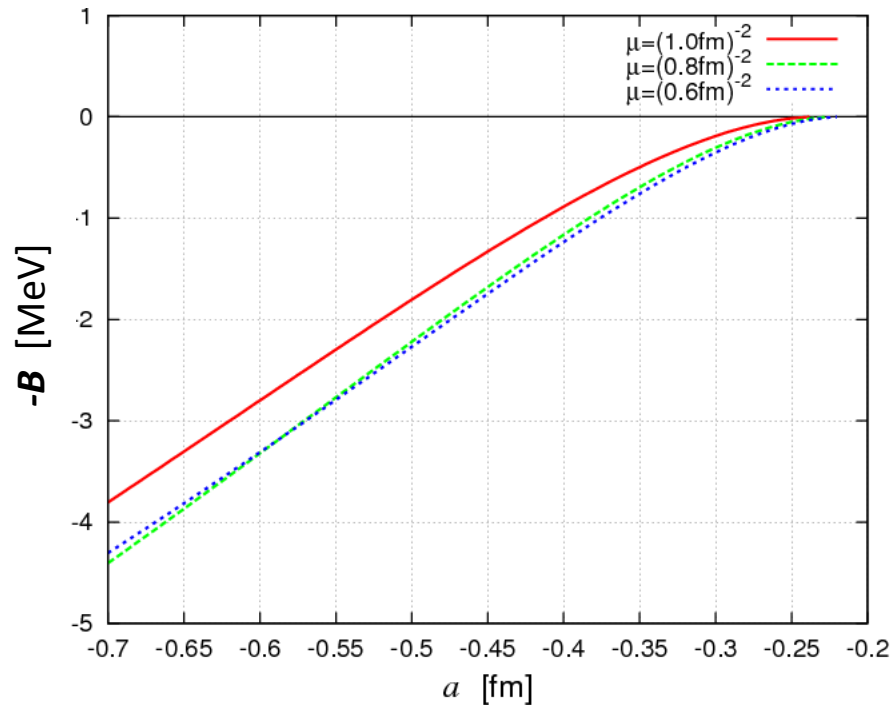
$$V_{\text{eff}}^{(0,0)} = \underline{v_0 - v_s} = v_{\text{eff}}(1/2) \quad (J = 0, S_{NN} = 1, T = 0)$$

$$V_{\text{eff}}^{(1,0)} = v_0 - \frac{1}{2}v_s \quad (J = 1, S_{NN} = 1, T = 0)$$

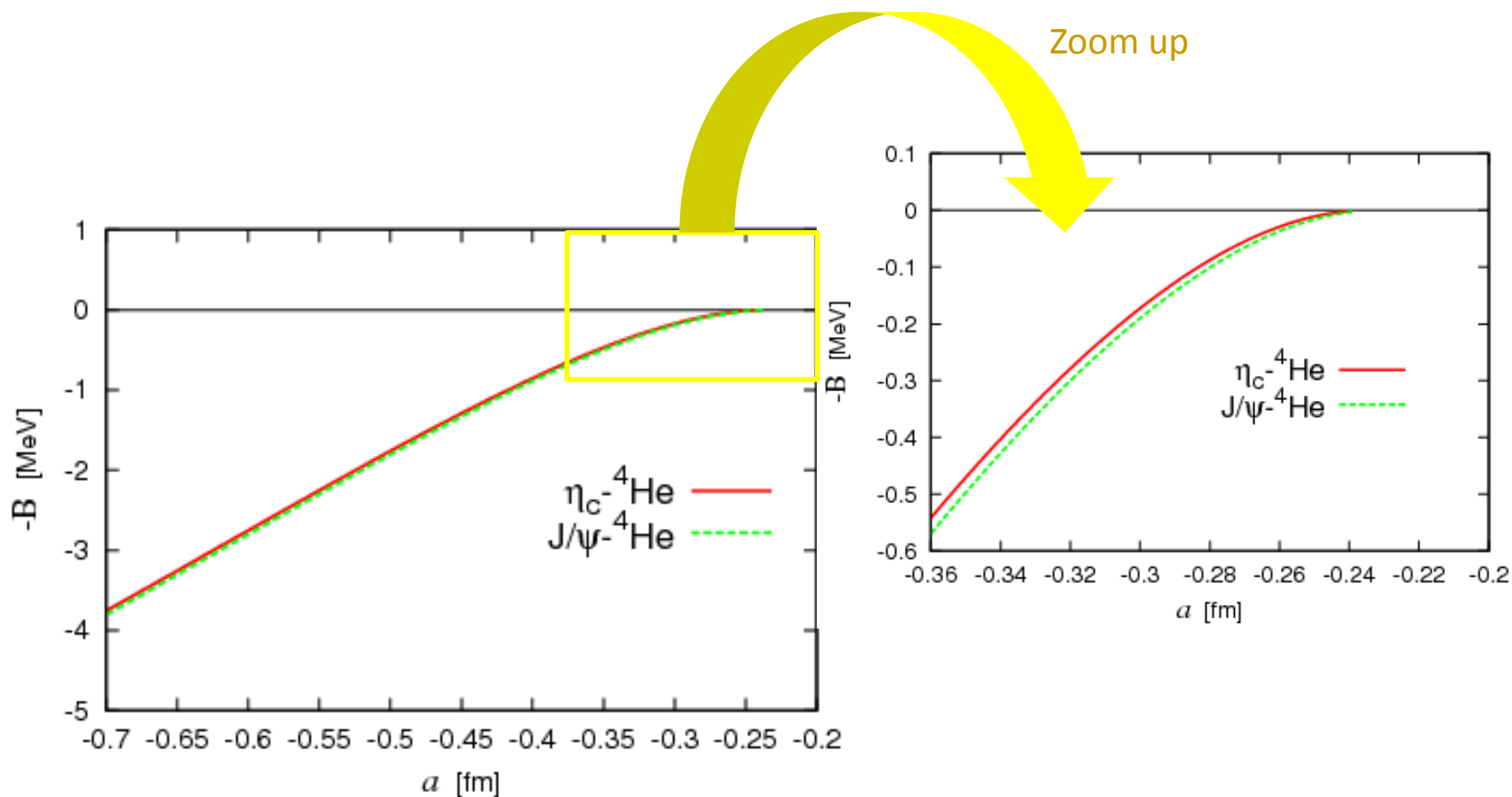
$$V_{\text{eff}}^{(2,0)} = \underline{v_0 + \frac{1}{2}v_s} = v_{\text{eff}}(3/2) \quad (J = 2, S_{NN} = 1, T = 0)$$

Range dependence of J/ ψ - ^4He binding energy

J/ ψ - ^4He binding energy



Difference between $\eta_c - ^4\text{He}$ and $J/\psi - ^4\text{He}$



Potential range parameter: $\mu = (1.0 \text{ fm})^{-2}$

$$J/\psi - \alpha - \alpha$$

Relations between scattering length a of $J/\psi - N$ and binding energy B of $J/\psi - {}^4\text{He}$ and $J/\psi - \alpha - \alpha$

