Fluctuations at the QCD phase transition from dynamical models

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— Insight into QCD matter from heavy-ion collisions —

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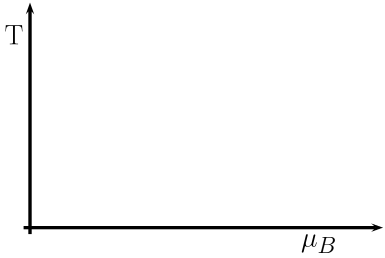
QCD phase diagram

Understanding the phase diagram of the strong interaction, QCD, is extremely difficult, because

- there is no global analytical approach to solve QCD, and
- one cannot produce a long-lived and controlable system of QCD matter to explore experimentally.

How can we still hope to fill this phase diagram?



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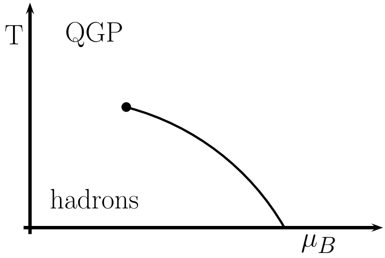
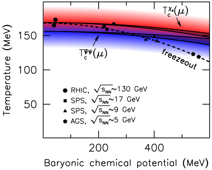


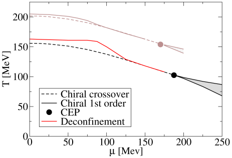
Approaches to the QCD phase diagram

- QCD calculations in the nonperturbative regime:

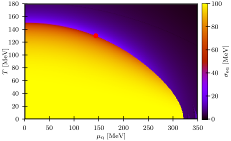
LQCD Wuppertal-Budapest JHEP **0203** (2002), JHEP **1104** (2011)



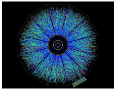
DSE C. Fischer, J. Luecker, PLB**718** (2013)



- Effective models of QCD:



- Heavy-ion collisions:

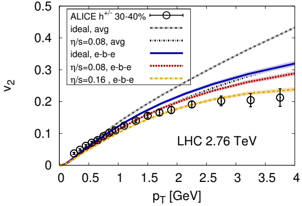


FAIR SHINE NA61 NICA

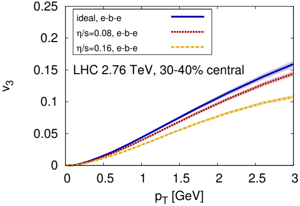
Fluid dynamical description of heavy-ion collisions

- The discovery of RHIC: The QGP is an almost ideal strongly coupled fluid.
- Early hydrodynamic calculations reproduce spectra and elliptic flow P. Kolb, U. Heinz, QGP (2003).
- Long road of improvements during the last decade:
  $(3+1\mathrm{d})$, viscosity, initial conditions, initial state fluctuations, hybrid models

elliptic flow at LHC



triangular flow at LHC



MUSIC by B. Schenke, S. Jeon, C. Gale PLB**702** (2011)

What is fluiddynamics?

- Two time scales:
  - fast processes $\Rightarrow$ local equilibration
  - slow processes $\Rightarrow$ change of conserved charges (energy, momentum, charge)
- General dynamics:

$$\partial_\mu T^{\mu\nu} = 0\,,$$
$$\partial_\mu N^\mu = 0$$

- Properties of the system enter via the equation of state and transport coefficients.

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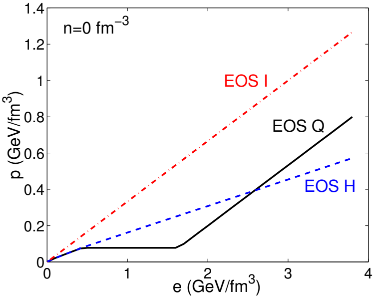
Phase transitions are easy to implement in fluiddynamics!

Equation of state - phase transition

- Build an equation of state from the QGP and the hadronic phase.

- Assume an noninteracting gas of hadronic resonances below $T_C \Rightarrow c_s^2 = \partial p / \partial e \sim 0.15$ soft
- QGP: gas of noninteracting quarks and gluons subject to an external bag pressure B $\Rightarrow$ eos: $p = 1/3e - 4/3B$ stiff
- Joined together by a Maxwell construction.



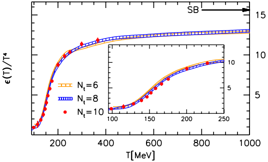
P. Kolb, U. Heinz, 2003

The speed of sound vanishes during the phase coexistence. However, no clear experimental signals were found corresponding to the “softest point” of the eos.

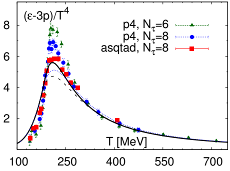
C. Hung, E. Shuryak, PRL**75** (1995)

Equation of state - lattice QCD

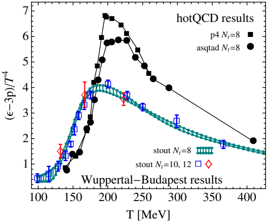
- At $\mu_B = 0$ the eos can be calculated on the lattice:



Wuppertal-Budapest, JHEP **1011** (2010)



P. Huovinen, P. Petreczky, NPA**837** (2010)



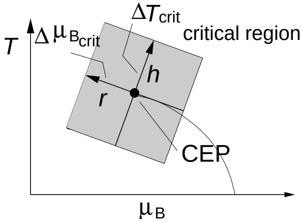
Wuppertal-Budapest, JHEP **1011** (2010)

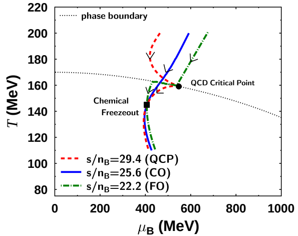
- Strong increase in the energy density around T_c due to the liberation of color dof.
- Compares well to the HRG eos below T_c $\Rightarrow$ allows for parametrizing the QCD eos for use in hydrodynamic simulations.
- Some differences between two lattice QCD groups.

Equation of state - critical point

- Construct an eos with CP from the universality class of the 3d Ising model.
- Map the temperature and the external magnetic field (r, h) onto the (T, μ)-plane $\Rightarrow$ critical part of the entropy density S_c.
- Match with nonsingular entropy density from QGP and the hadron phase:

$$s = 1/2(1 - \tanh S_c)s_H \\ + 1/2(1 + \tanh S_c)s_{QGP}$$





- Focussing of trajectories $\Rightarrow$ Different behavior of $\bar{p}/p$ yields.

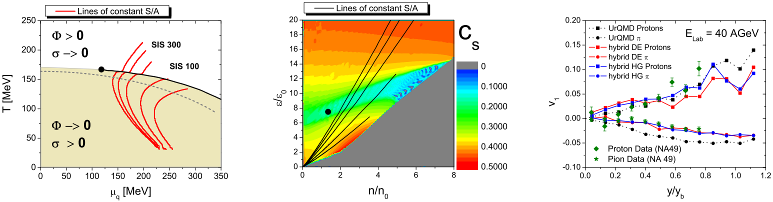
C. Nonaka, M. Asakawa PRC**71** (2005); M. Asakawa, S. Bass, B. Mueller, C. Nonaka PRL**101** (2008)

Equation of state - effective models

- Equations of state can be obtained from effective model Lagrangians.
- Hadronic SU(3) non-linear sigma model including quark degrees of freedom yields a realistic structure of the phase diagram and phenomenologically acceptable results for saturated nuclear matter.

V. Dexheimer, S. Schramm, PRC**81** (2010)

- The influence of the eos on the directed flow or mean transverse momentum spectra is negligible.



J. Steinheimer, V. Dexheimer, H. Petersen, M. Bleicher, S. Schramm, H. Stoecker, PRC**81** (2010)

Phase transitions are easy to implement in fluiddynamics on the level of the equation of state.

BUT:

Phase transitions are difficult to implement in fluiddynamics on the level of fluctuations!

- Conventional fluiddynamics locally propagates thermal averages
- Fluctuations really matter at the critical point...

Fluctuations at the critical point

- Coupling of the order parameter to pions $g\sigma\pi\pi$ and protons $G\sigma\bar{p}p \Rightarrow$ fluctuations in multiplicity distributions

$$\langle(\delta N)^2\rangle \propto \langle(\Delta\sigma)^2\rangle \propto \xi^2$$

ξ: correlation length, diverges at the CP

M. Stephanov, K. Rajagopal, E. Shuryak, PRL **81** (1998), PRD **60** (1999)

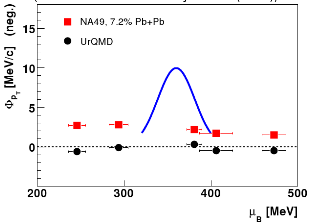
- Higher cumulants are more sensitive to the CP
skewness: $\langle(\delta N)^3\rangle \propto \xi^{4.5}$
kurtosis: $\langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2 \propto \xi^7$

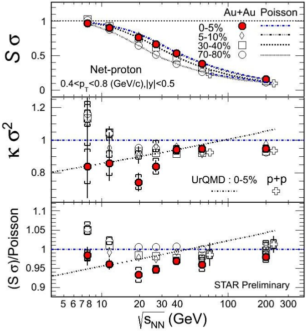
M. Stephanov, PLB **102** (2009), PRL **107** (2011)

- Experimental difficulties, baryon number conservation

MN et al. EPJ C**72** (2012)
A. Bzdak, V. Koch, PRC**86** (2012), PRC**87** (2013)

(NA49 collaboration J. Phys. G **35** (2008))





(STAR collaboration, QM2012)

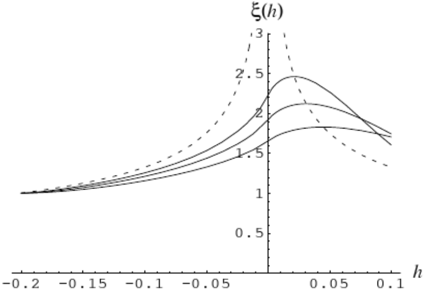
Fluctuations at the critical point

- Long relaxation times near a critical point
  $\Rightarrow$ the system is driven out of equilibrium (critical slowing down)!
- Phenomenological equation:

$$\frac{\mathrm{d}}{\mathrm{d}t} m_\sigma(t) = -\Gamma[m_\sigma(t)](m_\sigma(t) - \frac{1}{\xi_{\mathrm{eq}}(t)})$$

with $\Gamma(m_\sigma) = \frac{A}{\xi_0}(m_\sigma \xi_0)^z$
$z = 3$
(dynamic) critical exponent from model H in Hohenberg-Halperin

$\Rightarrow \xi \sim 1.5 - 2.5$ fm

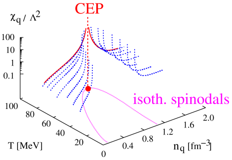


(B. Berdnikov and K. Rajagopal, PRD **61** (2000)); D.T.Son, M.Stephanov, PRD **70** (2004); M.Asakawa, C.Nonaka, Nucl. Phys. A774 (2006))

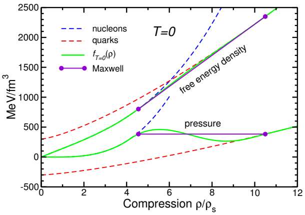
Fluctuations at the phase transition in heavy-ion collisions

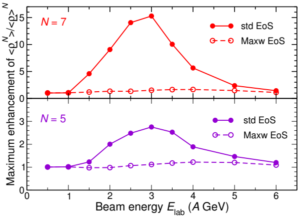
- Large nonstatistical fluctuations in nonequilibrium situations of single events.
- Instability of slow modes in the spinodal region (spinodal decomposition)

I. Mishustin, PRL **82** (1999)
C. Sasaki, B. Friman, K. Redlich, PRD **77** (2008)



- Significant amplification of initial density irregularities





J. Steinheimer, J. Randrup, PRL **109** (2012), arXiv:1302.2956

Heavy-ion collisions are

- inhomogeneous
- finite in space and time
- and highly dynamic.

? Can nonequilibrium effects become strong enough to develop signals of the first order phase transition?

? Do enhanced equilibrium fluctuations at the critical point survive the dynamics?

Goal:
Combine the fluid dynamical description of heavy-ion collisions with fluctuation phenomena at the phase transition!

- Explicit propagation of the order parameter(s) – NχFD
- Fluid dynamical fluctuations

Nonequilibrium chiral fluid dynamics - NχFD

- Langevin equation for the sigma field: damping and noise from the interaction with the quarks (QM model)

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

- For PQM: phenomenological dynamics for the Polyakov-loop

$$\eta_\ell \partial_t \ell T^2 + \frac{\partial V_{\mathrm{eff}}}{\partial \ell} = \xi_\ell$$

- Fluid dynamic expansion of the quark fluid = heat bath, including energy-momentum exchange

$$\partial_\mu T_{\mathrm{q}}^{\mu\nu} = S^\nu = -\partial_\mu T_\sigma^{\mu\nu}$$

$\Rightarrow$ includes a stochastic source term!

- Nonequilibrium equation of state $p = p(e, \sigma)$

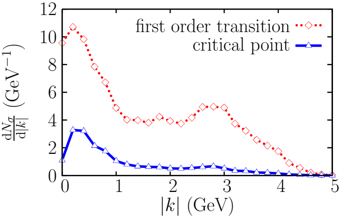
Selfconsistent approach within the 2PI effective action!

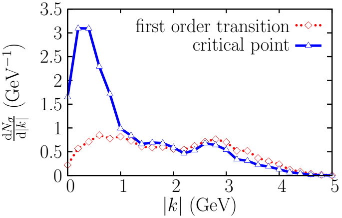
MN, S. Leupold, C. Herold, M. Bleicher, PRC **84** (2011); MN, S. Leupold, M. Bleicher, PLB **711** (2012);
MN, C. Herold, S. Leupold, I. Mishustin, M. Bleicher, arXiv:1105.1962; C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013)

Dynamics versus equilibration

- Quantify the fluctuations of the order parameter:

$$\frac{\mathrm{d}N_\sigma}{\mathrm{d}^3k} = \frac{(\omega_k^2|\sigma_k|^2 + |\partial_t\sigma_k|^2)}{(2\pi)^3 2\omega_k}$$



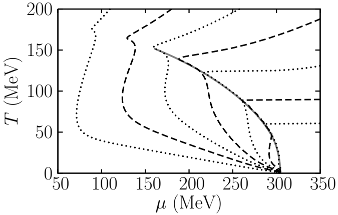


- Strong enhancement of the intensities for a first order phase transition **during the evolution**.
- Strong enhancement of the intensities for a critical point scenario **after equilibration**.

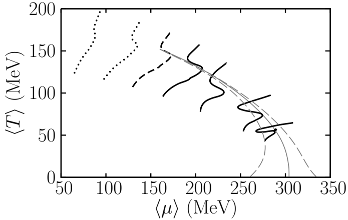
C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013)

Trajectories and isentropes at finite μ_B

Isentropes in the PQM model



Fluid dynamic trajectories

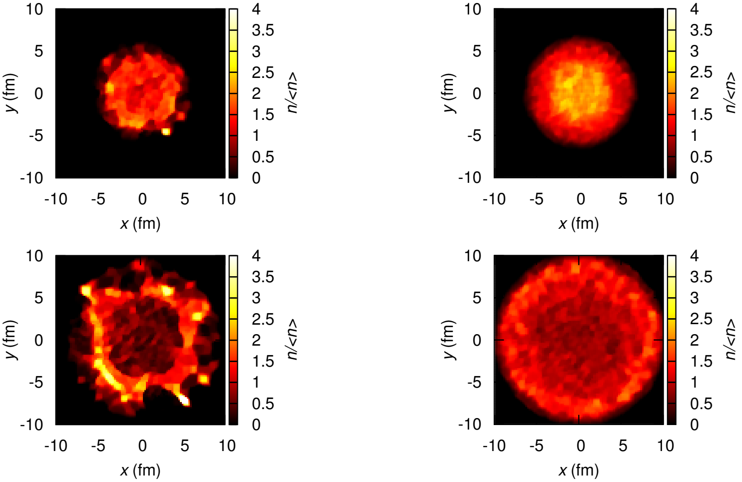


- Fluid dynamic trajectories differ from the isentropes due to interaction with the fields.
- No significant features in the trajectories left of the critical point.
- Right to the critical point: system spends significant time in the spinodal region! $\Rightarrow$ possibility of spinodal decomposition!

Irregularities in net-baryon densities

first-order phase transition

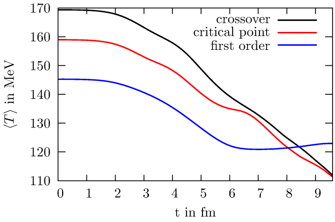
critical point



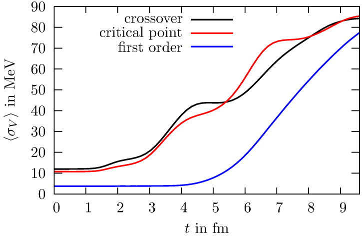
C. Herold, MN, I. Mishustin, M. Bleicher, arxiv:1304.5372

Dynamic enhancement of event-by-event fluctuations

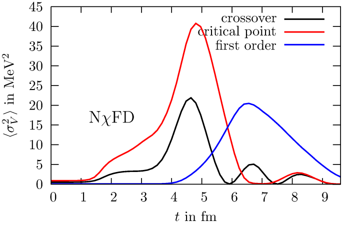
temperature



averaged sigma field



event-by-event fluctuations



- Initial fluctuations of the sigma field from initial fluctuations in T.
- Enhanced event-by-event fluctuations of the order parameter at T_c.

Relativistic theory of fluid dynamical fluctuations

Conventional fluiddynamics propagates thermal averages of the energy density, pressure, velocities, charge densities, etc.

However, ...

- ... already in equilibrium there are thermal fluctuations
- ... the fast processes, which lead to local equilibration also lead to noise!

Conventional ideal fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}}$$
$$N^{\mu} = N^{\mu}_{\mathrm{eq}}$$

Relativistic theory of fluid dynamical fluctuations

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Conventional viscous fluid dynamics:

$$
\begin{aligned}
T^{\mu\nu} &= T^{\mu\nu}_{\mathrm{eq}} + \Delta T^{\mu\nu}_{\mathrm{visc}} \\
N^{\mu} &= N^{\mu}_{\mathrm{eq}} + \Delta N^{\mu}_{\mathrm{visc}}
\end{aligned}
$$

Relativistic theory of fluid dynamical fluctuations

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Stochastic viscous fluid dynamics:

$$
\begin{aligned}
T^{\mu\nu} &= T^{\mu\nu}_{\text{eq}} + \Delta T^{\mu\nu}_{\text{visc}} + \Xi^{\mu\nu} \\
N^{\mu} &= N^{\mu}_{\text{eq}} + \Delta N^{\mu}_{\text{visc}} + \mathrm{I}^{\mu}
\end{aligned}
$$

Relativistic theory of fluid dynamical fluctuations

The noise terms are such that averaged quantities exactly equal the conventional quantities:

$$\langle T^{\mu\nu} \rangle = T^{\mu\nu}_{\mathrm{eq}} + \Delta T^{\mu\nu}_{\mathrm{visc}} \qquad \text{with} \quad \langle \Xi^{\mu\nu} \rangle = 0$$

$$\langle N^{\mu} \rangle = N^{\mu}_{\mathrm{eq}} + \Delta N^{\mu}_{\mathrm{visc}} \qquad \text{with} \quad \langle \mathrm{I}^{\mu} \rangle = 0$$

The two formulations will, however, differ when one calculates correlation functions:

$$\langle T^{\mu\nu}(x) T^{\mu\nu}(x') \rangle$$

$$\langle N^{\mu}(x) N^{\mu}(x') \rangle$$

Relativistic theory of fluid dynamical fluctuations

In linear response theory the retarded correlator $\langle T^{\mu\nu}(x)T^{\mu\nu}(x')\rangle$ gives the viscosities and $\langle N^{\mu}(x)N^{\mu}(x')\rangle$ the charge conductivities via the dissipation-fluctuation theorem (Kubo-formula)!

It means that when dissipation is included also fluctuations need to be included!

Relativistic theory of fluid dynamical fluctuations

recent interest in fluid dynamical fluctuations in relativistic (e.g heavy-ion collisions) and non-relativistic fluids (ultra-cold gases).

- J. Kapusta, B. Mueller, M. Stephanov PRC**85** (2012)
- P. Kovtun, J.Phys. A**45** (2012)
- J. Kapusta, J. Torres-Rincon PRC**86** (2012)
- C. Young, arXiv:1306.0472
- K. Murase and T. Hirano, arXiv:1304.3243
- C. Chafin and T. Schäfer, PRA**87** (2013)
- P. Romatschke and R. E. Young, PRA**87** (2013)

Relativistic theory of fluid dynamical fluctuations

As an example: consider only the conservation equation $\partial_\mu T^{\mu\nu} = 0$ (no baryon current).

Procedure:

- Write down the linearized fluid dynamical equations.
- Solve the coupled equations in linear response theory $\Rightarrow$ retarded Green's functions.
- Construct $\langle T^{\mu\nu}(x) T^{\mu\nu}(x') \rangle$ from the conservation equation and via the dissipation-fluctuation theorem.

Relativistic theory of fluid dynamical fluctuations

first-order (second-order) relativistic viscous fluid dynamics

$$
\begin{aligned}
T^{\mu\nu} &= e u^\mu u^\nu - (p+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \\
N^\mu &= n_B u^\mu + \nu_B^\mu \\
\Delta^{\mu\nu} &= g^{\mu\nu} - u^\mu u^\nu \\
\pi^\mu\nu &= \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3} g^{\alpha\beta}\partial_\mu u^\mu\right) - \tau_\eta \dot{\pi}^{\mu\nu} \\
\Pi &= \zeta \partial_\mu u^\mu - \tau_\zeta \dot{\Pi}
\end{aligned}
$$

In principle, one needs to solve the equations for (second-order) relativistic viscous fluid dynamics to preserve causality.

Here, as an example: first-order!

Relativistic theory of fluid dynamical fluctuations

- Linearized hydro equations: small fluctuations $\bar{e}+\delta e$, $\bar{p}+\delta p$ and δv^i
  with: $\delta T^{00}=\delta e$ and $\delta T^{ij}=m^i=(\bar{e}+\bar{p})v^i=\bar{w}v^i$

$$\begin{aligned}\partial_t \mathbf{m}_\perp + \eta/\bar{w}\mathbf{k}^2\mathbf{m}_\perp &= 0\\ \partial_t\delta e + i\mathbf{k}\cdot\mathbf{m}_{||} &= 0\\ \partial_t\mathbf{m}_{||} + iv_s^2\mathbf{k}\delta e + \gamma_v\mathbf{k}^2\mathbf{m}_{||} &= 0\end{aligned}$$

- Speed of sound v_s from the equation of state,
  $\gamma_v=(4/3\eta+\zeta)/\bar{w}$.
- Transverse momentum densities decouple, remaining equations can be written in compact form: $\partial_t\phi_a + M_{ab}=0$

Relativistic theory of fluid dynamical fluctuations

$\Rightarrow$ then the retarded Green's function can be calculated via:

$$G^{\mathrm{ret}}_{ab}(\omega, \mathbf{k}) = -(\mathbb{1} + i\omega(-i\omega\delta_{ab} + M)^{-1})\chi$$

- The static susceptibilities χ_{ab} - response to an external source.
- sources contribute time-dependent parts to the Hamiltonian, here:

$$\delta H = -\int \mathrm{d}^3x \left(\frac{\delta T}{T}\delta e + v_{||} m_{||}\right) \quad \Rightarrow \quad \lambda = (\delta T/T, v_{||})$$

- then $\chi = \frac{\partial \phi_a}{\partial \lambda_b} = \begin{pmatrix} c_v T & 0 \\ 0 & \bar{w} \end{pmatrix}$

Relativistic theory of fluid dynamical fluctuations

- retarded Green's function for δe and $\mathbf{m}_{||}$:

$$G^{\mathrm{ret}}_{ab}(\omega, \mathbf{k}) = \frac{\bar{w}}{\omega^2 - v_s^2\mathbf{k}^2 + i\omega\gamma_s\mathbf{k}^2} \begin{pmatrix} \mathbf{k}^2 & \omega|\mathbf{k}| \\ \omega|\mathbf{k}| & v_s^2\mathbf{k}^2 - i\omega\gamma_s\mathbf{k}^2 \end{pmatrix}$$

- including the transverse momentum density:

$$G^{\mathrm{ret}}_{m_i,m_j}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right) \frac{\eta\mathbf{k}^2}{i\omega - \gamma_\eta\mathbf{k}^2} + \frac{k_i k_j}{\mathbf{k}^2} \frac{\bar{w}(v_s^2\mathbf{k}^2 - i\omega\gamma_s\mathbf{k}^2)}{\omega^2 - v_s^2\mathbf{k}^2 + i\omega\gamma_s\mathbf{k}^2}$$

- Kubo-formulas for viscosities:

$$\eta = -\frac{\omega}{2\mathbf{k}^2}\left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right) \Im G^{\mathrm{ret}}_{m_i m_j}(\omega, \mathbf{k} \to 0)$$

$$\zeta + \frac{4}{3}\eta = -\frac{\omega^3}{\mathbf{k}^4} \Im G^{\mathrm{ret}}_{ee}(\omega, \mathbf{k} \to 0)$$

Relativistic theory of fluid dynamical fluctuations

$$
\begin{aligned}
\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle \Xi^{\mu 0}(x)\Xi^{\mu 0}(x')\rangle^S &= -\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle T^{\mu 0}(x)T^{\mu 0}(x')\rangle^S \\
&= \int \frac{\mathrm{d}\omega}{2\pi}\int\frac{\mathrm{d}^3k}{(2\pi)^3} e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')} e^{-i\omega(t-t')}\times \\
&\quad \times\left(\omega^2 \underbrace{G^S_{ee}(\omega,\mathbf{k})}_{\text{FDT}} - 2\omega|\mathbf{k}|\underbrace{G^S_{em_{||}}(\omega,\mathbf{k})}_{\text{FDT}} + \mathbf{k}^2\underbrace{G^S_{m_{||}m_{||}}(\omega,\mathbf{k})}_{\text{FDT}}\right) \\
&\qquad G^S_{ab}(\omega,\mathbf{k}) = -\frac{2T}{\omega}\Im G^{\mathrm{ret}}_{ab}(\omega,\mathbf{k}) \\
&= 0
\end{aligned}
$$

$\Rightarrow$ no noise term in the first fluid dynamical equation!

Relativistic theory of fluid dynamical fluctuations

many more terms in the second fluid dynamical equation, but same procedure:

$$\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle \Xi^{\mu i}(x)\Xi^{\mu j}(x')\rangle^S = -\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle T^{\mu i}(x)T^{\mu j}(x')\rangle^S$$

$$= 2T\left[\left(\zeta+\frac{4}{3}\eta\right)\partial_i\partial_j + \eta(\delta_{ij}\nabla^2 - \partial_i\partial_j)\right]\delta^4(x-x')$$

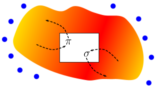
$\Rightarrow$

$$\langle \Xi^{ij}(x)\Xi^{kl}(x')\rangle^S = 2T\left[\left(\zeta-\frac{2}{3}\eta\right)\delta^{ij}\delta^{kl} + \eta(\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk})\right)\Big]\delta^4(x-x')$$

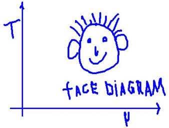
boost back to the Landau-Lifschitz frame

$$\langle \Xi^{\mu\nu}(x)\Xi^{\alpha\beta}(x')\rangle^S = 2T\left[\left(\zeta-\frac{2}{3}\eta\right)\Delta^{\mu\alpha}\Delta^{\nu\beta} + \eta(\Delta^{\mu\beta}\delta^{\nu\alpha}+\Delta^{\mu\nu}\Delta^{\alpha\beta})\right)\Big]\delta^4(x-x')$$

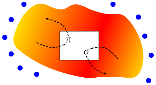
To do list



- Derive fluid dynamical fluctuations for the coupled system including the net-baryon number current in second-order viscous fluid dynamics and implement it numerically.
- Make a realistic choice for the equation of state and the transport parameters.
- For a comparison to experiment, acknowledge that there other sources of fluctuations and correlations in heavy-ion collisions, e.g. initial fluctuations, flow...
- Final state interactions. Would any signal survive?



Summary



- Heavy-ion collisions can successfully be described by fluiddynamics (possibly viscous and as a part of hybrid models)
- Equations of state from the universality class and effective model approaches.
- Nonequilibrium effects can become strong enough to develop signals of the first order phase transition.
- There are indications from NχFD for a dynamical enhancement of event-by-event-fluctuations of the order parameter (σ) at the critical point
- Fluid dynamical fluctuations play an important role at the critical point!