Induced Magnetic Moment in Effective Models of Quarks

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OUTLINE

EJF and V. de la Incera, Phys.Rev.Lett.102:050402,2009, NPB 824 (2010) 217-238
B. Feng, EJF and V. de la Incera, NPB B853 (2011) 213-239
EJF, V. de la Incera, I. Portillo and M. Quiroz, arXiv: 1311.3400 [nucl-th]

- ***Magnetic Moment: Its Early History***
- ***AMM in Massless QED***
- ***MM of Chiral Pairs***
- ***MM of Cooper Pairs***
- ***Concluding Remarks***

Magnetic Moment: Non-Relativistic Case

Schrodinger Eq. at H ≠ 0

$$H_S\psi = E_S\psi \ ,$$

$$H_S = \frac{1}{2m}(\widehat{\overrightarrow{p}} - e\overrightarrow{A})^2$$

$$E_S = \frac{p_z^2}{2m} + \frac{eH}{m}(n + \frac{1}{2}) \qquad n = 0, 1, 2, ...,$$

Pauli-Schrodinger Eq. at H ≠ 0

$$H_P = \frac{1}{2m}(\widehat{\overrightarrow{p}} - e\overrightarrow{A})^2 - \widehat{\overrightarrow{\mu}} \cdot \overrightarrow{H}$$

$$\widehat{\overrightarrow{\mu}} = g\mu_B \widehat{\overrightarrow{s}} \qquad g = 2 \qquad \mu_B = (e/2m) \qquad \widehat{\overrightarrow{s}} = \overrightarrow{\sigma}/2$$

$$E_P = \frac{p_z^2}{2m} + \frac{eH}{m}(n + \frac{1}{2}) - \mu_B\sigma H \qquad n = 0, 1, 2, ..., \qquad \sigma = \pm 1$$

$$E_P = \frac{p_z^2}{2m} + 2\mu_B(n + \frac{1}{2} - \frac{\sigma}{2})H = \frac{p_z^2}{2m} + 2\mu_B l H \qquad l = 0, 1, 2, ...$$

Magnetic Moment: Relativistic Case

Dirac Eq. at H ≠ 0

$$(m + \Pi_\mu \gamma^\mu)\psi = 0$$

where

$$\Pi_\mu = \partial_\mu - eA_\mu$$

$$E_R = \pm\sqrt{m^2 + p_z^2 + 2eH(n + \frac{1}{2} - \frac{\sigma}{2})} \qquad n = 0, 1, 2, ..., \qquad \sigma = \pm 1$$

In the Non-Relativistic limit: $2eH/m^2 \ll 1$ and $p_z^2/m^2 \ll 1$

$$E_R = m\sqrt{1 + \frac{p_z^2}{m^2} + \frac{2eH}{m^2}(n + \frac{1}{2} - \frac{\sigma}{2})} \simeq m + \frac{p_z^2}{2m} + \frac{eH}{m}(n + \frac{1}{2} - \frac{\sigma}{2}),$$

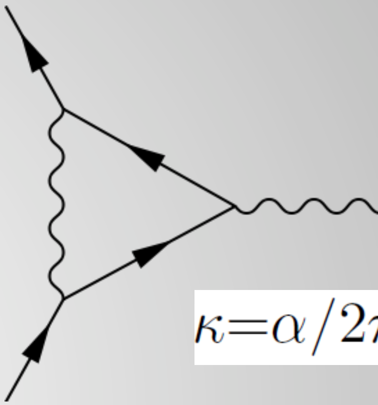
$$E_{NR} = E_R - m = \frac{p_z^2}{2m} + \frac{eH}{m}(n + \frac{1}{2} - \frac{\sigma}{2}) = \frac{p_z^2}{2m} + 2\mu_B l H$$

Electron Anomalous Magnetic Moment: QFT Case

Dirac Eq. with Radiative Corrections:

$$(m + \Pi_\mu \gamma^\mu - \kappa \mu_B H \Sigma_3)\psi = 0$$

$$\Sigma_3 = i\gamma_1\gamma_2 \text{ is the spin operator}$$



$$\kappa = \alpha/2\pi$$

$$E^2_{l,\sigma} = [(m^2 + 2eHl)^{1/2} - \mu_B \kappa H \sigma]^2 + p_z^2, \quad l = 0, 1, 2, ..., \quad \sigma = \pm 1$$

$$E_{NR} = \frac{p_z^2}{2m} + 2(n + \frac{1}{2})\mu_B H - 2(1+\kappa)\mu_B \frac{\sigma}{2} H, \quad n = 0, 1, 2, ..., \quad \sigma = \pm 1$$

$$= \frac{p_z^2}{2m} + 2(n + \frac{1}{2})\mu_B H - g' \mu_B \frac{\sigma}{2} H = \frac{p_z^2}{2m} + 2(l - \kappa)\mu_B H$$

Modified Lande *g*-factor:

$$\widehat{\vec{\mu}}' = g' \mu_B \widehat{\vec{s}} \qquad g' = 2(1 + \frac{\alpha}{2\pi})$$

Massless QED

For massless QED we cannot follow Schwinger's approach because an anomalous magnetic moment would break the chiral symmetry of the massless theory, but this symmetry is protected against perturbative corrections.

$$\Sigma(x,x') = (Z_{||}\Pi^{||}_{\mu}\gamma^{\mu}_{||} + Z_{\perp}\Pi^{\perp}_{\mu}\gamma^{\mu}_{\perp} + M + \frac{T}{2}\widehat{F}^{\mu\nu}\sigma_{\mu\nu})\delta^4(x-x')$$

However, the chiral symmetry can be broken dynamically via non-perturbative effects.

$$E_n(p_z) = \pm\sqrt{p_z^{\;2} + 2|eB|n + m^2},$$
p_z *is the momentum* $\parallel$ *to* B, *and*
$n = 0, 1, 2, \ldots$ *are the Landau levels.*

In the lowest Landau level n=0, so the energy become $E = \sqrt{p_z^{\;2} + m^2}$
and, if m=0, there is no gap between the vacuum and the LLL.

The pairing dynamics around the LLL is essentially 1+1 dimensional, so any attractive interaction between fermion excitations around the LLL becomes stronger.

Chiral Condensate $\langle \overline{\Psi}\Psi \rangle \neq 0 \Rightarrow m_d \neq 0$

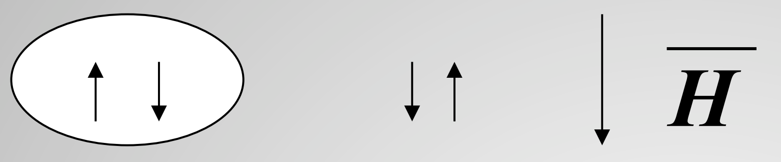
Magnetic Catalysis

Gusynin, Miransky and Shovkovy, NPB 462 (1996); PRL 349 (1995)

Chiral Condensate at $H \neq 0$

Cooper Pair

S=0

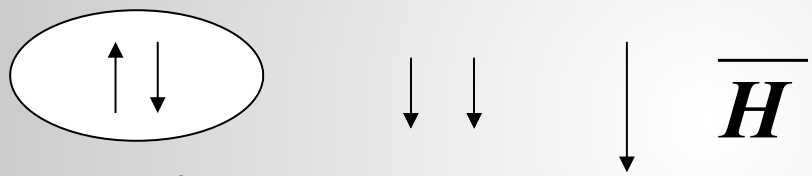


$\langle e^- e^- \rangle$

tends to break the condensate

Chiral Condensate

S=0



$\langle e^- e^+ \rangle$

tends to stabilize the condensate

H makes a pre-existing chiral condensate more stable.

H can induce the generation of the chiral condensate at weak coupling.

H can induce the generation of an anomalous magnetic moment.

Schwinger-Dyson Equation in the Quenched Ladder Approximation

$$\Sigma(x,x') = ie^2\gamma^\mu G(x,x')\gamma^\nu D_{\mu\nu}(x-x')$$

where

$$D_{\mu\nu}(x-x') = \int \frac{d^4}{(2\pi)^4}\frac{e^{iq\cdot(x-x')}}{q^2-i\epsilon}(g_{\mu\nu}-(1-\xi)\frac{q_\mu q_\nu}{q^2})$$

$$G(x,x') = \sum\!\!\!\!\!\!\int_l \frac{d^4p"}{(2\pi)^4}\overline{E}^{l"}_{p"}(x)\Pi(l")\widetilde{G}^{l"}(\overline{p}")E^{l"}_{p"}(x')$$

$$\widetilde{G}^l(\overline{p}) = \frac{1}{\overline{p}\cdot\gamma - \Sigma^l(\overline{p})}$$

$$\Sigma(p,p') = \int d^4x d^4y \overline{E}^l_p(x)\Sigma(x,y)E^l_p(y) = (2\pi)^4\widehat{\delta}^{(4)}(p-p')\Pi(l)\widetilde{\Sigma}^l(\overline{p})$$

$$\widetilde{\Sigma}^l(\overline{p}) = Z^l_{||}\overline{p}^\mu_{||}\gamma^{||}_\mu + Z^l_\perp\overline{p}^\mu_\perp\gamma^\perp_\mu + M^l I + iT^l\gamma^1\gamma^2$$

$$\Pi(l) = \Delta(+)\delta^{l0} + I(1-\delta^{l0})$$

$$I = \Delta(+) + \Delta(-)$$

Ritus' Transformation

$$E_p^l(x) = E_p^+(x)\Delta(+) + E_p^-(x)\Delta(-)$$

where

$$\Delta(\pm) = \frac{I \pm i\gamma^1\gamma^2}{2}$$

are the spin up (+) and down (-) projectors, and

$$E_p^+(x) = N(l)e^{i(p_0x^0+p_2x^2+p_3x^3)}D_l(\rho),$$
$$E_p^-(x) = N(l-1)e^{i(p_0x^0+p_2x^2+p_3x^3)}D_{l-1}(\rho)$$

are the eigenfunctions with normalization constant

$$N(l) = (4\pi|eH|)^{1/4}/\sqrt{l!}$$

and $D_l(\rho)$ the parabolic cylinder functions with argument $\rho = \sqrt{2|eH|}(x_1 - p_2/|eH|)$

Orthogonality Condition:

$$\int d^4x \overline{E}_p^l(x) E_{p'}^{l'}(x) = (2\pi)^4 \widehat{\delta}^{(4)}(p-p')\Pi(l)$$

with

$$\overline{E}_p^l \equiv \gamma^0 (E_p^l)^\dagger \gamma^0$$

$$\Pi(l) = \Delta(+)\delta^{l0} + I(1-\delta^{l0})$$

$$\widehat{\delta}^{(4)}(p-p') = \delta^{ll'}\delta(p_0-p_0')\delta(p_2-p_2')\delta(p_3-p_3')$$

The Full Fermion Propagator in Momentum Space

From

$$\widetilde{G}^l(\overline{p}) = \frac{1}{\overline{p}\cdot\gamma - \Sigma^l(\overline{p})}$$

where

$$\widetilde{\Sigma}^l(\overline{p}) = Z^l_{\|}(\Lambda^+_{\|} - \Lambda^-_{\|})|\overline{p}_{\|}| + iZ^l_{\perp}(\Lambda^-_{\perp} - \Lambda^+_{\perp})|\overline{p}_{\perp}| + (M^l + T^l)\Delta(+) + (M^l - T^l)\Delta(-)$$

with projectors

$$\Lambda^{\pm}_{\|} = \frac{1}{2}(1 \pm \frac{\gamma^{\|}\cdot\overline{p}_{\|}}{|\overline{p}_{\|}|}), \qquad \Lambda^{\pm}_{\perp} = \frac{1}{2}(1 \pm i\gamma^2)$$

It's obtained

$$\begin{aligned}\widetilde{G}^l(\overline{p}) = &\frac{N^l(T,V_{\|})}{D^l(T)}\Delta(+)\Lambda^+_{\|} + \frac{N^l(T,-V_{\|})}{D^l(-T)}\Delta(+)\Lambda^-_{\|}\\ &+\frac{N^l(-T,V_{\|})}{D^l(-T)}\Delta(-)\Lambda^+_{\|} + \frac{N^l(-T,-V_{\|})}{D^l(T)}\Delta(-)\Lambda^-_{\|}\\ &-iV^l_{\perp}(\Lambda^+_{\perp} - \Lambda^-_{\perp})[\frac{\Delta(+)\Lambda^+_{\|} + \Delta(-)\Lambda^-_{\|}}{D^l(T)} + \frac{\Delta(+)\Lambda^-_{\|} + \Delta(-)\Lambda^+_{\|}}{D^l(-T)}]\end{aligned}$$

Where we introduced the notation

$$N^l(T,V_{\|}) = M^l - T^l - V^l_{\|},$$
$$D^l(T) = (M^l)^2 - (V^l_{\|} + T^l)^2 + (V^l_{\perp})^2$$

with $V^l_{\|} = (1 - Z^l_{\|})|\overline{p}_{\|}|$ and $V^l_{\perp} = (1 - Z^l_{\perp})|\overline{p}_{\perp}| = (1 - Z^l_{\perp})\sqrt{2|eH|l}$

Schwinger-Dyson Equation in Momentum Space

$$\Sigma(x,x') = ie^2\gamma^\mu G(x,x')\gamma^\nu D_{\mu\nu}(x-x')$$

$$\int d^4x d^4x' \overline{E}^l_p(x)\Sigma(x,x')E^{l'}_{p'}(x') = ie^2 \int d^4x d^4x' \overline{E}^l_p(x)\gamma^\mu$$
$$(\sum\!\!\!\!\!\!\int_l \frac{d^4p"}{(2\pi)^4} E^{l"}_{p"}(x)\Pi(l")\widetilde{G}^{l"}(\overline{p}")\overline{E}^{l"}_{p"}(x'))\gamma^\nu E^{l'}_{p'}(x')D_{\mu\nu}(x-x')$$

$$\widetilde{\Sigma}^l(\overline{p})\Pi(l) = ie^2(2|eH|)\Pi(l)\int \frac{d^4\widehat{q}}{(2\pi)^4}\frac{e^{-\widehat{q}^2_\perp}}{\widehat{q}^2}[\gamma^\parallel_\mu \widetilde{G}^l(\overline{p-q})\gamma^\parallel_\mu$$
$$+\Delta(+)\gamma^\perp_\mu \widetilde{G}^{l+1}(\overline{p-q})\gamma^\perp_\mu \Delta(+) + \Delta(-)\gamma^\perp_\mu \widetilde{G}^{l-1}(\overline{p-q})\gamma^\perp_\mu \Delta(-)]$$

where

$$\overline{p-q} \equiv (p_0-q_0, 0, -sgn(eH)\sqrt{2|eH|n}, p_3-q_3)$$

$$n = l-1, l, l+1$$

Schwinger-Dyson Equation in the LLL

$$\widetilde{\Sigma}^0(\overline{p})\Delta(+) =$$

$$= ie^2(2|eH|)\Delta(+)\int \frac{d^4\widehat{q}}{(2\pi)^4}\frac{e^{-\widehat{q}_\perp^2}}{\widehat{q}^2}[\gamma_\mu^{||}\widetilde{G}^0(\overline{p-q})\gamma_\mu^{||} + \gamma_\mu^{\perp}\widetilde{G}^1(\overline{p-q})\gamma_\mu^{\perp}\Delta(+)]$$

$$(M^0 + T^0)\Delta(+) + Z_{||}^0\Delta(+)(\Lambda_{||}^+ - \Lambda_{||}^-)|\overline{p}_{||}| \simeq$$

$$\simeq ie^2(2|eH|)\Delta(+)\int \frac{d^4\widehat{q}}{(2\pi)^4}\frac{e^{-\widehat{q}_\perp^2}}{\widehat{q}^2}\gamma_\mu^{||}\widetilde{G}^0(\overline{p-q})\gamma_\mu^{||}$$

$$(M^0 + T^0)\Delta(+) + Z_{||}^0\Delta(+)(\Lambda_{||}^+ - \Lambda_{||}^-)|\overline{p}_{||}| = ie^2(2|eH|)$$

$$\times\Delta(+)\int \frac{d^4q}{(2\pi)^4}\frac{e^{-\widehat{q}_\perp^2}}{\widehat{q}^2}\frac{(M^0+T^0)}{(\overline{p}_{||}-\overline{q}_{||})^2 - (M^0+T^0)^2}$$

Solution of the Schwinger-Dyson Equation in the LLL

$$1 = ie^2(4|eH|)\int \frac{d^4\widehat{q}}{(2\pi)^4}\frac{e^{-\widehat{q}_\perp^2}}{\widehat{q}^2}\frac{1}{(M^{(0)}+T^{(0)})^2 - q_{||}^2}$$

$$M^0 + T^0 \simeq \sqrt{2|eH|}e^{-\sqrt{\frac{\pi}{\alpha}}}$$

Since the LLL propagator $G^0(\overline{p-q})$ only depends on the combination M^0+T^0 the solution of the LLL SD equation can only determine the sum of these parameters.

Then

$$E^0 = M^0 + T^0$$

represents the dynamically induced electron rest-energy.

Schwinger-Dyson Equation in Higher LL's

$$\widetilde{\Sigma}^{(l\neq 0)}(\overline{p}) =$$

$$= ie^2(2|eH|)\int \frac{d^4\widehat{q}}{(2\pi)^4}\frac{e^{-\widehat{q}_\perp^2}}{\widehat{q}^2}[\gamma_\mu^{\parallel}\widetilde{G}^l(\overline{p-q})\gamma_\mu^{\parallel} + \Delta(+)\gamma_\mu^{\perp}\widetilde{G}^{l+1}(\overline{p-q})\gamma_\mu^{\perp}\Delta(+)$$
$$+\Delta(-)\gamma_\mu^{\perp}\widetilde{G}^{l-1}(\overline{p-q})\gamma_\mu^{\perp}\Delta(-)]$$

In the first LL (l = 1) we have

$$\widetilde{\Sigma}^1(\overline{p}) = ie^2(2|eH|)\int \frac{d^4\widehat{q}}{(2\pi)^4}\frac{e^{-\widehat{q}_\perp^2}}{\widehat{q}^2}[\gamma_\mu^{\parallel}G^1(\overline{p}-q)\gamma_\mu^{\parallel} +$$
$$+\Delta(+)\gamma_\mu^{\perp}\widetilde{G}^2(\overline{p-q})\gamma_\mu^{\perp}\Delta(+) + \Delta(-)\gamma_\mu^{\perp}\widetilde{G}^0(\overline{p-q})\gamma_\mu^{\perp}\Delta(-)]$$

Solution of the Schwinger-Dyson Equation in the First LL

$$Z^1_\perp \gamma_2(2eH) + (M^1 + T^1)\Delta(+) + (M^1 - T^1)\Delta(-) =$$

$$= ie^2(4|eH|)\Delta(-)\int \frac{d^4\widehat{q}}{(2\pi)^4}\frac{e^{-\widehat{q}_\perp^2}}{\widehat{q}^2}\frac{E^0}{(E^0)^2 - q_\parallel^2}$$

Then

$$M^1 + T^1 = 0, \quad Z^1_\perp = 0$$

$$M^1 - T^1 = ie^2(4|eH|)\int \frac{d^4\widehat{q}}{(2\pi)^4}\frac{e^{-\widehat{q}_\perp^2}}{\widehat{q}^2}\frac{E^0}{(E^0)^2 - q_\parallel^2}$$

From where we obtain

$$M^1 = -T^1 = \frac{1}{2}E^0 = \sqrt{|eH|/2}\,e^{-\sqrt{\frac{\pi}{\alpha}}}$$

Dispersion Relation Equation in the Condensate Phase

Transforming to momentum space the QED Lagrangian in the condensate phase

$$\mathcal{L} = \int d^4x \overline{\psi}(x)(\Pi_\mu \gamma^\mu - \Sigma(x))\psi(x)$$

with the Ritus' transformation

$$\psi(x) = \sum\!\!\!\!\!\!\!\int_l \frac{d^4p}{(2\pi)^4} E_p^l(x)\psi_l(p) \qquad \overline{\psi}(x) = \sum\!\!\!\!\!\!\!\int_l \frac{d^4p'}{(2\pi)^4} \overline{\psi}_{l'}(p')\overline{E}_{p'}^{l'}(x)$$

we obtain the field equation

$$\Pi(l)[\gamma^\mu \overline{p}_\mu - \widetilde{\Sigma}^l(\overline{p})]\psi_l(p) = 0$$

In the LLL it is given by

$$[(\gamma^\mu \overline{p}_\mu - \widetilde{\Sigma}^0(\overline{p}))\Delta(+)]\psi_{LLL}(p) = 0$$

and the corresponding dispersion relation is

$$\det[(\gamma^\mu \overline{p}_\mu - \widetilde{\Sigma}^0(\overline{p}))\Delta(+)] = \det(\gamma^\mu \overline{p}_\mu - \widetilde{\Sigma}^0(\overline{p})) \cdot \det \Delta(+) = 0$$

Since $\det \Delta(+) = 0$ it means that the LLL dispersion relation has to be obtained in the reduced dimensional space of the spin-up LLL electrons.

Dispersion Relation Equation in the LLL

In the (1+1) Dimensional space of the spin-up electrons of the LLL we have that the field equation is given by

$$[\widetilde{\gamma}^{\parallel} \cdot p_{\parallel} - E^0]\ \psi_{LLL}(p) = 0$$

where

$$\widetilde{\gamma}^0 = \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \widetilde{\gamma}^3 = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \widetilde{\gamma}^5 = -i\sigma_1\sigma_2 = \sigma_3$$

These matrices in (1+1)-D satisfy the algebra

$$\widetilde{\gamma}^{\mu}\widetilde{\gamma}^{\nu} = g^{\mu\nu} + \epsilon^{\mu\nu}\widetilde{\gamma}^5, \qquad \widetilde{\gamma}^{\mu}\widetilde{\gamma}^5 = -\epsilon^{\mu\nu}\widetilde{\gamma}_{\nu}$$

$$g^{\mu\nu} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \quad \epsilon^{\mu\nu} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

The corresponding dispersion relation is

$$p_0 = \pm\sqrt{p_3^2 + (E^0)^2}$$

Notice that in the LLL the energy of the electrons depends on the induced rest energy already obtained.

Dispersion Relation in the First LL

For electrons in LL's different from $l = 0$, the field equation is

$$[\overline{p}\cdot\gamma - M^l I - iT^l\gamma^1\gamma^2]\psi_l = 0$$

From where it is obtained the dispersion relation

$$det\ \ [\overline{p}\cdot\gamma - M^l I - iT^l\gamma^1\gamma^2] =$$

$$= [(M^l)^2 - (\overline{p}_{||} - T^l)^2 + \overline{p}_{\perp}^2][(M^l)^2 - (\overline{p}_{||} + T^l)^2 + \overline{p}_{\perp}^2] = 0$$

yielding

$$p_0^2 = p_3^2 + [\sqrt{(M^l)^2 + 2eHl} \pm T^l]^2$$

Since $M^{(1)}/\sqrt{2|eH|}\ , T^{(1)}/\sqrt{2|eH|} \ll 1$ we have in the leading approximation

$$p_0^2 = p_3^2 + 2eH + (M^1)^2 + (T^1)^2 \pm 2T^{(1)}\sqrt{2eH}$$

Hence the energy of the electron in the first LL depends on its spin orientation with respect to the field direction.

Energy Shift in the First LL

Starting from the energy square

$$p_0^2 = p_3^2 + 2eH + (M^1)^2 + (T^1)^2 \pm 2T^1\sqrt{2eH}$$

We have that the energy leading contribution in the infrared limit $(p_3/\sqrt{2|eH|} \ll 1)$ is

$$p_0 \simeq \pm[\sqrt{2|eH|} + \frac{p_3^2}{2\sqrt{2|eH|}} + \frac{(M^1)^2 + (T^1)^2}{2\sqrt{2|eH|}} \pm T^{(1)}]$$

Hence, the energy splitting in the first LL is

$$\Delta E = |2T^1| = \widetilde{g}\widetilde{\mu}_B H$$

Where the non-perturbative correction to the *g*-factor is given by

$$\widetilde{g} = 2e^{-2\sqrt{\pi/\alpha}}$$

and the non-perturbative Bohr Magneton is

$$\widetilde{\mu}_B = \frac{e}{2M^1}$$

Magnetic Moment of the Chiral Pairs

Model

$$\mathcal{L} = \bar{\psi} i \gamma^\mu D_\mu \psi + \mathcal{L}_{int}^{(1)} + \mathcal{L}_{int}^{(2)}$$

$$\mathcal{L}_{int}^{(1)} = \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma^5 \psi)^2],$$

$$\mathcal{L}_{int}^{(2)} = \frac{G'}{2}[(\bar{\psi}\Sigma^3\psi)^2 + (\bar{\psi} i \gamma^5 \Sigma^3 \psi)^2],$$

$$\eta_{\|}^{\mu\nu} = \eta^{\mu\nu} - \widehat{F}^{\mu\rho}\widehat{F}_\rho^\nu, \quad \eta_{\perp}^{\mu\nu} = \widehat{F}^{\mu\rho}\widehat{F}_\rho^\nu$$

$$\widehat{F}_{\mu\nu} = F_{\mu\nu}/|B|$$

$$\gamma^{\|} = \eta_{\|}^{\mu\nu}\gamma_\nu, \quad \gamma^{\perp} = \eta_{\perp}^{\mu\nu}\gamma_\nu$$

Equivalently

$$\mathcal{L}_{int} = \frac{g_{\|}^2}{2\Lambda^2}(\bar{\psi}\gamma_{\|}^{\mu}\psi)(\bar{\psi}\gamma_{\mu}^{\|}\psi) + \frac{g_{\perp}^2}{2\Lambda^2}(\bar{\psi}\gamma_{\perp}^{\mu}\psi)(\bar{\psi}\gamma_{\mu}^{\perp}\psi).$$

$$G = (g_{\|}^2 + g_{\perp}^2)/2\Lambda^2, \qquad G' = (g_{\|}^2 - g_{\perp}^2)/2\Lambda^2$$

Fierz Identities with Rotational Symmetry Breaking

$$
\begin{pmatrix}
(1)_{ij}(1)_{kl} \\
(\gamma^{\parallel})_{ij}(\gamma_{\parallel})_{kl} \\
(\gamma^{\perp})^{ij}(\gamma_{\perp})_{kl} \\
(\sigma^{30})_{ij}(\sigma_{30})_{ij} \\
(\sigma^{\perp\parallel})_{ij}(\sigma_{\perp\parallel})_{kl} \\
\frac{1}{2}(\sigma^{\perp\perp})_{ij}(\sigma_{\perp\perp})_{kl} \\
(\gamma^{\parallel}\gamma_5)_{ij}(\gamma_{\parallel}\gamma_5)_{kl} \\
(\gamma^{\perp}\gamma_5)_{ij}(\gamma_{\perp}\gamma_5)_{kl} \\
(i\gamma_5)_{ij}(i\gamma_5)_{kl}
\end{pmatrix}
=
\begin{pmatrix}
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\
\frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\
\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\
1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & -1 \\
\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\
-\frac{1}{2} & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} \\
-\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} \\
-\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4}
\end{pmatrix}
\begin{pmatrix}
(1)_{ij}(1)_{kl} \\
(\gamma^{\parallel})_{ij}(\gamma_{\parallel})_{kl} \\
(\gamma^{\perp})^{ij}(\gamma_{\perp})_{kl} \\
(\sigma^{30})_{ij}(\sigma_{30})_{ij} \\
(\sigma^{\perp\parallel})_{ij}(\sigma_{\perp\parallel})_{kl} \\
\frac{1}{2}(\sigma^{\perp\perp})_{ij}(\sigma_{\perp\perp})_{kl} \\
(\gamma^{\parallel}\gamma_5)_{ij}(\gamma_{\parallel}\gamma_5)_{kl} \\
(\gamma^{\perp}\gamma_5)_{ij}(\gamma_{\perp}\gamma_5)_{kl} \\
(i\gamma_5)_{ij}(i\gamma_5)_{kl}
\end{pmatrix}
$$

Vertices

$$(\gamma^{\mu}_{\parallel})_{il}\left(\gamma^{\parallel}_{\mu}\right)_{kj} = \frac{1}{2}\left\{(1)_{il}\,(1)_{kj} + (i\gamma_5)_{il}(i\gamma_5)_{kj} + \frac{1}{2}\,(\sigma^{\mu\nu}_{\perp})_{il}\,(\sigma^{\perp}_{\mu\nu})_{kj} - (\sigma^{03})_{il}\,(\sigma_{03})_{kj} + \ldots\right\},$$

$$(\gamma^{\mu}_{\perp})_{il}\,(\gamma^{\perp}_{\mu})_{kj} = \frac{1}{2}\left\{(1)_{il}\,(1)_{kj} + (i\gamma_5)_{il}(i\gamma_5)_{kj} - \frac{1}{2}\,(\sigma^{\mu\nu}_{\perp})_{il}\,(\sigma^{\perp}_{\mu\nu})_{kj} + (\sigma^{03})_{il}\,(\sigma_{03})_{kj} + \ldots\right\}$$

$$G = (g_{\parallel}^2 + g_{\perp}^2)/2\Lambda^2, \qquad G' = (g_{\parallel}^2 - g_{\perp}^2)/2\Lambda^2$$

Mean-Field Approximation

Condensates

$$\langle\bar\psi\psi\rangle = -\frac{\sigma}{G}, \qquad \langle\bar\psi i\gamma^5\psi\rangle = -\frac{\Pi}{G},$$

$$\langle\bar\psi i\gamma^1\gamma^2\psi\rangle = -\frac{\xi}{G'}, \qquad \langle\bar\psi i\gamma^0\gamma^3\psi\rangle = -\frac{\xi'}{G'},$$

Action

$$S(\sigma,\Pi,\xi,\xi') = \int d^4x\bar\psi(x)(i\gamma^\mu D_\mu - \sigma - i\gamma^5\Pi - i\gamma^1\gamma^2\xi - i\gamma^0\gamma^3\xi')\psi(x) -$$

$$-\frac{V}{2G}(\sigma^2+\Pi^2) - \frac{V}{2G'}(\xi^2+\xi'^2).$$

Mean-Field Effective Potential

$$\Omega(\sigma,\Pi,\xi,\xi') = \frac{\sigma^2+\Pi^2}{2G} + \frac{\xi^2+\xi'^2}{2G'} +$$

$$+\frac{i}{V}\mathrm{Trln}(iD\cdot\gamma - \sigma - i\gamma^5\Pi - i\gamma^1\gamma^2\xi - i\gamma^0\gamma^3\xi')$$

Inverse Propagator in Momentum Space

Inverse Propagator

$$G_l^{-1}(p,p') =$$

$$= \int d^4x d^4x' \overline{E}_p^l(x)[iD\cdot\gamma - \sigma - i\gamma^5\Pi - i\gamma^1\gamma^2\xi - i\gamma^0\gamma^3\xi')]\delta^{(4)}(x-x')E_{p'}^{l'}(x') =$$

$$= (2\pi)^4\widehat{\delta}^{(4)}(p-p')\Theta(l)\widetilde{G}_l^{-1}(\overline{p})$$

where

$$\widetilde{G}_l^{-1}(\overline{p}) = [\overline{p}\cdot\gamma - \sigma - i\gamma^5\Pi - i\gamma^1\gamma^2\xi - i\gamma^0\gamma^3\xi'],$$

$$\overline{p}^\mu = (p^0, 0, -\mathrm{sgn}(qB)\sqrt{2|qB|l}, p^3)$$

$$\Theta(l) = \Delta(+)\delta^{l0} + I(1-\delta^{l0})$$

Quasiparticle Spectra

Model

$$\Omega(\sigma, \Pi, \xi, \xi') = \frac{\sigma^2 + \Pi^2}{2G} + \frac{\xi^2 + \xi'^2}{2G'} -$$

$$- \frac{N_c qB}{4\pi^2} \int_0^\infty |\varepsilon_0| dp_3 - \frac{N_c 2qB}{4\pi^2} \sum_{l=1}^{\infty} \int_0^\infty |\varepsilon_l| dp_3,$$

Quasiparticle Spectrum

$$\varepsilon_0^2 = p_3^2 + (\sigma + \xi)^2 + (\Pi + \xi')^2, \quad l = 0,$$

$$\varepsilon_l^2 = p_3^2 + \Pi^2 + \xi'^2 + \sigma^2(1 - X) + 2lqB(1 - X') + \left(\sqrt{\sigma^2 X + 2lqB} \pm \xi\right)^2, \quad l \geq 1,$$

$$X = \left(1 + \frac{\Pi}{\sigma} \frac{\xi'}{\xi}\right)^2, \qquad X' = \left(1 + \frac{\xi'^2}{\xi^2}\right)$$

Gap Equations

Gap Equations for the four condensates

$$\frac{\partial\Omega(\sigma,\Pi,\xi,\xi')}{\partial\sigma} = \frac{\sigma}{G} - (\sigma+\xi)\mathcal{I} = 0, \qquad \frac{\partial\Omega(\sigma,\Pi,\xi,\xi')}{\partial\xi} = \frac{\xi}{G'} - (\sigma+\xi)\mathcal{I} = 0,$$

$$\frac{\partial\Omega(\sigma,\Pi,\xi,\xi')}{\partial\Pi} = \frac{\Pi}{G} - (\Pi+\xi')\mathcal{I} = 0, \qquad \frac{\partial\Omega(\sigma,\Pi,\xi,\xi')}{\partial\xi'} = \frac{\xi'}{G'} - (\Pi+\xi')\mathcal{I} = 0,$$

with

$$\mathcal{I} = \frac{N_c qB}{2\pi^2}\int_0^{\Lambda}\frac{dp_3}{\varepsilon_0} + \frac{N_c qB}{\pi^2}\sum_{l=1}^{l_{max}}\int_0^{\sqrt{\Lambda^2-2qBl}}\frac{dp_3}{\varepsilon_l}$$

$$l_{max} = [\Lambda^2/2qB], \text{ with } [n+\eta] = n, \text{ for } n \in Z$$

Condensate Solutions

$$\overline{\xi} = \frac{G'}{G}\overline{\sigma}, \qquad \overline{\xi'} = \frac{G'}{G}\overline{\Pi}$$

Gap Equation in the LLL

$$\int_0^{\Lambda} \frac{dp_3}{\sqrt{p_3^2 + (1+\frac{G'}{G})^2(\overline{\sigma}^2 + \overline{\Pi}^2)}} = \frac{4\pi^2}{(G+G')N_c qB}$$

Solutions

$$\overline{\sigma} = \left(\frac{2G\Lambda}{G+G'}\right) \exp - \left[\frac{2\pi^2}{(G+G')N_c qB}\right]$$

$$\overline{\xi} = \left(\frac{2G'\Lambda}{G+G'}\right) \exp - \left[\frac{2\pi^2}{(G+G')N_c qB}\right]$$

Dynamical Mass

Model

$$M_\xi = \overline{\sigma} + \overline{\xi} = 2\Lambda \exp - \left[\frac{2\pi^2}{(G+G')N_c qB} \right]$$

$$\ln\left(\frac{M_\xi}{M_{\xi=0}}\right) = \frac{2\pi^2}{GN_c qB}\left(\frac{\eta}{1+\eta}\right)$$

$$G' = \eta G$$

Then

$$\ln\left(\frac{M_\xi}{M_{\xi=0}}\right) \simeq 18,$$

Where

$$B/\Lambda^2 \sim 1, \quad G\Lambda^2 = 1.835,$$

$$N_c = 3, \quad q = |e|/3 \simeq 0.1$$

Critical Temperature

Thermodynamic Potential in the LLL

$$\Omega_0^T(\overline{\sigma},\overline{\xi}) = -N_c qB \int_0^{\Lambda} \frac{dp_3}{2\pi^2}\left[\varepsilon + \frac{2}{\beta}\ln\left(1+e^{-\beta\varepsilon}\right)\right] + \frac{\overline{\sigma}^2}{2G} + \frac{\overline{\xi}^2}{2G'}$$

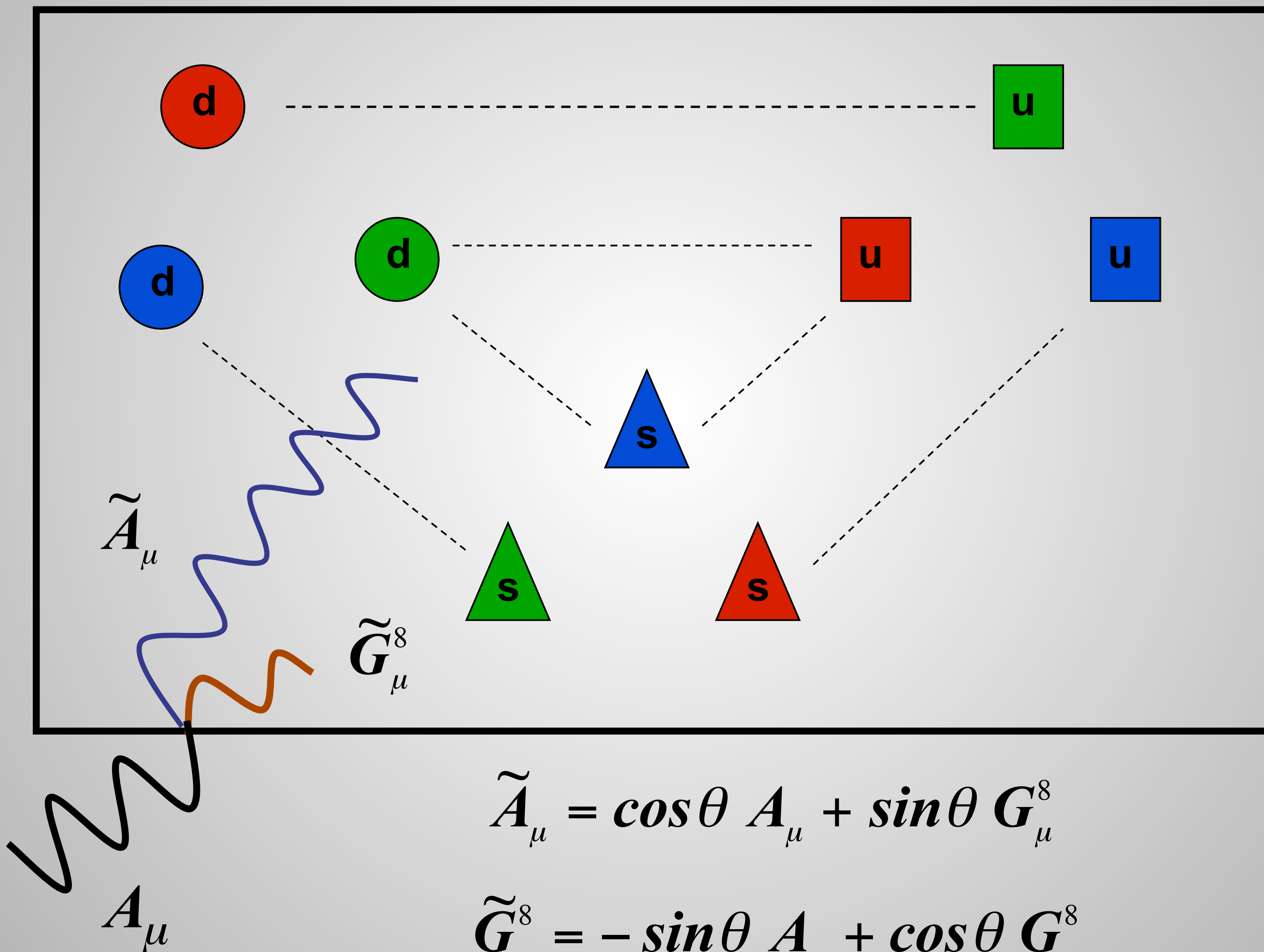
$$\frac{\partial^2 \Omega_0^{T_{C_\chi}}}{\partial\overline{\sigma}^2}\Big|_{\overline{\sigma}=\overline{\xi}=0} = -\frac{N_c qB}{2\pi^2}\left[\frac{G+G'}{G}\int_0^{\Lambda}\frac{dp_3}{p_3}\tanh\left(\frac{\beta_{C_\chi} p_3}{2}\right) + \frac{2\pi^2}{GN_c qB}\right] = 0$$

$$\int_0^{\Lambda/T_{C_\chi}} \frac{dp_3}{p_3}\tanh\left(\frac{p_3}{2}\right) = \frac{2\pi^2}{(G+G')N_c qB},$$

Critical Temperature

$$T_{C_\chi} = 1.6\Lambda \exp - \left[\frac{2\pi^2}{(G+G')N_c qB}\right] = 0.8 M_\xi$$

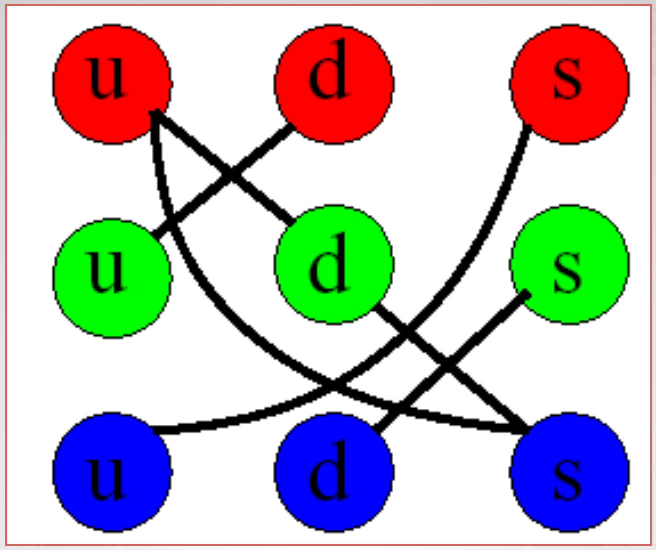
Magnetic Field Penetration in Color Superconductivity



$$\tilde{A}_\mu = \cos\theta\, A_\mu + \sin\theta\, G^8_\mu$$

$$\tilde{G}^8_\mu = -\sin\theta\, A_\mu + \cos\theta\, G^8_\mu$$

Rotated Charges



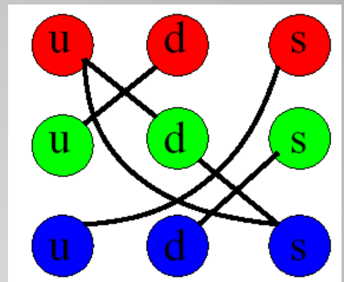
The pairs are all $\tilde{Q}$-neutral, but the quarks can be neutral or charged

$\tilde{Q}$ - CHARGES

s	s	s	d	d	d	u	u	u
0	0	-1	0	0	-1	1	1	0

All $\tilde{Q}$ -charged quarks have integer charges

Magnetic CFL



B=*0*   ***CFL***

$$\Delta_1 = \Delta_2 = \Delta_3$$

$$\mathbf{SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_{e.m}}$$

$$\downarrow$$

$$SU(3)_{C+L+R} \times \tilde{U}(1)_{em} \rightarrow \textbf{9 GB}$$

B ≠ *0*   ***MCFL*** $\Delta_1 = \Delta_2 \neq \Delta_3$

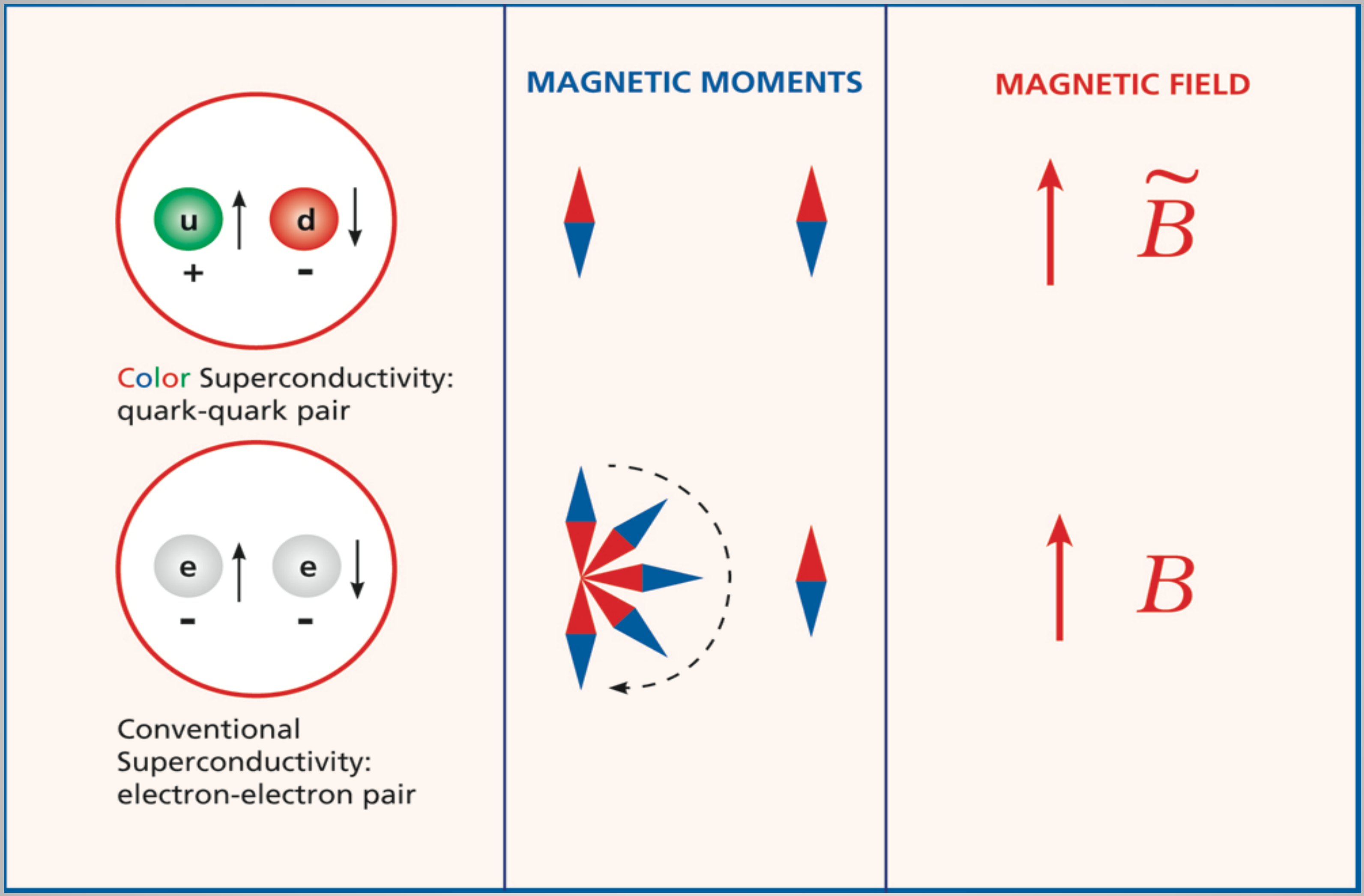
E.J. Ferrer, V.I. and C. Manuel, PRL '05; NPB'06

$$\mathbf{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_B \times U^{(-)}(1)_A \times U(1)_{e.m.}}$$

$$\downarrow$$

$$SU(2)_{C+L+R} \times \tilde{U}(1)_{em} \rightarrow \textbf{5 GB}$$

Color Superconductivity & B



A magnetic field reinforces color superconductivity!

The Cooper pairs have net magnetic moments!

Dirac Structure of the New Condensate

B. Feng, EJF, V. de la Incera, NPB 853 (2011) 213-239

$$\mathcal{L}_{int} = -G(\bar{\psi}\Gamma^a_\mu\psi)(\bar{\psi}\Gamma^\mu_a\psi),$$

with

$$\Gamma^a_\mu = \gamma_\mu\lambda^a$$

The magnetic field explicitly breaks the rotational symmetry O(3) $\longrightarrow$ O(2)

$$g^{\mu\nu}_\perp = \widehat{F}^{\mu\rho}\widehat{F}^\nu_\rho \qquad g^{\mu\nu}_{\|} = g^{\mu\nu} - g^{\mu\nu}_\perp \qquad \widehat{F}^{\mu\nu} = \widehat{F}^{\mu\nu}/|B|$$

$$\gamma^\mu\gamma_\mu = [au_\mu u_\nu + bg^\perp_{\mu\nu} + c(g^{\|}_{\mu\nu} - u_\mu u_\nu)]\gamma^\mu\gamma^\nu$$

$$\begin{aligned}\mathcal{L}_{int} = &- g_E(\bar{\psi}\gamma_0\lambda^a\psi)(\bar{\psi}\gamma_0\lambda^a\psi) - \\ &- g^\perp_M(\bar{\psi}\gamma^\perp\lambda^a\psi)(\bar{\psi}\gamma_\perp\lambda^a\psi) \\ &- g^3_M(\bar{\psi}\gamma^3\lambda^a\psi)(\bar{\psi}\gamma_3\lambda^a\psi).\end{aligned}$$

Fierz Identities for particle-particle channel at $\vec{B} \neq 0$

B. Feng, EJF, V. de la Incera, NPB 853 (2011) 213-239

$$\Delta_M = \langle \psi^T \mathcal{C} \gamma^5 \sigma_{12} \psi \rangle$$

$$\begin{pmatrix} (1)_{ij}(1)_{kl} \\ (\gamma_0)_{ij}(\gamma_0)_{kl} \\ (\gamma^a)_{ij}(\gamma_a)_{kl} \\ (\gamma^3)_{ij}(\gamma_3)_{kl} \\ (\sigma^{a0})_{ij}(\sigma_{a0})_{kl} \\ (\sigma^{30})_{ij}(\sigma_{30})_{kl} \\ \frac{1}{2}(\sigma^{ab})_{ij}(\sigma_{ab})_{kl} \\ (\sigma^{3a})_{ij}(\sigma_{3a})_{kl} \\ (\gamma_0\gamma_5)_{ij}(\gamma_0\gamma_5)_{kl} \\ (\gamma^a\gamma_5)_{ij}(\gamma_a\gamma_5)_{kl} \\ (\gamma^3\gamma_5)_{ij}(\gamma^3\gamma_5)_{kl} \\ (i\gamma_5)_{ij}(i\gamma_5)_{kl} \end{pmatrix} =$$

$$= \begin{pmatrix}
-\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\
\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & \frac{1}{2} \\
\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} \\
\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\
\frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} \\
\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4}
\end{pmatrix}
\begin{pmatrix} (\mathcal{C})_{il}(\mathcal{C})_{kj} \\ (\gamma_0\mathcal{C})_{il}(\mathcal{C}\gamma_0)_{kj} \\ (\gamma^a\mathcal{C})_{il}(\mathcal{C}\gamma_a)_{kj} \\ (\gamma^3\mathcal{C})_{il}(\mathcal{C}\gamma_3)_{kj} \\ (\sigma^{a0}\mathcal{C})_{il}(\mathcal{C}\sigma_{a0})_{kj} \\ (\sigma^{30}\mathcal{C})_{il}(\mathcal{C}\sigma_{30})_{kj} \\ \frac{1}{2}(\sigma^{ab}\mathcal{C})_{il}(\mathcal{C}\sigma_{ab})_{kj} \\ (\sigma^{3a}\mathcal{C})_{il}(\mathcal{C}\sigma_{3a})_{kj} \\ (\gamma_0\gamma_5\mathcal{C})_{il}(\mathcal{C}\gamma_0\gamma_5)_{kj} \\ (\gamma^a\gamma_5\mathcal{C})_{il}(\mathcal{C}\gamma_a\gamma_5)_{kj} \\ (\gamma^3\gamma_5\mathcal{C})_{il}(\mathcal{C}\gamma^3\gamma_5)_{kj} \\ (i\gamma_5\mathcal{C})_{il}(\mathcal{C}i\gamma_5)_{kj} \end{pmatrix}$$

Color-Flavor Structure of the New Condensate

- Strongest Attractive Channel → **Color antisymmetric channel**
- $(\mathcal{C}\gamma^5\sigma_{ab})$ → **Symmetric in Dirac indices**
- Pauli Principle → **Total antisymmetric**

↓

Symmetric in Flavor

$$\Phi^{\alpha\beta}_{ij} = \Delta\epsilon^{\alpha\beta 3}\epsilon_{ij3} + \Delta_H(\epsilon^{\alpha\beta 1}\epsilon_{ij1} + \epsilon^{\alpha\beta 2}\epsilon_{ij2}) + T[\epsilon^{\alpha\beta 1}(\delta_{i2}\delta_{j3} + \delta_{i3}\delta_{j2}) + \epsilon^{\alpha\beta 2}(\delta_{i1}\delta_{j3} + \delta_{i3}\delta_{j1})],$$

$$\hat{\Delta} = \Delta\mathcal{C}\gamma_5, \qquad \hat{\Delta}_B = \Delta_B\mathcal{C}\gamma_5 \qquad \hat{\Delta}_M = \Delta_M\mathcal{C}\gamma_5\sigma_{12}$$

Gaps in the MCFL Phase

$$\mathcal{L}_{\text{int}} = G'(\bar{\psi} P_\eta \bar{\psi}^t)(\psi^t \bar{P}_\eta \psi) + G''(\bar{\psi} M_\rho \bar{\psi}^t)(\psi^t \bar{M}_\rho \psi)$$

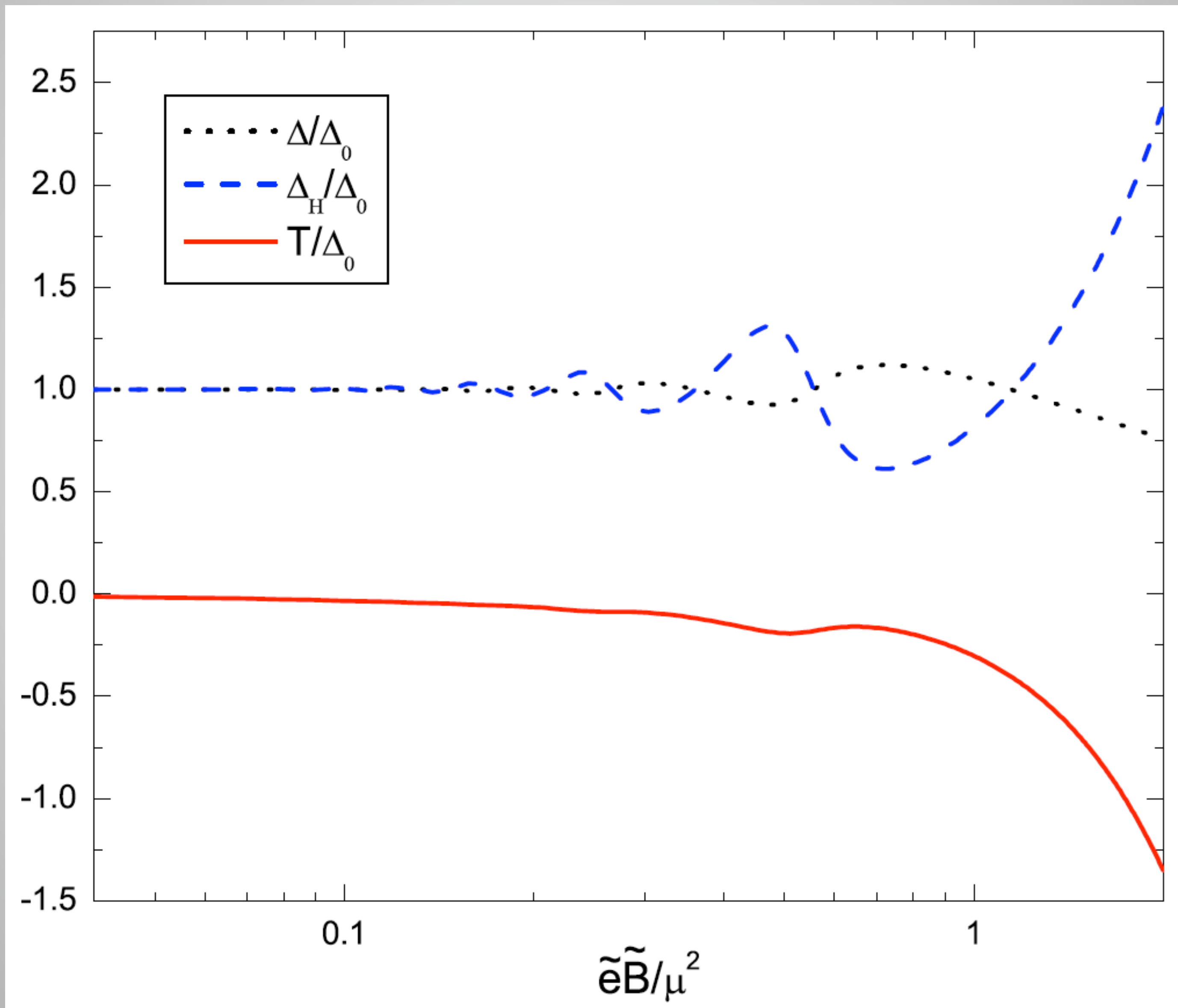
$$(P_\eta)_{ij}^{\alpha\beta} = i\mathcal{C}\gamma_5 \epsilon^{\alpha\beta\eta}\epsilon_{ij\eta}, \qquad (M_1)_{ij}^{\alpha\beta} = \mathcal{C}\sigma_{ab}\epsilon^{\alpha\beta 1}(\delta_{i2}\delta_{j3} + \delta_{i3}\delta_{j2})$$

$$\begin{pmatrix}
0 & \Delta & \Delta_H + T & 0 & 0 & 0 & 0 & 0 & 0 \\
\Delta & 0 & \Delta_H + T & 0 & 0 & 0 & 0 & 0 & 0 \\
\Delta_H - T & \Delta_H - T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\Delta & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\Delta & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Delta_H + T & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Delta_H + T \\
0 & 0 & 0 & 0 & 0 & -\Delta_H - T & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\Delta_H - T & 0 & 0
\end{pmatrix}$$

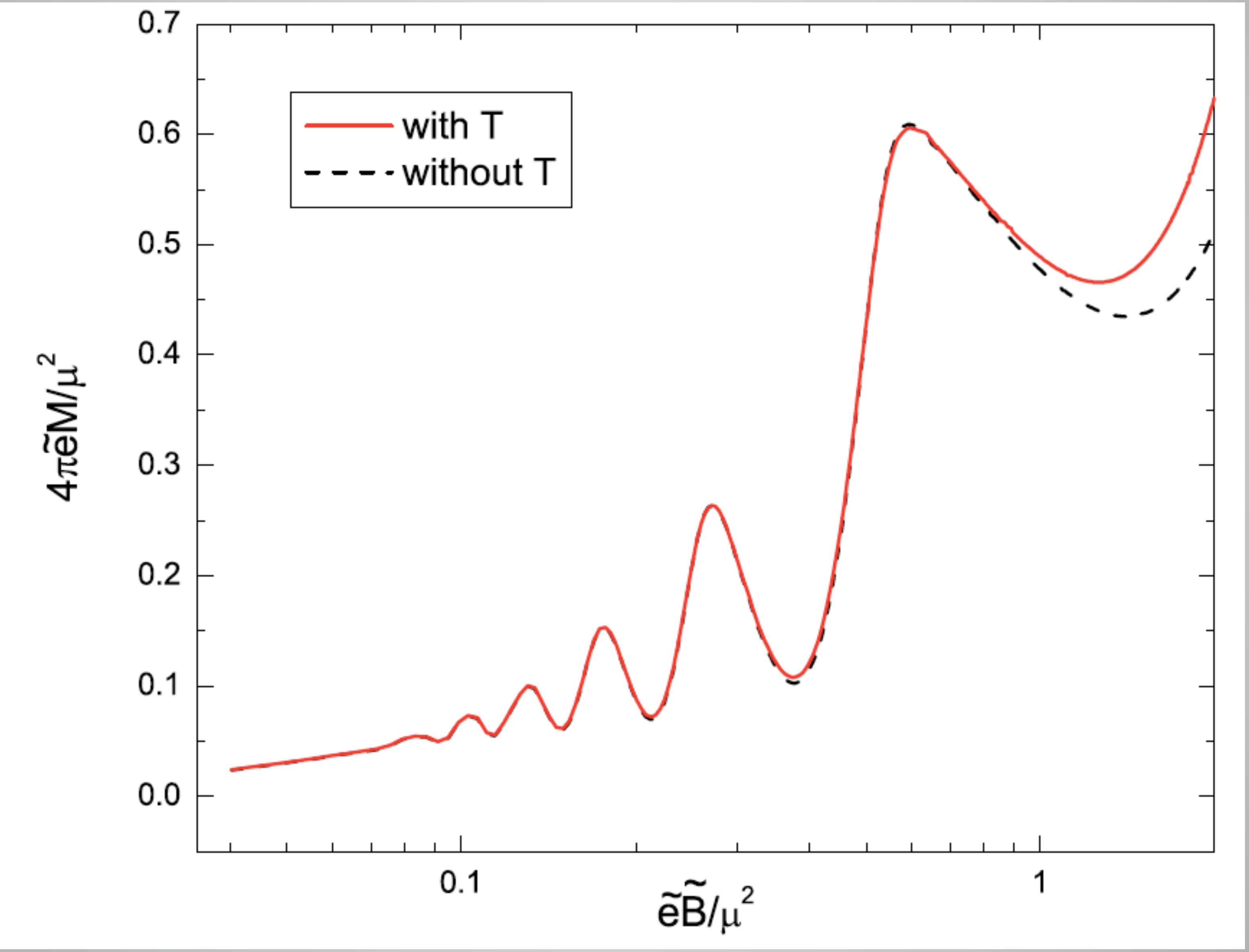
$$\frac{\partial \Omega}{\partial \Delta} = \frac{\partial \Omega}{\partial \Delta_H} = \frac{\partial \Omega}{\partial T} = 0.$$

Gap Parameters vs $\tilde{e}\tilde{B}/\mu^2$

Feng, EJF & de la Incera, NPB B853 (2011) 213-239



Effect of T on the Magnetization



Concluding Remarks

1. In the phenomenon of *Magnetic Catalysis of Chiral Symmetry Breaking*, the induction of an *anomalous magnetic moment* has to be considered along with the generation of the *dynamical mass*.
2. For particles in the LLL, the dynamical anomalous magnetic moment simply redefines the system's rest energy.
3. For higher LL's, the induction of an anomalous magnetic moment produces an energy splitting of each LL. The AMM depends non-perturbatively on both, the magnetic field and the fine-structure constant.
4. The magnetic catalysis produces non-perturbative *Lande g-factor* and *Bohr Magneton*.
5. The NJL model at B≠0, has new interaction channels that give rise to a condensate associated to the MM of the chiral pairs.
6. The induction of a MM for the chiral pairs in a magnetic field increases the critical temperature of the chiral phase transition in 8 orders!
7. In magnetized color superconductivity the Cooper pairs acquire MM.
8. The MCFL phase exhibits a richer condensate landscape with two spin zero condensates and one spin one condensate.
9. The spin one condensate increases the system energy gap and magnetization