CHIRAL SPIRALS
vs
CRYSTALLINE P-H GROUND STATES

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Insights into QCD Matter from Heavy-Ion Collisions, Nov. 18-22, 2013

Outline

- Motivation
- Inhomogeneous Condensates in QCD Effective Models
- Inhomogeneous Condensates in NJL Models
- Real (soliton/crystalline) Solution vs Chiral Spiral
- Relations between the two approaches
- Outlook

Collaborators:
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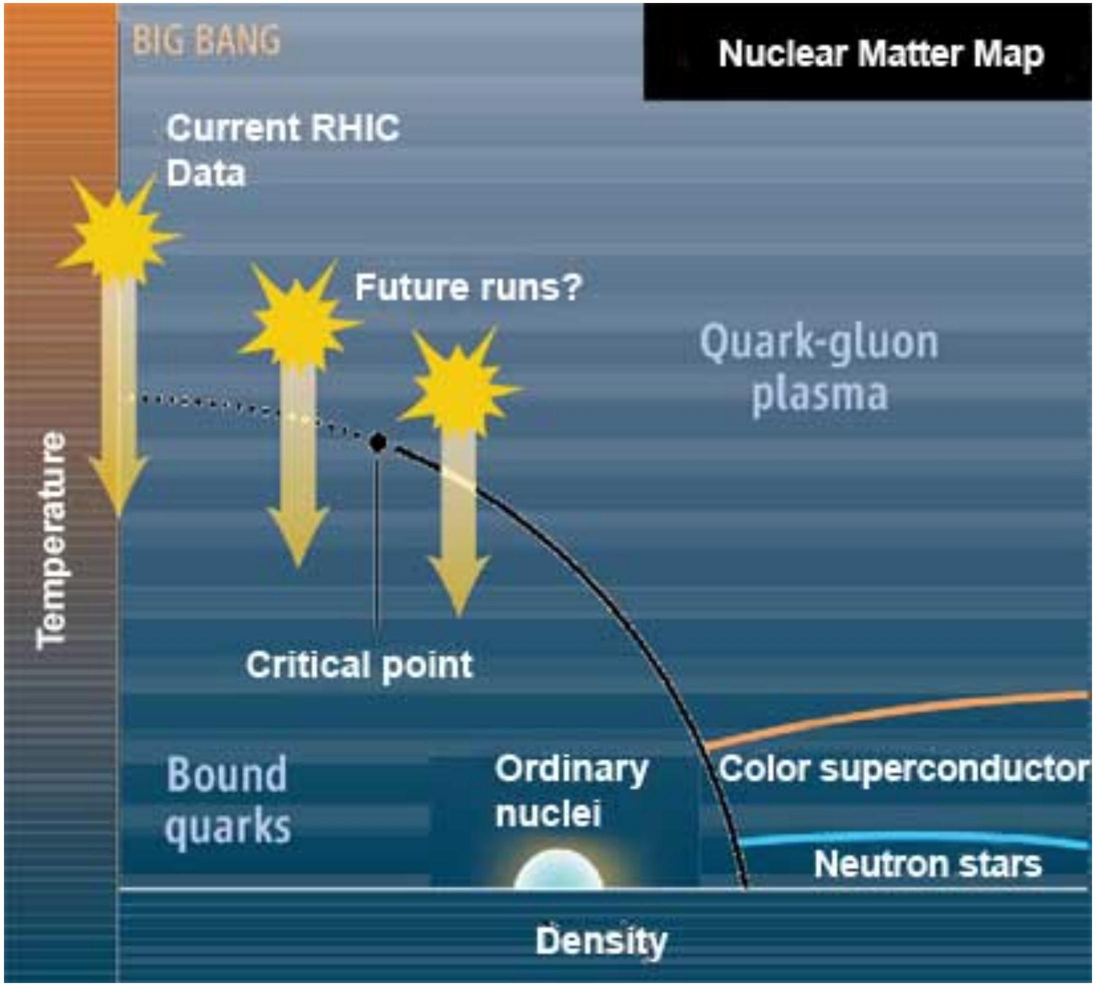
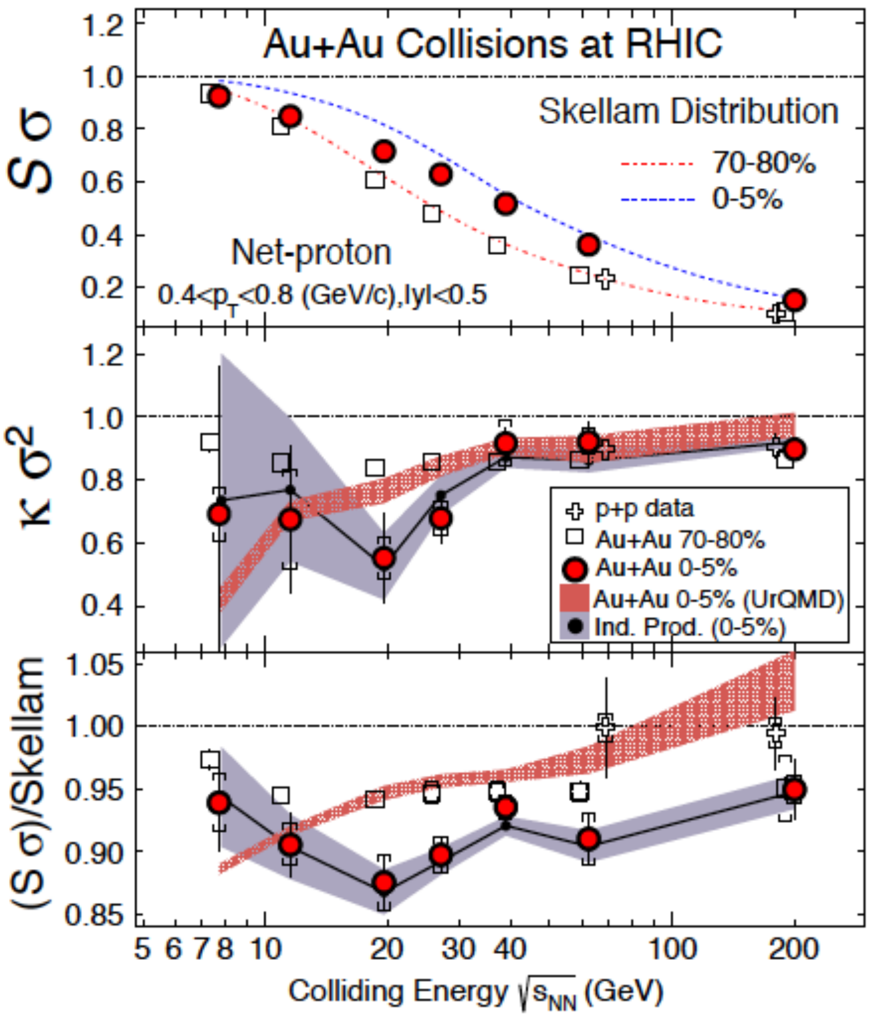
RHIC Energy Scans



$\sqrt{s_{NN}}$ (GeV)	$< \mu_B >$ (MeV)
7.7	421
11.5	316
19.6	206
39	112

Cleymans et al. PRC 73, 034905

Are We Observing the CP?

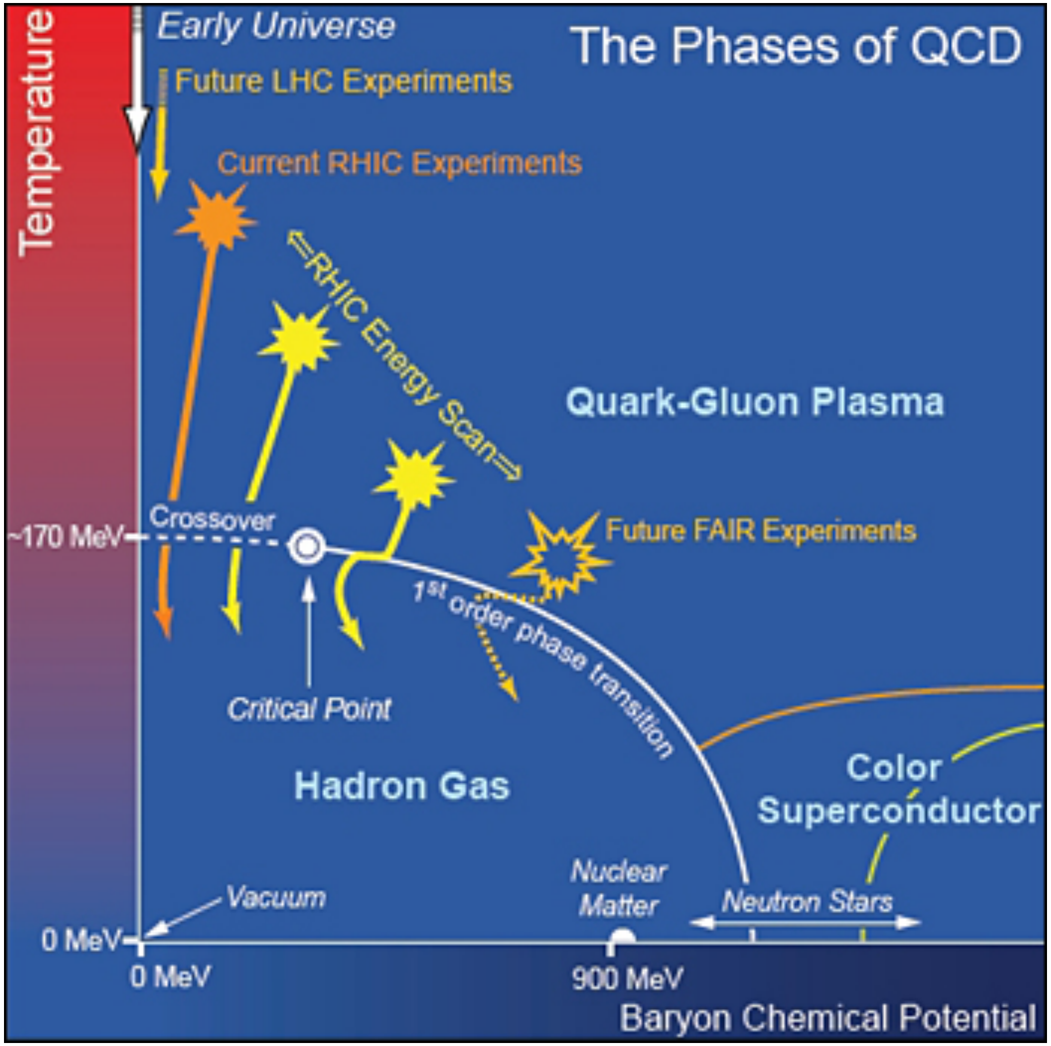


STAR collaboration arXiv:1309.5681

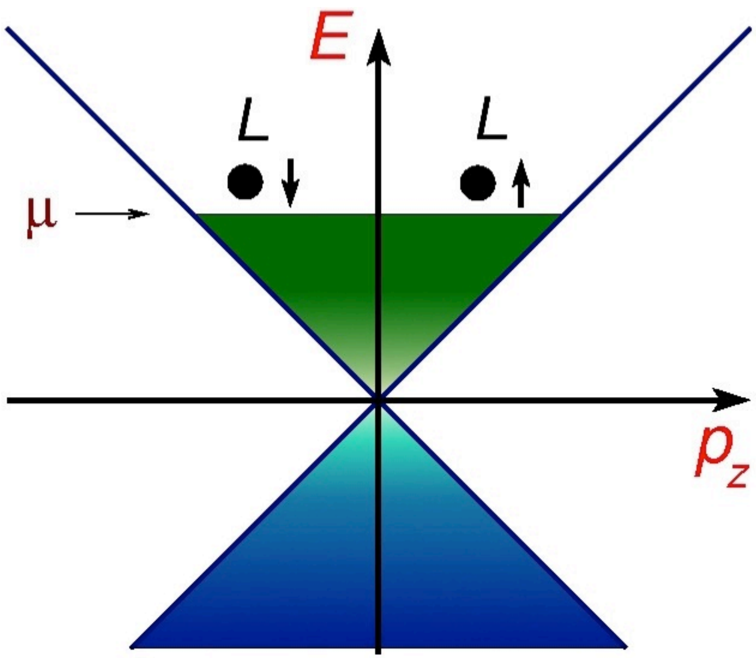
The products of the moments $S\sigma$ and $\kappa\sigma^2$ are significantly below Skellam expectation, a behavior qualitatively consistent with a QCD based model which includes a CP.

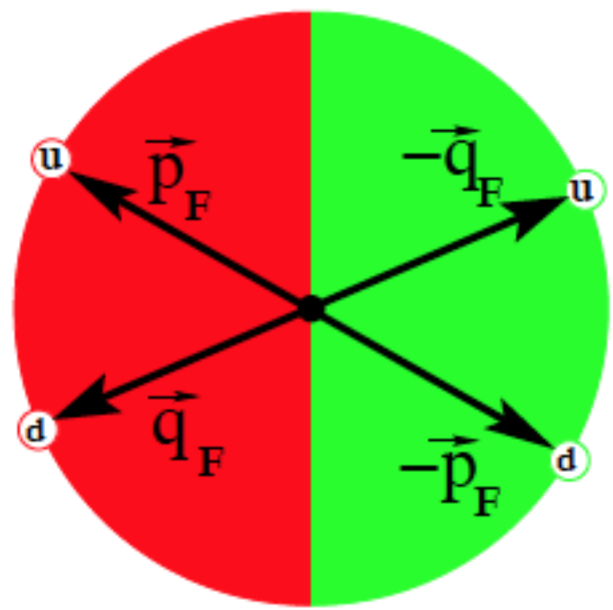
No definite conclusion can be made until a comparison with QCD calculations with CP behavior which include the dynamics of HI collisions as freeze-out effects and finite correlation lengths is done.

Future Probes



What do We Expect at Very Large Density?



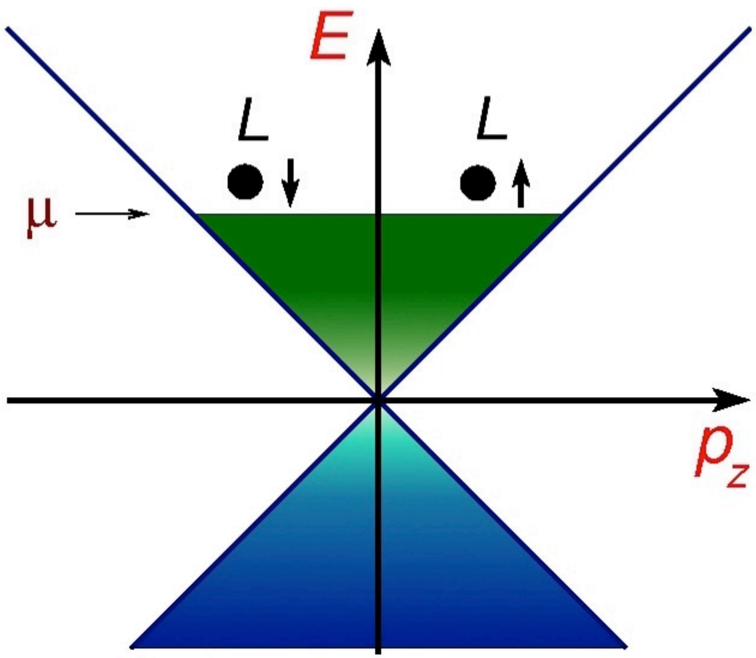


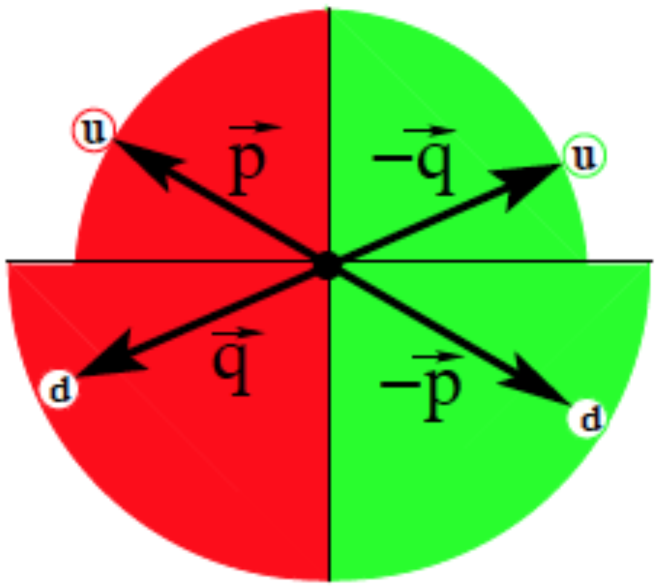
Cooper Pairing via the color anti-triplet channel

Color Superconductivity:
Favored at asymptotically large densities

Intermediate Densities:

No crystal-clear picture

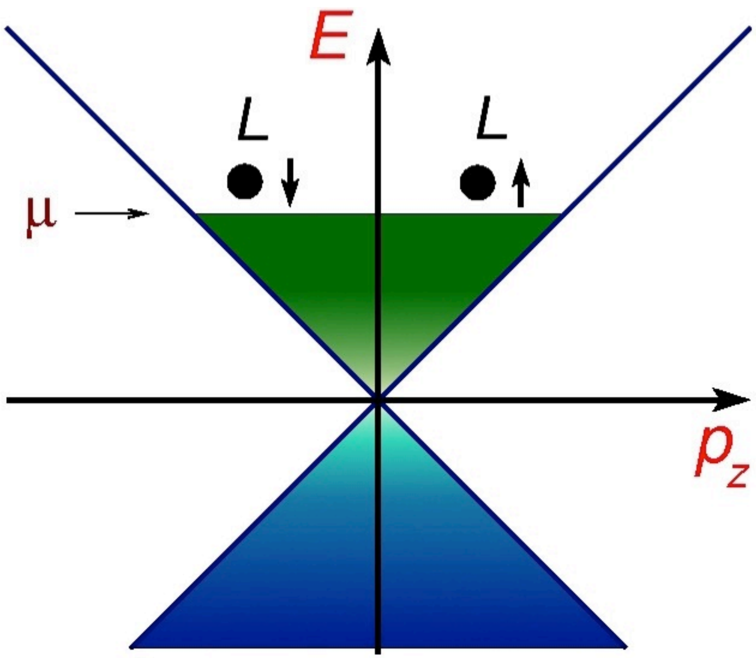




Cooper pairing is distorted by the mismatch of the flavors' Fermi surfaces. It leads to gapless CS and chromomagnetic instabilities.

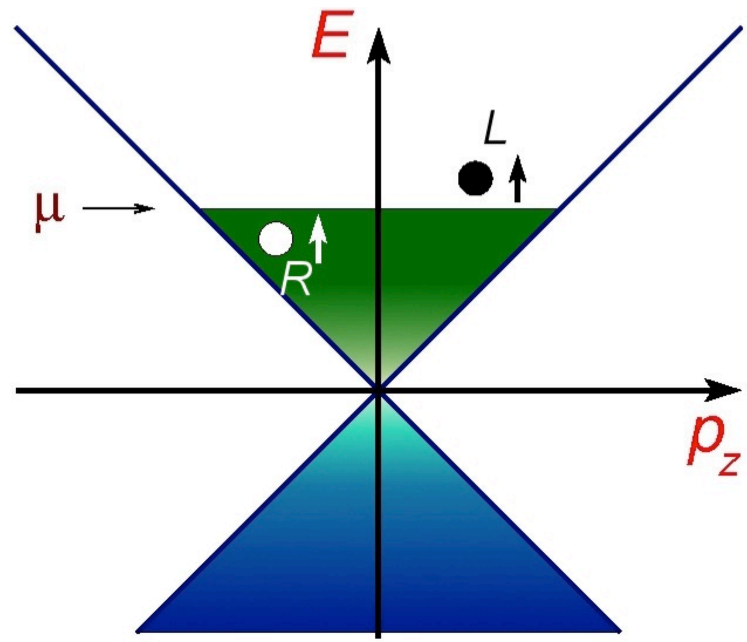
Possible solution: inhomogeneous CS phases

Competing Pairings at Intermediate Density



Cooper Pairing

Becomes less favored with decreasing densities. Color nonsinglet, hence not favored at large Nc.



Particle-Hole Pairing

No Fermi surface mismatch issue because it pairs fermions with holes. Favored over CS at large Nc.

DGR Solution at Intermediate Density

Deryagin, Grigoriev, Rubakov IJMP7,1992

$$\mu >> \Lambda_{QCD}$$

Large Nc, $g_s^2 N_c$ <<1
(perturbative regime)

→

Use an effective action that neglects gluon self-interactions and ghosts.
Fermi surface unstable towards the formation of an
inhomogeneous chiral condensate

Integrating the gluons:

$$S_\psi = \frac{1}{2}\int d^4x d^4y J^a_\rho(x) \triangle_{\rho\sigma}(x-y) J^a_\sigma(y) + \int dx^4 \overline{\psi}\partial_\nu\gamma_\nu\psi$$

$$\triangle_{\rho\sigma}(x-y) = \frac{1}{4\pi^2(x-y)^2}\delta_{\rho\sigma} \qquad J^a_\rho = g\overline{\psi}\gamma_\nu\lambda^a\psi$$

Density Wave Condensate

$$\boxed{\langle\overline{\psi}(x)\psi(y)\rangle \sim F(x-y)\cos\frac{\mathbf{p}}{2}(\mathbf{x}+\mathbf{y})} \qquad |\mathbf{p}| = \mu$$

But for the instability to develop they needed Nc → ∞

Does the DGR instability still develop at finite Nc?

Shuster & Son, NPB 573, 2000

$$\sim \frac{1}{p^2}\overline{\psi}\psi\overline{\psi}\psi$$

DGR instability develops because the effective 4-fermion interaction is singular. The gluon propagator behaves as $1/p^2$. The instability survives only if the screening effects are not too large.

$$m^2 \sim g^2\mu^2 \sim O(1/N_c)$$

SS main question:

For each chemical potential what is the smaller N_c at which the screening is not large enough to cut off the instability?

Dimensional Reduction

Shuster & Son, NPB 573, 2000

The theory becomes equivalent to a (1+1)-D Thirring-like model

$$L_{\rm eff} = i\overline{\Psi}\gamma^\mu\partial_\mu\Psi - \lambda_0(q_\|)\left(\overline{\Psi}\gamma^0\frac{T^a}{2}\Psi\right)^2 + \lambda_1(q_\|)\left(\overline{\Psi}\gamma^1\frac{T^a}{2}\Psi\right)^2$$

Has different couplings for electric and magnetic interactions

$$\lambda_0(q_\|) = \frac{g^2}{4\pi}\ln\frac{\Delta}{\max(q_\|, m)}$$

$$\lambda_1(q_\|) = \frac{g^2}{4\pi}\ln\frac{\Delta}{\max(q_\|, m^{2/3}q_\|^{1/3})}$$

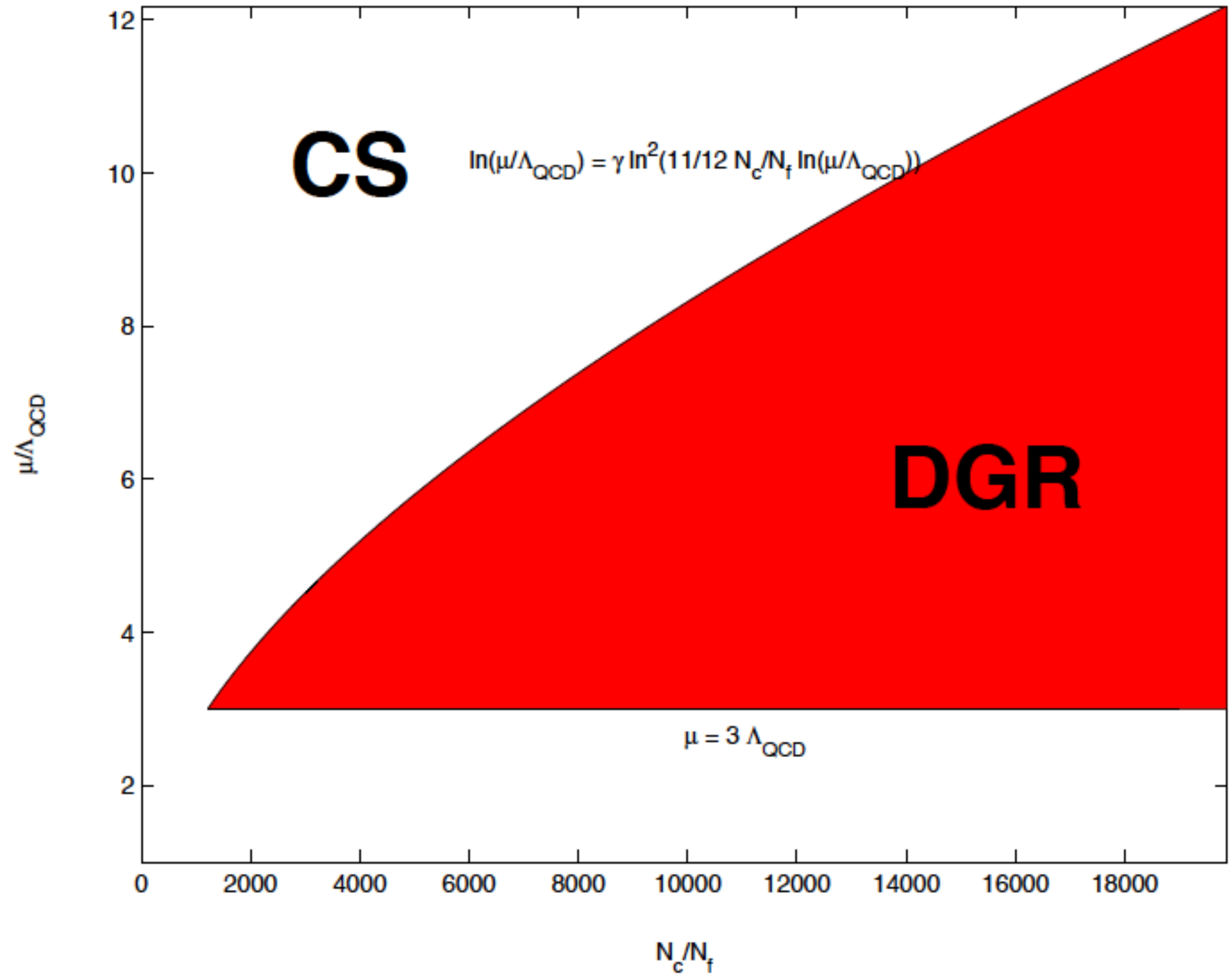
The DGR instability exists only if

$$N_{\rm c} \gtrsim 2N_{\rm f}h^2e^{2c/h}$$

$$h^2 = \frac{6}{11\ln\frac{\mu}{\Lambda_{\rm QCD}}}$$

No Favored DW in the Perturbative Regime!

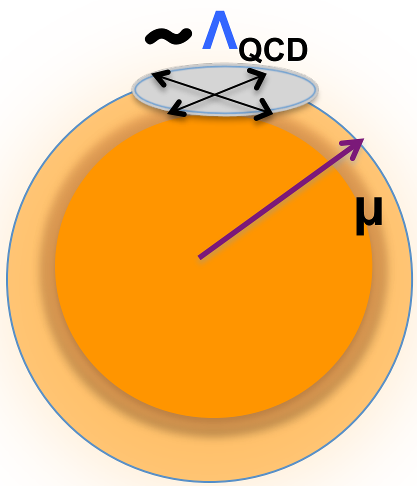
Shuster & Son, NPB 573, 2000



In the perturbative regime the DW is very sensitive to screening

Quarkyonic Matter

McLerran & Pisarski, NPA'07



Relevant at high density & large N_c , where screening effects are negligible

$$m_D << \Lambda_{QCD} << \mu$$

Bulk Properties: perturbative

Excitations at the Fermi surface: confined

Large N_c : Gluon Propagator (same as in confined vacuum). Gribov-Zwanziger propagator:

$$\boxed{D_{44}^{AB}(k) = -\frac{8\pi}{C_F} \times \frac{\sigma}{(\vec{k}^2)^2}\,\delta^{AB}}$$

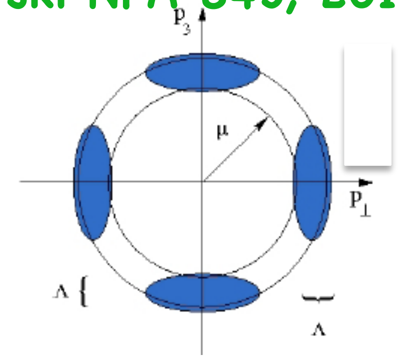
$$D_{ik}(\omega, \mathbf{q}) = \frac{\delta_{ik} - q_i q_k/q^2}{\omega^2 + \mathbf{q}^2}$$

perturbative

Valid in the Coulomb gauge and for $|p| \leq \Lambda_{QCD}$

SD-equation in Quarkyonic Matter

Kojo, Hidaka, McLerran & Pisarski NPA 843, 2010



$$\Sigma(p) = -\int \frac{d^4k}{(2\pi)^4}\, D^{AB}_{\mu\nu}(p-k)\,(\gamma_\mu t_A)\, S(k)\,(\gamma_\nu t_B)$$

$$\Sigma(p) = \int \frac{d^4k}{(2\pi)^4}\, \frac{8\pi\sigma}{((\vec{p}-\vec{k})^2)^2}\, \gamma_4\, S(k)\, \gamma_4$$

$$\Sigma(p_4, p_z, \vec{0}_\perp) \simeq \frac{N_c g_{2D}^2}{2} \int \frac{dk_4\, dk_z}{(2\pi)^2}\, \gamma_4\, S(k_4, k_z, \vec{0}_\perp)\, \gamma_4\, \frac{1}{(k_z - p_z)^2}$$

4D QCD in Coulomb gauge reduces to
1+1 D QCD in axial gauge $A_z=0$

Quarkyonic Chiral Spirals

Kojo, et al NPA'10

Equivalent to the (1+1)D theory

$$\mathcal{L}_{\rm eff}^{\rm 2D} = \overline{\Phi}\left[i\,\Gamma^\mu\left(\partial_\mu + ig_{\rm 2D}\,A_\mu\right) + \mu\,\Gamma^0\right]\Phi = \overline{\Phi}'\left[i\,\Gamma^\mu\left(\partial_\mu + ig_{\rm 2D}\,A_\mu\right)\right]\Phi'$$

Minimum Solution: Quarkyonic Chiral Spirals

$$\langle\overline{\Phi}\Phi\rangle = \langle\overline{\Phi}'\Phi'\rangle\cos(2\mu z)$$

$$\langle\overline{\Phi}i\Gamma^5\Phi\rangle = \langle\overline{\Phi}'\Phi'\rangle\sin(2\mu z)$$

In terms of the 4D fermions the two condensates are:

$$\langle\overline{\psi}\psi\rangle \qquad \langle\overline{\psi}\gamma_0\gamma_3\psi\rangle$$

No Need for “Almost” Infinite Nc

Kojo, et al NPA’10; Kojo, arXiv: 1106.2117

$$\langle \overline{\psi} exp(2i\mu\gamma_0\gamma_3)\psi \rangle$$

QCS forms thanks to the nonperturbative dynamics of the excitations near the Fermi surface.

Can occur at smaller values of N_c than the perturbative DW because the screening becomes relevant only when quark fluctuations become comparable to those of gluons

$$N_c \times \frac{\mu^2}{\Lambda_{QCD}^2} \sim N_c^2$$

NJL Approach: Dual Chiral Density Wave

Nakano & Tatsumi, PRD71, 2005

Start with NJL model

$$\mathcal{L}_{NJL} = \bar{\psi}(i\partial\!\!\!/ - m_c)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2]$$

Consider the condensates

$$\begin{aligned} \langle\bar{\psi}\psi\rangle &= \Delta\cos(\mathbf{q}\cdot\mathbf{r}) \\ \langle\bar{\psi}i\gamma_5\tau_3\psi\rangle &= \Delta\sin(\mathbf{q}\cdot\mathbf{r}), \end{aligned}$$

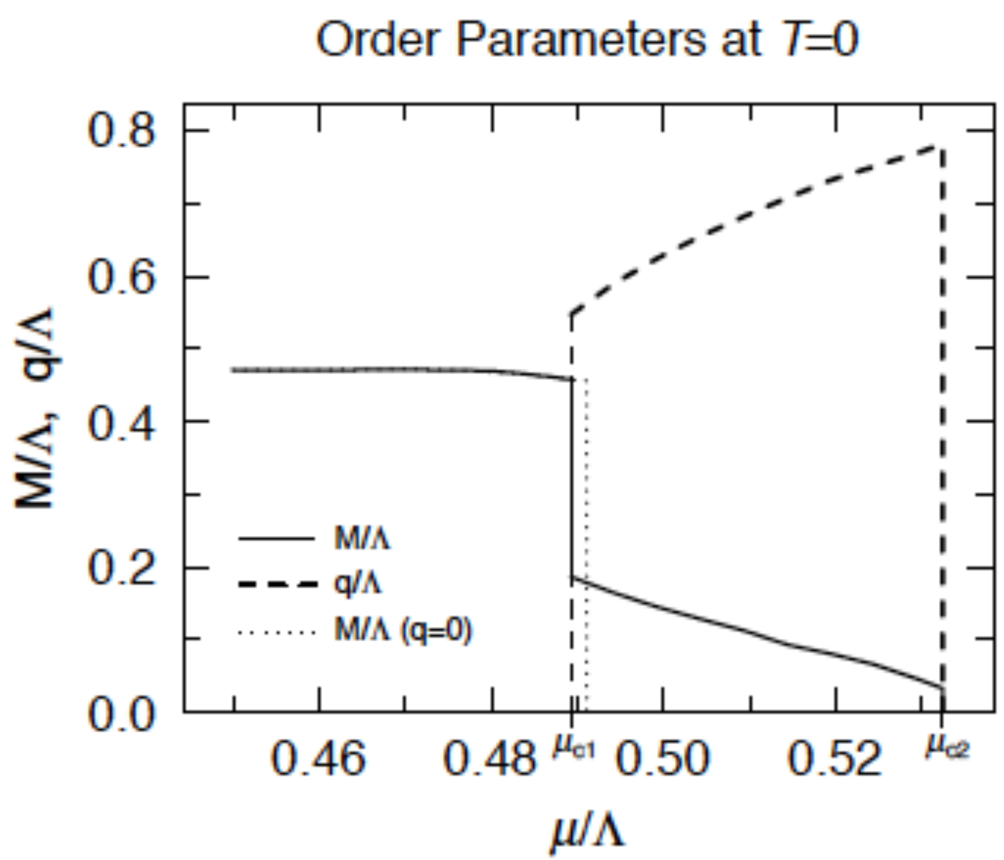
The mean-field Lagrangian then is

$$\mathcal{L}_{MF} = \bar{\psi}\left[i\partial\!\!\!/ + \mu\gamma_0 - M\{\cos(\mathbf{q}\cdot\mathbf{r}) + i\gamma_5\tau_3\sin(\mathbf{q}\cdot\mathbf{r})\}\right]\psi - \frac{M^2}{4G}$$

NJL Approach: Dual Chiral Density Wave

Nakano & Tatsumi, PRD71, 2005

Solution is a DCDW $\langle\overline{\psi} exp(2i\mu\gamma_5)\psi\rangle$



NJL Approach: Soliton/Crystalline Condensate

Nickel, PRL103, 2009; PRD80, 2009

Considered NJL model with discrete chiral symmetry

$$\mathcal{L} = \bar{\psi} i \gamma^{\mu} \partial_{\mu} \psi + G(\bar{\psi} \psi)^2$$

With mean-field Lagrangian

$$\mathcal{L}_{\mathrm{MF}} = \bar{\psi}[i\gamma^{\mu}\partial_{\mu} - M(\mathbf{x})]\psi - \frac{M(\mathbf{x})^2}{4G}$$

$$\langle \bar{\psi}(\mathbf{x}) \psi(\mathbf{x}) \rangle = -\tfrac{1}{2G} M(\mathbf{x})$$

Crystalline Condensate

Nickel, PRL103, 2009; PRD80, 2009

The GL expansion has the same structure of the GN (1+1)D model

$$\Omega_{\mathrm{GL}}[T,\mu;M(\mathbf{x})] = \frac{\alpha_2}{2}M(\mathbf{x})^2 + \frac{\alpha_4}{4}\{M(\mathbf{x})^4 + [\nabla M(\mathbf{x})]^2\} + \frac{\alpha_6}{6}\Big(M(\mathbf{x})^6 + 5[\nabla M(\mathbf{x})]^2 M(\mathbf{x})^2 + \frac{1}{2}[\Delta M(\mathbf{x})]^2\Big)$$

For a single modulated condensate the solution is

$$M_{1\mathrm{D}}(z) = \sqrt{\nu}q\ \mathrm{sn}(qz,\nu)$$

It behaves as a sinusoidal function close to the transition:

Crystalline condensate

Crystalline or Spiral Solution?

Nickel, PRD80, 2009

Consider NJL model with continuous chiral symmetry

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - \hat{m})\psi + G_s((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2)$$

and two condensates

$$\langle\bar{\psi}\psi\rangle = S(\mathbf{x}) \qquad \langle\bar{\psi}i\gamma^5\tau^a\psi\rangle = P_a(\mathbf{x})$$

Then, the mean-field Hamiltonian is

$$\tilde{H}_{\mathrm{MF}} = -i\gamma^0\gamma^i\partial_i + \gamma^0(m - 2G_sS(\mathbf{x}) - 2iG_s\gamma^5\tau^3P(\mathbf{x}))$$

So we have now a complex condensate

$$H_{\mathrm{MF},+} = \begin{pmatrix} i\sigma^i\partial_i & M(\mathbf{x}) \\ M(\mathbf{x})^* & -i\sigma^i\partial_i \end{pmatrix}. \quad M(\mathbf{x}) = m - 2G_s(S(\mathbf{x}) + iP(\mathbf{x}))$$

The thermodynamic potential is

$$\Omega(T,\mu;S(\mathbf{x}),P(\mathbf{x})) = -\frac{TN_c}{V}\sum \mathrm{Tr}_{D,f,V}\,\mathrm{Log}\Big(\frac{1}{T}(i\omega_n + \tilde{H}_{\mathrm{MF}} - \mu)\Big) + \frac{G_s}{V}\int_V (S(\mathbf{x})^2 + P(\mathbf{x})^2)$$

For single-modulated condensates, $P_\perp$ commute with H and the eigenvalue spectrum can be constructed from the subspace spanned by the eigenfunctions at zero $P_\perp$, thus H reduces to

$$H'_{\mathrm{MF};1D} = \begin{pmatrix} H_{1D}(M(z)) & \\ & H_{1D}(M(z)^*) \end{pmatrix}$$

Leads to the GL expansion of the Gross-Neveu model

$$H_{1D}(M^*(z)) = \begin{pmatrix} -i\partial_z & M^*(z) \\ M(z) & i\partial_z \end{pmatrix}$$

Bogoliubov-De Gennes Hamiltonian

GL Expansion

$$H'_{\mathrm{MF};1D} = \begin{pmatrix} H_{1D}(M(z)) & \\ & H_{1D}(M(z)^*) \end{pmatrix}$$

Each H_{1D} in the direct product leads to the GL expansion of the NJL2 model:

$$\Omega_{\mathrm{GL,GN}}(M) - \Omega_{\mathrm{GL,GN}}(0) = \frac{\alpha_2}{2}|M|^2 + \frac{\alpha_3}{3}\mathrm{Im}(MM'^*) + \frac{\alpha_4}{4}(|M|^4 + |M'|^2)$$
$$+ \frac{\alpha_5}{5}\mathrm{Im}((M'' - 3|M|^2 M)M'^*)$$
$$+ \frac{\alpha_6}{6}\left(|M|^6 + 3|M|^2|M'|^2 + 2|M|^2|M^2|' + \frac{1}{2}|M''|^2\right) + \ldots$$

Bassar, Dunnes, Thies, PRD79, 2009

The terms with odd coefficients cancel out in the sum because one H is evaluated in M and the other in M*. The result is again the GN Hamiltonian (related to the theory with discrete chiral symmetry). The minimum solution is again the real, soliton solution! The chiral spiral is not favored.

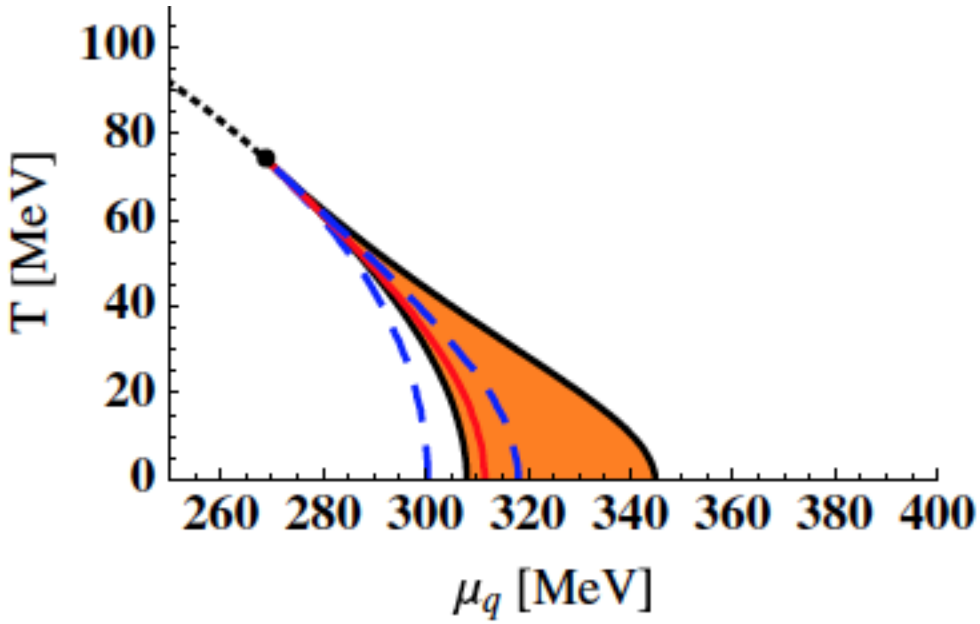
$$\Omega_{\mathrm{GL,GN}}(M) - \Omega_{\mathrm{GL,GN}}(0) = \frac{\alpha_2}{2}|M|^2 + \frac{\alpha_4}{4}(|M|^4 + |M'|^2)$$

$$+ \frac{\alpha_6}{6}\left(|M|^6 + 3|M|^2|M'|^2 + 2|M|^2|M^2|' + \frac{1}{2}|M''|^2\right) + \ldots$$

Nickel, PRD80, 2009

No CP with the Crystalline Minimum

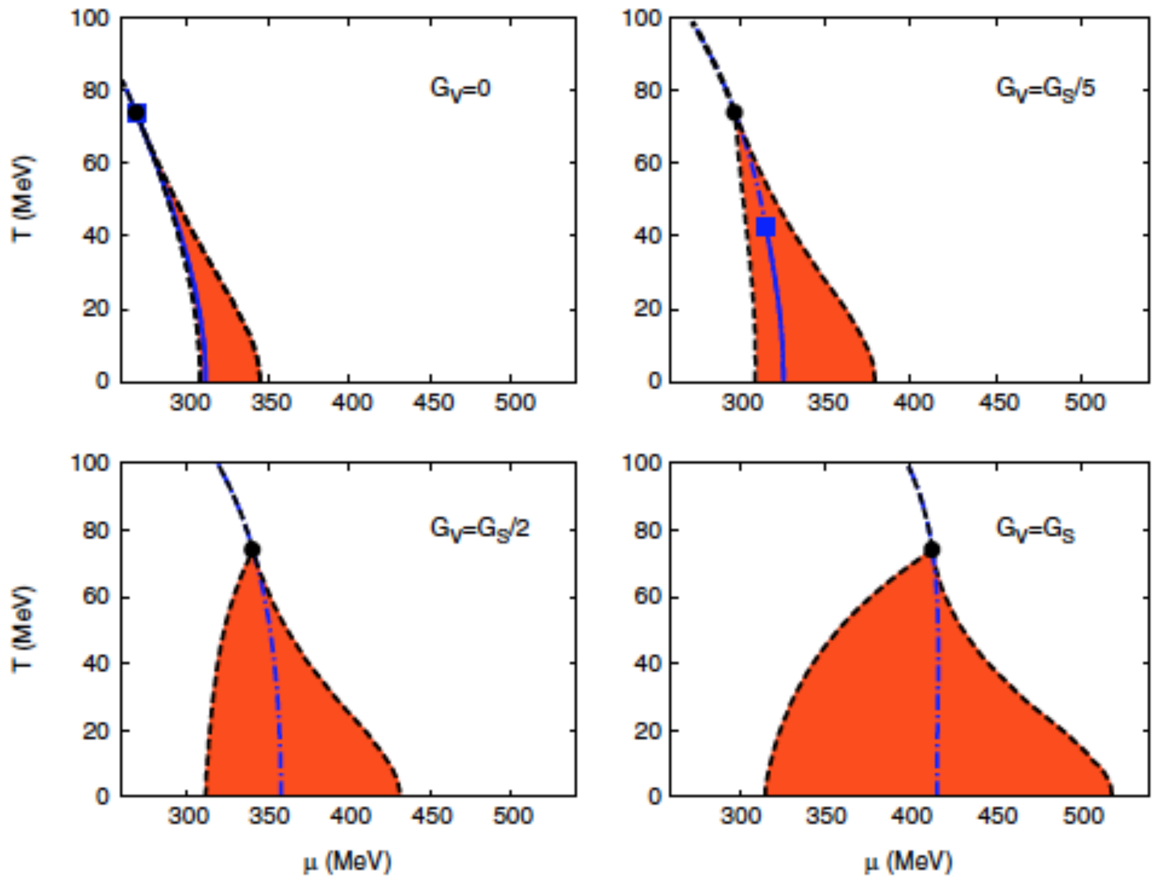
Nickel, PRD80, 2009



The critical point is now a Lifshitz point. Hence, the critical region that surrounds the CP disappears. No divergent susceptibilities in this phase diagram.

Vector Interactions Effects

Carignano, Nickel, & Buballa PRD82, 2010



No much effect on the inhomogeneous phase. The LP stays at the same T when vector interactions are included. Their effect on the homogeneous phases in contrast is significant, as they shift the CP position to lower temperatures and larger chemical potentials

A New Look to P-H Condensates in the NJL Approach

Feng, Ferrer, & VI, arXiv. 1304.0256

Consider an NJL model with continuous chiral symmetry

$$\mathcal{L} = \bar{\psi}\left(\gamma^\mu(i\partial_\mu + \mu\delta_{\mu 0}\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2\right]$$

A New Look to P-H Condensates in the NJL Approach

Feng, Ferrer, & VI, arXiv. 1304.0256

Consider an NJL model with continuous chiral symmetry and tensor channels

$$\mathcal{L} = \bar{\psi}\left(\gamma^{\mu}(i\partial_{\mu} + \mu\delta_{\mu 0}\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2\right]$$

$$+G'\left[(\bar{\psi}\sigma^{\mu\nu}\psi)^2 + (\bar{\psi}i\tau\gamma_5\sigma^{\mu\nu}\psi)^2\right]$$

At finite density the Fierz identities open up tensor channels that were absent in the Lorentz invariant theory

$$\gamma^0\gamma^0 = \frac{1}{4}\{(1)(1) + (i\gamma_5)(i\gamma_5) + \sigma^{oi}\sigma^{oi} + \ldots\}$$

Single modulated condensates:

$$\langle\bar{\psi}\psi\rangle = S(z) \qquad \langle\bar{\psi}\sigma^{03}\psi\rangle = D(z)$$

$$H_{MF} = -i\gamma^0\gamma^i\partial_i + \gamma^0\left[d_+ M(z) + d_- M^*(z)\right]$$

$$-\frac{M(z)}{2G} = S(z) + iD(z) \qquad d_\pm = \frac{1\pm\gamma^0\gamma^3}{2}$$

$$\Omega(T,\mu;M) = -\frac{TN_cN_f}{V}\sum_n \mathrm{Tr}\ln\frac{1}{T}\left[i\omega_n + H_{MF} - \mu\right] + \frac{1}{V}\int_V \frac{|M(z)|^2}{4G}$$

$$H'_{MF;1D} = \begin{pmatrix} H_{1D}(M^*(z)) & 0 \\ 0 & H_{1D}(M^*(z)) \end{pmatrix}$$

Leads to the NJL2 GL expansion

$$H_{1D}(M^*(z)) = \begin{pmatrix} -i\partial_z & M^*(z) \\ M(z) & i\partial_z \end{pmatrix}$$

Bogoliubov-De Gennes Hamiltonian

GL Expansion in the NJL with tensor channel

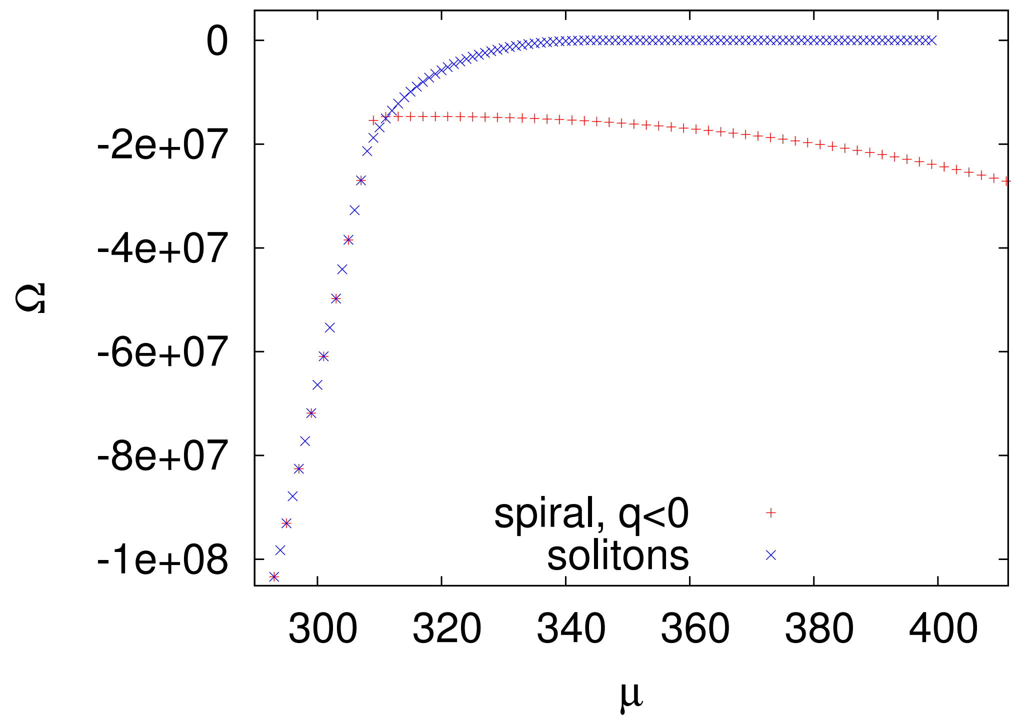
$$H'_{MF;1D} = \begin{pmatrix} H_{1D}(M^*(z)) & 0 \\ 0 & H_{1D}(M^*(z)) \end{pmatrix}$$

No cancellation of the odd terms in the GL expansion

$$\Omega_{\mathrm{GL,GN}}(M) - \Omega_{\mathrm{GL,GN}}(0) = \frac{\alpha_2}{2}|M|^2 + \frac{\alpha_3}{3}\mathrm{Im}(MM'^*) + \frac{\alpha_4}{4}(|M|^4 + |M'|^2)$$
$$+ \frac{\alpha_5}{5}\mathrm{Im}((M'' - 3|M|^2 M)M'^*)$$
$$+ \frac{\alpha_6}{6}\left(|M|^6 + 3|M|^2|M'|^2 + 2|M|^2|M^2|' + \frac{1}{2}|M''|^2\right) + \dots$$

The symmetry of this theory is the same as in the NJL_2

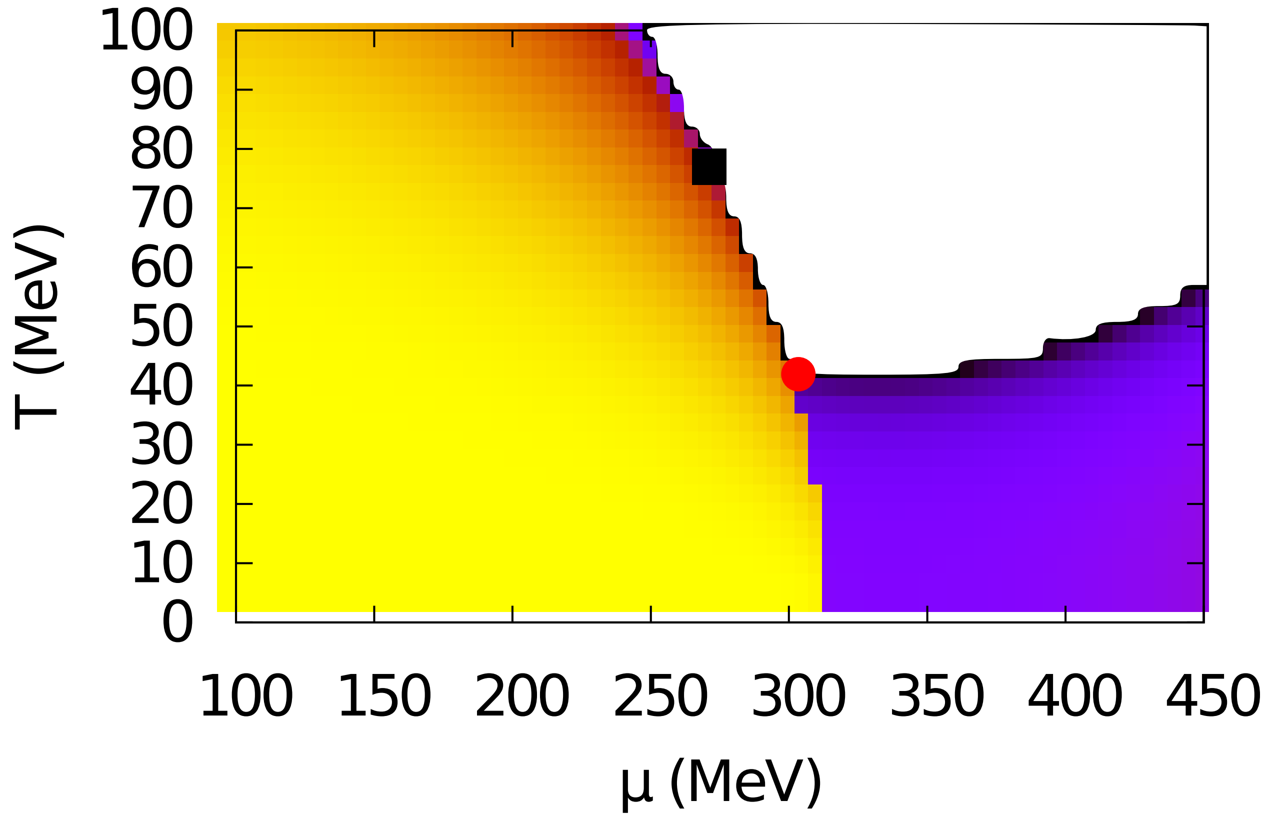
Carignano, Ferrer, & VI, in prep



The most favored solution is now the chiral spiral, not the real crystalline condensate!

Symmetry is also different from the model with scalar and pseudoscalar channels only because now the ground state breaks parity!

Carignano, Ferrer, & VI, in prep



Experimental Implications (for current and future experiments):

- Two regions of critical behavior: the phase diagram has the expected CP plus another one at lower T.
- Parity violation should also lead to experimental signals.

NJL-QCD Effective Model Connections

Real (sinusoidal) solution (GN symmetry)

DGR

$$J_\rho^a(x)J_\sigma^a(y) = G_{ABCD}\overline{\psi}_A(x)\psi_B(x)\overline{\psi}_C(y)\psi_D(y) \qquad J_\rho^a = g\overline{\psi}\gamma_\rho\lambda^a\psi$$

Nickel's

$$\mathcal{L} \;=\; \bar{\psi}\,(\gamma^\mu(i\partial_\mu)\,\psi + +G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2\right]$$

Chiral Spiral solution (NJL_2 symmetry)

Kojo et al. (quarkyonic chiral spiral)

$$\Sigma(p) = \int \frac{d^4k}{(2\pi)^4}\,\frac{8\pi\sigma}{((\vec{p}-\vec{k})^2)^2}\,\gamma_4\,S(k)\,\gamma_4 \qquad J_0^a(x)J_0^a(y)$$

Feng, Ferrer, and VI

$$\begin{aligned}\mathcal{L} \;&=\; \bar{\psi}\,(\gamma^\mu(i\partial_\mu)\,\psi + +G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2\right] \\ &+\; G\left[(\bar{\psi}\sigma^{0i}\psi)^2 + (\bar{\psi}i\tau\gamma_5\sigma^{0i}\psi)^2\right]\end{aligned}$$

To be Done…

- Finding the exact location of the new CP
- Calculating the susceptibilities
- Effects of vector interactions
- Comparing with CS condensates
- …