Non-local chiral quark models in medium - a case study

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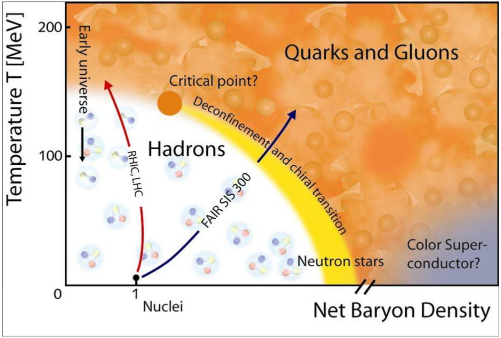
Overview

- introduction
- non-local Nambu–Jona-Lasinio model
- mean field
- propagator singularities
- thermodynamics
- correlations
- conclusions

QCD phase transition

- two "basis" of states in QCD - hadrons & quarks (+gluons)
- hadrons: e. g. ChPT
- quarks+gluons: e. g. pQCD
- describe the phase transition of one world to another..
- ..in terms of hadrons dissolving into quarks (+gluons)



Illustration

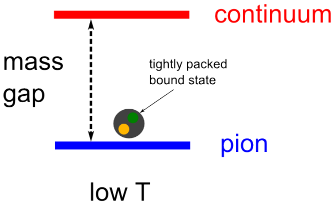
1. field theory picture: melting condensates

$$\langle \bar{q}q \rangle \, , \quad \langle G^2 \rangle \, , \ldots$$

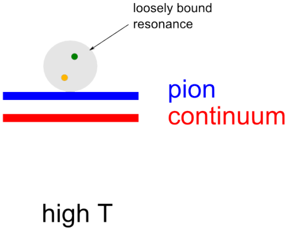
2. chemical picture: hadron dissociation

- reconcile the two pictures? $\rightarrow$ e. g. pion

- condensates create mass gap



- condensates melt, interaction screens



- caveat: chiral physics $\rightarrow$ confinement?

Task

- construct hadrons from substructure
- lowest lying excitations first to appear in thermodynamics: pions, kaons, etas
- tools: QCD?

  → chiral quark models beyond mean-field

  1. Nambu–Jona-Lasinio (NJL)

     Hüfner, Klevansky, Zhuang, Voss, Annals Phys. **234**, 225 (1994)
     Zhuang, Hüfner, Klevansky, Nucl. Phys. A **576**, 525 (1994)
     Wergieluk, Blaschke, Kalinovsky, Friesen, arXiv:1212.5245 [nucl-th]
     Yamazaki, Matsui, Nucl. Phys. A **913**, 19 (2013)
     Blaschke, Zablocki, Buballa, Röpke, arXiv:1305.3907 [hep-ph]
     Yamazaki, Matsui, arXiv:1310.4960 [hep-ph]

  2. non-local NJL

     Blaschke, Buballa, Radzhabov, Volkov, Yad. Fiz. **71**, 2012 (2008)
     Hell, Roessner, Cristoforetti, Weise, Phys. Rev. D **79**, 014022 (2009)
     Hell, Rossner, Cristoforetti, Weise, Phys. Rev. D **81**, 074034 (2010)
     Radzhabov, Blaschke, Buballa, Volkov, Phys. Rev. D **83**, 116004 (2011)

Non-local Nambu-Jona-Lasinio model

- initial studies

  Schmidt, Blaschke, Kalinovsky, Phys. Rev. C **50**, 435 (1994)
  Bowler, Birse, Nucl. Phys. A **582** 655 (1995)
  Ripka, “Quarks bound by chiral fields” Oxford, UK: Clarendon Pr. (1997) 205 p
  Golli, Broniowski, Ripka, Phys. Lett. B **437** 24 (1998)
  General, Gomez Dumm, Scoccola, Phys. Lett. B **506**, 267 (2001)

- benefits of the non-local approach

  close contact with Dyson-Schwinger methods (via separable kernels) and QCD inspired approaches

  Roberts, Schmidt,Prog. Part. Nucl. Phys. **45** (2000) S1
  Alkofer, von Smekal, Phys. Rept. **353**, 281 (2001)
  Blaschke, Burau, Kalinovsky, Maris, Tandy, Int. J. Mod. Phys. A **16**, 2267 (2001)
  Horvatic, Blaschke, Klabucar, Kaczmarek, Phys. Rev. D **84** (2011) 016005
  Blaschke, Horvatic, Klabucar, Radzhabov, hep-ph/0703188 [HEP-PH]
  Schäfer, Shuryak, Rev. Mod. Phys. **70** 323 (1998)
  Kondo, Phys. Rev. D **82**, 065024 (2010)

  model confinement via positivity violation

  Roberts, Schmidt,Prog. Part. Nucl. Phys. **45** (2000) S1

  strong running of quark self energy as seen on the lattice

  Parappilly, Bowman, Heller, Leinweber, Williams, Zhang, Phys. Rev. D **73**, 054504 (2006)
  Kamleh, Bowman, Leinweber, Williams, Zhang, Phys. Rev. D **76**, 094501 (2007)

  covariant regularization $\rightarrow$ no ad-hoc prescriptions in treating anomalies

  Blaschke, Kalinovsky, Röpke, Schmidt, Volkov, Phys. Rev. C **53** 2394 (1996)
  Holdom, Terning, Verbeek, Phys. Lett. B **232** 351 (1989)
  Ruiz Arriola, Salcedo, Phys. Lett. B **450** 225 (1999)

Non-local NJL model for $N_f = 2$

$$S_E = \int_x \left\{ \bar{q}(-i\partial\!\!\!/ + m)q - \frac{G_S}{2} \left[j_a^S(x) j_a^S(x) + j_p(x) j_p(x) \right] \right\}$$

Blaschke, Buballa, Radzhabov, Volkov, Yad. Fiz. **71**, 2012 (2008)
Contrera, Orsaria, Scoccola, Phys. Rev. D **82**, 054026 (2010)
Hell, Kashiwa, Weise, Phys. Rev. D **83** (2011) 114008
S. B., Blaschke, Contrera, Horvatic, arXiv:1306.0588 [hep-ph]

$$j_a^S(x) = \int_z g(z) \bar{q}\left(x + \frac{z}{2}\right) \Gamma_a q \left(x - \frac{z}{2}\right) , \quad j_p(x) = \int_z f(z) \bar{q}\left(x + \frac{z}{2}\right) \frac{i \overleftrightarrow{\partial\!\!\!/}}{2\kappa_p} q \left(x - \frac{z}{2}\right)$$

$$\Gamma_a = (1, i\gamma_5 \boldsymbol{\tau})$$

- mean-field approximation

$$j^2 \approx -\langle j \rangle^2 + 2\langle j \rangle j$$

$$\Omega = \frac{{\sigma_1}^2 + \kappa_p^2 {\sigma_2}^2}{2G_S} - N_f N_c \int_p \mathrm{tr} \log[S^{-1}(p)]$$

- non-locality yields dynamical mass and wave function renormalization

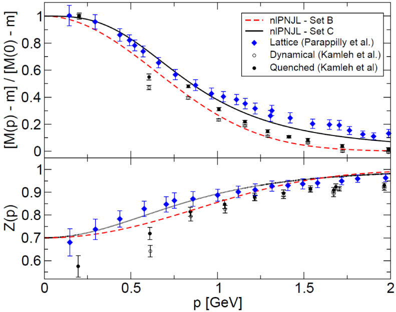
$$S^{-1}(p) = -p\!\!\!/ A(p^2) + B(p^2)$$

$$B(p^2) = m + \sigma_1 g(p^2) , \qquad A(p^2) = 1 + \sigma_2 f(p^2)$$

Lattice fits

- alternative form

$$M(p^2) = \frac{B(p^2)}{A(p^2)}\,, \qquad Z(p^2) = \frac{1}{A(p^2)}$$



Parappilly, Bowman, Heller, Leinweber, Williams, Zhang, Phys. Rev. D **73**, 054504 (2006)
Kamleh, Bowman, Leinweber, Williams, Zhang, Phys. Rev. D **76**, 094501 (2007)
Noguera, Scoccola, Phys. Rev. D **78** (2008) 114002
Contrera, Grunfeld, Blaschke, arXiv:1207.4890 [hep-ph]

- many shapes of form-factors $g(p^2)$ and $f(p^2)$ possible

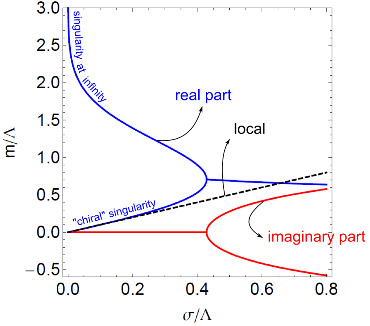
Spectrum of the quark propagator

- want to describe meson in-medium dissociation

  what are the states in which meson will decay to?

  → study analytic structure of the quark propagator
- simplest case: chiral limit, no WFR term, Gaussian form-factor

$$f(p^2) = 0\,, \quad g(p^2) = e^{-p^2/\Lambda^2}$$



- strong interactions ($\sigma > \sigma_c = \Lambda/(\sqrt{2}e)$)
  → complex propagator singularities

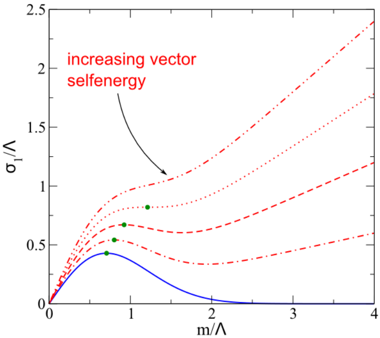
  → positivity violation → confinement

Roberts, Schmidt,Prog. Part. Nucl. Phys. **45** (2000) S1
Alkofer, von Smekal, Phys. Rept. **353**, 281 (2001)
Fischer, J. Phys. G **32** (2006) R253

Spectrum of the quark propagator

- chiral limit with WFR term, two Gaussians

$$f(p^2) = e^{-p^2/\Lambda^2}, \quad g(p^2) = e^{-p^2/\Lambda^2}$$



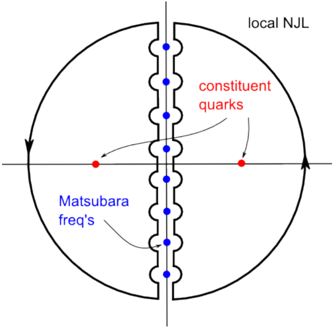
- conclusion

no WFR	mass gap overcritical
WFR	mass gap undercritical

S. B., Blaschke, Contrera, Horvatic, arXiv:1306.0588 [hep-ph]

Impact on thermodynamics

- Matsubara technique for $T > 0$: $p_4 \to \omega_n = (2n+1)\pi T$
- contour for thermal sums sees the complex singularities $\mathcal{E} = \epsilon + i\gamma$



$$\Omega \sim -\int_{\mathbf{p}} \left\{ \log[1 + e^{-\beta\mathcal{E}}] + \log[1 + e^{-\beta\mathcal{E}^*}] \right\} = -\int_{\mathbf{p}} \log[1 + 2\cos(\beta\gamma) e^{-\beta\epsilon} + e^{-2\beta\epsilon}]$$

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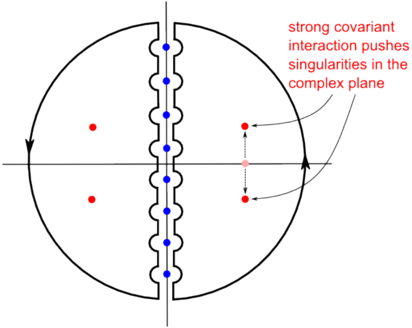

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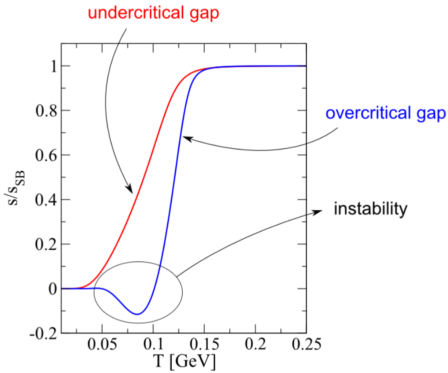

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Stabilizing the thermodynamics

- if these states represent confined particles should we count them in thermodynamics?

  omit them by hand? $\to$ but some of these singularities are continously connected to physical states in the UV

  $\to$ possible mechanism: destructive interference with the Polyakov loop

$$\exp(-\beta\mathcal{E}) = \exp(-\beta(\epsilon + i\gamma)) \to \exp(-\beta(\epsilon + i\gamma + i\phi_3))$$

$$\Omega \sim -3\int_{\mathbf{p}} \log\left[1 + e^{-\beta\mathcal{E}}\right] + \text{c.c.} \to -\int_{\mathbf{p}} \log\left[1 + 3\Phi e^{-\beta\mathcal{E}} + 3\Phi e^{-2\beta\mathcal{E}} + e^{-3\beta\mathcal{E}}\right] + \text{c.c.}$$

- similar to expelling constituent quarks in NJL $\to$ PNJL

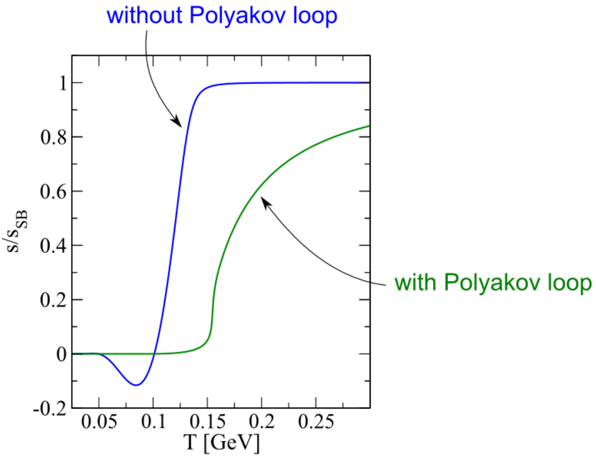
Fukushima, Phys. Lett. B **591**, 277 (2004)
Ratti, Thaler, Weise, Phys. Rev. D **73** 014019 (2006)

$$\Omega \sim -\int_{\mathbf{p}} \log\left\{1 + 6\Phi\left[e^{-\beta\epsilon}\cos(\beta\gamma)e^{-4\beta\epsilon}\cos(2\beta\gamma)\right] + 6\Phi\left[e^{-2\beta\epsilon}\cos(\beta\gamma) + e^{-5\beta\epsilon}\cos(\beta\gamma)\right]\right.$$
$$\left. + 9\Phi^2 e^{-2\beta\epsilon} + 9\Phi^2 e^{-4\beta\epsilon} + 18\Phi^2 e^{-2\beta\epsilon}\cos(\beta\gamma) + 2e^{-3\beta\epsilon}\cos(3\beta\gamma) + e^{-6\beta\epsilon}\right\}$$

S. B., Blaschke, Buballa, Phys.Rev. D **86** 074002 (2012)

Stabilizing the thermodynamics

- explicit calculation



S. B., Blaschke, Buballa, Phys.Rev. D **86** 074002 (2012)
S. B., Blaschke, Contrera, Horvatic, arXiv:1306.0588 [hep-ph]

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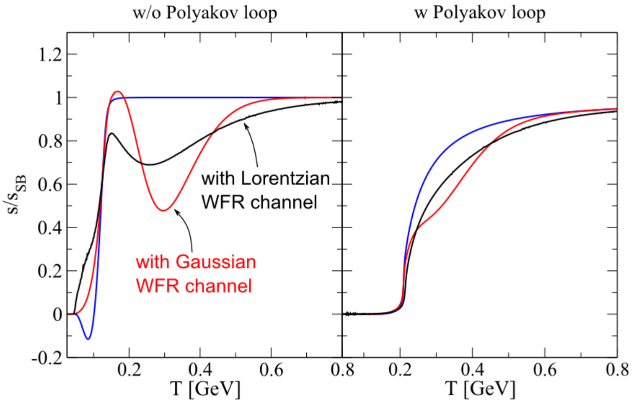
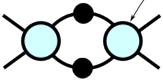


S. B., Blaschke, Buballa, Phys.Rev. D **86** 074002 (2012)
S. B., Blaschke, Contrera, Horvatic, arXiv:1306.0588 [hep-ph]

Non-local NJL vs. Instanton liquid models (ILM)



- same on the mean-field level, but..
- ..for correlations different momentum partitioning for the form-factor in the quark-antiquark polarization loop
- nl-NJL

$$\Pi_M(q) = \frac{8N_c}{3} \int_p g^2(p^2) \frac{K_M(p^2, q^2, p \cdot q)}{\mathcal{D}(p_+^2)\mathcal{D}(p_-^2)}$$

- ILM

$$\Pi_M(q) = \frac{8N_c}{3} \int_p r^2(p_+^2) r^2(p_-^2) \frac{K_M(p^2, q^2, p \cdot q)}{\mathcal{D}(p_+^2)\mathcal{D}(p_-^2)}$$

$(p_\pm = p \pm q/2)$

- consequence in vacuum

nl-NJL	mass gap overcritical
ILM	mass gap undercritical

Correlations in medium

- correct degrees of freedom at low T: mesons
- in local NJL: mesons dissociate when their mass hits the continuum threshold - Mott transition
- in non-local models non-trivial: gap needs to lower sufficiently to create a threshold
- start by studying the quark-antiquark polarization loop in medium

$$\Pi_M(\nu_m, |\mathbf{q}|) = \frac{8N_c}{3} T \sum_{n=-\infty}^{\infty} \int_{\mathbf{p}} \mathrm{tr}_C \left[g^2(\tilde{p}_n^2) \frac{K_M(\tilde{\omega}_n^2, \mathbf{p}^2, \nu_m^2, \mathbf{q}^2)}{\mathcal{D}((\tilde{\omega}_n^+)^2, (\mathbf{p}^+)^2)\mathcal{D}((\tilde{\omega}_n^-)^2, (\mathbf{p}^-)^2)} \right]$$

- real part of Π_M: all singularities contribute
  1. Gaussian model: infinite number of them
  2. Lorentzian model: finite number
- imaginary part of Π_M: only real singularities contribute

Plant, Birse, Nucl.Phys. A **628** 607-644 (1998)

Meson width-approximations

$$\Gamma_M(q_0) \simeq g^2_{M\bar{q}q}(q_0) \frac{\mathrm{Im}[\Pi_M(-iq_0, 0)]}{q_0}$$

1. low T $\rightarrow$ all singularities complex
2. high T $\rightarrow$ some singularities become real
3. calculate the threshold only for singularities that continuously evolve to current quarks as T increases
4. assume the Mott temperature is high enough so that the singularities are those given at zero momentum, m_{qp}

$$\mathrm{Im}[\Pi_M(-iq_0, 0)] = \frac{d_q}{16\pi} \left[1 - n_+^{\Phi}(q_0/2) - n_-^{\Phi}(q_0/2)\right] \sqrt{1 - \left(\frac{2m_{\mathrm{qp}}}{q_0}\right)^2}$$
$$\times\, g^2\left(\frac{q_0^2}{4} - m_{\mathrm{qp}}^2\right) \frac{K_M\left(0, \frac{q_0^2}{4} - m_{\mathrm{qp}}^2, -q_0^2, 0\right)}{\left[\mathcal{D}'\left(-\frac{q_0^2}{4}, \frac{q_0^2}{4} - m_{\mathrm{qp}}^2\right)\right]^2} \theta\left(\frac{q_0}{2} - m_{\mathrm{qp}}\right)$$

S. B., Blaschke, Contrera, Horvatic, arXiv:1306.0588 [hep-ph]

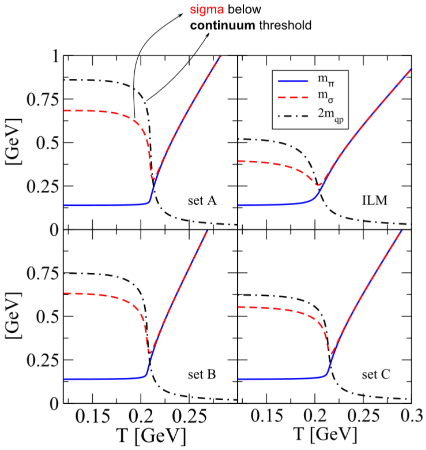
Meson width

5. assume Golberger-Treiman relation holds in medium

$$g_{\pi\bar{q}q} \simeq m_\pi/f_\pi \simeq \text{const.}$$

6. for q_0 put screening (spatial) masses (pole masses much more difficult to calculate)

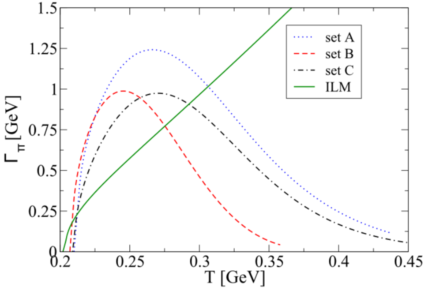
$$1 - G_S \Pi_M(0, -i m_M^{\text{spat}})$$



Meson width - results

- form-factor in the imaginary part

nl-NJL	$g^2(q_0^2 - \frac{m_{\text{qp}}^2}{4})$
ILM	$r^4(-m_{\text{qp}}^2)$



- widths extremely sensitive to the way the non-local interaction is introduced

S. B., Blaschke, Contrera, Horvatic, arXiv:1306.0588 [hep-ph]

Conclusions

- QCD phase transition through meson dissociation
- thermodynamics in the mean-field unstable $\rightarrow$ Polyakov loop
- mesons as bound states in medium
- approximate form of the width in the non-local model
- widths extremely sensitive to the way the non-locality is introduced

Thank you very much for your attention!