Inhomogeneous phases in the Quark-Meson model (or “why I hate regularization and what am I trying to do about it”)

Stefano Carignano

The University of Texas at El Paso

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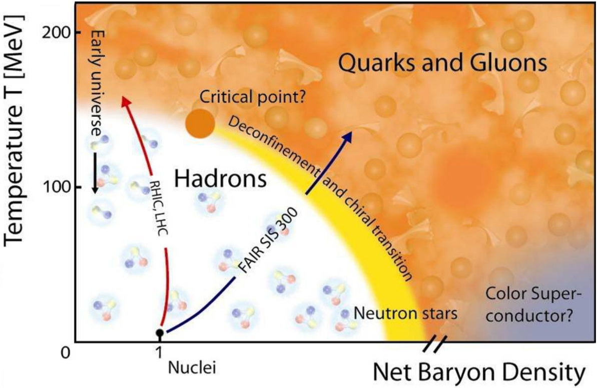
Motivation: The QCD phase diagram



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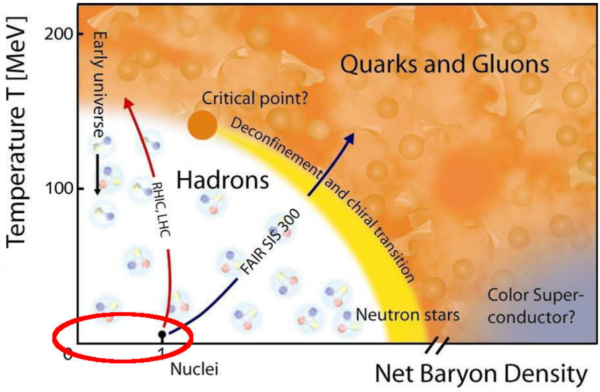

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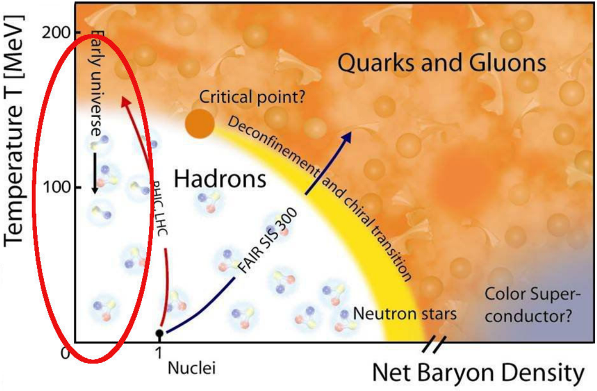

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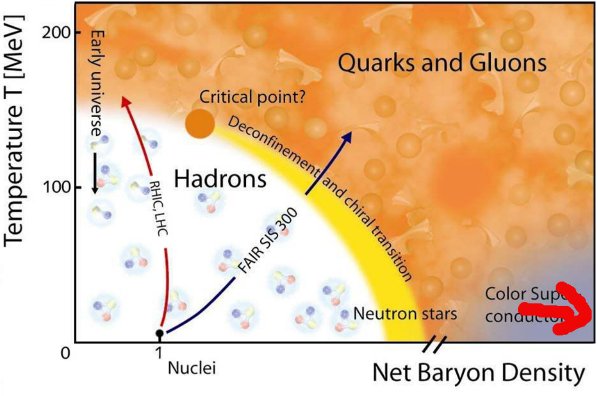

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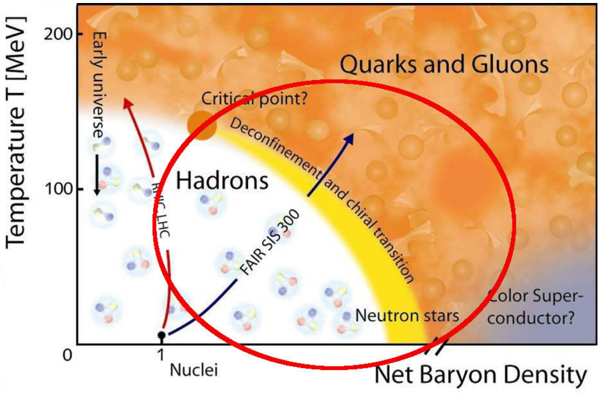

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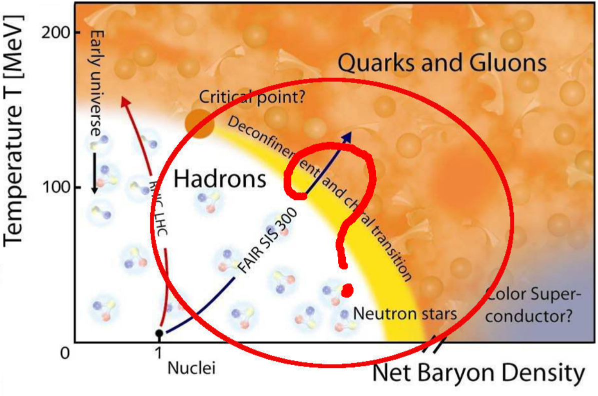

A popular approach: NJL

- Nambu–Jona-Lasinio model

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi + G\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi} i\gamma^5 \tau^a \psi\right)^2\right)$$

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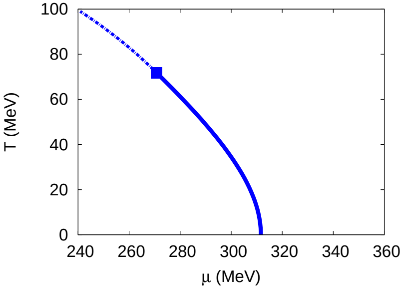
- Inhomogeneous phases:
  retain spatial dependence of the condensates

$$\langle\bar{\psi}\psi\rangle = S(\mathbf{x}), \qquad \langle\bar{\psi}i\gamma^5\tau_3\psi\rangle = P(\mathbf{x})$$

- Minimize thermodynamic potential

First NJL results: Inhomogeneous islands



- Homogeneous only:
- First order phase transition
- ending at a critical point

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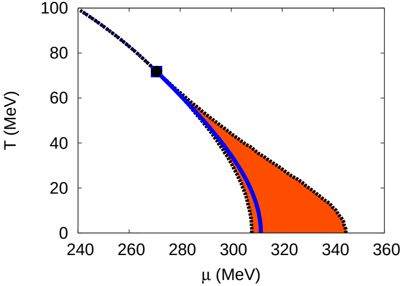
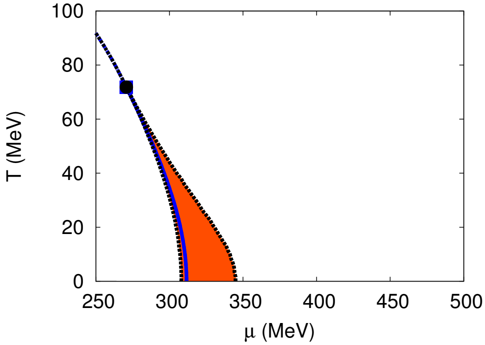


- Allow for spatial modulations of the chiral condensate
- First order transition line covered by inhomogeneous phase
- Critical point → Lifschitz point

(Broniowski et al., Nakano and Tatsumi, Nickel, . . .)

However...

- What happens at higher densities ?



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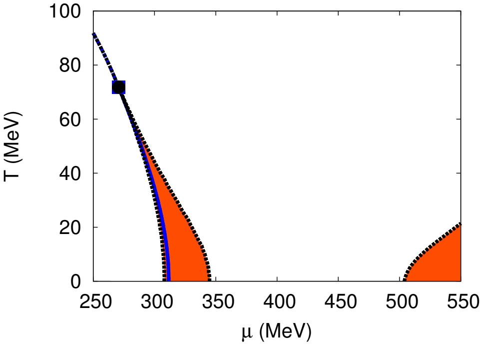

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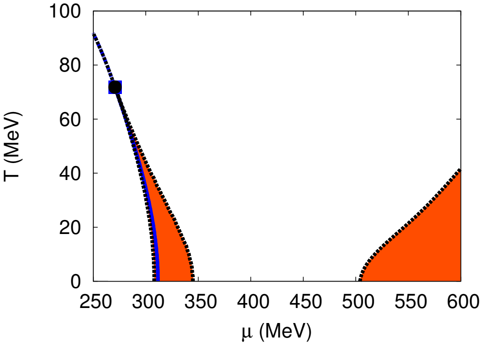

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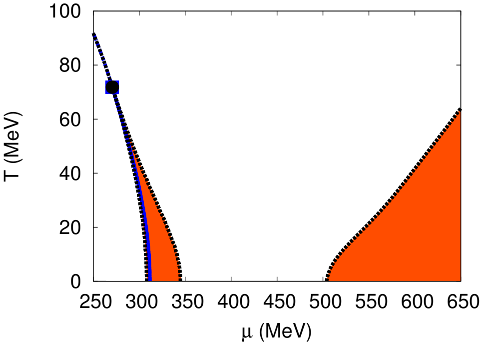
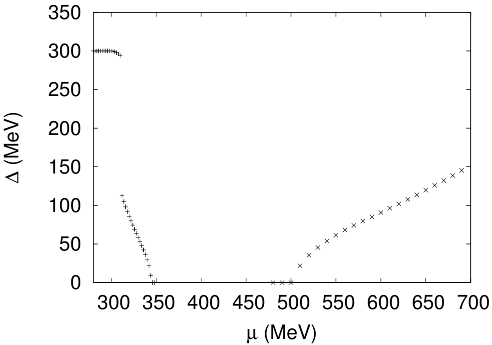
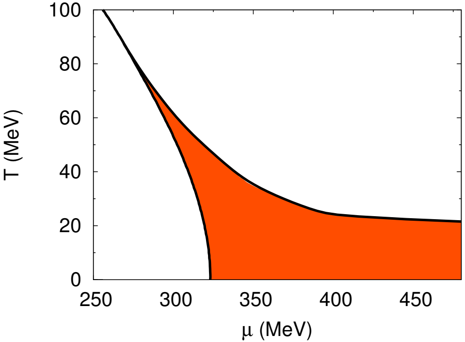


Order parameter ($T = 0$)



With a stronger coupling...



The inhomogeneous “continent”

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- But: vacuum contribution must be regularized.
- Difficult to disentangle a model "feature" from a regularization artifact

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  - Vector interactions
  - Coupling with Polyakov loop (PNJL)
    (SC, D.Nickel and M.Buballa, Phys.Rev.D82)
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- However: requires regularization!
  - No universal "good" prescription
  - Some model "features" may be regularization artifacts

Try something different: Quark-Meson model

$$\mathcal{L}_{QM} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - g(\sigma + i\gamma_5\tau^a\pi^a)\right)\psi + \mathcal{L}_{mes}^{kin} - U(\sigma, \pi^a)$$

- Meson kinetic contributions:

$$\mathcal{L}_{mes}^{kin} = \frac{1}{2}\left(\partial_{\mu}\sigma\partial^{\mu}\sigma + \partial_{\mu}\pi^a\partial^{\mu}\pi^a\right)$$

- Meson potential

$$U(\sigma, \pi^a) = \frac{\lambda}{4}\left(\sigma^2 + \pi^a\pi^a - v^2\right)^2 - c\sigma$$

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- First step: neglect Dirac sea contribution

Model parameters

- In the following: chiral limit! $m = c = m_\pi = 0$

- NJL:
  - Coupling constant G
  - Cutoff Λ
- Fixed using
  - Pion decay constant f_π
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- QM:
  - Quark-meson coupling g
  - Vacuum expectation value v
  - Quartic coupling λ
- Fixed using
  - Pion decay constant f_π
  - Constituent quark mass M_q in vacuum
  - Sigma meson mass m_σ

Inhomogeneous phases in QM

Just like in NJL:

- Allow for spatially modulated mean-fields

$$\sigma \to \sigma(\mathbf{x})\,, \qquad \pi \to \pi(\mathbf{x})$$

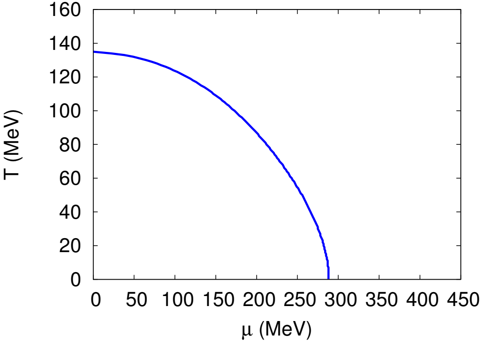
- Simplest ansatz: chiral density wave:

$$M(\mathbf{x}) = g(\sigma + i\pi) \to M(z) = \Delta e^{iQz}$$

- Minimize thermodynamic potential $\Omega(\Delta, Q)$

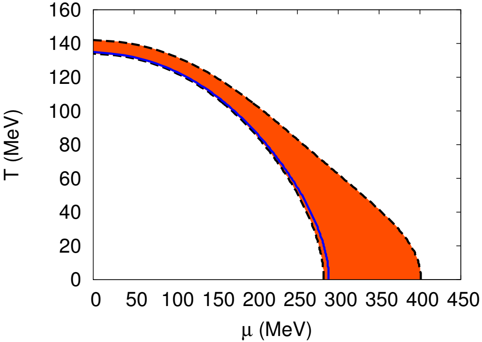
QM phase diagram - without Dirac sea

- Homogeneous phases:
  first-order everywhere
- No critical point



QM phase diagram - without Dirac sea

- Inhomogeneous phase up to $\mu = 0$!
- No critical/Lifshitz point
- On the bright side: no continent!



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  - Vary the cutoff Λ
  - Refit parameters
  - Calculate the model phase diagram

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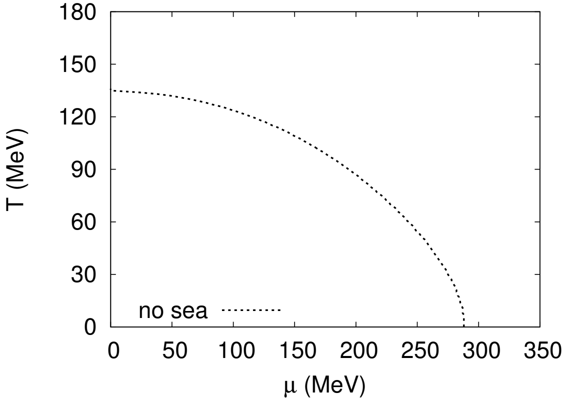
- Two different schemes considered:
  - Sharp three-momentum cutoff
  - Pauli-Villars type regularization

Phase diagram with Dirac sea (homogeneous phases)



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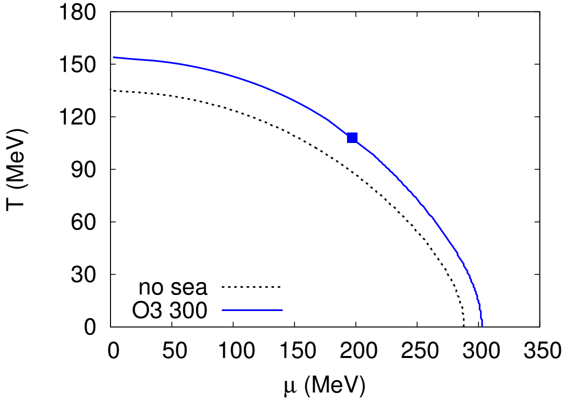

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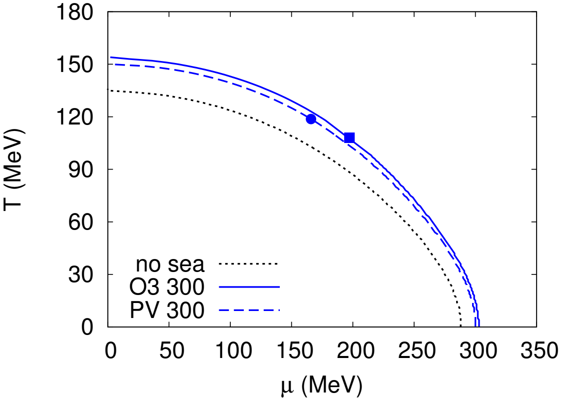

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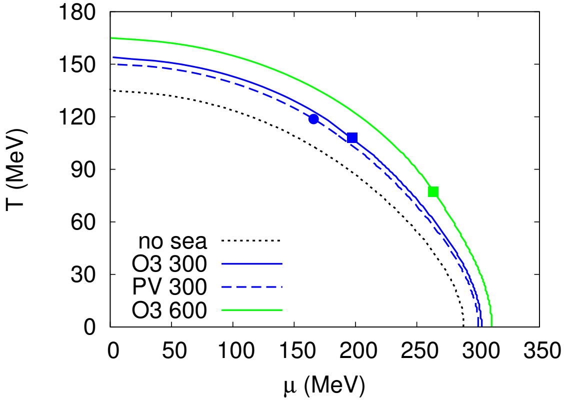

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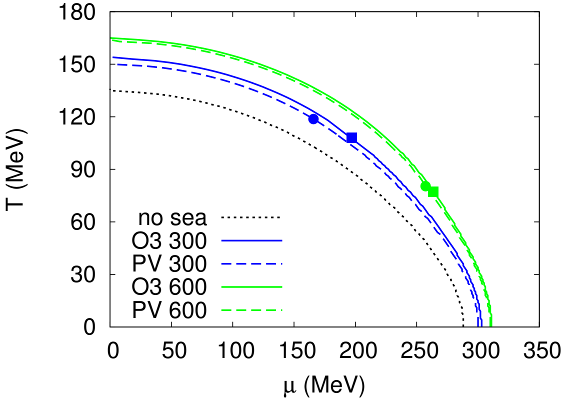

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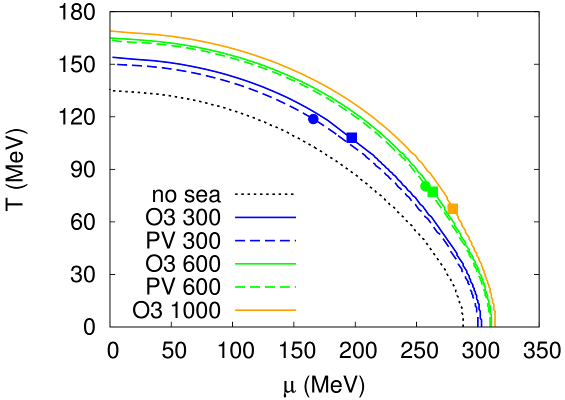

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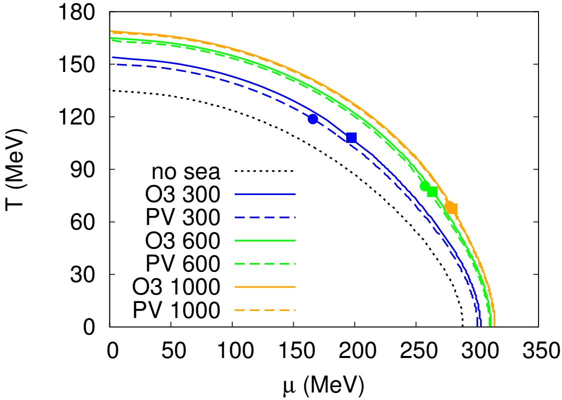

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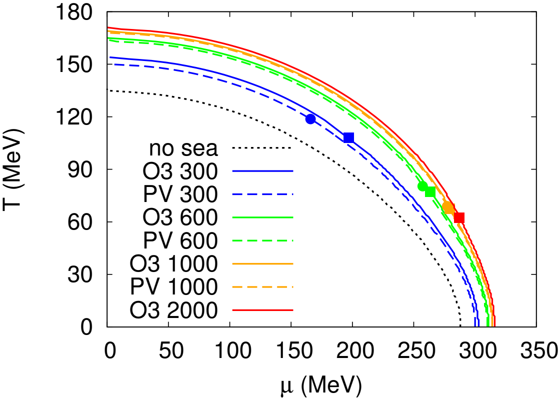

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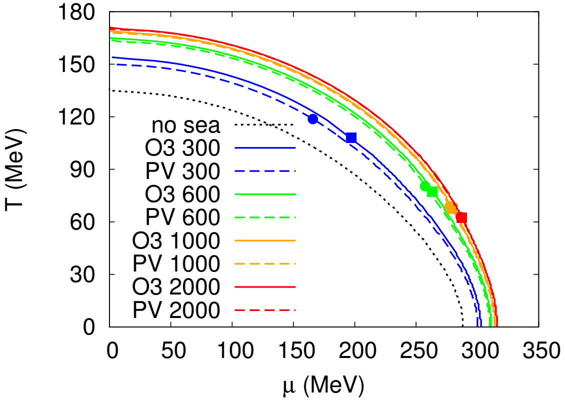

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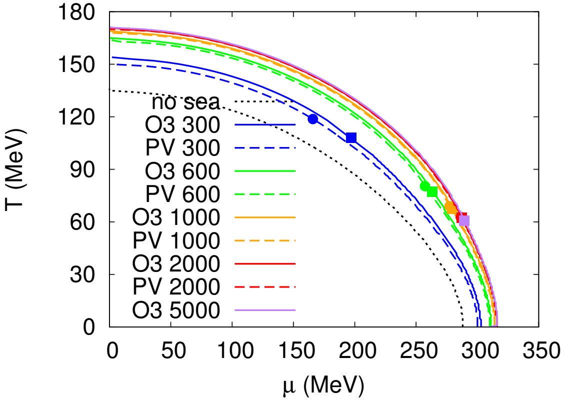

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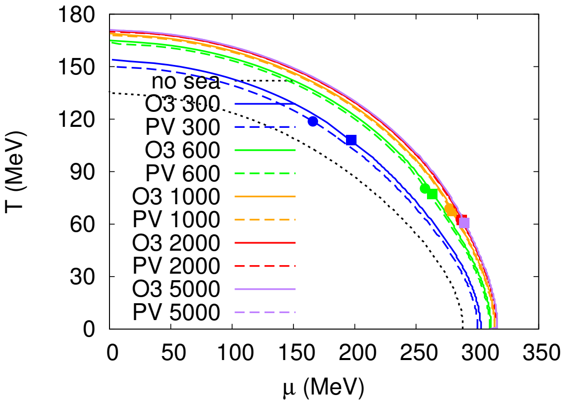
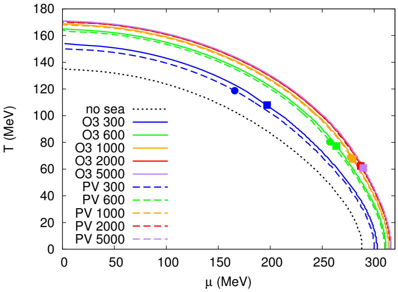


Phase diagram with Dirac sea (homogeneous phases)

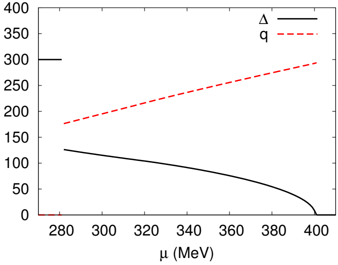
- Critical point re-appears
- Larger cutoffs move the CP to lower temperatures
- Convergence of the results at higher cutoffs



Chiral density wave: order parameters ($T = 0$)

Now allow for inhomogeneous phases...

- Without Dirac sea



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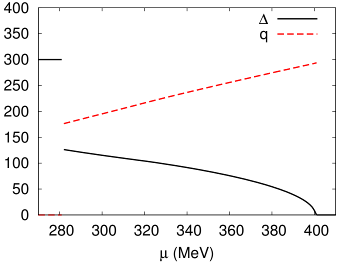
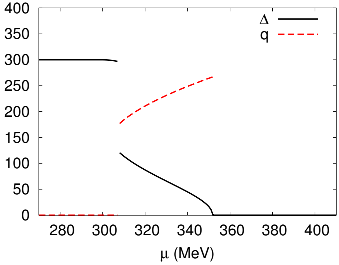


- With sea ($\Lambda_{PV} = 600$ MeV)



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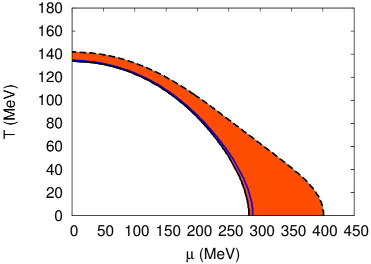

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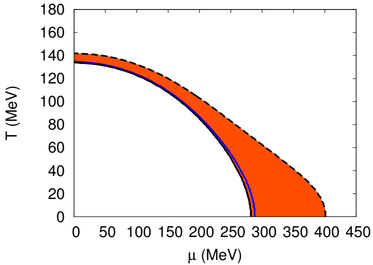

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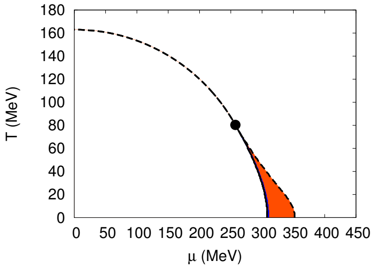

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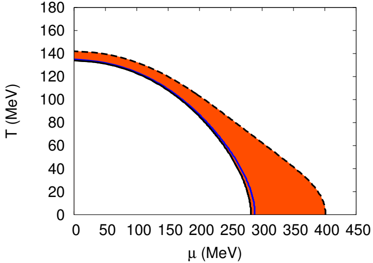

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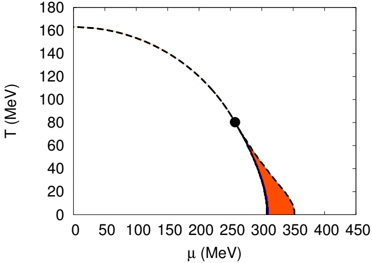
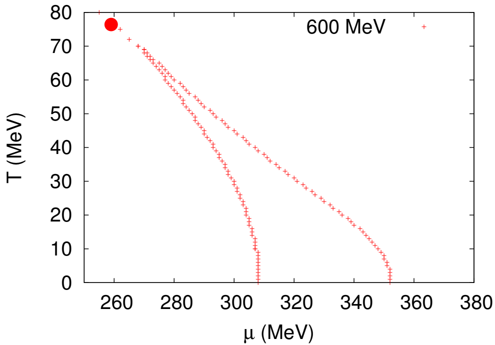


- More realistic phenomenology (no inhom. phase at zero density)
- CP coincides with LP as long as $m_\sigma = 2M_q$

Cranking up the cutoff...



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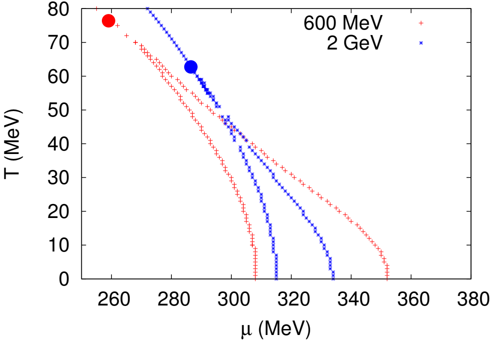

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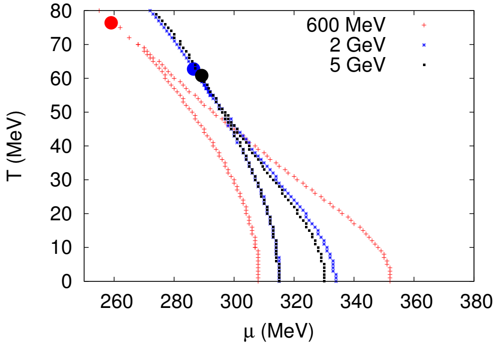
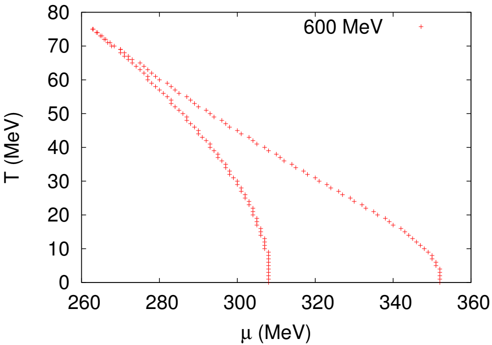


- The inhomogeneous phase shrinks
- Results stabilize at higher cutoffs

Changing the σ mass

- So far: $m_\sigma = 2M_q = 600$ MeV
- What happens if we vary m_σ ?



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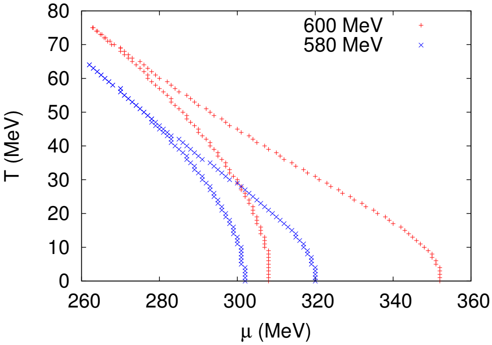

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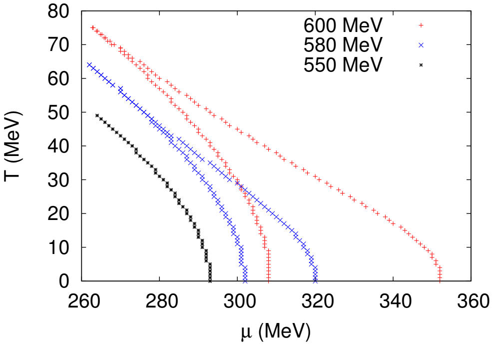

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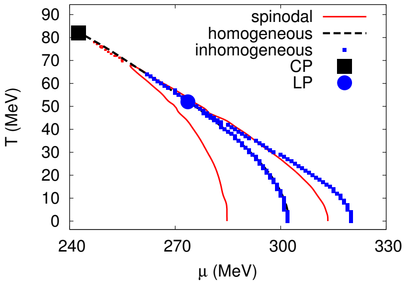
  ... and eventually disappears!

CP vs LP at $m_\sigma < 2M_q$

- Consider the intermediate case $m_\sigma = 580$ MeV



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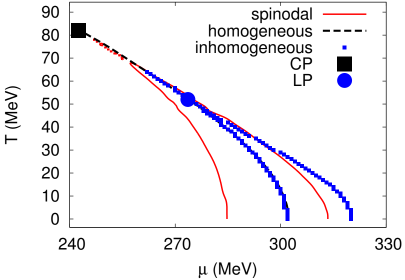

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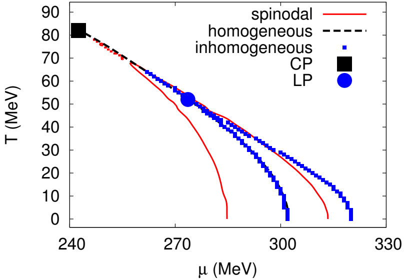

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- Coming soon: $m_\sigma > 2M_q$

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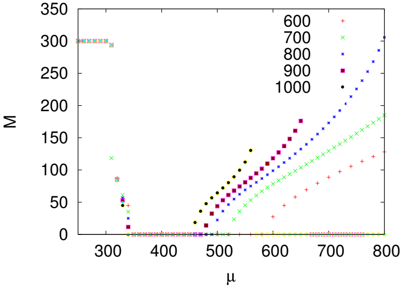

The inhomogeneous "continent" is back!

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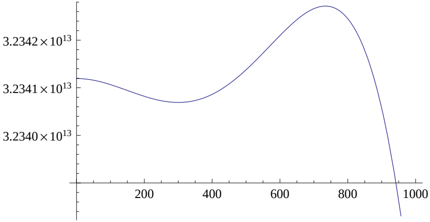

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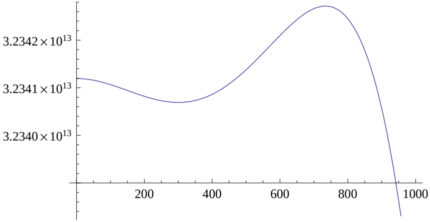
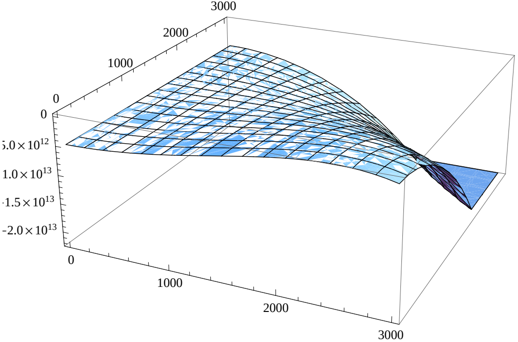


- Believed to be a one-loop artifact!
  (V. Skokov et al., Phys. Rev. D82)

With inhomogeneous phases

- With inhomogeneous phases,
  "dip" occurs for large values of Δ and q



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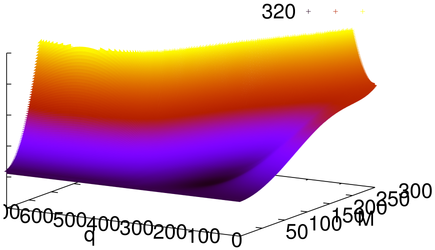

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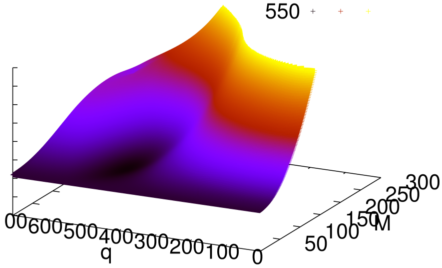

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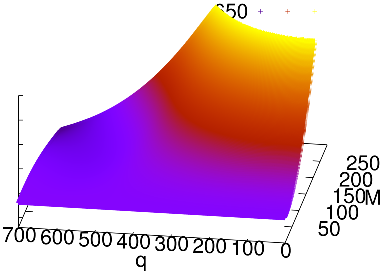
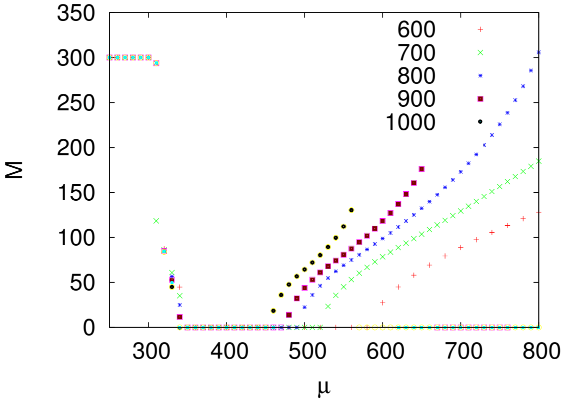


Looking again at the continent



Conclusions

- Qualitative picture we are starting to believe:
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- Regularization issues in effective models
- Possible solution: pick a renormalizable model

Conclusions

- We investigated the cutoff and m_σ dependence of QM results:
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- Next step: what happens beyond mean-field?