Sub-leading order heavy quark potential and jet quench parameter from AdS/CFT

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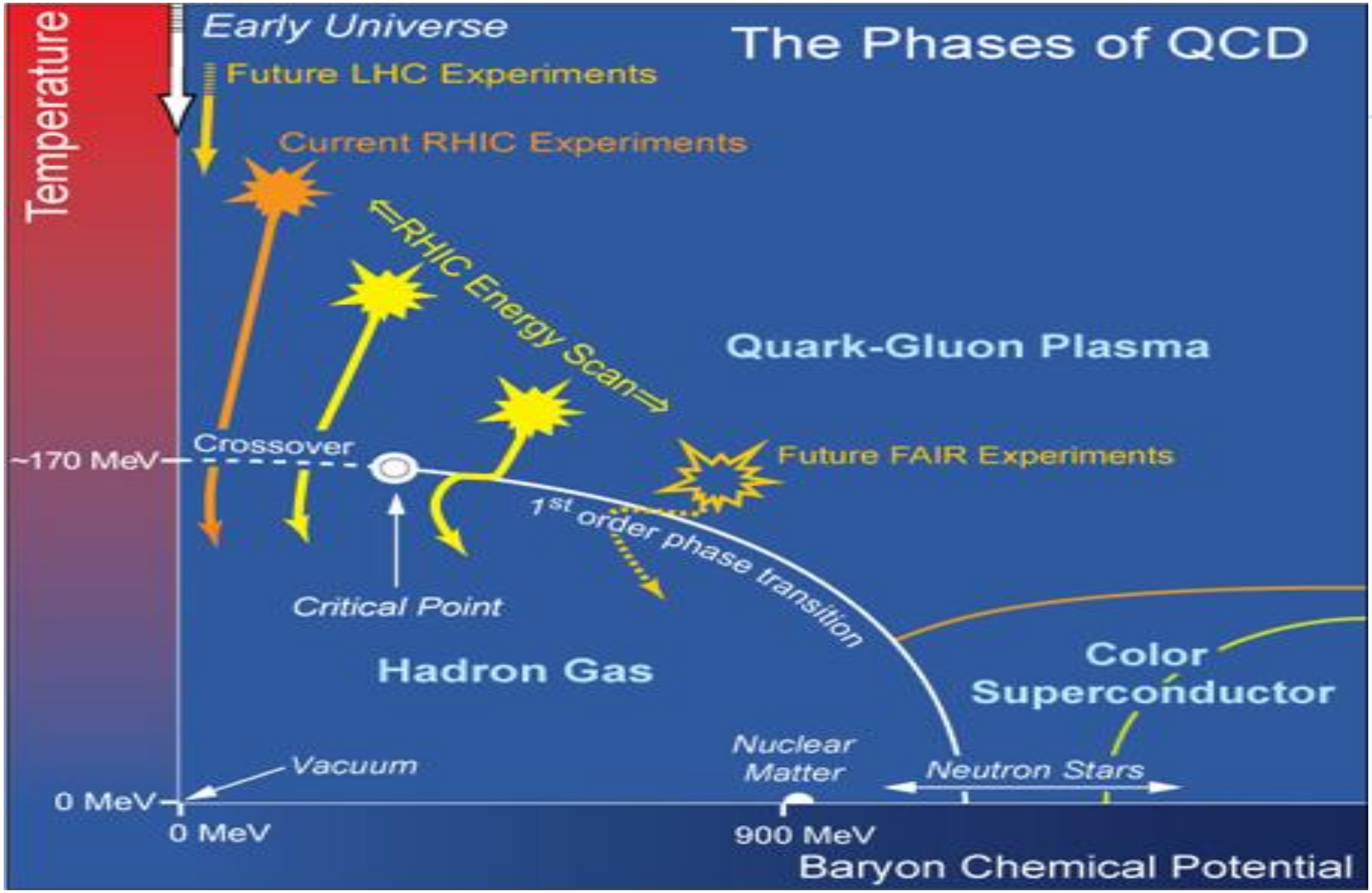
Outlines

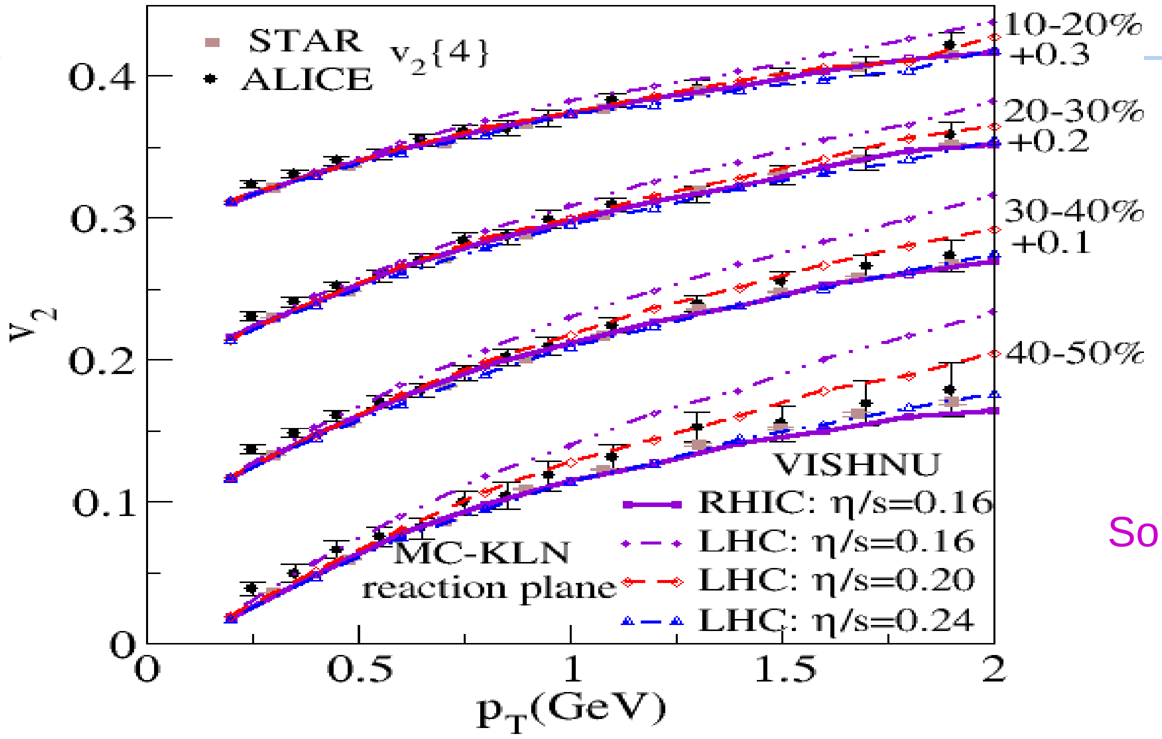
* **Introductory review of gauge/string duality**
* **NL Heavy quark potential from AdS/CFT and heavy quarkonium melting**
* **NL Jet quenching parameter from AdS/CFT**
* **Summary**

Zhang, Hou, Ren, JHEP1301 (2013) 03
Wu, Hou , Ren , PRC 87 (2013),025203
Zhang,Hou, Ren,Yin, JHEP07:035（2011）
Hou, Ren, JHEP01:029（2008）
Chu, Hou , Ren, JHEP08: 004 (2009)



Many interesting phenomena in QCD lie in the strongly coupled region

Experiment side



Song, Bass & Heinz, PRC2011

Robust v2 well described by hydro => sQGP

sQGP seems to be the almost perfect fluid known η/s>= .1-.2<<1

New theoretical techniques needed!

Lattice QCD

difficulty with Finite baryon density, Real time dynamics

Continuum

(1) Phenomenological models: (p)NJL、(p)QMC…

(2) Field Theory: HD(T)L , pQCD , Chiral Perturbation, Renormalization Group

DS equations ….

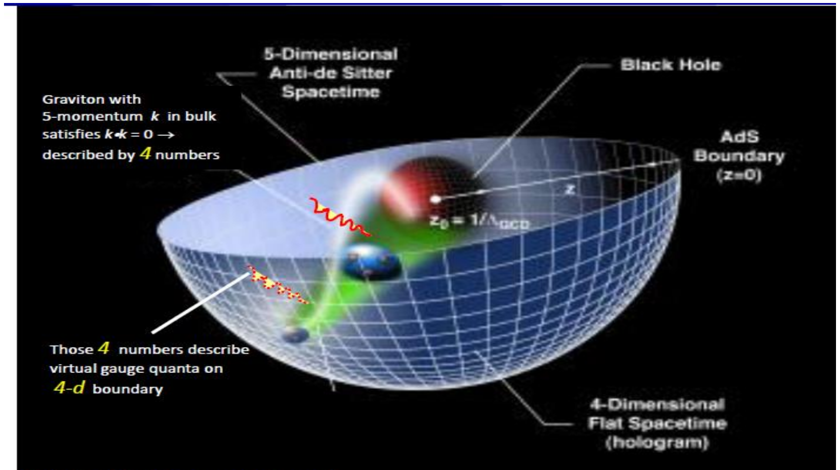
(3) AdS/CFT

AdS/CFT Correspondence

4D Large-Nc strongly coupled SU(Nc) N=4 SYM (finite T).

Maldacena '97 ↕ **conjecture** Witten '98

Type II B Super String theory on AdS5-BH × S5



(Adopted from S. Brodsky figure)

Maldacena conjecture: *Maldacena, Witten*

$N=4$ SUSY YM on the boundary $\Leftrightarrow$ Type IIB string theory in the bulk

't Hooft coupling $\lambda \equiv N_c g_{YM}^2 = \frac{1}{\alpha'^2}$ (string tension $= \frac{1}{2\pi\alpha'}$)

$$\frac{\lambda}{N_c} = 4\pi g_s$$

$$< e^{\int d^4x \phi_0(x) O(x)} > = Z_{\text{string}}[\phi(x,0) = \phi_0(\text{x})]$$

In the limit $N_c \to \infty$ and $\lambda \to \infty$

$$Z_{\text{string}}[\phi(x,0) = \phi_0(x)] = e^{-I_{\text{sugra}}[\phi]} |_{\phi(x,0)=\phi_0(x)}$$

$$I_{\text{sugra}}[\phi] = \text{classical supergravity action}$$

AdS/CFT applied to heavy-ion physics

* **Viscosity ratio, η/s.** $$\frac{\eta}{s} = \frac{1}{4\pi}$$ Policastro, Son and Starinet
* **Thermodynamics.** $$s = \frac{3}{4} s^{(0)}$$ Gubser
* **Jet quenching**

$$\hat{q} = \pi^{\frac{3}{2}} \frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)} \sqrt{\lambda} T^3$$ Liu, Rajagopal and Wiederman

* **Photon production** Yaffe et al
* **Heavy quarkonium (hard probe)** Maldacena
* **Thermalization , phase transition**
* **Hardron spectrum (AdS/QCD)**
* **AdS/CDM**

Heavy quark potential from AdS/CFT

The gravity dual of a Wilson loop at large N_c and large λ

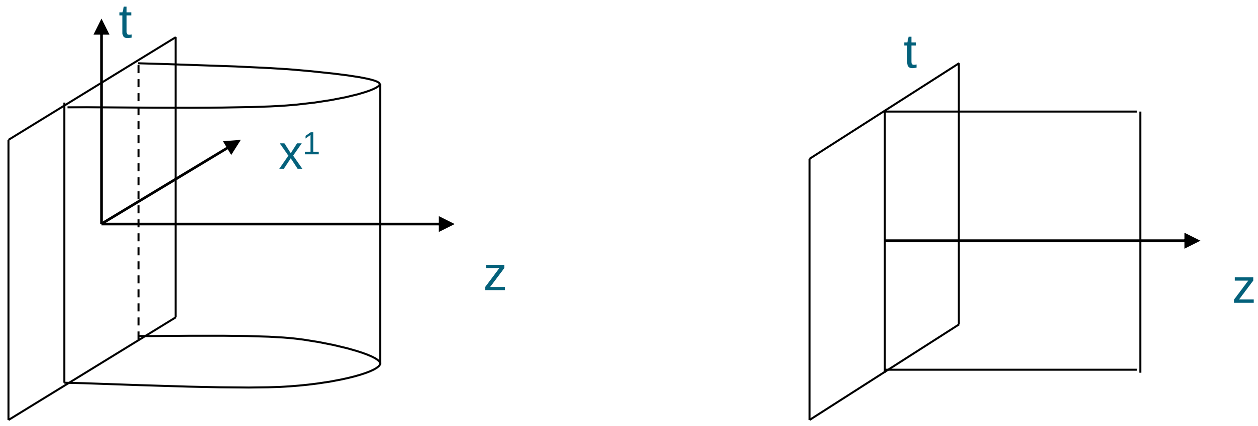
$$\mathrm{tr} < W(C) > = e^{-\sqrt{\lambda} S_{\min}[C]}$$

the min.area of string world sheet in the AdS_5

$$W(C) = P e^{-i \oint_C dx^{\mu} A_{\mu}(x)}$$

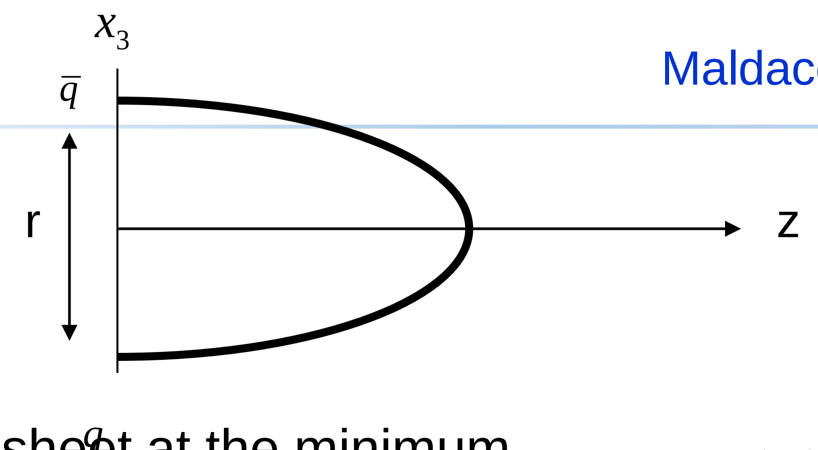
Heavy quark potential probes confinement hadronic phase and meson melting in plasma

$$F(r,T) = T\left(S_{\min}[\text{parallel lines}] - 2S_{\min}[\text{single line}]\right)$$



Heavy quark potential at zero temperature

Maldacena



The world sheet at the minimum

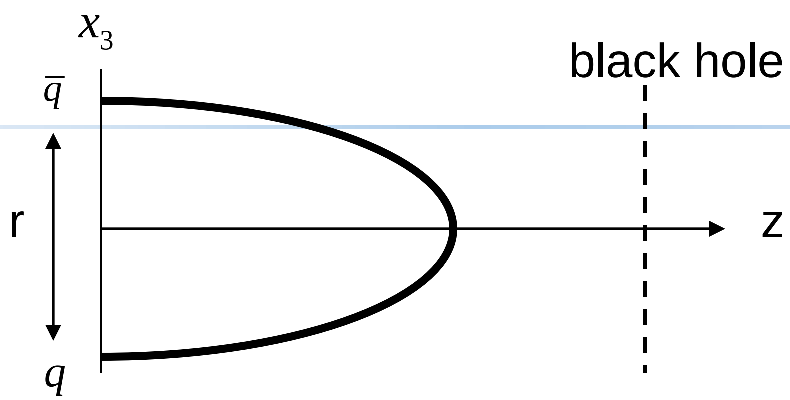
$$x_3 = \pm \int_z^{z_0} d\varsigma \frac{\varsigma^2}{\sqrt{z_0^4 - \varsigma^4}} \qquad \text{with} \qquad z_0 = \frac{\Gamma^2\left(\frac{1}{4}\right)}{(2\pi)^{\frac{3}{2}}} r$$

The potential

$$V(r,0) = F(r,0) = -\frac{4\pi^2\sqrt{2N_{c\Gamma} g_{YM}^2}}{\Gamma^4\left(\frac{1}{4}\right) r}$$

No confinement in N=4 SYM!

Heavy quark potential at a nonzero temperature



Rey, Theisen and Yee

The world sheet at the minimum

$$x_3 = \pm\sqrt{z_h^4 - z_0^4}\int_z^{z_0} d\varsigma \frac{\varsigma^2}{\sqrt{(z_0^4 - \varsigma^4)(z_h^4 - \varsigma^4)}}$$

$$\frac{r}{2} = \pm\sqrt{z_h^4 - z_0^4}\int_0^{z_0} d\varsigma \frac{\varsigma^2}{\sqrt{(z_0^4 - \varsigma^4)(z_h^4 - \varsigma^4)}}$$

$$z_h = \frac{1}{\pi T}$$

r_c

Free energy:

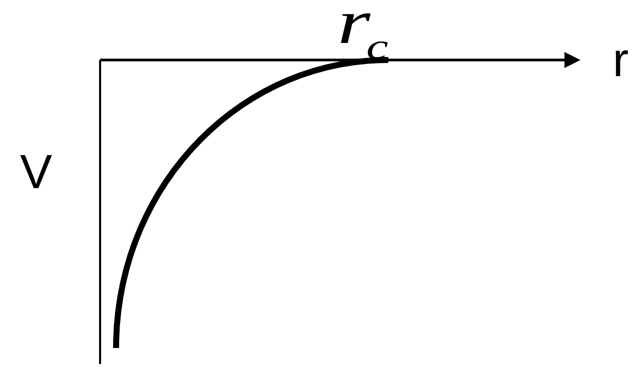
$$F(r,T) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma^4\left(\frac{1}{4}\right)r}\phi(\pi T r)\theta(r_c - r) \qquad \phi(\pi T r_c) = 0$$

$$r_c \cong \frac{0.7541}{\pi T}$$

Potential:

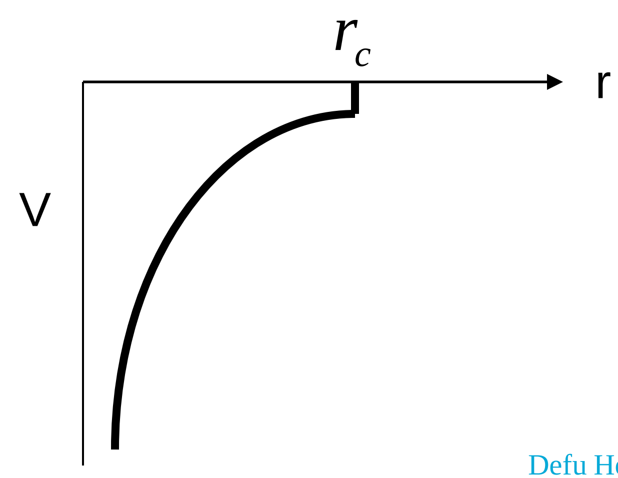
F-ansatz

$$V(r,T) = F(r,T)$$



U-ansatz

$$V(r,T) = F(r,T) + TS(r,T) = -T^2\frac{\partial}{\partial T}\left(\frac{F}{T}\right)$$



Non Yukawa screening!

Heavy quarkonium Dissociate Temperature

Hou , Ren, JHEP0801:029

ansatz	$J/\psi(1S)$	$J/\psi(2S)$	$J/\psi(1P)$	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(1P)$
F	67-124	15-28	13-25	197-364	44-81	40-73
U	143-265	27-50	31-58	421-780	80-148	92-171

With deformed metric

ansatz	J/ψ	Υ
F	NA	235-385
U	219-322	459-780

T_d/T_c

ansatz	J/ψ(holographic)	J/ψ(lattice)	Υ(holographic)	Υ(lattice)
F	NA	1.1	1.3-2.1	2.3
U	1.2-1.7	2.0	2.5-4.2	4.5

Relativistic correction

Wu, Hou , Ren , PRC 87 (2013),025203

	$c\bar{c}$		$b\bar{b}$	
	$\lambda = 5.5$	$\lambda = 6\pi$	$\lambda = 5.5$	$\lambda = 6\pi$
$1s$	162.54	387.54	478.76	1139.11
$2s$	29.15	62.75	85.67	184.44
$1p$	32.04	62.14	94.18	182.66

This lists the results of $T_0 + \delta_1 T$ in MeV's, that we just considered the correction of the p^4 term, which increased the dissociation temperature.

Relativistic correction

Wu, Hou , Ren , PRC 87 (2013),025203

	$c\bar{c}$		$b\bar{b}$	
	$\lambda = 5.5$	$\lambda = 6\pi$	$\lambda = 5.5$	$\lambda = 6\pi$
$1s_0^1$	130.79	188.65	385.63	555.58
$1s_1^3$	130.79	188.65	385.63	555.58
$2s_0^1$	26.71	48.16	79.15	142.59
$2s_1^3$	26.71	48.16	79.15	142.59
$1p_1^1$	31.53	61.33	93.54	180.79
$1p_0^3$	32.65	68.48	96.85	201.80
$1p_1^3$	32.09	64.90	95.20	191.30
$1p_2^3$	30.96	57.76	91.89	170.29

We wrote the state as: nL_J^{2S+1}

For J/Psi,the magnitude of the correction ranges from 8% to 30%!

Higher order corrections

Leading orders are strictly valid when $N_c \to \infty$, $\lambda \to \infty$

- **For real QCD. The t'Hooft coupling is not infinity**

$$5.5 < \lambda < 6\pi.$$

- **The super gravity correction to the AdS-Schwarschild metric is of order** $O(\lambda^{-\frac{3}{2}})$

,

- **The fluctuation around the minimum world sheet presents at all T, and is of order** $O(\lambda^{-\frac{1}{2}})$ **(more important)**

Gravity dual of a Wilson loop at finite coupling

$$W[C] \equiv \left\langle \exp\left(i \oint_C dx^\mu A_\mu \right) \right\rangle = \int [dX][d\theta] \exp\left[\frac{i}{2\pi\alpha'} S(X,\theta) \right]$$

Strong coupling expansion ⇄ Semi-classical expansion

$$\ln W[C] = i\sqrt{\lambda}\left[S\left(\bar{X}, 0\right) + \frac{b[C]}{\sqrt{\lambda}} + \ldots \right]$$

$\bar{X}$ = the solution of the classical equation of motion;

b[C] comes from the fluctuation of the string world sheet around $\bar{X}$

more significant than α'^3 -correction for Wilson loops.

$$\frac{1}{2\pi\alpha'} S(X,\theta) = \text{the superstring action in } AdS_5 \times S^5$$

Metsaev and Tseytlin

With fluctuations:

$$X^{\mu} = \bar{X}^{\mu} + \delta X^{\mu}, \qquad \theta \neq 0 \qquad g_{ij} = \bar{g}_{ij} + \delta g_{ij}$$

$$S(X,\theta) = S(\bar{X},0) + S_B^{(2)}(\delta X) + S_F^{(2)}(\theta) + \ldots$$

Bosonic and fermionic fluctuations decouple.

$$W[C] = e^{iS(\bar{X},0)} Z \qquad Z = Z_B Z_F$$

Partition function at finite T with fluctuations underlying the potential

Hou, Liu, Ren, PRD80,2009

Straight line：

$$Z = Z_B Z_F = \frac{\det^2\left(-\nabla_+^2 + 1 + \frac{1}{4}R^{(2)}\right)\det^2\left(-\nabla_-^2 + 1 + \frac{1}{4}R^{(2)}\right)}{\det^{\frac{3}{2}}\left(-\nabla^2 + \frac{8}{3} + \frac{1}{2}R^{(2)}\right)\det^{\frac{5}{2}}(-\nabla^2)}$$

Parallel lines：

$$Z = \frac{\det^2\left(-\nabla_+^2 + 1 + \frac{1}{4}R^{(2)}\right)\det^2\left(-\nabla_-^2 + 1 + \frac{1}{4}R^{(2)}\right)}{\det^{\frac{1}{2}}\left(-\nabla^2 + 4 + R^{(2)} - 2\delta\right)\det(-\nabla^2 + 2 + \delta)\det^{\frac{5}{2}}(-\nabla^2)}$$

Next leading order Results

Chu,Hou, Ren,JHEP0908,(2009)

$$V(r) \approx -\frac{4\pi^2}{\Gamma^4(\frac{1}{4})}\frac{\sqrt{\lambda}}{r}[1-\frac{1.33460}{\sqrt{\lambda}}+O(\frac{1}{\lambda})] \quad \text{for } \lambda \gg 1$$

Confirmed by Forini JHEP 1011 (2010) 079

$$\begin{aligned} a_1 &= \frac{5\pi}{12} - 3\ln 2 + \frac{2\mathbb{K}}{\pi}\left(\mathbb{K} - \sqrt{2}\,(\pi + \ln 2) + \mathcal{I}^{\text{num}}\right) \\ &= -1.3345953052806007 7364\ldots\ , \end{aligned}$$

$$V_{\text{ladder}}(r) = -\frac{\sqrt{\lambda}}{\pi r}\left(1 - \frac{\pi}{\sqrt{\lambda}}\right).$$

Erickson etc. NPB582,(2000)

$$-\frac{\lambda}{4\pi r}\left[1 - \frac{\lambda}{2\pi^2}\left(\ln\frac{2\pi}{\lambda} - \gamma_E + 1\right) + O(\lambda^2)\right] \quad \text{for } \lambda \ll 1$$

Next leading order potential at finite T

Zhang,Hou, Ren,Yin　JHEP07:035（2011）

$$V(r) \simeq -\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\frac{\sqrt{\lambda}}{r}\left[g_0(rT) - \frac{1.33460 g_1(rT)}{\sqrt{\lambda}} + O\left(\frac{1}{\lambda}\right)\right]$$

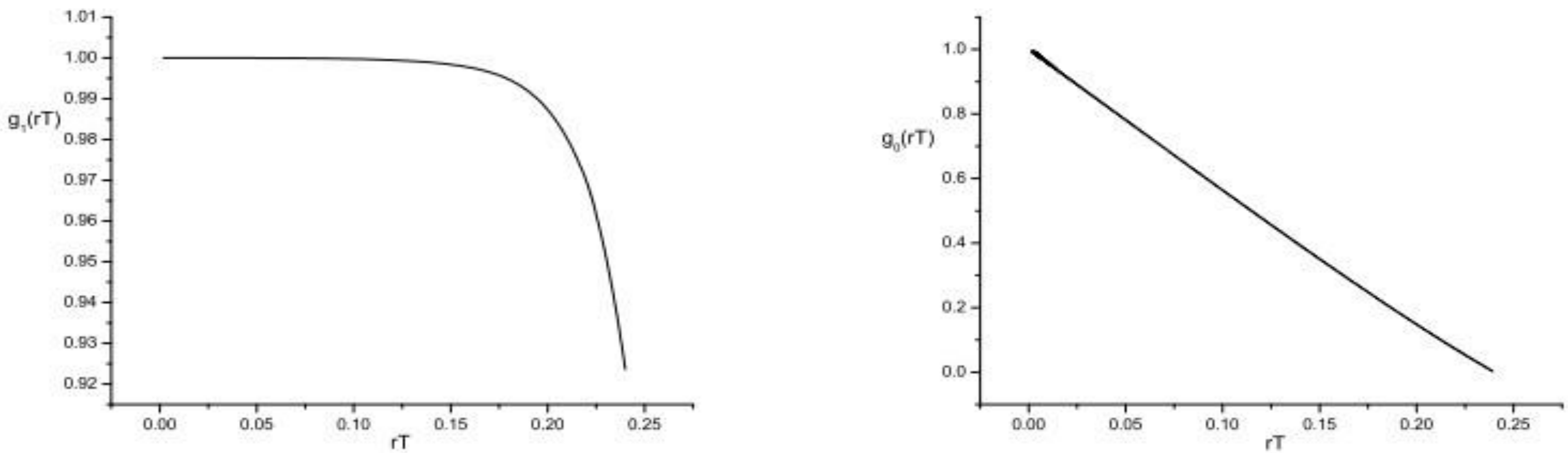
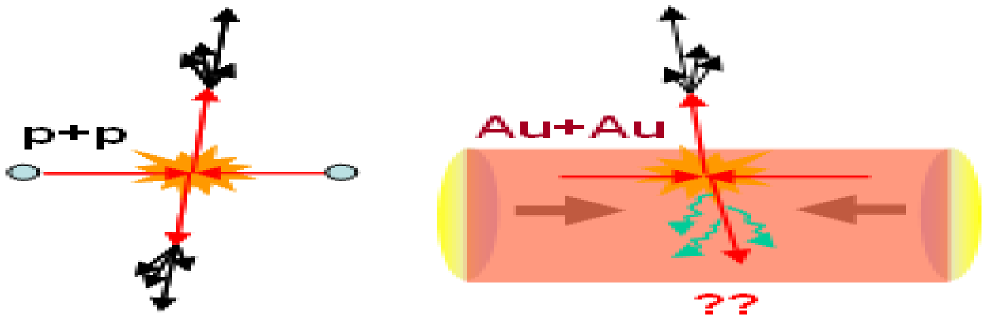


Figure 3. The left curve represents $g_1(rT)$, while the right represents $g_0(rT)$.

Jet quenching in QGP

Energy loss/scattering (Gyulassy and XNW'94)



$$\Delta E \approx -\frac{\alpha_S}{2\pi} N_C \, \hat{q} \, L^2$$

Baier, Dokshitzer, Mueller, Peigne, Schiff (1996):

$\hat{q} = \frac{\mu^2}{\lambda}$ jet transport coefficient

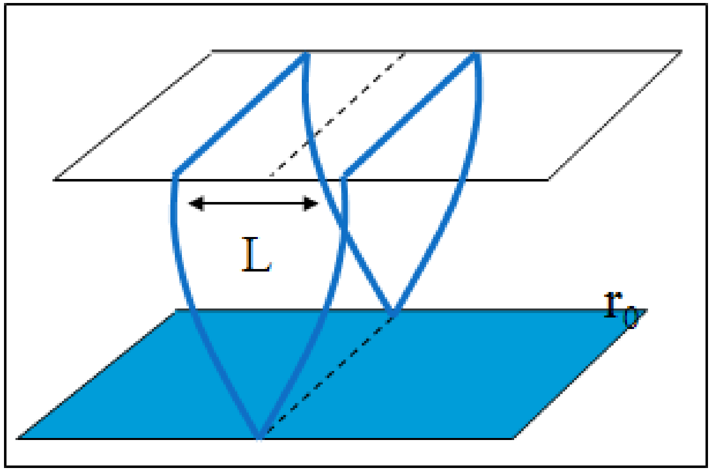
$\hat{q}$ reflects the ability of the medium to "quench" jets.

jet quenching parameter from AdS/CFT

Liu, Rajagopal & Wiedemann, PRL,97,182301(2006)

$$\hat{q}_0 = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3$$

Dipole amplitude: two parallel Wilson lines in the light cone:



$$W^A[C] = \exp(-\frac{\hat{q} L_- L^2}{4\sqrt{2}})$$

NL correction to jet quenching parameter

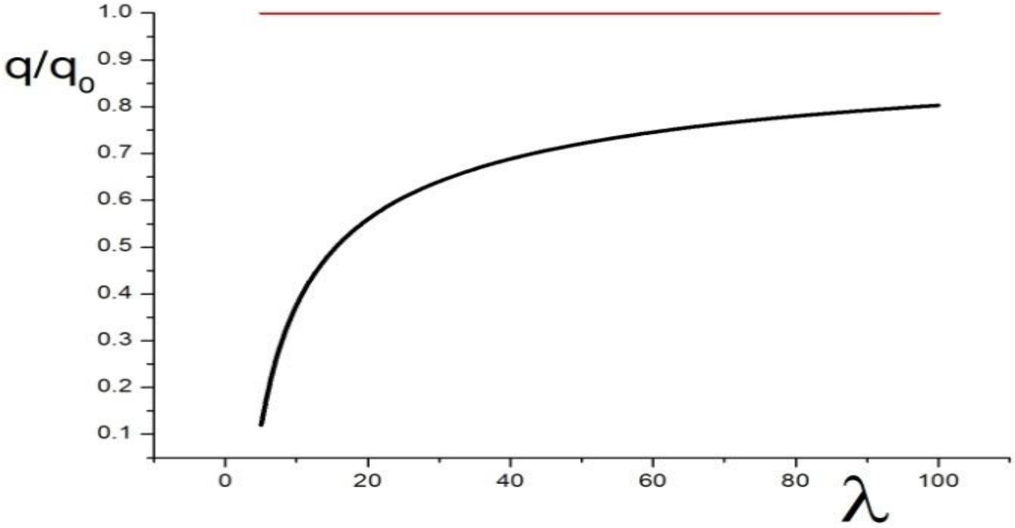
Zhang, Hou, Ren, JHEP1301 (2013) 032

$$\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3[1-1.97\lambda^{-1/2}+O(\lambda^{-1})]$$

dominant

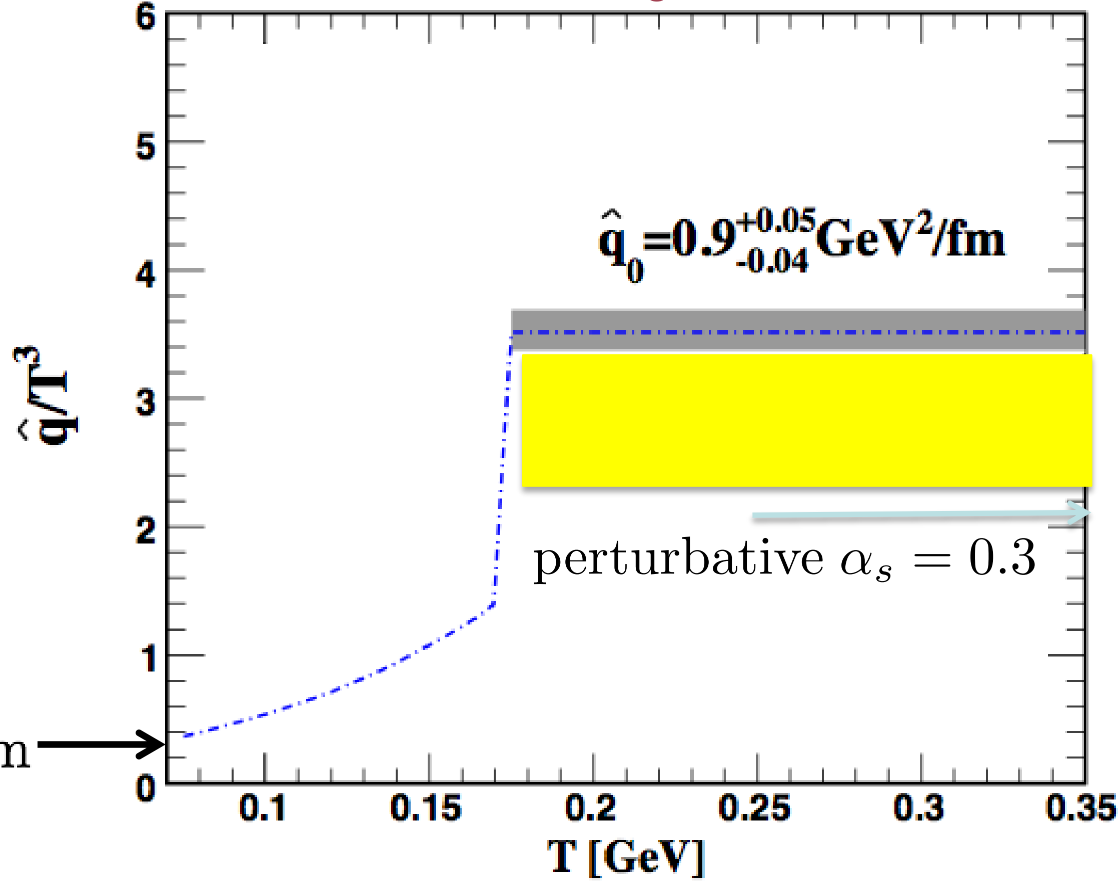
$$1 - 1.765\lambda^{-3/2}.$$

N. Armesto et al JHEP09 (06)



Jet quenching in QGP & hadronic phase

Chen, Greiner, Wang, XNW, Xu （2010）



DIS & Drell-Yan

$\hat{q}_N = 0.02 \ \ \mathrm{GeV}^2/\mathrm{fm}$

30% quenching from hadronic phase

Jet quenching parameter discussion

$\hat{q}_{\exp} = 1 \rightarrow 15 GeV^2 / fm$ Nestor Armesto ,et, al, JHEP 0609 (2006) 039

Take Nc=3, α=0.3 ⟺ λ=3.6π

Choose T=300MeV

$$\hat{q}_0 = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3$$ =3.45 GeV^2/fm

$\hat{q}$ = 1.37 GeV^2/fm

1 Sub-leading order gives rise to 40% reduction from the leading order .

2.The negative sign of sub-leading order is consistent with a monotonic behavior from strong coupling to weak coupling.

Summary and discussion

AdS/CFT provides a useful way to address the physics at strong coupling .

The partition function of Wilson loop with fluctuations in strongly coupling N=4 SYM plasma are derived.

We computed the jet quenching parameter and heavy quark potential up to sub-leading orders.

We estimated the melting T with holographic potential and its relativitic correction

The applicability of these AdS/QCD results demands phenomenological work to explain them in a way which can be translated to real QCD.

QCD versus
N=4 Super Yang-Mills from gravity dual

	QCD	Super YM
N_c	3	>>1
t'Hooft coupling	5.5-18.8	>>1
Quarks	Fundamental	Adjoint
Conformal symmetry	No	Yes at zero T No at nonzero T
Supersymmetry	No	Yes at zero T No at nonzero T

Thanks