QCD ϑ-vacua from Chiral Limit to Quenched Limit

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θ-vacuum

QCD action + ϑ-term $$S_\theta = \int d^4x \mathcal{L}_{\mathrm{QCD}} + iQ\theta$$

Chern-Simons Number

$$Q = \frac{g^2}{32\pi^2} \int d^4x \epsilon_{\mu\nu\sigma\rho} F_a^{\mu\nu} F_a^{\sigma\rho} = \frac{g^2}{8\pi^2} \int d^4x \boldsymbol{E}^a \cdot \boldsymbol{B}^a$$

ϑ-vacuum

$$\boxed{|\theta\rangle = \sum_n e^{in\theta} |n\rangle}$$

Phase Transition at θ=π

Chern Simons term breaks CP-symmetry ⇒ $Q \to -Q$

$$S_\theta = \int d^4x \mathcal{L}_{\mathrm{QCD}} + iQ\theta \qquad Q = \frac{g^2}{32\pi^2}\int d^4x \epsilon_{\mu\nu\sigma\rho} F_a^{\mu\nu} F_a^{\sigma\rho}$$

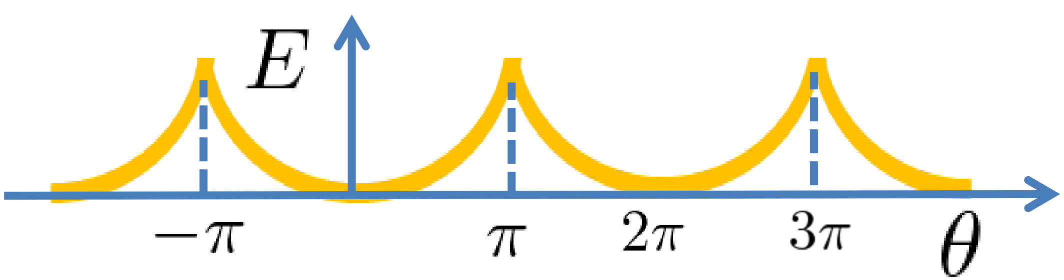
Only at θ=0,π CP-symmetry is unbroken

Strong-CP Problem

$\theta = 0$ ⇒ trivial ***No Priori Reason***

$\theta = \pi$ ⇒ $e^{i\pi Q} \to e^{-i\pi Q} = e^{i\pi Q}$

First order phase transition at ϑ=π



Chiral $m \to 0$

Witten 1980 (Chiral Model)

Boer 2008 (NJL Model)

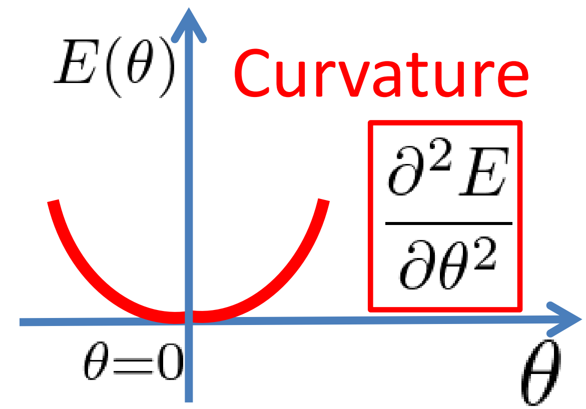
Quenched $m \to \infty$

Witten 1998 (Holographic)

D'Elia 2012 (Lattice)

Topological Susceptibility

$$\chi_{\rm top} = \frac{1}{Z_\theta V} \frac{\partial^2 Z_\theta}{\partial \theta^2}\bigg|_{\theta=0} = \frac{\langle Q^2 \rangle_{\theta=0}}{V}$$

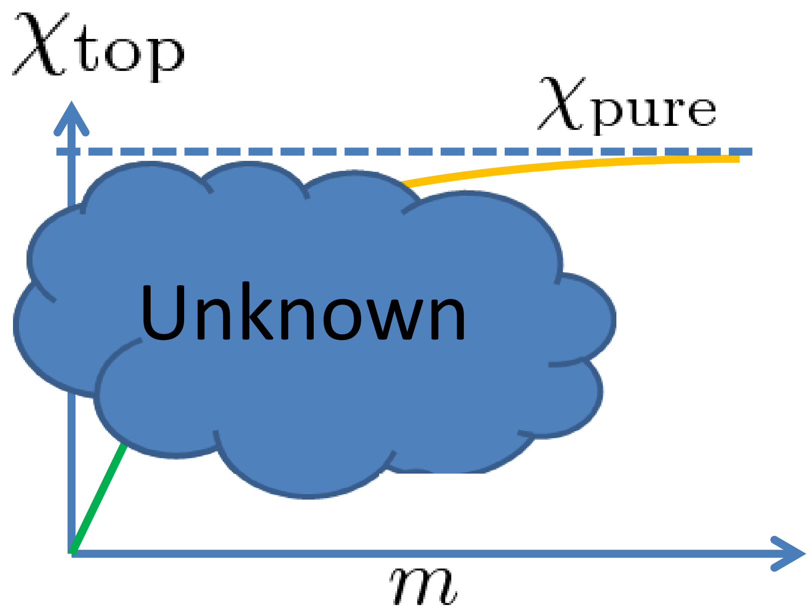


Chiral

$m \to 0$

$\chi_{\rm top} \propto m$

Chiral Model



Quenched

$m \to \infty$

$\chi_{\rm top} \to \chi_{\rm pure}$
$\sim (170 {\rm MeV})^4$

Lattice QCD

**There is NO model to be used for a quark mass*

**Lattice QCD cannot be used with dynamical quark*

Where does the mass-dependence of ϑ-vacuum come from?

Mass-dependence of θ-vacuum

U(1) Axial Transformation $\psi \rightarrow e^{i\alpha\gamma^5}\psi \qquad \bar{\psi} \rightarrow \bar{\psi}e^{i\alpha\gamma^5}$

*Mass-term $m\bar{\psi}\psi \rightarrow m\bar{\psi}e^{i2\alpha\gamma^5}\psi$

*Measure $\mathcal{D}\bar{\psi}\mathcal{D}\psi \rightarrow \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-i2\alpha Q}$ Axial Anomaly

$$S_\theta = \int d^4x \left(-\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \bar{\psi}(i\partial_\mu\gamma^\mu - m)\psi + gA_\mu\gamma^\mu\bar{\psi}\psi\right) + iQ\theta$$

$$\rightarrow S_\theta = \int d^4x \left(-\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \bar{\psi}(i\partial_\mu\gamma^\mu - me^{i2\alpha\gamma^5})\psi + gA_\mu\gamma^\mu\bar{\psi}\psi\right) + iQ(\theta - 2\alpha)$$

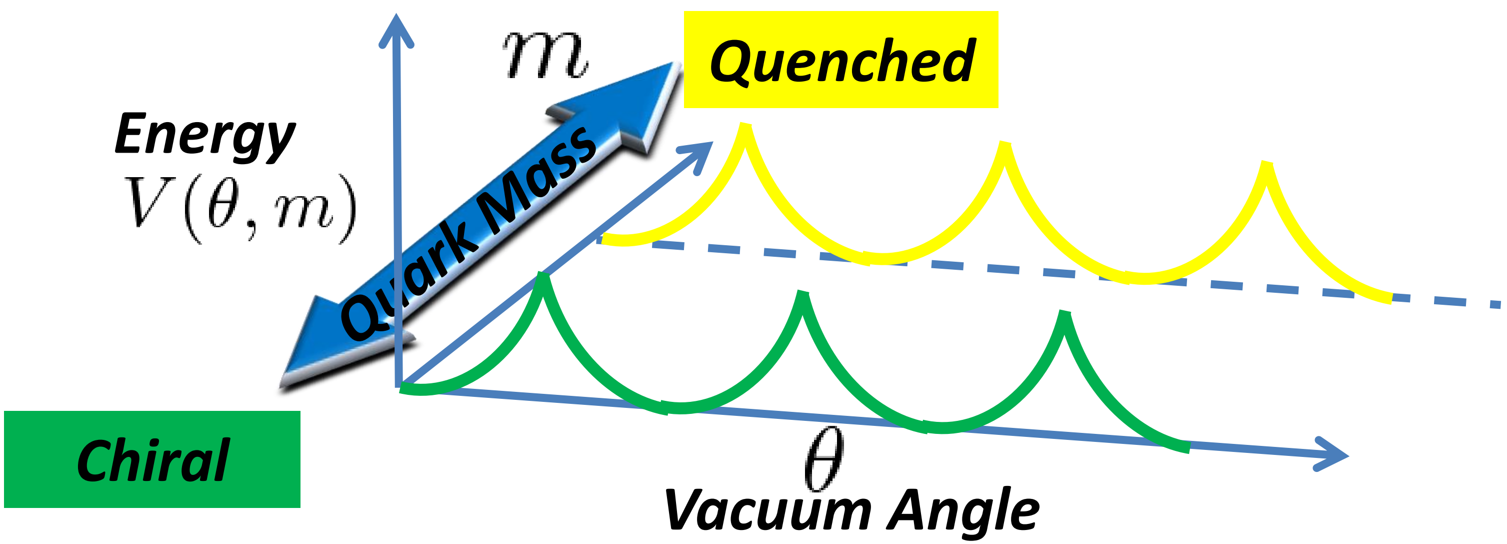
$$\xrightarrow{\alpha = \theta/2} \boxed{S_\theta = \int d^4x \left(-\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \bar{\psi}(i\partial_\mu\gamma^\mu - me^{i\theta\gamma^5})\psi + gA_\mu\gamma^\mu\bar{\psi}\psi\right)}$$

In chiral limit ($m \rightarrow 0$) ϑ-dependence is gone

Light Quark VS Heavy Quark

	Order of Phase Transition at ϑ=π	*Phase Transition results from*	*Topological Susceptibility*
Light/Chiral	First	*Quark*	~Quark Mass
Heavy/Quenched	First	*Gluon*	(170MeV)^4

These seem to be similar, but actually the origins of these are completely different.



How can we obtain a unified understanding about the mass-dependence of ϑ-vacuum?

Chiral Effective Lagrangian

Di Vechhia-Veneziano (1980)

$$\mathcal{L}_\chi = \underline{\frac{f_\pi^2}{4}\mathrm{tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] + \frac{|\langle \bar{q} q\rangle|}{2}\mathrm{tr}\left[M(U+U^\dagger)\right]} + \underline{\frac{\chi_{\rm pure}}{2}\left[\theta - \frac{i}{2}\mathrm{tr}(\ln U - \ln U^\dagger)\right]^2}$$

$$\Rightarrow \frac{1}{2}\partial_\mu \pi^a \partial^\mu \pi^a - \frac{1}{2}m_\pi^2 \pi^a \pi^a + \cdots \qquad \Rightarrow \theta \to \theta - 2\alpha$$

Meson Field

$$U = e^{i\pi^a\tau^a/f_\pi}$$
$$\simeq 1 + i\pi^a\tau^a/f_\pi - \frac{1}{2f_\pi^2}\pi^a\tau^a\pi^b\tau^b + \cdots$$

U(1) Axial Rotation

$$\psi_R \to e^{i\alpha}\psi_R$$
$$\psi_L \to e^{-i\alpha}\psi_L \Rightarrow U \to e^{i\alpha} U e^{i\alpha}$$
$$U \sim \psi_R \psi_L^\dagger$$

Potential

$$U(\phi_1,\phi_2) = \begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{i\phi_2} \end{pmatrix} \qquad M = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$V(\phi_1,\phi_2) = -m|\langle \bar{q} q\rangle|(\cos\phi_1 + \cos\phi_2) + \frac{\chi_{\rm pure}}{2}(\theta + \phi_1 + \phi_2)^2$$

Global Minimum of the Potential = ϑ-Vacuum

Minimization

Potential

$$\boxed{V(\phi_1, \phi_2) = -m|\langle\bar{q}q\rangle|(\cos\phi_1 + \cos\phi_2) + \frac{\chi_{\rm pure}}{2}(\theta + \phi_1 + \phi_2)^2}$$

EOM $$m|\langle\bar{q}q\rangle|\sin\phi_i + \chi_{\rm pure}(\theta + \phi_1 + \phi_2) = 0$$

$$\boxed{\begin{array}{c} \phi_1 = \phi_2 + 2n\pi = \phi_n \\ m|\langle\bar{q}q\rangle|\sin\phi_n + \chi_{\rm pure}(\theta - 2n\pi + 2\phi_n) = 0 \end{array}}$$

$$\phi_1 = -\phi_2 + (2n+1)\pi = \phi_n$$

It does not give the ground state

Phase Transition at θ=π (Light)

$$\boxed{\begin{array}{l}\phi_1 = \phi_2 + 2n\pi = \phi_n \\ m|\langle\bar{q}q\rangle|\sin\phi_n + \chi_{\rm pure}(\theta - 2n\pi + 2\phi_n) = 0\end{array}}$$

$$\boxed{m \ll \chi_{\rm pure}/|\langle\bar{q}q\rangle|}$$

$$\boxed{2n = 0, \pm 4, \pm 8, \ldots}$$

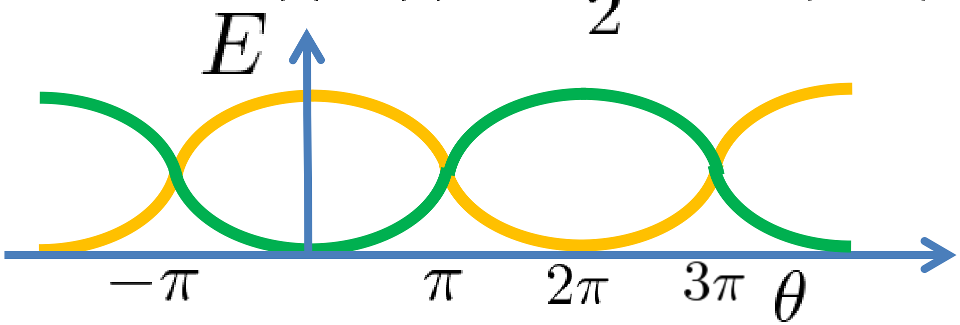
$$\phi_0 = -\frac{\theta}{2} + \mathcal{O}(m) \qquad V = -m|\langle\bar{q}q\rangle|\cos\frac{\theta}{2} + \mathcal{O}(m^2)$$

$$\boxed{2n = \pm 2, \pm 6, \pm 10, \ldots} \qquad \boxed{\theta \to \theta - 2\pi}$$

$$\phi_0 = -\frac{\theta}{2} + \pi + \mathcal{O}(m) \qquad V = m|\langle\bar{q}q\rangle|\cos\frac{\theta}{2} + \mathcal{O}(m^2)$$

First order phase transition at ϑ=π

Witten (1980)



Minimization

Potential

$$\boxed{V(\phi_1,\phi_2) = -m|\langle\bar{q}q\rangle|(\cos\phi_1 + \cos\phi_2) + \boxed{\frac{\chi_{\mathrm{pure}}}{2}(\theta+\phi_1+\phi_2)^2}}$$

The potential has an information in quenched limit as the topological term, although the model is the chiral model

⇓

Can I use it in quenched limit?

Phase Transition at θ=π (Heavy!)

$$\phi_1 = \phi_2 + 2n\pi = \phi_n$$
$$m|\langle\bar{q}q\rangle|\sin\phi_n + \chi_{\text{pure}}(\theta - 2n\pi + 2\phi_n) = 0$$

$$m \gg \chi_{\text{pure}}/|\langle\bar{q}q\rangle|$$

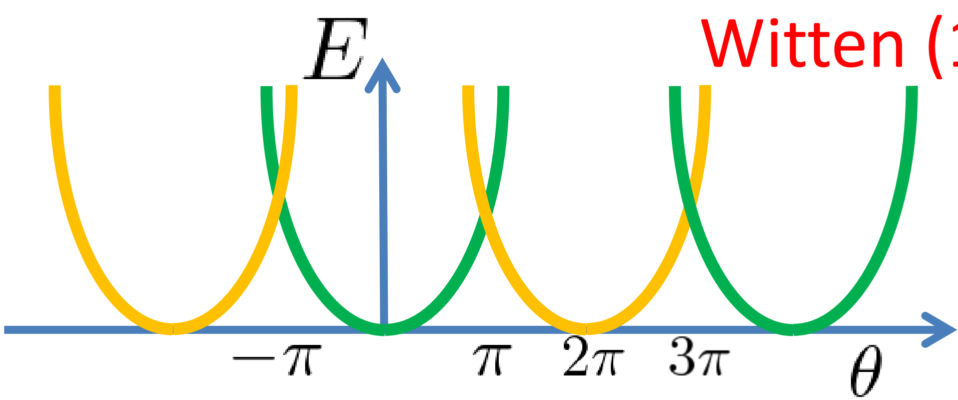
Solution
$$\phi_0 = -\frac{\chi_{\text{pure}}}{m|\langle\bar{q}q\rangle|}\theta + 2n\pi + \mathcal{O}(m^{-2})$$

$$V = -2m|\langle\bar{q}q\rangle| + \frac{\chi_{\text{pure}}}{2}\underline{(\theta + 2n\pi)^2} + \mathcal{O}(m^{-1})$$

same result in pure YM theory

Witten (1998)

First order phase transition at ϑ=π



For any mass

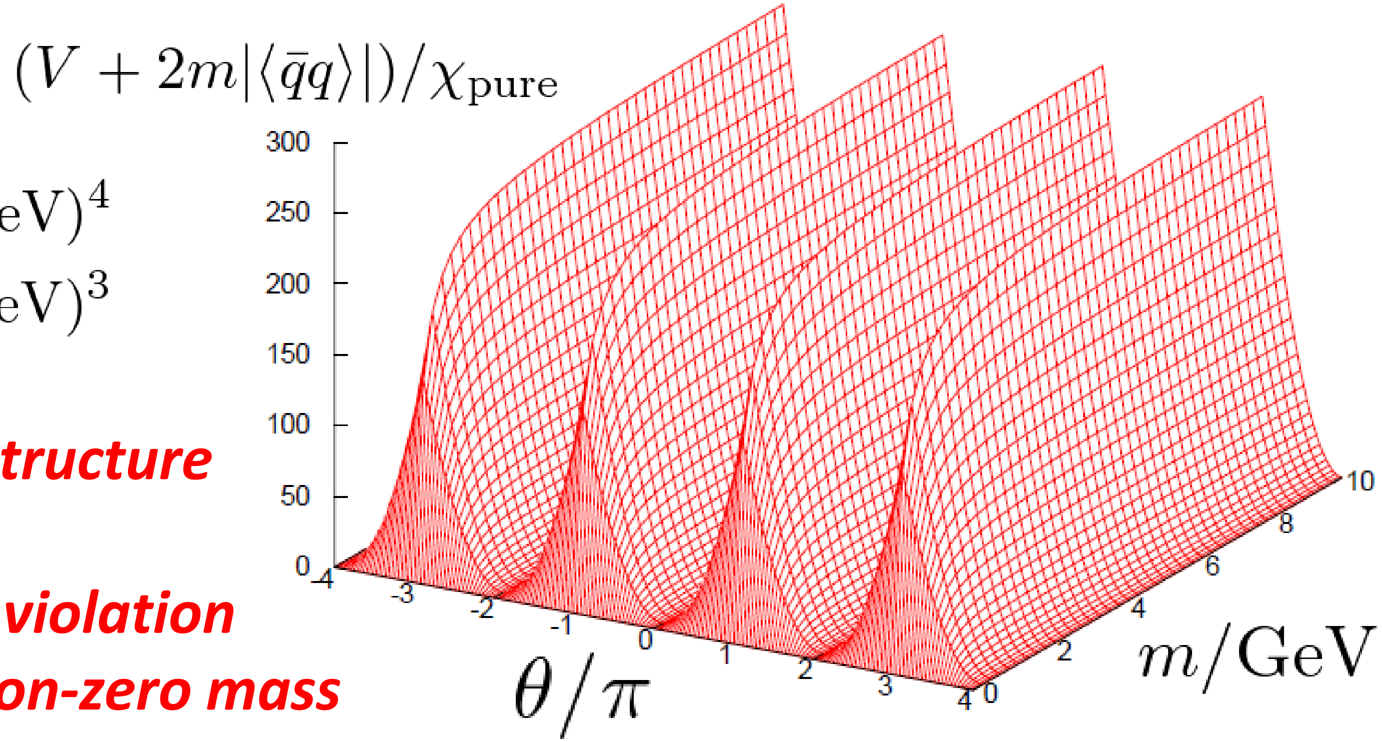
Potential

$$V(\phi_1, \phi_2) = -\frac{|\langle \bar{q}q \rangle| m}{2}(\cos\phi_1 + \cos\phi_2) + \frac{\chi_{\text{pure}}}{2}(\theta + \phi_1 + \phi_2)^2$$

$$\chi_{\text{pure}} = (170\text{MeV})^4$$

$$|\langle \bar{q}q \rangle| = (250\text{MeV})^3$$

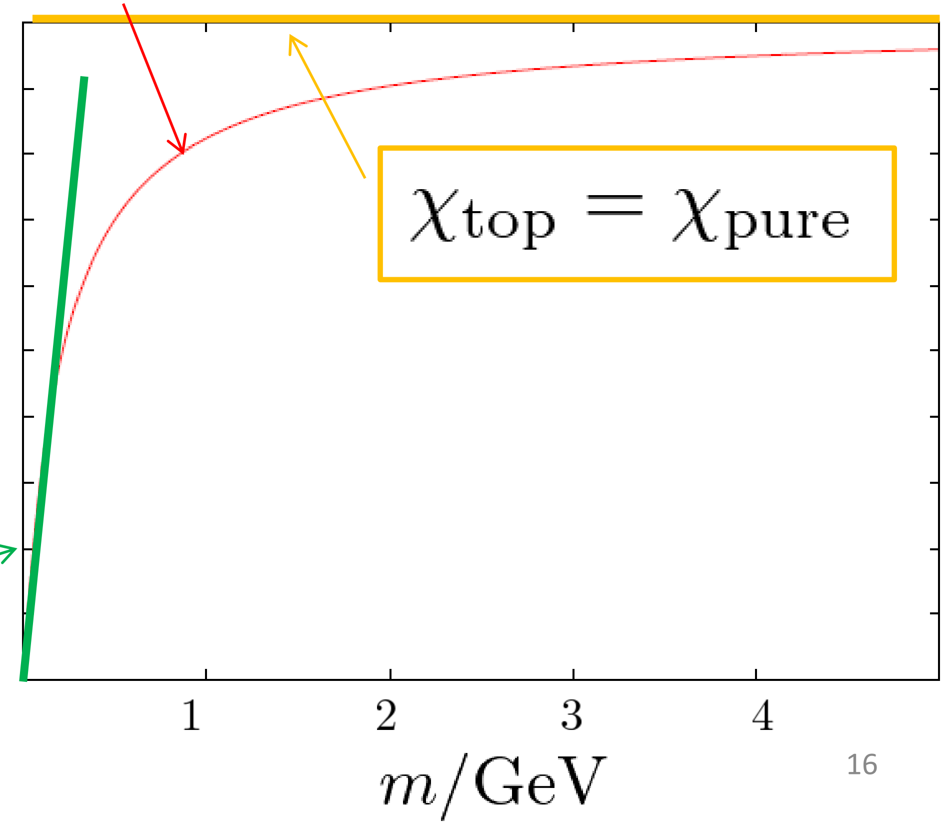
the continuous structure of ϑ-vacua with spontaneous CP violation at ϑ=π for any non-zero mass



Landscape of ϑ-vacua

Topological Susceptibility

$$\chi_{\text{top}} = \frac{\partial^2 V}{\partial \theta^2}\bigg|_{\theta=0} = \boxed{\frac{m\chi_{\text{pure}}}{m + 2\langle \bar{q}q\rangle/\chi_{\text{pure}}}}$$



In spite of the chiral model we can obtain the shape of topological susceptibility, which we expected, for an arbitrary quark mass

$$\boxed{\chi_{\text{top}} = \frac{m\langle \bar{q}q\rangle}{2}}$$

Role of Quark Mass

Potential

$$\boxed{V(\phi_1, \phi_2) = \boxed{-m|\langle\bar{q}q\rangle|(\cos\phi_1 + \cos\phi_2)} + \frac{\chi_{\mathrm{pure}}}{2}(\theta + \phi_1 + \phi_2)^2}$$

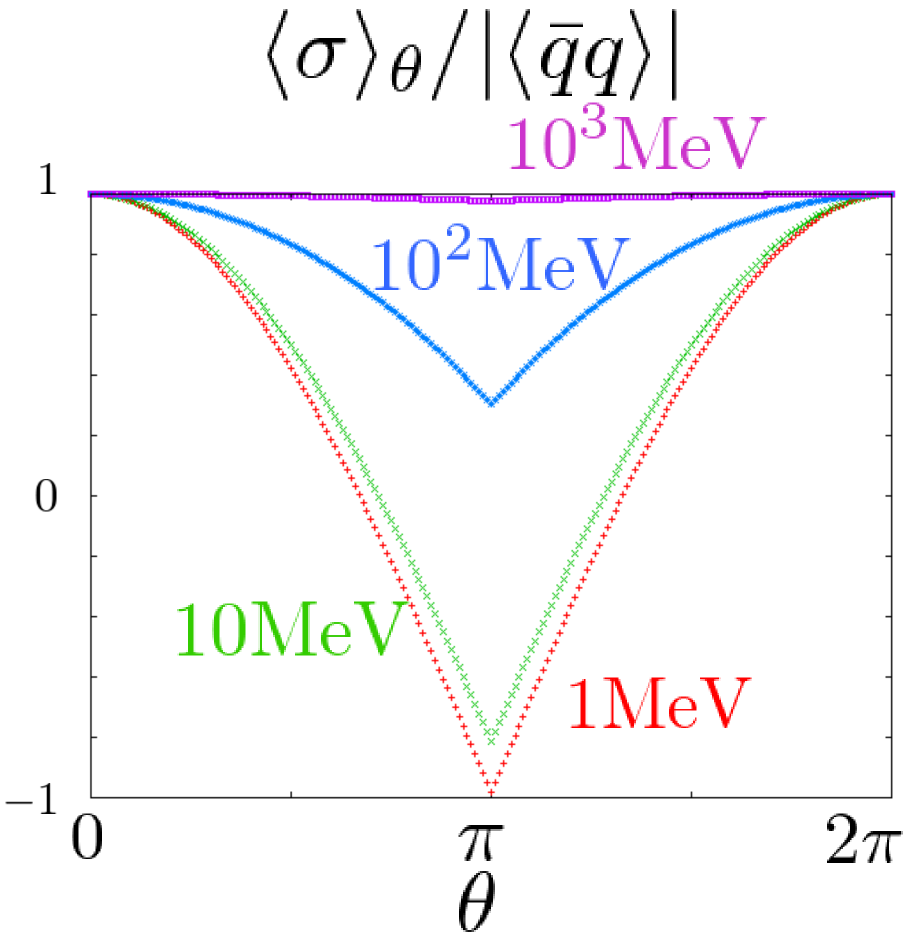
The larger the mass becomes, the stronger the fields have to be fixed to zero so that mass term can be the smallest.

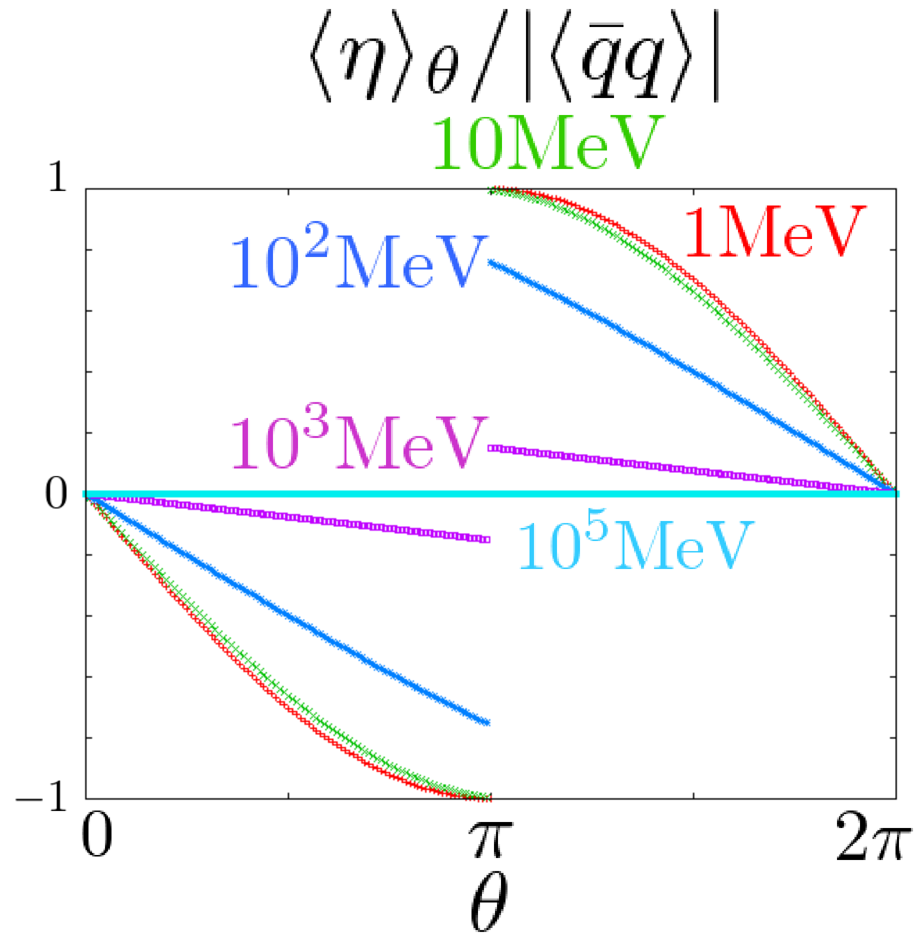
$$\phi_1 = \phi_2 \sim 0$$

Since these two phi-fields mean phases of mass term or chiral condensate, the value is fixed to real value.

$$\bar{q}q \sim \mathrm{tr}[U(\phi_1, \phi_2)] \qquad \langle\bar{q}q\rangle = \langle\sigma\rangle + \langle\eta\rangle$$

Mass-dependence of Chiral Condensate





There exists a discontinuity of the condensate for small mass

Boer 2008 (NJL Model)

Summary

- The chiral model which contains the topological effect could give a understanding of θ-vacuum even in the theory with heavy quark mass.
- This model Lagrangian is quite pedagogical since through the model, we can understand how quark mass act on the phase of mass term/chiral condensate.