Analysis of the chiral effective model of QCD using Non-perturbative Renormalization Group at finite temperature and finite density

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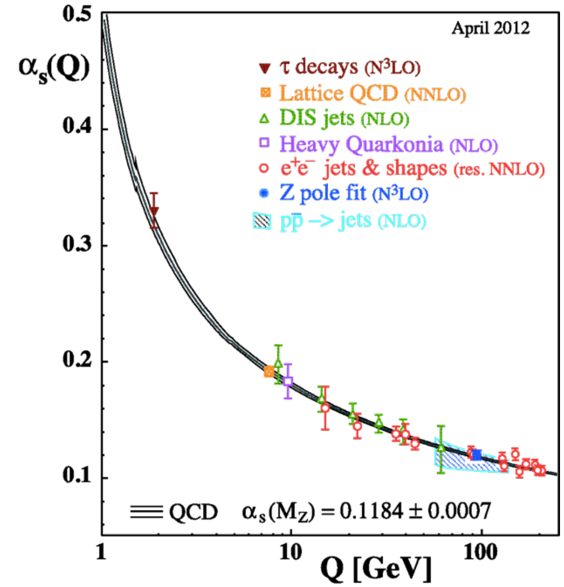
New Frontiers in QCD 2013
--- Insight into QCD matter from heavy-ion collisions ---

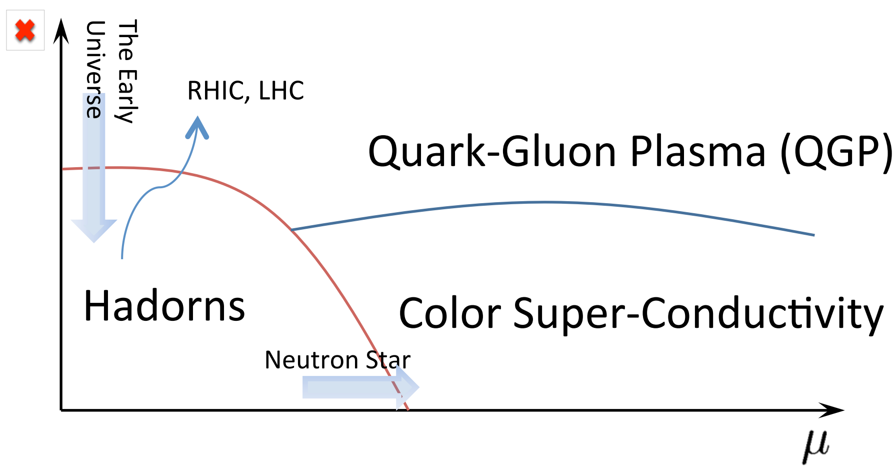
Introduction



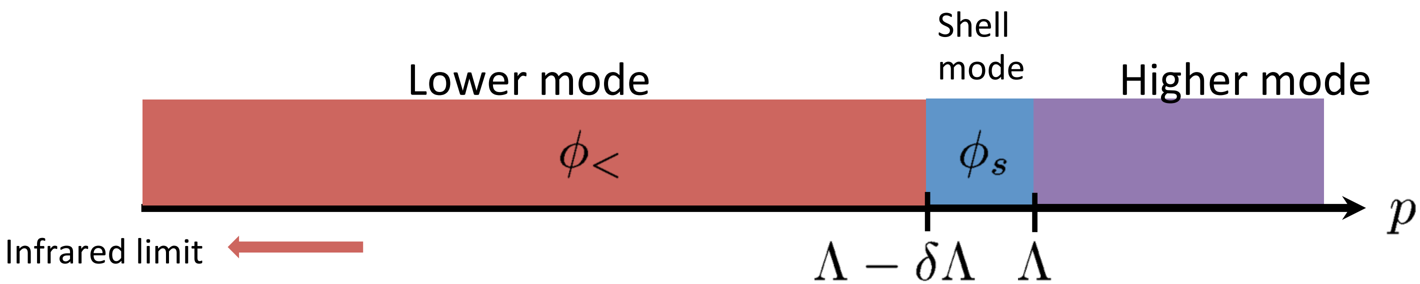
- Quantum Chromodynamics (QCD)
  - Strong Coupling at low energy scale $\alpha_s \gg 1$
  - Hadron mass $\mathcal{O}(10^3)$ MeV
  - ➡ Constituent quark mass 300 MeV
  - Current quark mass $\mathcal{O}(10)$ MeV

Dynamical Chiral Symmetry Breaking(DχSB) $\langle \bar{\psi}\psi \rangle \neq 0$

- QCD at finite temperature and finite density
  - Phase diagram



Introduction
Non-Perturbative Renormalization Group(NPRG)



$$Z = \int^{\Lambda_0} \mathcal{D}\phi \; e^{-S_0} = \int^{\Lambda} \mathcal{D}\phi_< \int_{\Lambda}^{\Lambda_0} \mathcal{D}\phi_s \; e^{-S_0[\phi_< + \phi_s]}$$

$$= \int^{\Lambda} \mathcal{D}\phi_< \; e^{-S_{\text{eff}}[\phi_<;\Lambda]}$$

$$e^{-S_{\text{eff}}[\phi_<;\Lambda]} = \left(\int_{\Lambda}^{\Lambda_0} \mathcal{D}\phi_s \; e^{-S_0[\phi_s]} \right)$$

Wilsonian effective action

⇨ NPRG equation

$$\frac{dS_{\text{eff}}}{d\Lambda} = \beta$$

NPRG

- Legendre effective action with IR cutoff $\Gamma_\Lambda[\Phi]$

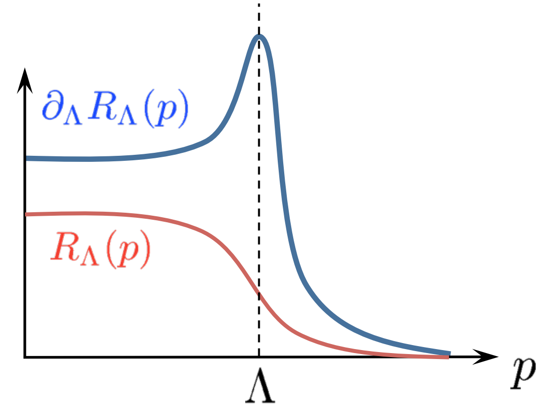
  ➢ Propagator with IR cutoff function $R_\Lambda(p)$

$$G_{0,\Lambda} = \frac{1}{G_0^{-1} + R_\Lambda(p)} \sim \frac{1}{\not{p} + R_\Lambda(p)}$$

⟹ The IR cutoff function suppresses the lower modes with $p < \Lambda$.

⟹ The higher modes with $\Lambda < p < \Lambda_0$ are integrated out.

Λ is regarded as an IR cutoff scale.



ex: Optimized cutoff function

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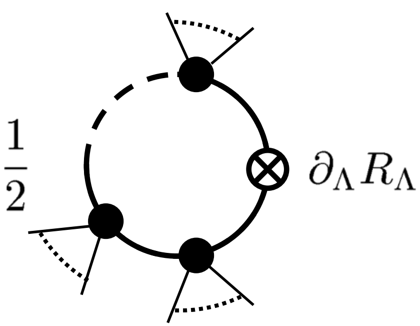
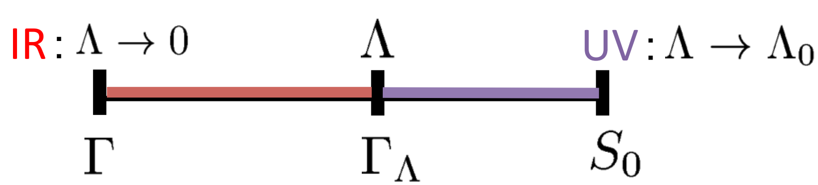
$$R_\Lambda(p) = \not{p}\left(\frac{\Lambda}{|p|} - 1\right)\theta(1 - \frac{p^2}{\Lambda^2})$$

- Wetterich flow equation

$$\partial_\Lambda \Gamma_\Lambda[\Phi] = \frac{1}{2}\mathrm{STr}\left\{\left[\frac{\overrightarrow{\delta}}{\delta\Phi}\Gamma_\Lambda[\Phi]\frac{\overleftarrow{\delta}}{\delta\Phi} + R_\Lambda\right]^{-1} \cdot (\partial_\Lambda R_\Lambda)\right\} = \frac{1}{2}$$





Approximations for NPRG

- Approximation methods
  - Derivative expansion

    Ex: ϕ^4 theory with Z_2 symmetry

$$\Gamma_{\Lambda_0}[\phi] = S_0[\phi] = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2!}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4\right]$$

$$\Gamma_\Lambda[\phi] = \int d^4x \left[V_\Lambda(\phi) + \frac{1}{2}Z_\Lambda(\phi)(\partial_\mu \phi)^2 + \frac{1}{2}Y_\Lambda(\phi)(\partial^2\phi)^2 + \cdots\right]$$

Local Potential Approximation(LPA) $\phi(p) = (2\pi)^4\delta^4(p)\phi \quad Z_\Lambda = 1$

$$\Rightarrow \Gamma_\Lambda[\phi] = \int d^4x \left[V_\Lambda(\phi) + \frac{1}{2}(\partial_\mu \phi)^2\right]$$

  - Truncation

$$V_\Lambda(\phi) = \frac{1}{2!}m_\Lambda^2\phi^2 + \frac{1}{4!}\lambda_\Lambda\phi^4 + \cdots$$

The potential function is spanned by the polynomials of field.

We need to truncate the expansion to some finite order.

Nambu—Jona-Lasinio Model

- 4-fermi interaction

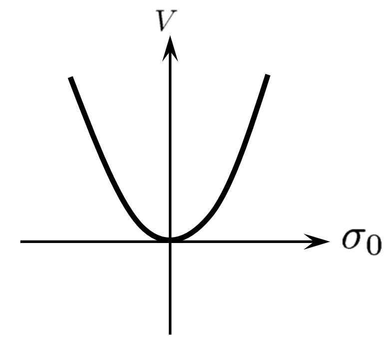
$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \frac{G}{2}\{(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \psi)^2\}$$

- Invariant under Chiral global U(1) transformation

$$\psi(x) \to e^{i\gamma_5\theta}\psi(x), \quad \bar{\psi}(x) \to \bar{\psi}(x)e^{i\gamma_5\theta}$$

⇒ Prohibit the mass term $m\bar{\psi}\psi$

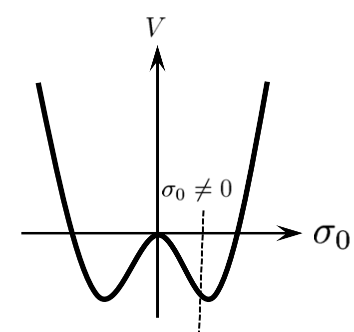
$$\bar{\psi}\psi \to \bar{\psi}e^{2i\gamma_5\theta}\psi \neq \bar{\psi}\psi$$



- Describe the DχSB of QCD

- 4-fermi coupling constant is fluctuation of chiral order parameter.

$$\mathcal{L} + m_0\bar{\psi}\psi$$



$$G \sim \langle(\bar{\psi}\psi)^2\rangle \sim \frac{\partial\langle\bar{\psi}\psi\rangle}{\partial m_0}$$

NPRG in LPA and NJL model at finite temperature and density

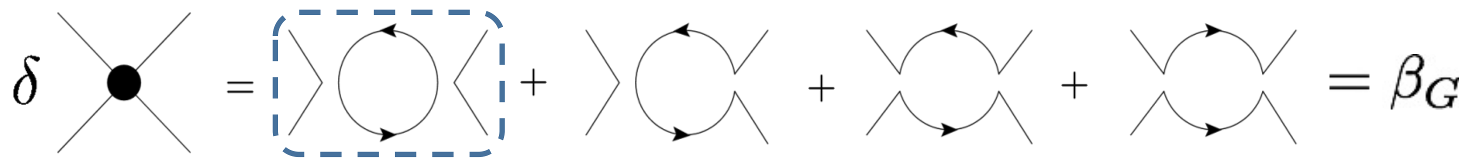
- Bare action

$$S_0 = \int d^4x \left[\bar{\psi}\partial\!\!\!/\psi + \mu\bar{\psi}\gamma_0\psi - \frac{G_0}{2}\{(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\} \right]$$

- Effective action

$$\Gamma_\Lambda = \int d^4x \left[\bar{\psi}\partial\!\!\!/\psi + \mu\bar{\psi}\gamma_0\psi - \frac{G_\Lambda}{2}\{(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\} \right]$$

- Generate the 4-fermi interaction



Large-*N* leading part

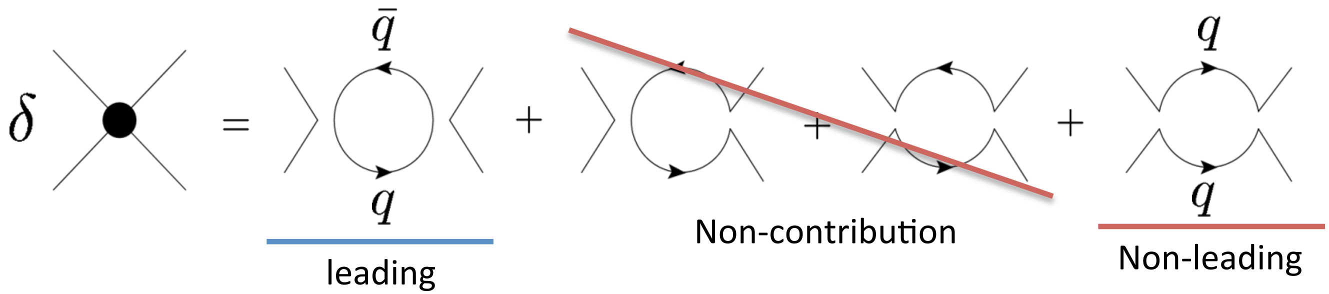
Large-*N* leading NPRG eq. ⇔ Mean field approximation

⇒ Large-*N* non-leading NPRG eq.

- 3d optimized cutoff function

$$R_\Lambda(\boldsymbol{p}) = \boldsymbol{p}\!\!\!/ \left(\frac{\Lambda}{|\boldsymbol{p}|} - 1 \right) \theta(1 - \frac{\boldsymbol{p}^2}{\Lambda^2}) = \boldsymbol{p}\!\!\!/ \, r(\boldsymbol{p}/\Lambda)$$

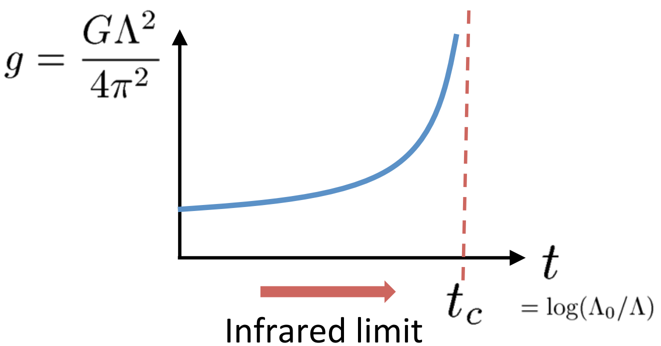
NPRG equations



$$\begin{cases} \partial_t g = -2g + \frac{1}{3} g^2 (4I_0 - I_1) \\ \partial_t \tilde{T} = \tilde{T} \\ \partial_t \tilde{\mu} = \tilde{\mu} \end{cases}$$

Negative sign: **restore chiral symmetry**

$$\tilde{T} = T/\Lambda, \ \tilde{\mu} = \mu/\Lambda \qquad n_\pm = \frac{1}{e^{\beta\epsilon_\pm} + 1}$$



$$I_0 = \left[\left(\frac{1}{2} - n_+ \right) + \left(\frac{1}{2} - n_- \right) + \frac{\partial}{\partial \omega} (n_+ + n_-) \right] \Bigg|_{\omega \to 1}$$

$$I_1 = \left[\frac{1}{(1+\mu)^2} \left(\frac{1}{2} - n_+ \right) + \frac{1}{(1-\mu)^2} \left(\frac{1}{2} - n_- \right) + \frac{1}{1+\mu} \frac{\partial}{\partial \omega} n_+ + \frac{1}{1-\mu} \frac{\partial}{\partial \omega} n_- \right] \Bigg|_{\omega \to 1}$$

- $T \to 0$ $(\beta \to \infty)$ limit

$$I_0 = 1 - \theta(\tilde{\mu} - 1) + \delta(\tilde{\mu} - 1)$$

$$I_1 = \frac{1}{2(1+\tilde{\mu})^2} + \frac{1}{(1-\tilde{\mu})^2} \left(\frac{1}{2} - \theta(\tilde{\mu} - 1) \right) + \frac{1}{1-\tilde{\mu}} \delta(\tilde{\mu} - 1)$$

I_1 has singularity at $\mu = \Lambda$

Analysis Method

$$\begin{cases} \partial_t g = -2g + \frac{4}{3} g^2 I_0 & \text{: leading} \\ \partial_t g = -2g + \frac{1}{3} g^2 (4I_0 - I_1) & \text{: non-leading} \end{cases}$$

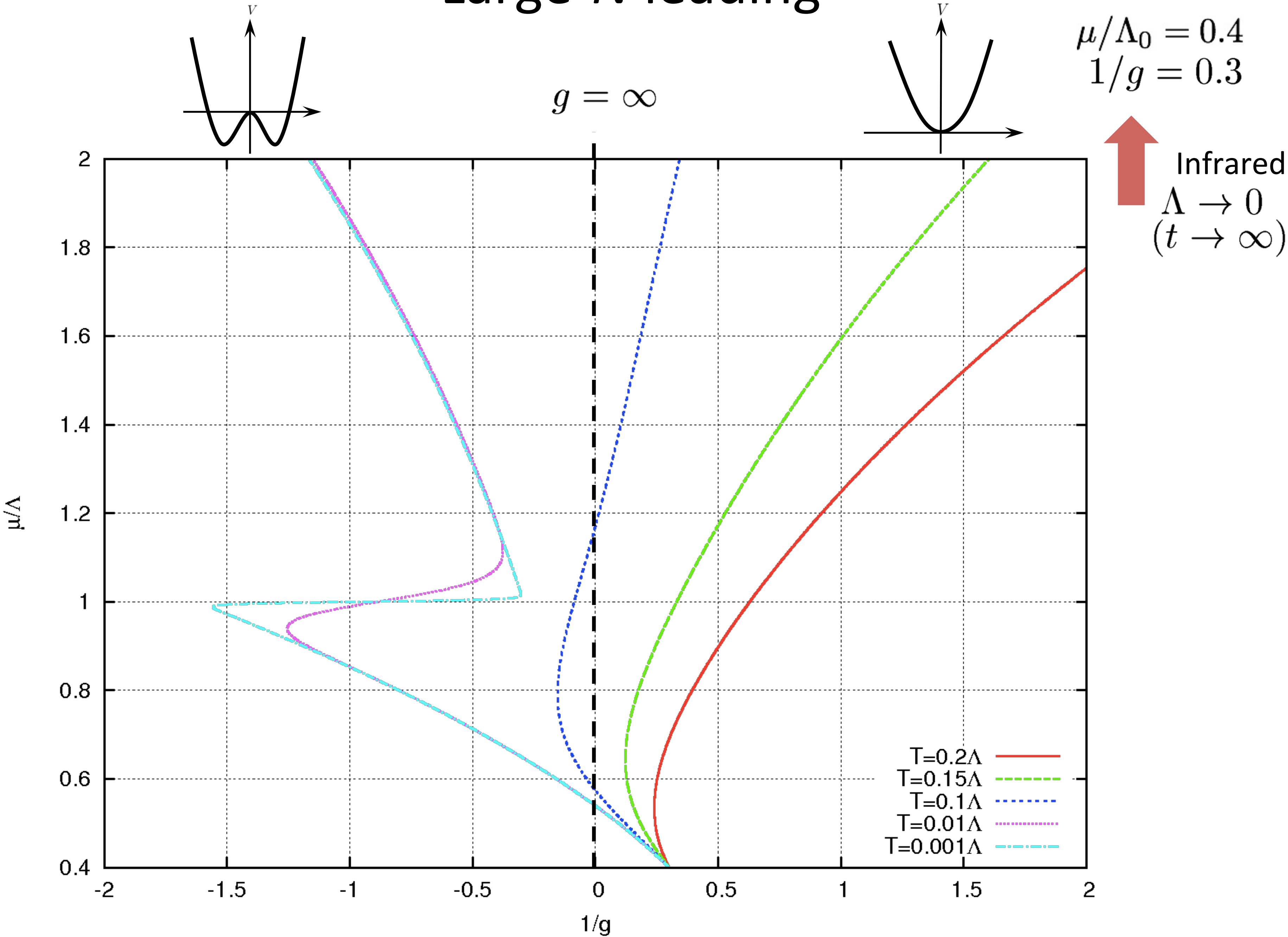
$$\tilde{g} = \frac{1}{g}$$

$$\Rightarrow \begin{cases} \partial_t \tilde{g} = 2\tilde{g} - \frac{4}{3} I_0 & \text{: leading} \\ \partial_t \tilde{g} = 2\tilde{g} - \frac{1}{3} (4I_0 - I_1) & \text{: non-leading} \end{cases}$$

$g = \infty$ chiral symmetry braking

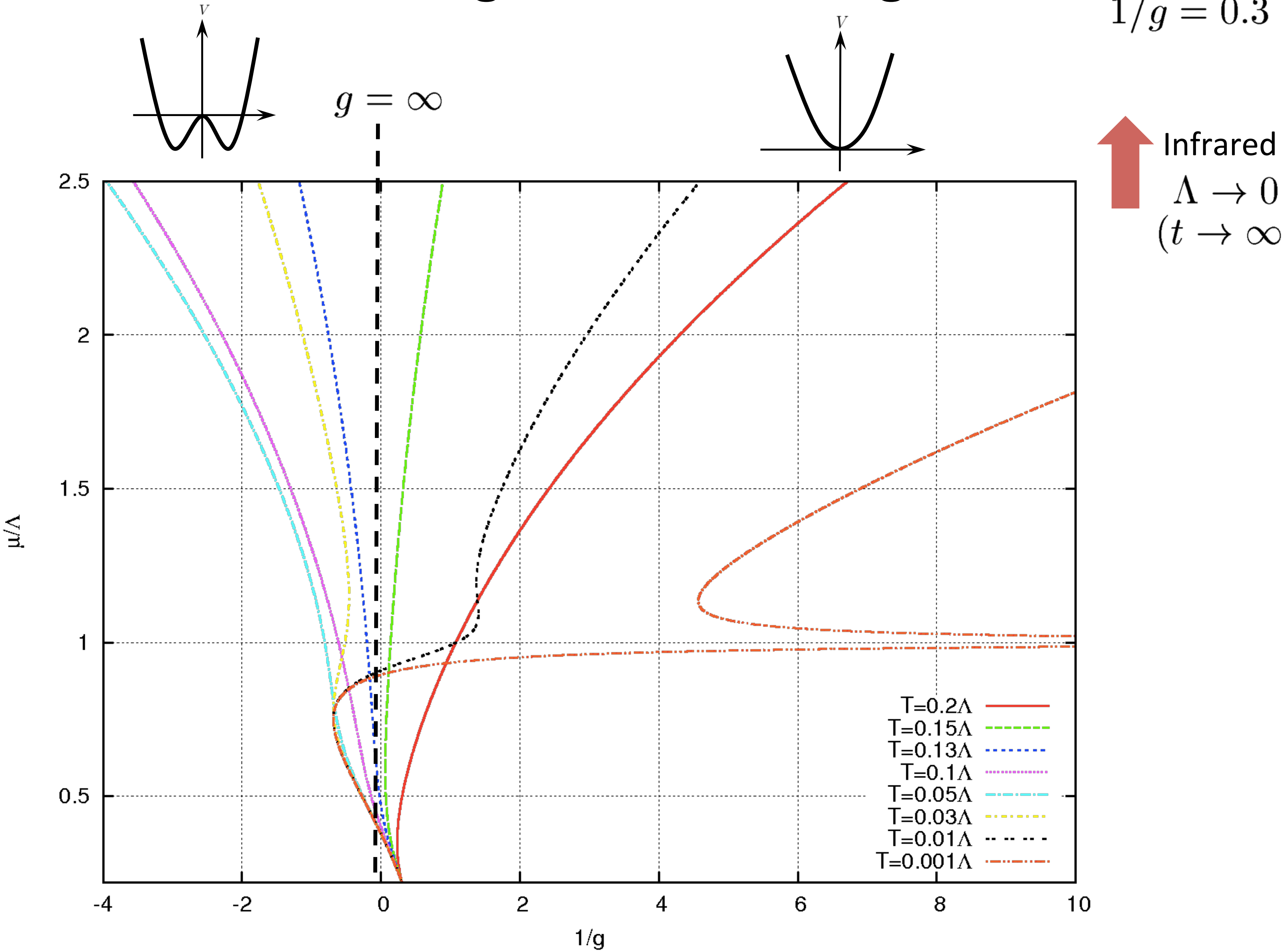
$\Rightarrow \quad \tilde{g} = 0$

Large-N leading



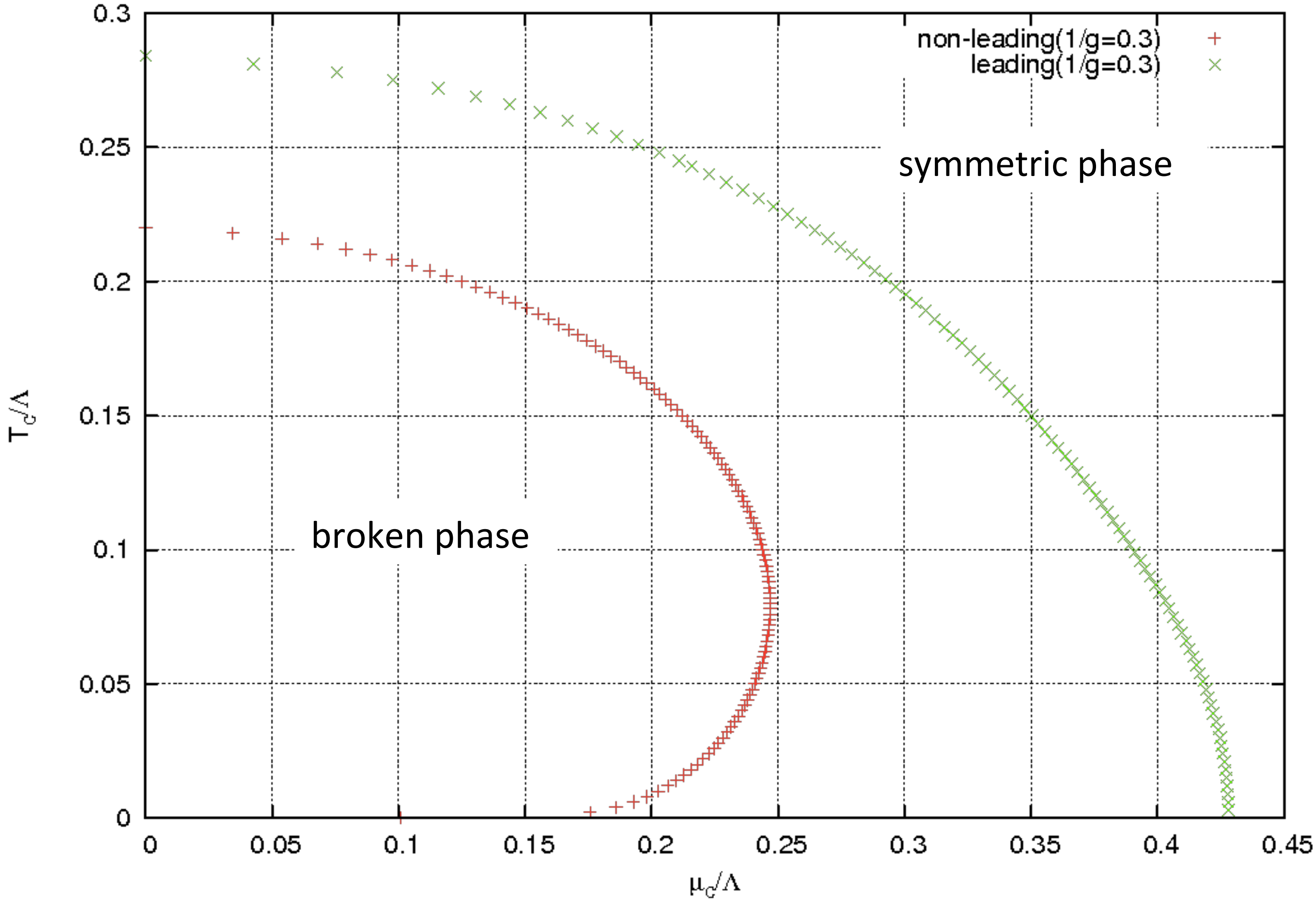
Large-N non-leading

$\mu/\Lambda_0 = 0.22$
$1/g = 0.3$



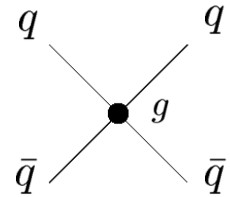
Leading vs. non-leading

- Phase diagram



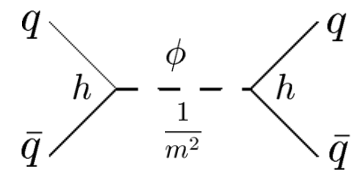
Leading vs. non-leading

4-fermi coupling constant:

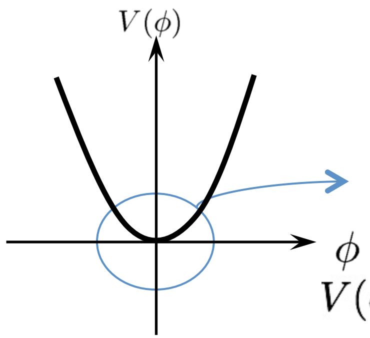


$$V(\phi) = m^2\phi^2 + \lambda\phi^4 + \cdots$$

Inverse curvature of meson field (auxiliary field) at the origin

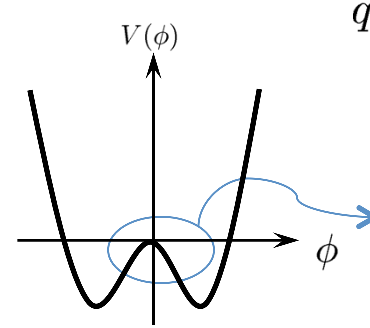


$$\frac{1}{g} \sim m^2$$



$m^2\phi^2;\ m^2 > 0$
$(1/g > 0)$

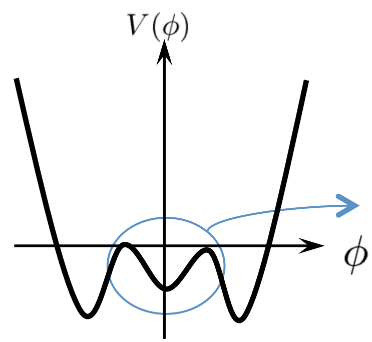
$$V(\phi) = +|m^2|\phi^2 + \lambda\phi^4 + \cdots$$



$m^2\phi^2;\ m^2 < 0$
$(1/g < 0)$

2nd order transition

$$V(\phi) = -|m^2|\phi^2 + \lambda\phi^4 + \cdots$$



$m^2\phi^2;\ m^2 > 0$
$(1/g > 0)$

1st order transition

Bosonized NJL Model

- Effective action of the bosonized NJL model at finite temperature and density

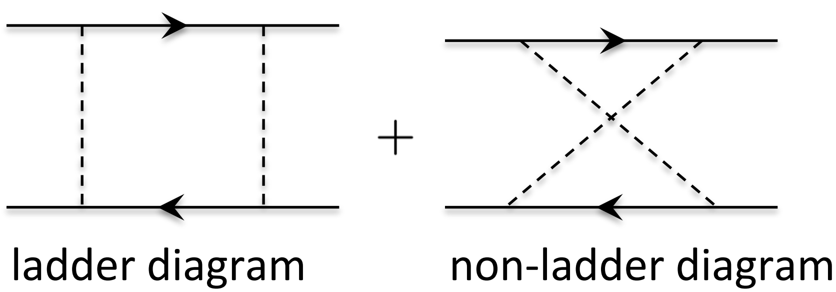
$$\Gamma_\Lambda[\Phi] = \int_0^{1/T} d\tau \int d^3x \left\{ \bar{\psi}[Z_\psi^{||}\gamma^0(\partial_0 + \mu) + Z_\psi^{\perp}\gamma^i\partial_i + \bar{h}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)]\psi + \frac{\bar{G}}{2}\{(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\} \right.$$
$$\left. + \frac{Z_\phi^{||}}{2}(\partial_0\sigma)^2 + \frac{Z_\phi^{\perp}}{2}(\partial_i\sigma)^2 + \frac{Z_\phi^{||}}{2}(\partial_0\vec{\pi})^2 + \frac{Z_\phi^{\perp}}{2}(\partial_i\vec{\pi})^2 + U_\Lambda(\sigma^2 + \vec{\pi}^2) \right\}$$

- Bare action

$$S_{\Lambda_0}[\Phi] = \int_0^{1/T} d\tau \int d^3x \left\{ \bar{\psi}[\gamma^\mu\partial_\mu + \gamma^0\mu + \bar{h}_0(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)]\psi + \frac{1}{2}m^2\phi^2 \right\}$$

$$U_{\Lambda_0} = 0, G_0 = 0, Z_\phi = 0, Z_\psi = 1 \text{ at } \Lambda = \Lambda_0$$

$\partial_t G \sim$



$\sim h^4$

⇒ $\frac{1}{\Lambda - \mu}$ at $T = 0$

- Yukawa coupling constant generates 4-fermi interaction.
- The non-ladder diagram has the singularity.

QM model and NPRG

$$N_c = 3 \quad N_f = 2$$

- Effective action of Quark Meson model

$$\Gamma_\Lambda[\Phi] = \int_0^{1/T} d\tau \int d^3x \left\{ \bar{\psi}[Z_\psi^{||}(\phi)\gamma^0\partial_0 + Z_\psi^\perp(\phi)\gamma^i\partial_i + \frac{\bar{h}(\phi)}{\sqrt{2}}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)]\psi\} \right.$$
$$\left. + \frac{Z_\phi^{||}(\phi)}{2}(\partial_0\sigma)^2 + \frac{Z_\phi^\perp(\phi)}{2}(\partial_i\sigma)^2 + \frac{Z_\phi^{||}(\phi)}{2}(\partial_0\vec{\pi})^2 + \frac{Z_\phi^\perp(\phi)}{2}(\partial_i\vec{\pi})^2 + U_\Lambda(\sigma^2 + \vec{\pi}^2) \right\}$$

- Bare action

$$S_{\Lambda_0}[\Phi] = \int_0^{1/T} d\tau \int d^3x \left\{ \bar{\psi}[\gamma^\mu\partial_\mu + \frac{\bar{h}_0}{\sqrt{2}}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)]\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + U_{\Lambda_0}(\sigma^2 + \vec{\pi}^2) \right\}$$

$$Z_\phi = 1, Z_\psi = 1 \text{ at } \Lambda = \Lambda_0$$

- We use 3d optimized cutoff function.

$$R_\Lambda^\psi(\boldsymbol{p}) = Z_\psi^\perp \not{\boldsymbol{p}} \left(\frac{\Lambda}{|\boldsymbol{p}|} - 1 \right) \theta(1 - \frac{\boldsymbol{p}^2}{\Lambda^2}) = Z_\psi^\perp \not{\boldsymbol{p}}\, r_\psi(\boldsymbol{p}/\Lambda)$$

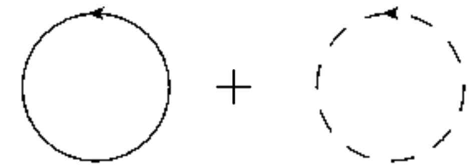
$$R_\Lambda^B(\boldsymbol{p}) = Z_\phi^\perp \boldsymbol{p}^2 \left(\frac{\Lambda^2}{\boldsymbol{p}^2} - 1 \right) \theta(1 - \frac{\boldsymbol{p}^2}{\Lambda^2}) = Z_\phi^\perp \boldsymbol{p}^2\; r_B(\boldsymbol{p}/\Lambda)$$

- $Z_\psi^{||} \approx Z_\psi^\perp \quad Z_\phi^{||} \approx Z_\phi^\perp$

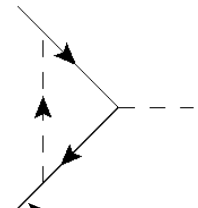
QM model and NPRG

- Renormalization equations

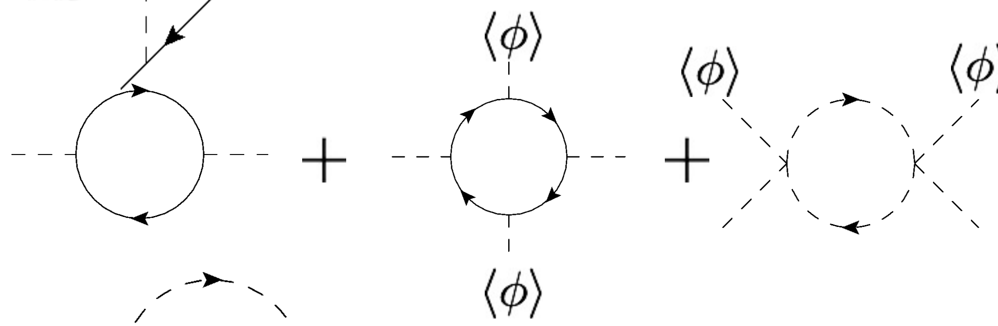
$$\partial_\Lambda U_\Lambda = \beta_U(\phi, U, \partial_\phi U, \partial_\phi^2 U, T, \mu, \Lambda) \sim$$



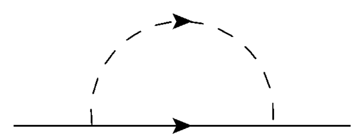
$$\partial_\Lambda h_\Lambda = \beta_h(h, T, \mu, \Lambda) \sim$$



$$\partial_\Lambda Z_{\phi,\Lambda} = -\frac{\eta_\phi}{\Lambda} Z_{\phi,\Lambda} \sim$$



$$\partial_\Lambda Z_{\psi,\Lambda} = -\frac{\eta_\psi}{\Lambda} Z_{\psi,\Lambda} \sim$$



- Initial values

$$U_{\Lambda_0},\ h_{\Lambda_0},\ Z_{\phi,\Lambda_0} = 1,\ Z_{\psi,\Lambda_0} = 1$$

$$\Leftarrow f_\pi \sim 83\ \mathrm{MeV},\ m_q \sim 300\ \mathrm{MeV}$$

at infrared scale and zero temperature and density.

Numerical results

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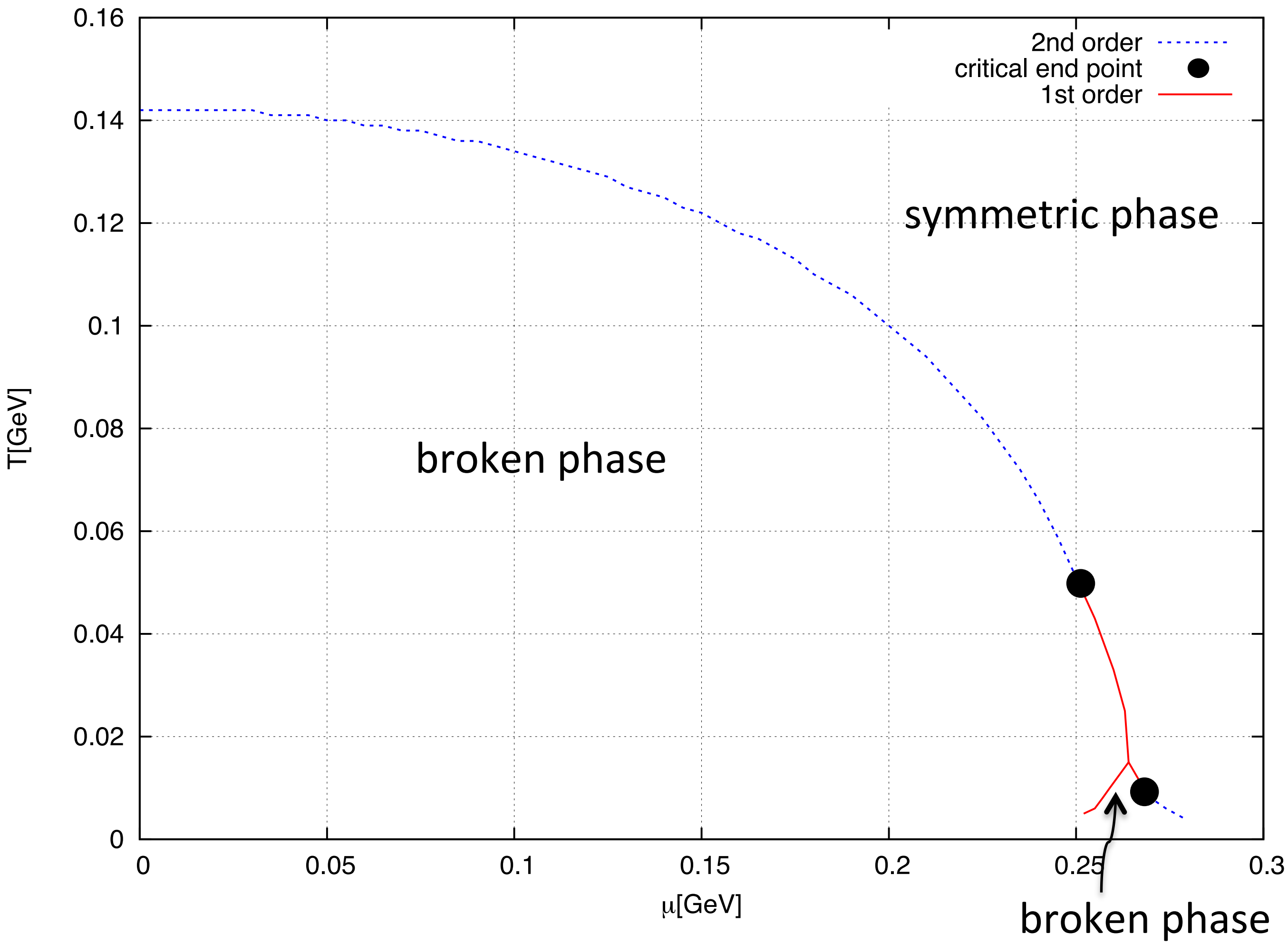
- Phase diagram

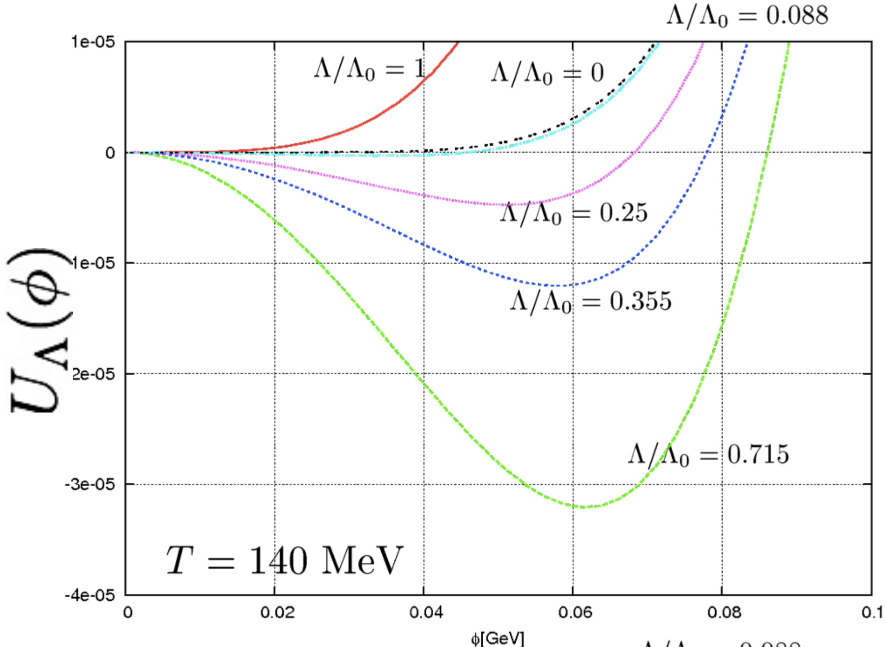
$$\partial_\Lambda h_\Lambda = 0,\ \eta_\phi = \eta_\psi = 0$$

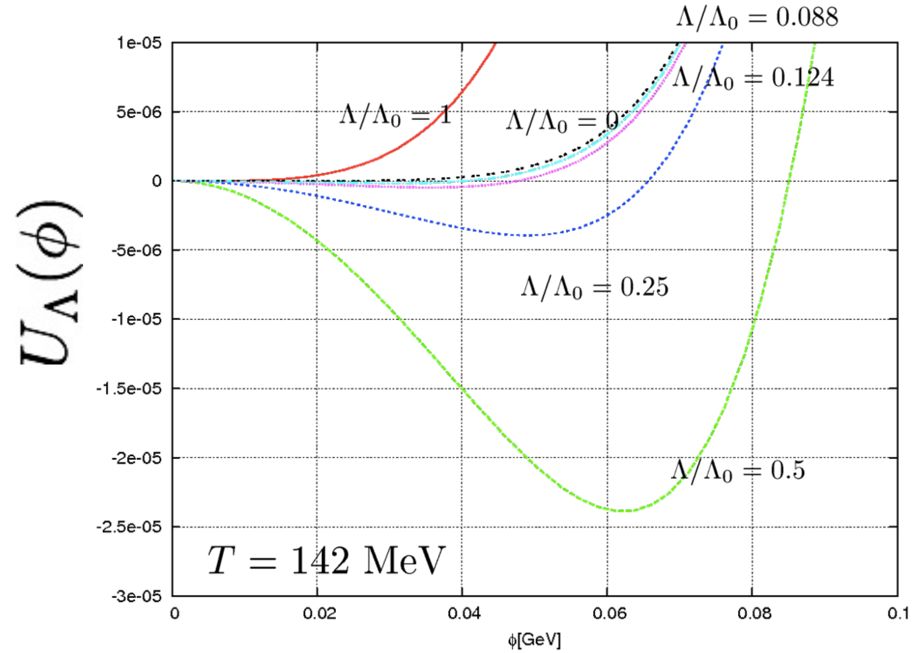


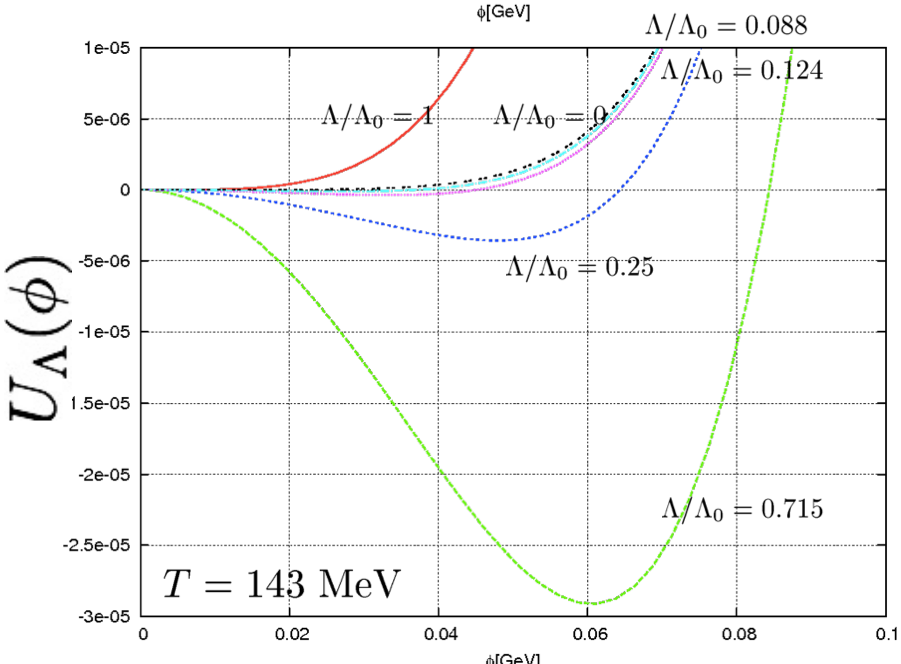
$$T_{\rm cri} = 52\ \mathrm{MeV}$$
$$\mu_{\rm cri} = 251\ \mathrm{MeV}$$

Numerical results

- 2^{nd} order transition







$$\mu = 0$$

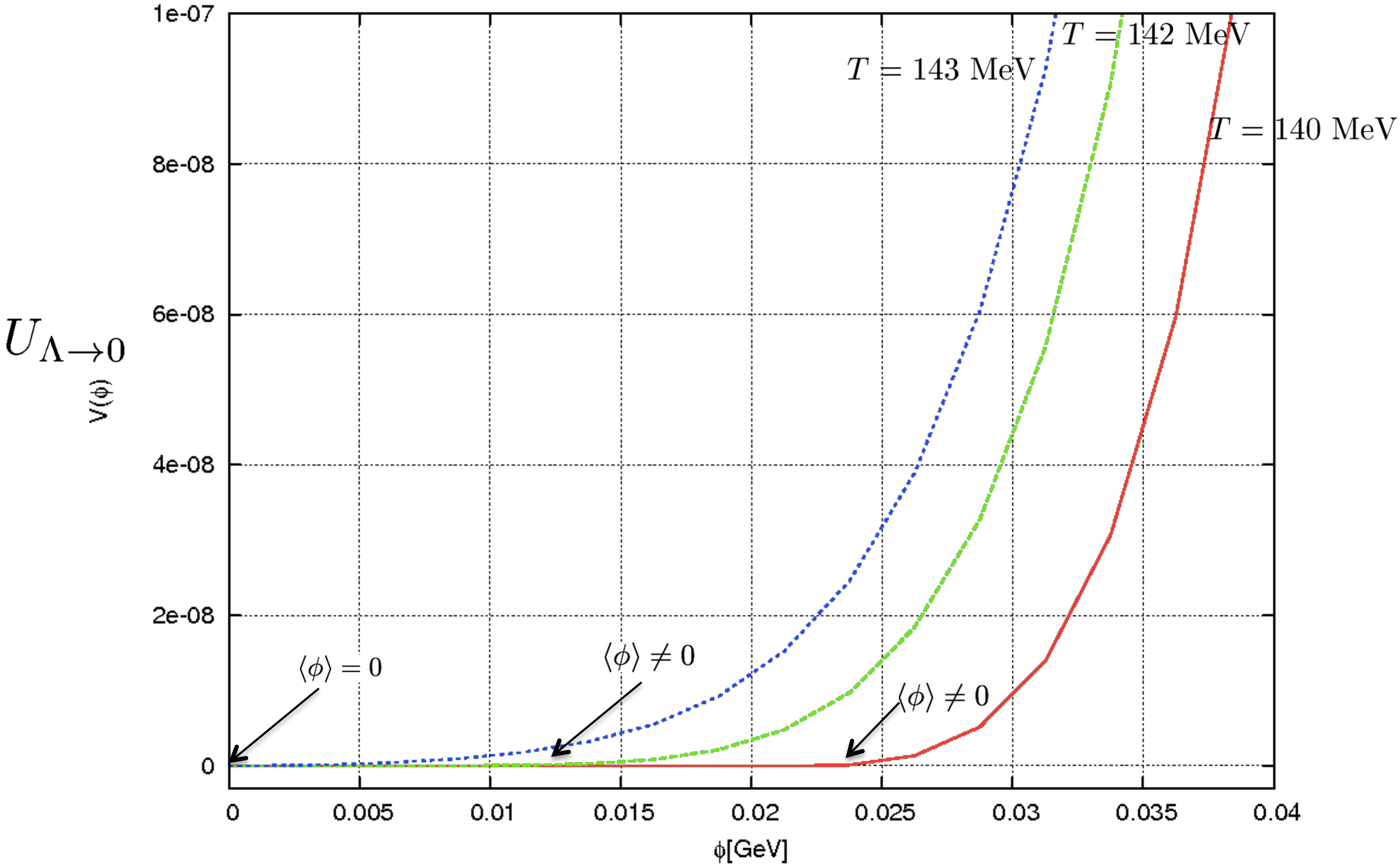
$$U_{\Lambda_0}(\phi^2) = \frac{\lambda}{4}(\phi^2)^2$$

$$\Lambda_0 = 500 \text{ MeV}$$
$$h = 3.2 \quad \lambda = 10$$

Numerical results

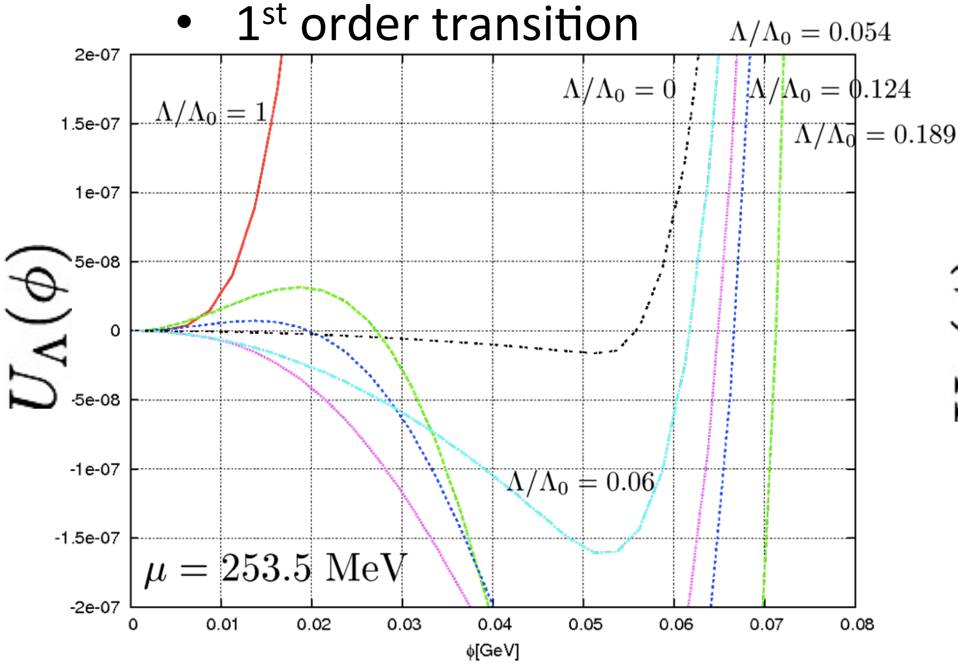
- 2[nd] order transition

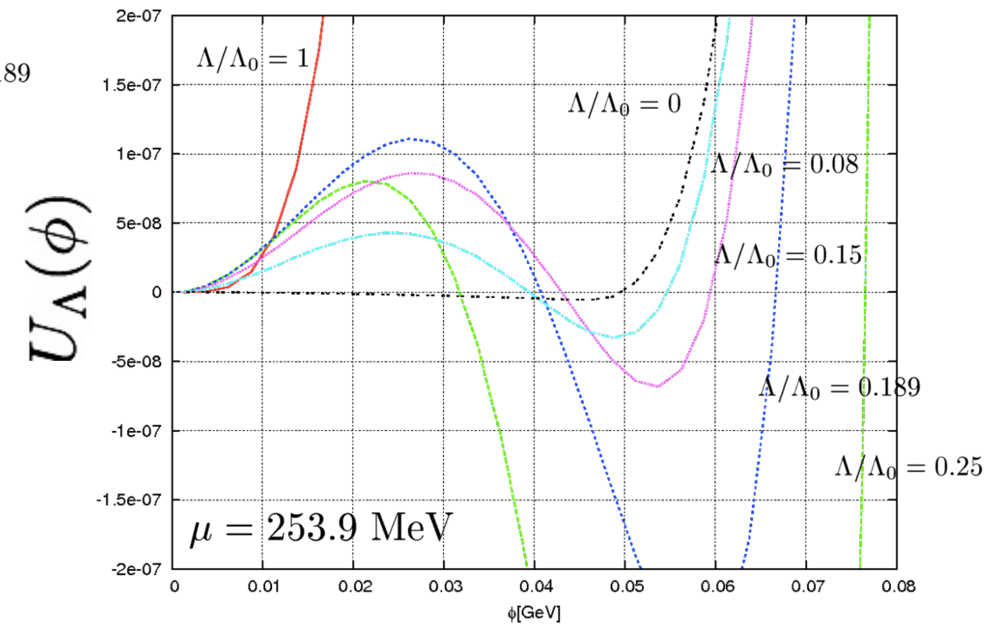
$\mu = 0$

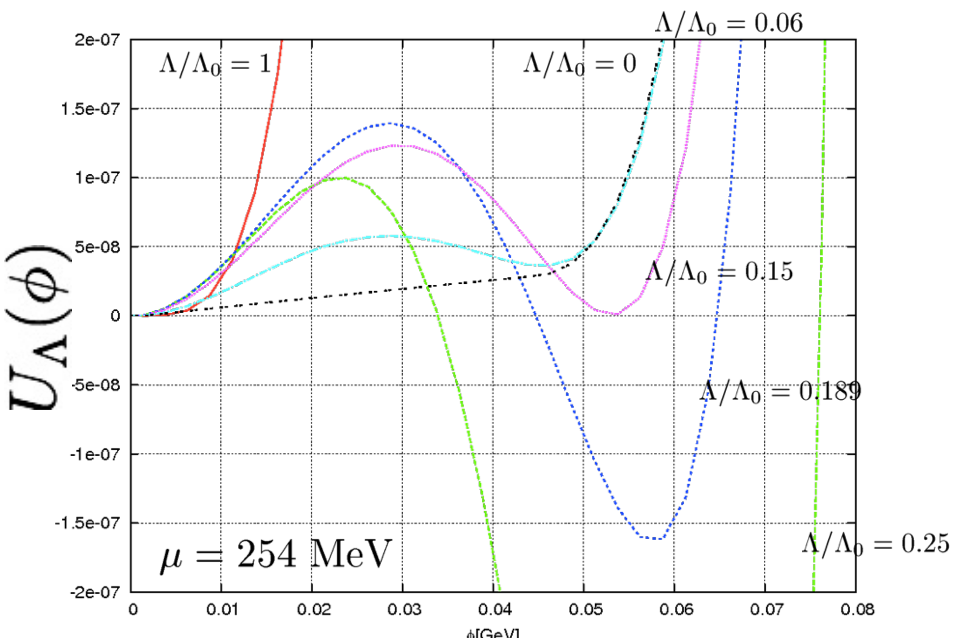


Numerical results

- 1st order transition





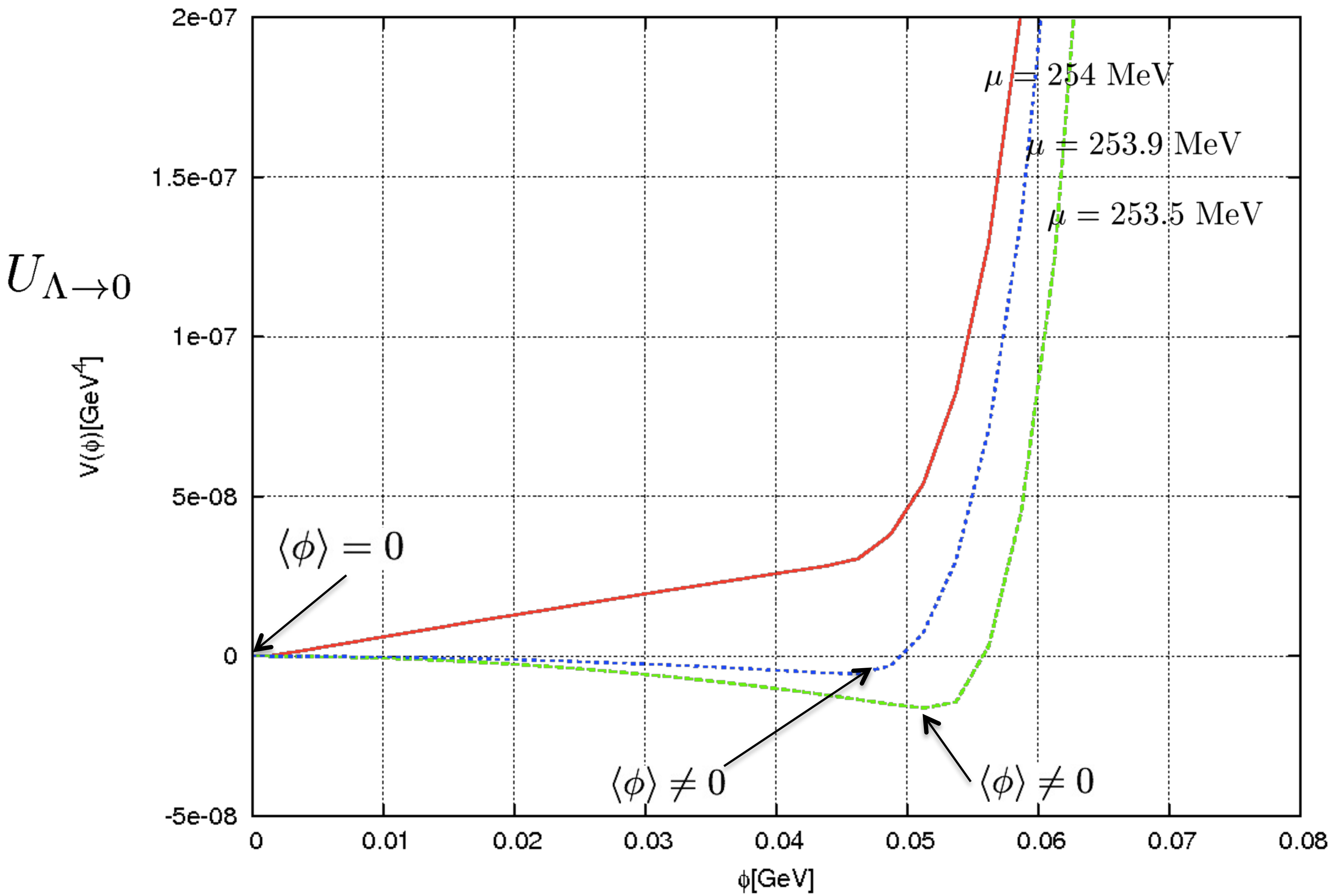


$T = 45$ MeV

Numerical results

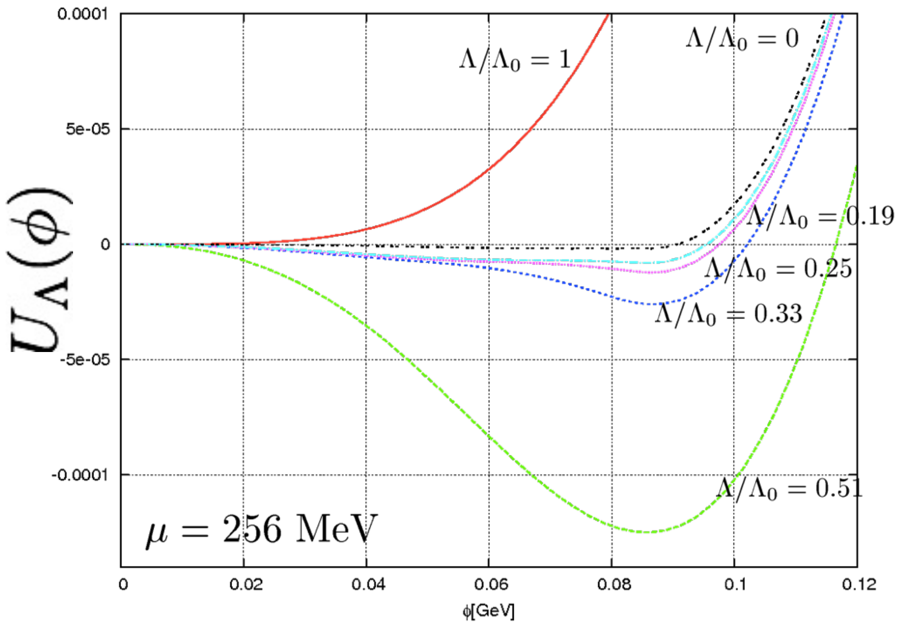
$T = 45$ MeV

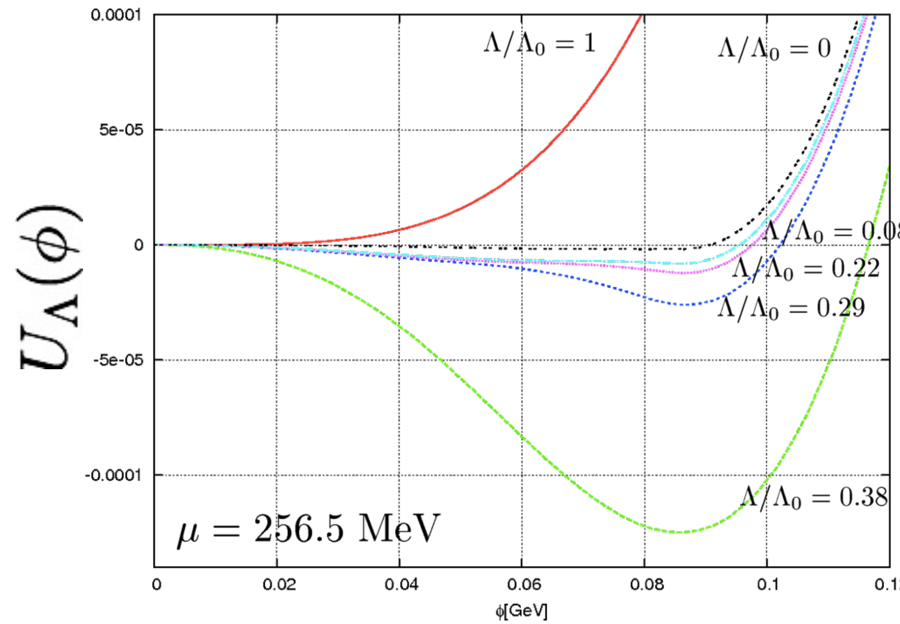
- 1^{st} order transition

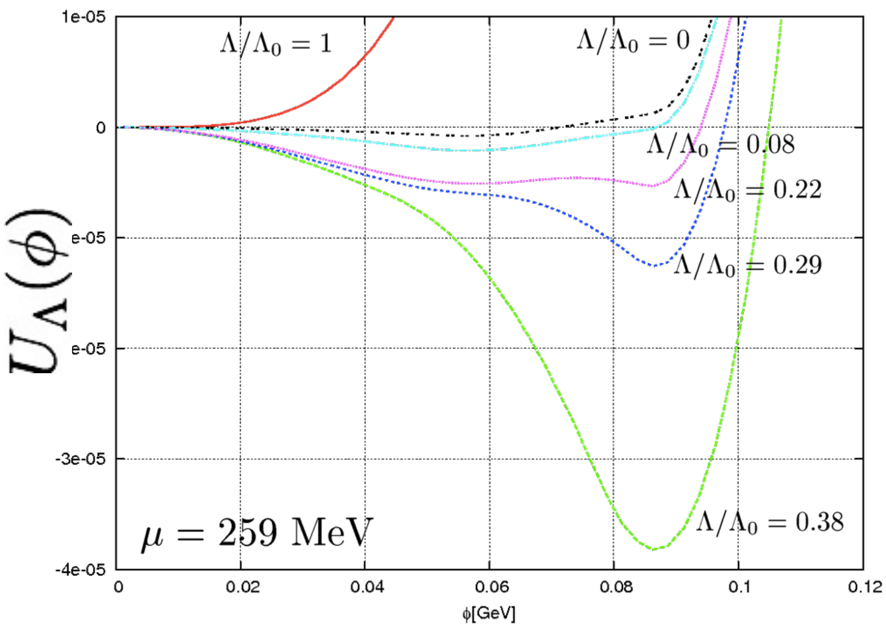


Numerical results

- 1st order transition at low temperature and high density





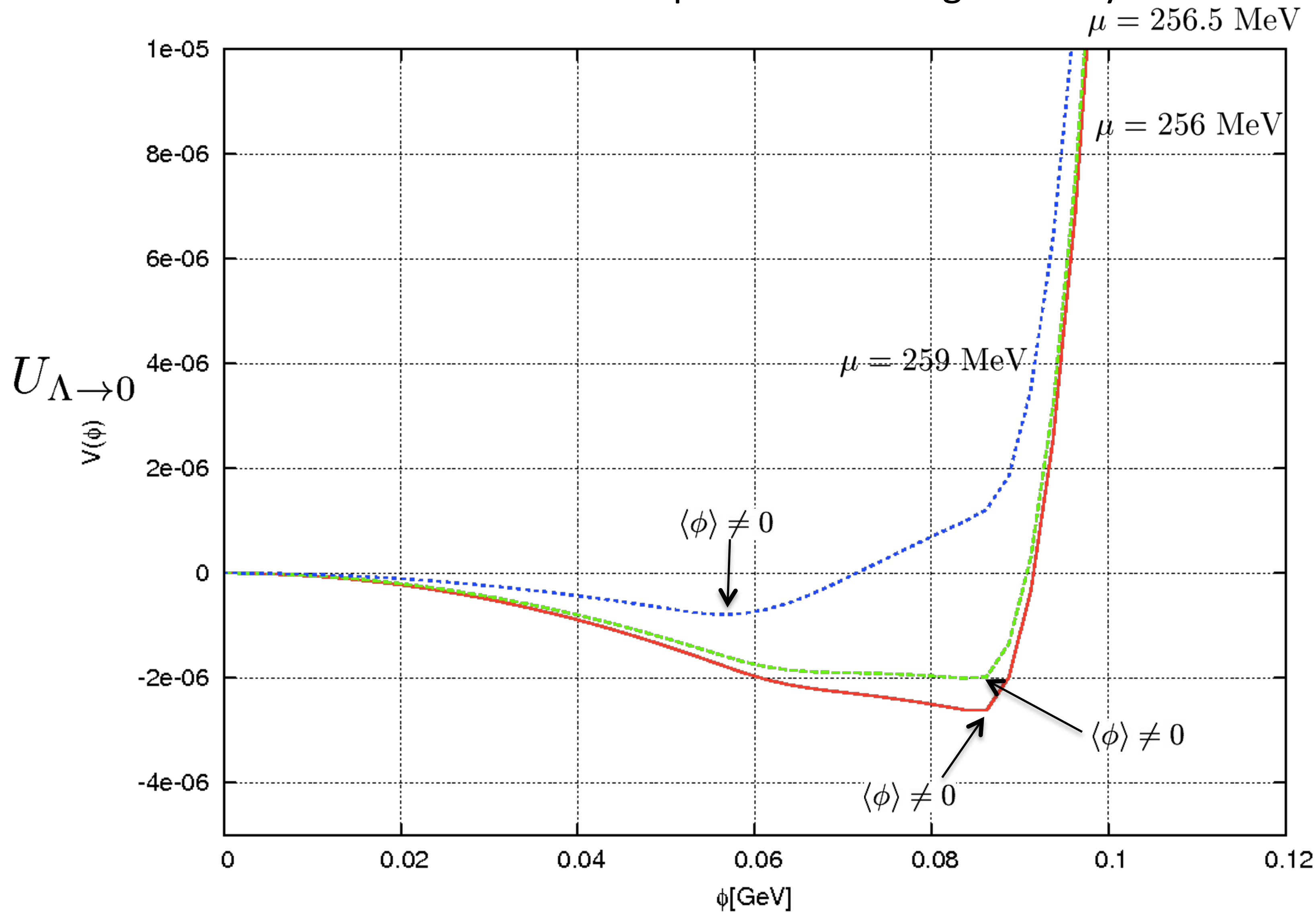


$T = 8$ MeV

Numerical results

$T = 8\ \text{MeV}$

- 1[st] order transition at low temperature and high density

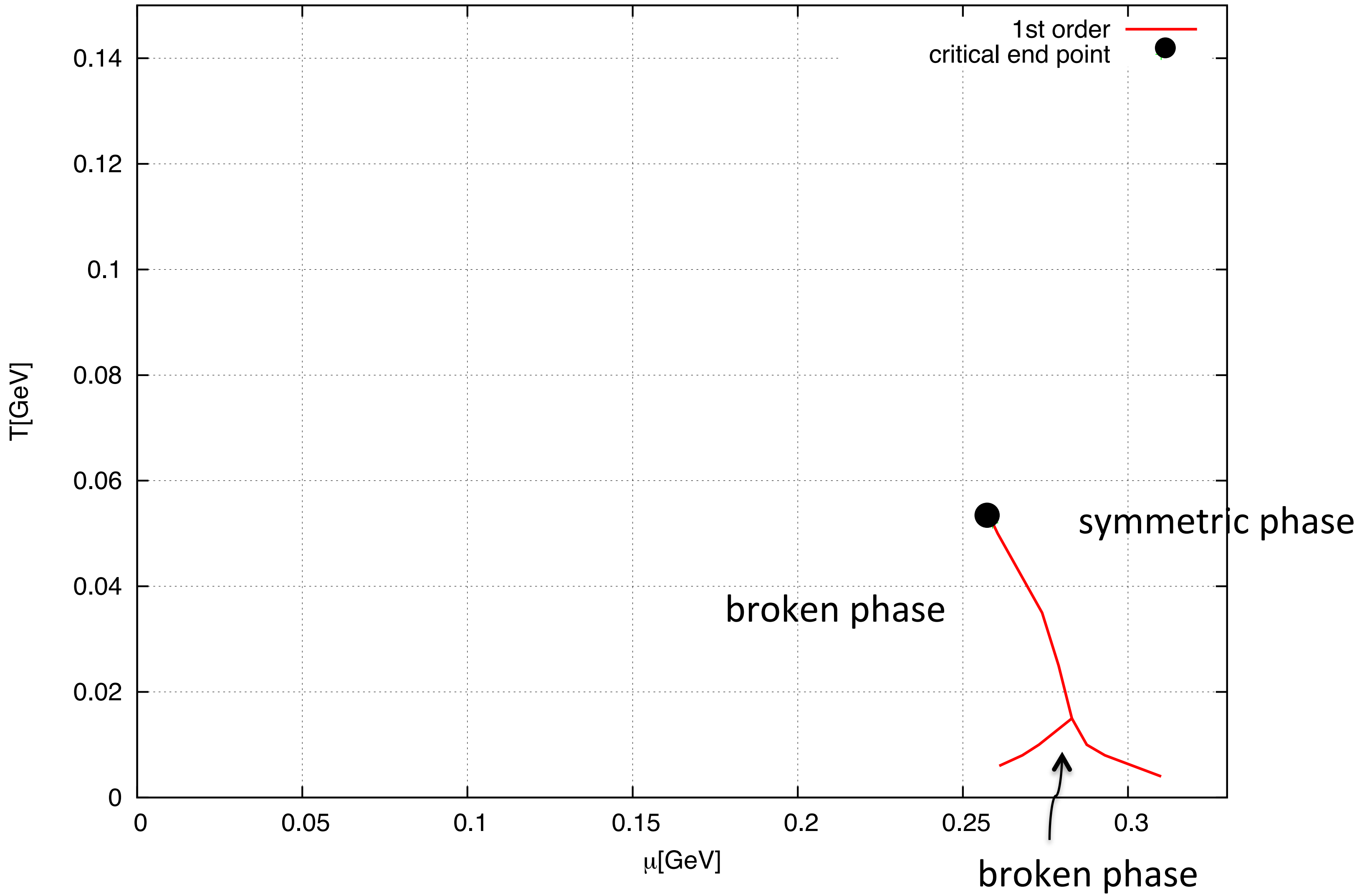


Numerical results

$$\partial_\Lambda Z_{\phi,\Lambda} = -\frac{\eta_\phi}{\Lambda} Z_{\phi,\Lambda}$$

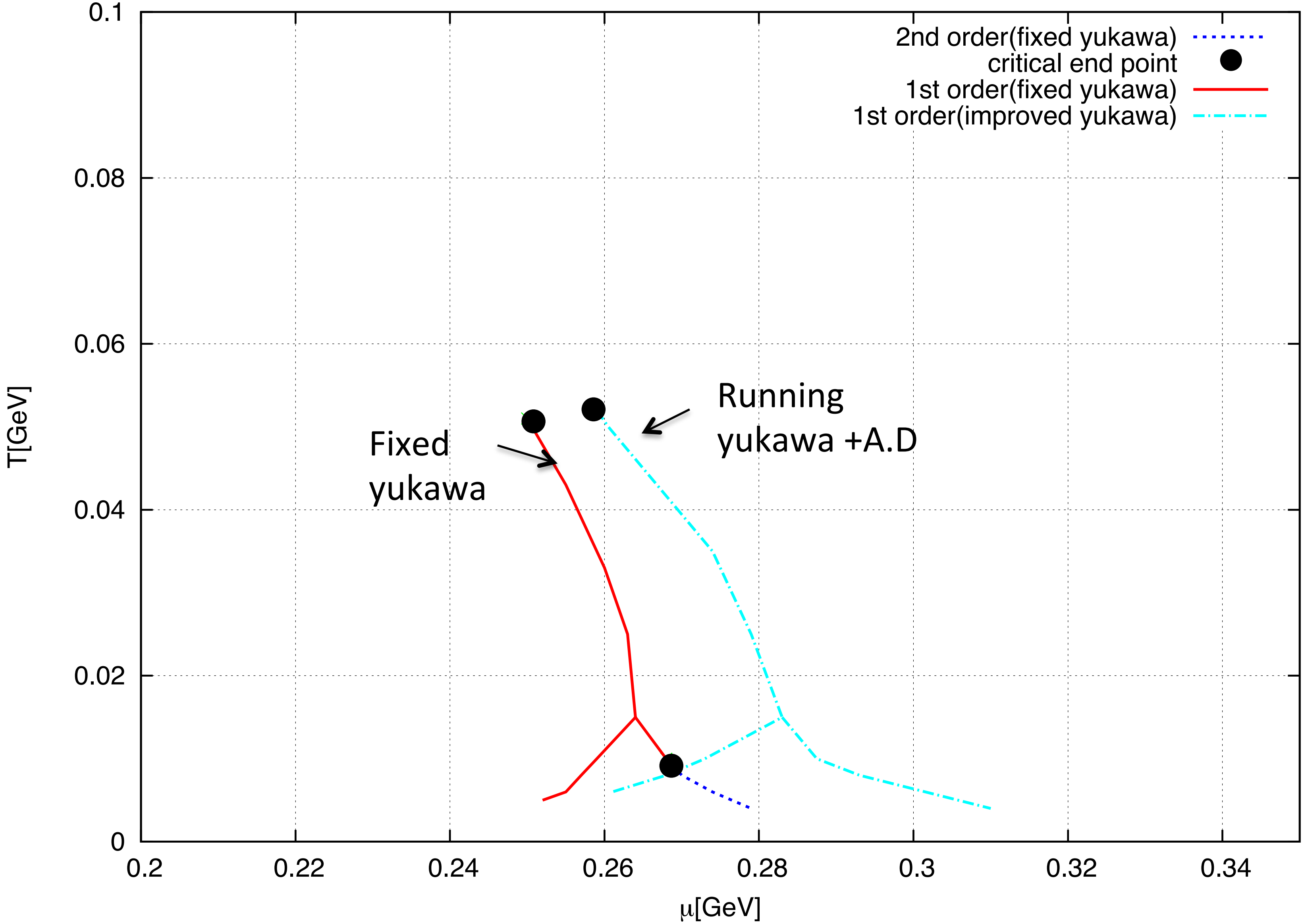
- Phase diagram

$$\partial_\Lambda h_\Lambda = \beta_h \quad \partial_\Lambda Z_{\psi,\Lambda} = -\frac{\eta_\psi}{\Lambda} Z_{\psi,\Lambda}$$



Numerical results

- Phase diagram



Results and Prospects

- We analyzed NJL model and Quark-Meson model at finite temperature and finite density.

- In NJL model, the large-N non-leading effects become large at low temperature & high density region, which makes the system more symmetric.

- In Quark-Meson model, we newly took account of RG running of the yukawa coupling constant and meson/quark anomalous dimensions.
  - ➢ The triangular intermediate phase still exists after this improvement.
  - ➢ However, the critical end point at higher density side boundary vanishes.
  - ➢ The chiral restoration temperature/density become higher, thus the system shifts to be less symmetric.
- We proceed to include the large-N non-leading effects.
  - ➢ By adopting the "re-bosonization" method.

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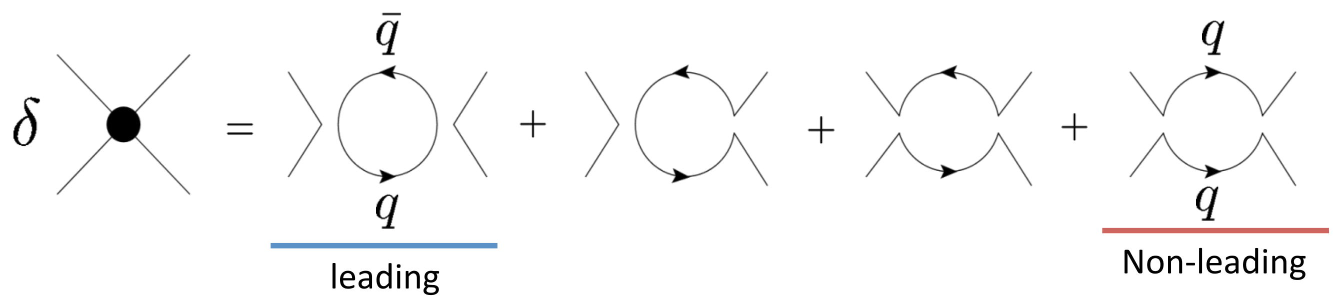
  - ➢ Study how does the chiral phase structure change at high density.

Appendix

NPRG equations



$$\begin{cases} \partial_t g = -2g + \frac{1}{3} g^2 (4I_0 - I_1) \\ \partial_t \tilde{T} = \tilde{T} \\ \partial_t \tilde{\mu} = \tilde{\mu} \end{cases}$$

$\tilde{T} = T/\Lambda,\ \tilde{\mu} = \mu/\Lambda$

$$\partial_t = -\Lambda \frac{\partial}{\partial \Lambda},\quad g = \frac{G\Lambda^2}{4\pi^2},\quad n_\pm = \frac{1}{e^{\beta\epsilon_\pm} + 1}$$

$$I_0 = \left[\left(\frac{1}{2} - n_+ \right) + \left(\frac{1}{2} - n_- \right) + \frac{\partial}{\partial \omega} (n_+ + n_-) \right] \Bigg|_{\omega \to 1}$$

$$I_1 = \left[\frac{1}{(1+\mu)^2} \left(\frac{1}{2} - n_+ \right) + \frac{1}{(1-\mu)^2} \left(\frac{1}{2} - n_- \right) + \frac{1}{1+\mu} \frac{\partial}{\partial \omega} n_+ + \frac{1}{1-\mu} \frac{\partial}{\partial \omega} n_- \right] \Bigg|_{\omega \to 1}$$

- $T \to 0\ (\beta \to \infty)$ limit

$$I_0 = 1 - \theta(\tilde{\mu} - 1) + \delta(\tilde{\mu} - 1)$$

$$I_1 = \frac{1}{2(1+\tilde{\mu})^2} + \frac{1}{(1-\tilde{\mu})^2} \left(\frac{1}{2} - \theta(\tilde{\mu} - 1) \right) + \frac{1}{1-\tilde{\mu}} \delta(\tilde{\mu} - 1)$$

I_1 has singularity at $\mu = \Lambda$

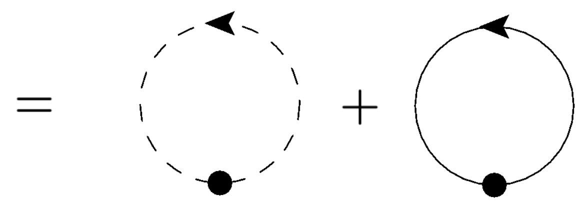
NPRG flow equations

- NPRG eq. of the scale-dependent grand canonical thermodynamic potential $\Omega(T,\mu;\phi)$

$$N_\mathrm{c}=3 \qquad N_\mathrm{f}=2$$

$$-\Lambda\frac{\partial}{\partial\Lambda}\Omega_\Lambda(T,\mu;\phi) = -\frac{\Lambda^5}{12\pi^2}\left[\frac{3}{E_\pi}\coth\left(\frac{E_\pi}{2T}\right)+\frac{1}{E_\sigma}\coth\left(\frac{E_\sigma}{2T}\right)\right.$$
$$\left.-\frac{2N_\mathrm{c}N_\mathrm{f}}{E_q}\left\{\tanh\left(\frac{E_q-\mu}{2T}\right)+\tanh\left(\frac{E_q+\mu}{2T}\right)\right\}\right]$$

$$\partial_t = -\Lambda\frac{\partial}{\partial\Lambda}$$



$$h^2=\frac{\bar{h}^2}{Z_\psi^2 Z_\phi}$$

$$E_q = \sqrt{\Lambda^2+\bar{M}_q^2/Z_\psi^2} \qquad \frac{\bar{M}_q^2}{Z_\psi^2}=\frac{1}{2}\frac{\bar{h}^2}{Z_\psi^2}\phi^2=\frac{1}{2}h^2(Z_\phi\phi^2)$$

$$E_\sigma = \sqrt{\Lambda^2+\bar{M}_\sigma^2/Z_\phi} \qquad \bar{M}_\sigma = 2\frac{\partial\Omega_\Lambda}{\partial\phi^2}+4\phi^2\frac{\partial^2\Omega_\Lambda}{\partial(\phi^2)^2}$$

$$E_\pi = \sqrt{\Lambda^2+\bar{M}_\pi^2/Z_\phi} \qquad \bar{M}_\pi = 2\frac{\partial\Omega_\Lambda}{\partial\phi^2}$$

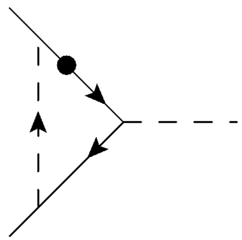
NPRG flow equations

$$h^2 = \frac{\bar{h}^2}{Z_\psi^2 Z_\phi}$$

- Yukawa coupling

$$\partial_t h^2(\phi) = -(\eta_\phi + 2\eta_\psi) + \frac{4h^4}{8\pi^2}\{(N_f^2 - 1)L_{1,1}^{(FB),(4)}(T,\mu,M_q^2,M_\pi^2;\eta_\psi,\eta_\phi) - L_{1,1}^{(FB),(4)}(T,\mu,M_q^2,M_\sigma^2;\eta_\psi,\eta_\phi)\}$$

$$L_{1,1}^{(FB),(d)}(T,\mu,M_q^2,M_B^2;\eta_\psi,\eta_\phi) = -\frac{T}{2}\sum_{n=-\infty}^{\infty}\int_0^\infty d|\boldsymbol{p}|^2\ |\boldsymbol{p}|^{d-3}\tilde{\partial}_t \times G_\psi\left((\omega_n + i\mu)^2, M_q^2\right) G_B\left(\omega_n^2, M_B^2\right)$$

$=$ 

$$\frac{\partial}{\partial\phi}h,\ \frac{\partial}{\partial\phi}Z_\psi,\ \frac{\partial}{\partial\phi}Z_\phi$$

fermion

$$G_\psi\left((\omega_n + i\mu)^2, M_q^2\right) = \frac{1}{(\omega_n + i\mu)^2 + |\boldsymbol{p}|^2(1 + r_\psi)^2 + M_q^2/Z_\psi^2}$$

$$\tilde{\partial}_t|_\psi = \left(\frac{\Lambda}{|\boldsymbol{p}|} - \eta_\psi\left(\frac{\Lambda}{|\boldsymbol{p}|} - 1\right)\right)\theta(1 - \boldsymbol{p}^2/\Lambda^2)\frac{\partial}{\partial r_\psi}$$

boson

$$G_B\left(\omega_n^2, M_B^2\right) = \frac{1}{\omega_n^2 + |\boldsymbol{p}|^2(1 + r_B) + M_B^2/Z_\phi}$$

$$\tilde{\partial}_t|_B = \left(2\frac{\Lambda^2}{\boldsymbol{p}^2} - \eta_\phi\left(\frac{\Lambda^2}{\boldsymbol{p}^2} - 1\right)\right)\theta(1 - \boldsymbol{p}^2/\Lambda^2)\frac{\partial}{\partial r_B}$$

NPRG flow equations

- Wave-function of fermion

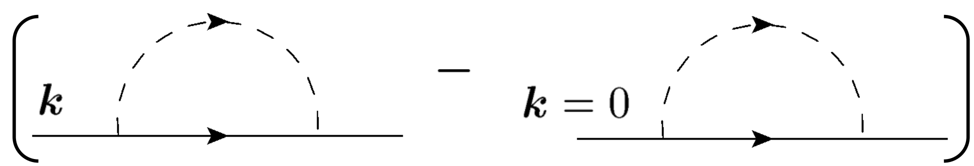
$$\eta_\psi = -\frac{\partial_t Z_\psi(\phi)}{Z_\psi(\phi)}$$

$$= \frac{4h^2}{3(8\pi^2)}\{\mathcal{M}_{1,2}^{(FB),(4)}(T,\mu,M_q^2,M_\sigma^2;\eta_\psi,\eta_\phi) + (N_\mathrm{f}^2-1)\mathcal{M}_{1,2}^{(FB),(4)}(T,\mu,M_q^2,M_\pi^2;\eta_\psi,\eta_\phi)\}$$

$$\mathcal{M}_{1,2}^{(FB),(d)}(T,\mu,M_q^2,M_B^2;\eta_\psi,\eta_\phi) = \frac{T}{2}\sum_{n=-\infty}^{\infty}\int_0^\infty d\boldsymbol{p}^2\ |\boldsymbol{p}|^{d-1}\tilde{\partial}_t$$

$$\times\left\{(1+r_\psi)G_\psi\left((\omega_n+i\mu)^2,M_q^2)\right)\frac{d}{d\boldsymbol{p}^2}G_B\left(\omega_n^2,M_B^2\right)\right\}$$

$$= \frac{d}{d\boldsymbol{k}}$$



NPRG flow equations

- Wave-function of boson

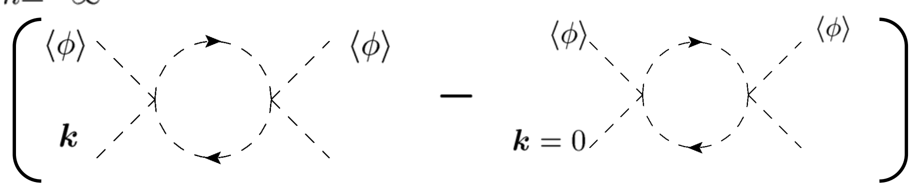
$$\eta_\phi = -\frac{\partial_t Z_\phi(\phi)}{Z_\phi(\phi)}$$

$$= \frac{16(\Omega'')^2\rho}{3(8\pi^2)Z_\phi^3}\{\mathcal{M}_{2,2}^{(B),(4)}(T, M_\sigma^2, M_\pi^2; \eta_\phi)\}$$

$$+ \frac{N_\mathrm{c}N_\mathrm{f}h^2}{8\pi^2}\left\{\frac{40}{9}\mathcal{M}_4^{(F),(4)}(T,\mu,M_2^2;\eta_\psi) + \frac{16}{3}M_q^2\mathcal{M}_2^{(F),(4)}(T,\mu,M_q^2;\eta_\psi)\right\}$$

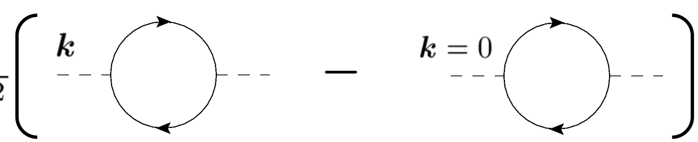
$$\mathcal{M}_{2,2}^{(B),(d)}(T, M_{B1}^2, M_{B2}^2; \eta_\phi) = -\frac{T}{2}\sum_{n=-\infty}^{\infty}\int_0^\infty d\boldsymbol{p}^2|\boldsymbol{p}|^{d-1}\tilde{\partial}_t\left(\frac{d}{d\boldsymbol{p}^2}G_B\left(\nu_n^2, M_{B1}^2\right)\right)\left(\frac{d}{d\boldsymbol{p}^2}G_B\left(\nu_n^2, M_{B2}^2\right)\right)$$

$$= \frac{d}{d\boldsymbol{k}^2}$$



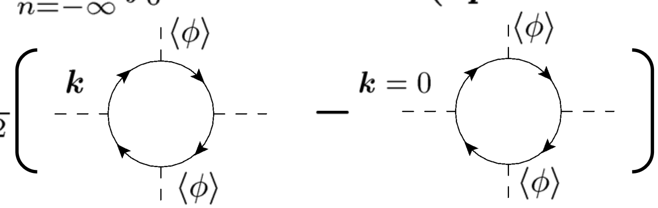
$$\mathcal{M}_2^{(F),(d)}(T,\mu,M_q^2;\eta_\psi) = -\frac{T}{2}\sum_{n=-\infty}^{\infty}\int_0^\infty d\boldsymbol{p}^2|\boldsymbol{p}|^{d-1}\tilde{\partial}_t\left(\frac{d}{d\boldsymbol{p}^2}G_\psi\left((\omega_n+i\mu)^2, M_q^2\right)\right)^2$$

$$= \frac{d}{d\boldsymbol{k}^2}$$



$$\mathcal{M}_4^{(F),(d)}(T,\mu,M_q^2;\eta_\psi) = -\frac{T}{2}\sum_{n=-\infty}^{\infty}\int_0^\infty d\boldsymbol{p}^2|\boldsymbol{p}|^{d+1}\tilde{\partial}_t\left(\frac{d}{d\boldsymbol{p}^2}(1+r_\psi)G_\psi\left((\omega_n+i\mu)^2, M_q^2\right)\right)^2$$

$$= \frac{d}{d\boldsymbol{k}^2}$$



QM model and NPRG

- NPRG eq. of the scale-dependent grand canonical thermodynamic potential $\Omega(T,\mu;\phi)$

$$-\Lambda\frac{\partial}{\partial\Lambda}\Omega_\Lambda(T,\mu;\phi) = -\frac{\Lambda^5}{12\pi^2}\left[\frac{3}{E_\pi}\coth\left(\frac{E_\pi}{2T}\right) + \frac{1}{E_\sigma}\coth\left(\frac{E_\sigma}{2T}\right) - \frac{2N_\mathrm{c}N_\mathrm{f}}{E_q}\left\{\tanh\left(\frac{E_q-\mu}{2T}\right) + \tanh\left(\frac{E_q+\mu}{2T}\right)\right\}\right]$$

$$E_i = \sqrt{\Lambda^2 + M_i^2} \qquad i = q, \sigma, \pi$$

$$M_q^2 = h^2\phi^2 \quad M_\sigma = 2\frac{\partial\Omega_\Lambda}{\partial\phi^2} + 4\phi^2\frac{\partial^2\Omega_\Lambda}{\partial(\phi^2)^2} \quad M_\pi = 2\frac{\partial\Omega_\Lambda}{\partial\phi^2}$$

- Initial conditions

$$U_{\Lambda_0}(\phi^2) = \frac{\lambda}{4}(\phi^2)^2$$

$$\Lambda_0 = 500\ \mathrm{MeV} \qquad \lambda = 10 \qquad h = 3.2$$