Quark-Hadron Phase Transition in the PNJL model for interacting quarks

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in collaboration with **Tetsuo Matsui**

1. Motivations : quark-hadron transition

2. 2-flavor PNJL model

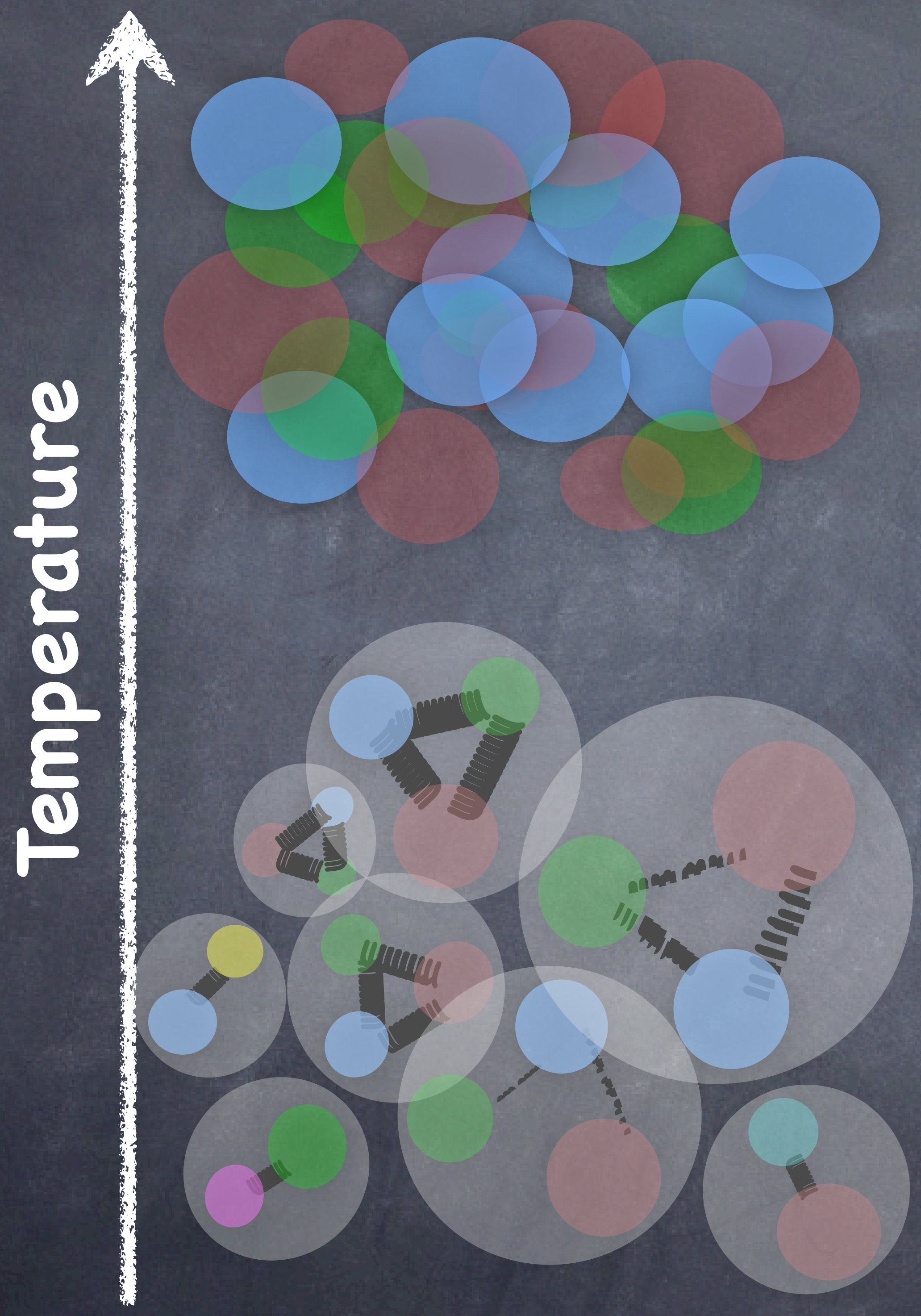
   KY and T. Matsui, Nucl. Phys. A913 (2013) 19.

3. 3-flavor PNJL model

   KY and T. Matsui, arXiv:1310.4960.

4. Numerical results

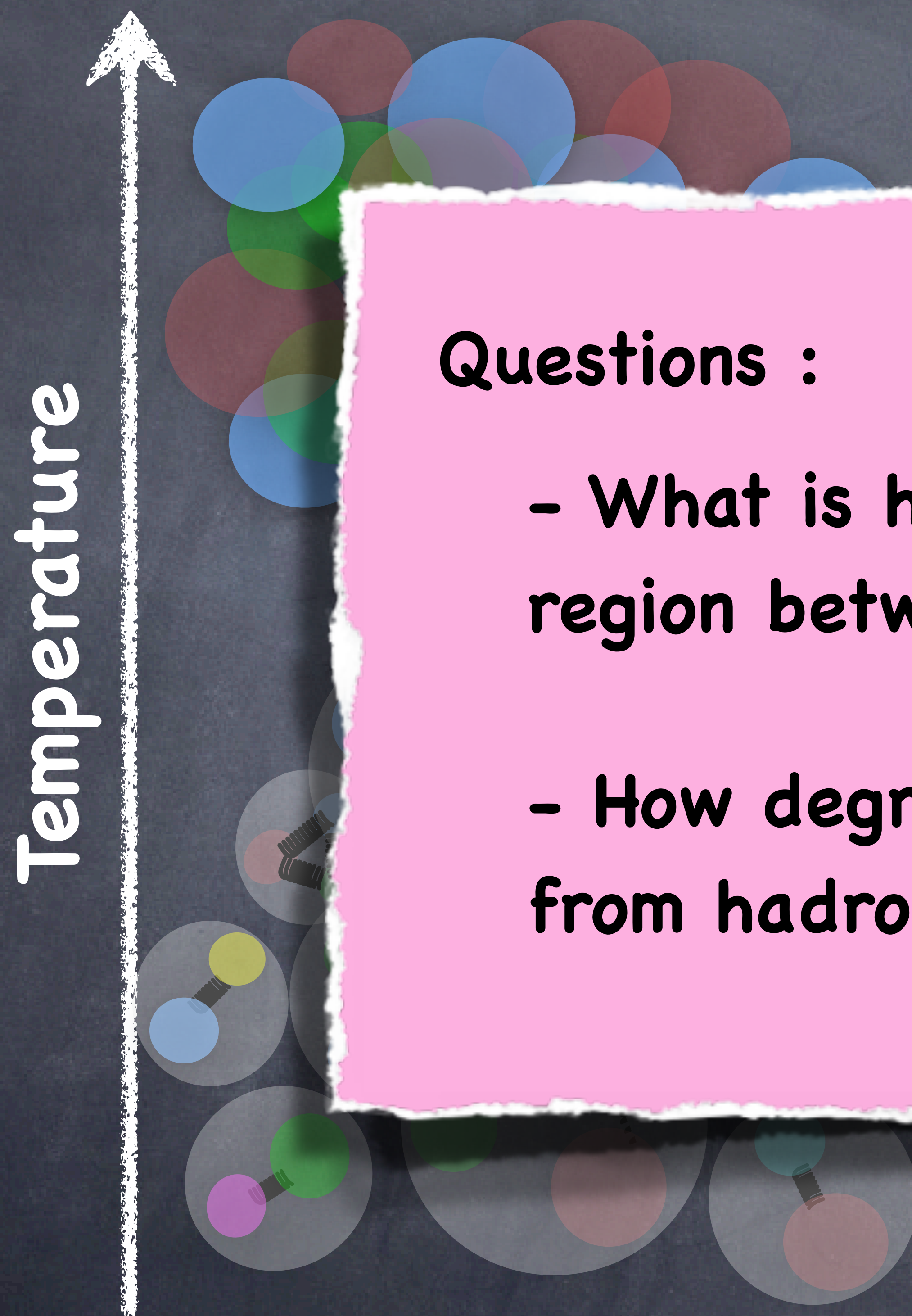
Quark-Hadron Phase Transition



- Chiral symmetry **restoration**
- Color **de-confinement**

- Chiral symmetry **breaking**
- Color **confinement**

Quark-Hadron Phase Transition



- Chiral symmetry restoration

Questions :

- What is happening in intermediate region between two phases ?

- How degrees of freedom change from hadrons to quarks ?

Method

- Calculating partition function in path integral method
- Model choosing
  - Chiral phase transition

    → Nambu-Jona-Lasinio (NJL)

    Y. Nambu, G. Jona-Lasinio, 1961

    T. Hatsuda, T. Kunihiro, 1994
  - De-confining phase transition

    → Polyakov loop

    A. M. Polyakov, 1978

  } PNJL model

  K. Fukushima, 2004
- Bosonization
  - inserting dummy integrals
  - 4- and 6-point interactions --> bosonic fields
- Mean field approximation + Mesonic correlations

2-flavor PNJL model

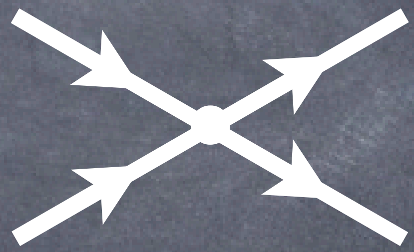
Model setup

Partition function

$$Z(T, A_4) = \int [dq][d\bar{q}] \exp\left[\int_0^{\beta} d\tau \int d^3x \mathcal{L}_{\mathrm{NJL}}(q, \bar{q}, A_4)\right]$$

$$\mathcal{L}_{\mathrm{NJL}}(q, \bar{q}, A_4) = \bar{q}(i\gamma^{\mu} D_{\mu} - m_0)q + G\left[(\bar{q}q)^2 + (i\bar{q}\gamma_5\tau q)^2\right]$$

$$D_{\mu} = \partial_{\mu} + g\boxed{A_0}\delta_{\mu,0} \quad , \quad A_4 = iA_0$$



- m_0 : bare quark mass
  breaks chiral symmetry explicitly

- A_4 : temporal component of gauge field
  treated as external field

Bosonization

Hubbard-Stratonovich transformation

J. Hubbard, 1956
R. L. Stratonovich, 1957

- introducing auxiliary bosonic fields : $\phi_i = (\sigma, \boldsymbol{\pi})$

- eliminating 4 point interaction by inserting dummy integral over boson fields

$$Z(T, A_4) = \int [dq][d\bar{q}][d\phi] \exp\left[\int_0^\beta d\tau \int d^3x \mathcal{L}_{\text{eff}}(q, \bar{q}, \phi, A_4)\right]$$

$$\mathcal{L}_{\text{eff}}(q, \bar{q}, \phi\,, A_4) = \bar{q}\left[i\gamma^\mu D_\mu + \sigma\, + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}\right] q - \frac{1}{4G}((\sigma\, - m_0)^2 + \pi_i^2).$$

four fermi interaction --> Yukawa interaction

Thermodynamic potential

- integrating over **fermion fields**

$$Z(T, A_4) = \int [d\phi] e^{-I(\phi, A_4)}$$

- expand effective action up to second order of fluctuation

$$Z(T, A_4) \simeq e^{-I_0} \int [d\varphi] \exp\left[-\frac{1}{2} \frac{\delta^2 I}{\delta\phi_i \delta\phi_j}\bigg|_{\phi = \phi_0} \varphi_i \varphi_j \right]$$

$$\varphi_i = \phi_i - \phi_0$$

determined by **stationary condition** :

$$\frac{\delta I}{\delta \phi'_i}\bigg|_{\phi' = \phi'_0} = 0.$$

$$\Omega(T, A_4) = T \left(I_0 + \frac{1}{2} \mathrm{Tr_M} \ln \frac{\delta I}{\delta\phi_i \delta\phi_j} \right)$$

$$\boxed{\begin{aligned} \sigma_0 &= -M_0 \\ \pi_0 &= 0 \end{aligned}}$$

contribution from **mean field** contribution from **mesonic excitations**

Mean field approximation

$$\Omega_{\rm MF}(T, A_4) = T I_0 = -p_{MF} V.$$

- Taking statistical average over the external color gauge field A4

- Replacing A4 with the thermal average of Polyakov loop Φ

$$p_{\rm MF}(T, \Phi, M_0) = p^0_{\rm MF}(M_0) - \Delta p_{\rm vac} + 4 \times 3 \times \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_p} f_\Phi(E_p) \boxed{- \mathcal{U}(T, \Phi)}$$

effective potential of Φ

represent dynamics of gluon phenomenologically

$$\mathcal{U}(T, \Phi)/T^4 = -\frac{1}{2} b_2(T) \bar{\Phi}\Phi - \frac{1}{6} b_3(\Phi^3 + \bar{\Phi}^3) + \frac{1}{4} b_4(\bar{\Phi}\Phi)^2$$

Mean field approximation

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modified quark distribution function

$$f_\Phi(E_p) = \frac{\bar{\Phi} e^{2\beta E_p} + 2\Phi e^{\beta E_p} + 1}{e^{3\beta E_p} + 3\bar{\Phi} e^{2\beta E_p} + 3\Phi e^{\beta E_p} + 1}$$

$$\Phi = \frac{1}{3}\langle {\rm tr}_c L \rangle, \qquad \bar{\Phi} = \frac{1}{3}\langle {\rm tr}_c L^\dagger \rangle$$

Quark distribution function

Two extreme cases

- De-confining phase

$$\Phi = \bar{\Phi} = 1 \quad \Rightarrow \quad f_\Phi(E_p)|_{\Phi=1} = \frac{1}{e^{\beta E_p} + 1}$$

quark distribution function

- Confining phase

$$\Phi = \bar{\Phi} = 0 \quad \Rightarrow \quad f_\Phi(E_p)|_{\Phi=0} = \frac{1}{e^{3\beta E_p} + 1}$$

triad three quark distribution function

$$E_p = \sqrt{p^2 + M_0^2}$$

M_0 is determined by Gap equation

Gap equation

- M_0 is determined by stationary condition

$$\left.\frac{\delta I}{\delta \phi'_i}\right|_{\phi'=\phi'_0} = 0.$$

$$\boxed{M_0} - m_0 = 8GN_f \sum_n \int \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{\boxed{M_0}}{(\epsilon_n - gA_4)^2 + p^2 + \boxed{M_0^2}}$$

self-consistent equation : **Gap equation**

M_0 : **constituent quark mass**

m_0 : **bare quark mass**

Mesonic Correlations

- Contribution of mesonic correlations

$$p_M = -\sum_n \int \frac{d^3q}{(2\pi)^3}\left\{3\ln\mathcal{M}_\pi(\omega_n,q) + \ln\mathcal{M}_\sigma(\omega_n,q)\right\}$$

- **chiral limit** : $m_0=0$

$$\mathcal{M}_\pi = \underline{(\omega_n^2+q^2)}\,\underline{F(\omega_n,q)}\ , \quad \mathcal{M}_\sigma = \underline{(\omega_n^2+q^2+4M_0^2)}\,\underline{F(\omega_n,q)}$$

$$p_M = \boxed{p_\pi^{\rm free} + p_\sigma^{\rm free}} + \boxed{p_{non-coll.}}$$

- **breaking chiral symmetry** with $m_0 \neq 0$

$$\mathcal{M}_\pi = (\omega_n^2+q^2)F(\omega_n,q) + \frac{m_0}{2GM_0}\ , \quad \mathcal{M}_\sigma = (\omega_n^2+q^2+4M_0^2)F(\omega_n,q) + \frac{m_0}{2GM_0}$$

3-flavor PNJL model

3-flavor PNJL model

Partition function

$$Z(T, A_4) = \int [dq][d\bar{q}] \exp\left[\int_0^\beta d\tau \int d^3x \mathcal{L}_{\rm NJL}(q, \bar{q}, A_4)\right]$$

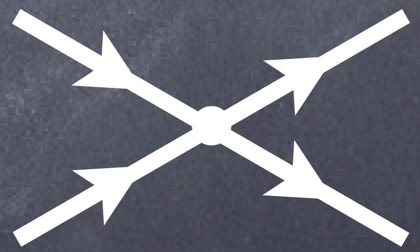
$$\mathcal{L}_{NJL} = \sum_{i,j=1}^{3} \bar{q}_i (i \not{D} - \hat{m})_{i,j} q_j + \boxed{\mathcal{L}_4} + \boxed{\mathcal{L}_6}$$

$$D_\mu = \partial_\mu + g \boxed{A_0} \delta_{\mu,0}$$

- 4 point interaction

$$\mathcal{L}_4 = G \sum_{a=0}^{8} \left[(\bar{q}\lambda^a q)^2 + (\bar{q} i \gamma_5 \lambda^a q)^2 \right]$$

$a = 0 \sim 8$



- 6 point interaction

U(1)$_A$ breaking

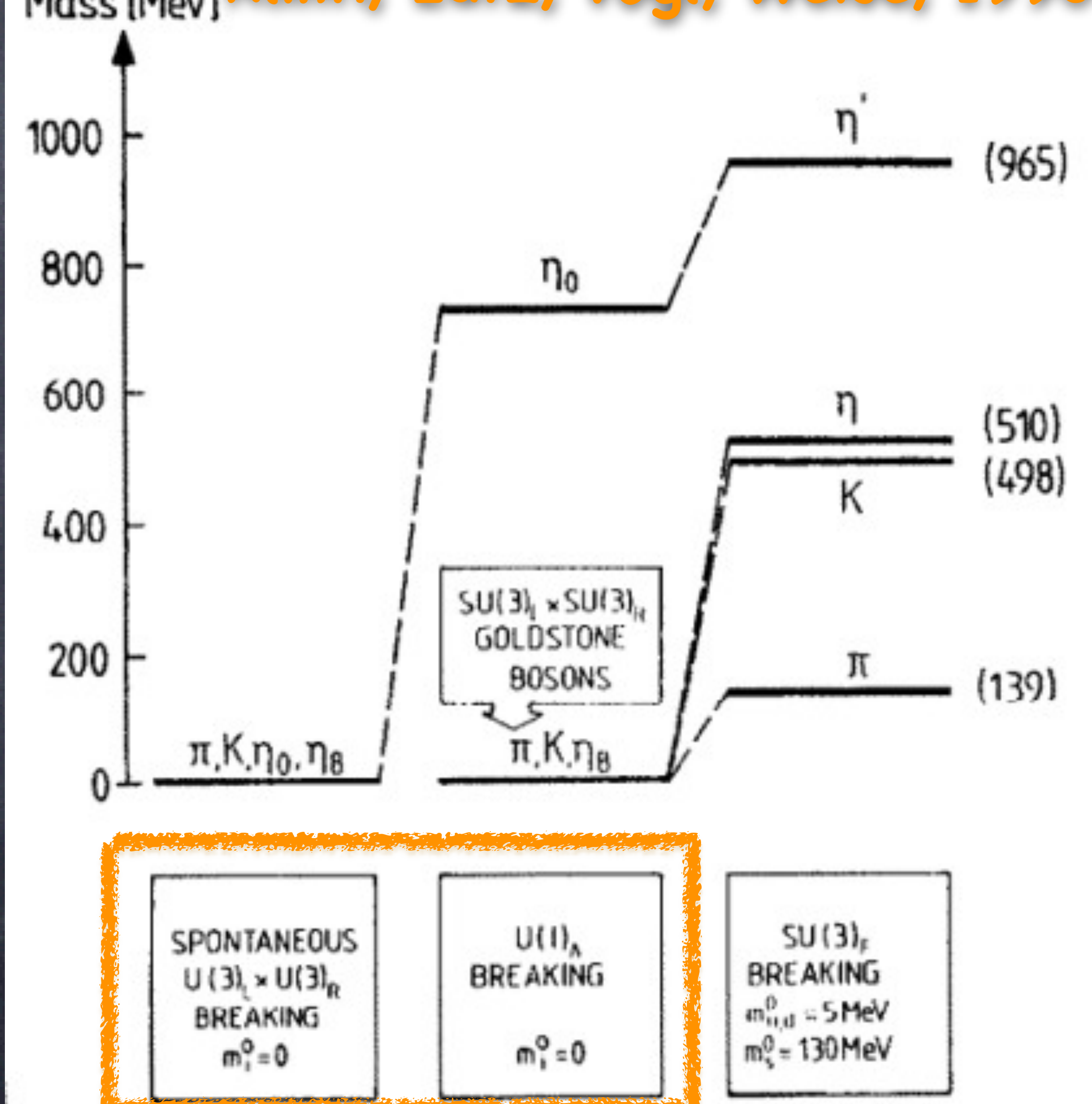
$$\mathcal{L}_6 = -K \left[\det \bar{q}(1 + \gamma_5) q + \det \bar{q}(1 - \gamma_5) q \right]$$

Meson nonets

Pseudo scalar meson

π, K, η, η'

Klimt, Lutz, Vogl, Weise, 1990



$\mathcal{L}_6$ plays a role in **mass splitting**

Scalar meson

σ, κ, f0, a0

Ishida, 1998

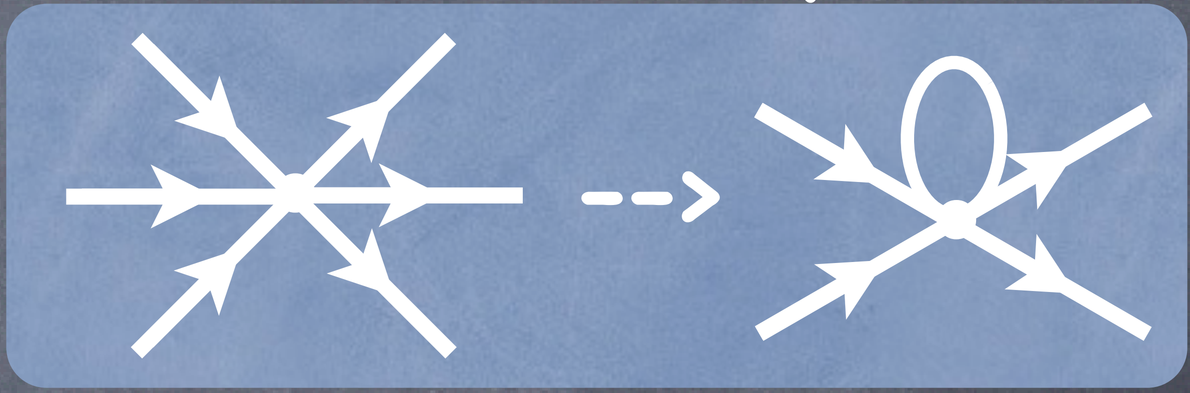
Fariborz, Jora, Schechter, 2009

	Mass [MeV]	Width [MeV]
σ	~550	400 - 700
κ	~800	
$f_0(980)$	~980	40 - 100
$a_0(980)$	~980	50 - 100

Rewrite P.F. by auxiliary fields

- Lagrangian contains **4th power** and **6th power** of fermion fields.

- 6th power can be effectively rewritten to 4th power by **replacing with condensate**.



- eliminating 4point interactions to bosonic fields : ϕ^a, π^a

- We get partition function **as a function of auxiliary bosonic fields** :

$$Z(T, A_4) = \int [d\phi][d\pi] \exp\left[-I_{eff}(\phi^a, \pi^a, A_4)\right]$$

Thermodynamic potential

- Expanding effective action up to **second order of fluctuation** around **stationary point**

- Stationary point is determined by **stationary condition** :

$$\left.\frac{\delta I}{\delta \phi_a}\right|_{\phi=\phi_0} = 0.$$

- Performing **Gaussian integrals** over bosonic fields

- Thermodynamic potential

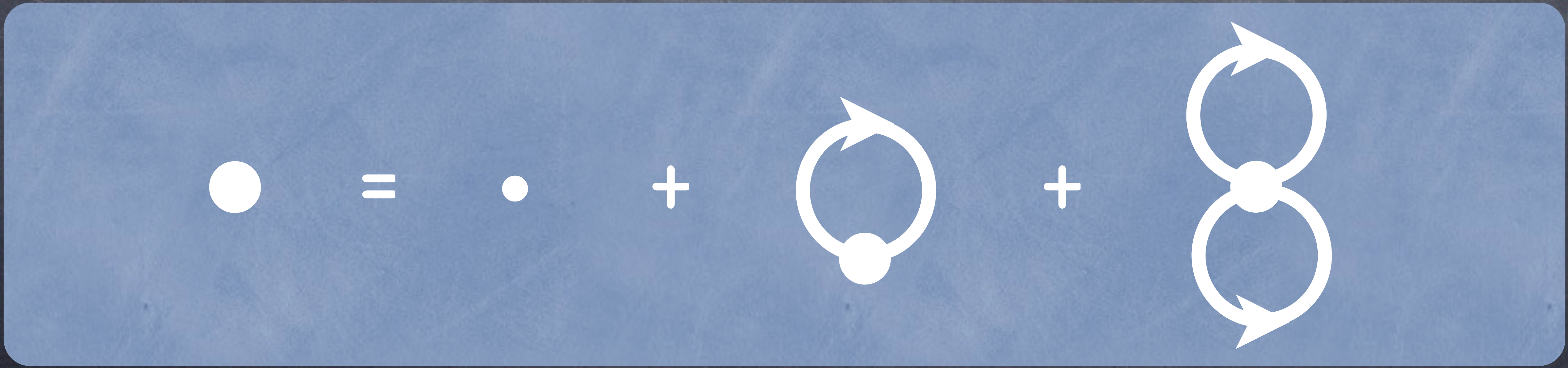
$$\Omega(T, A_4) = T\Big(\underbrace{I_0}_{\text{mean field}} + \underbrace{\frac{1}{2}\mathrm{Tr}_M \ln \frac{\delta^2 I}{\delta\phi_a \delta\phi_b} + \frac{1}{2}\mathrm{Tr}_M \ln \frac{\delta^2 I}{\delta\pi_a \delta\pi_b}}_{\text{mesonic excitations}}\Big)$$

Constituent quark mass

- Pressure depends on **constituent quark masses**.

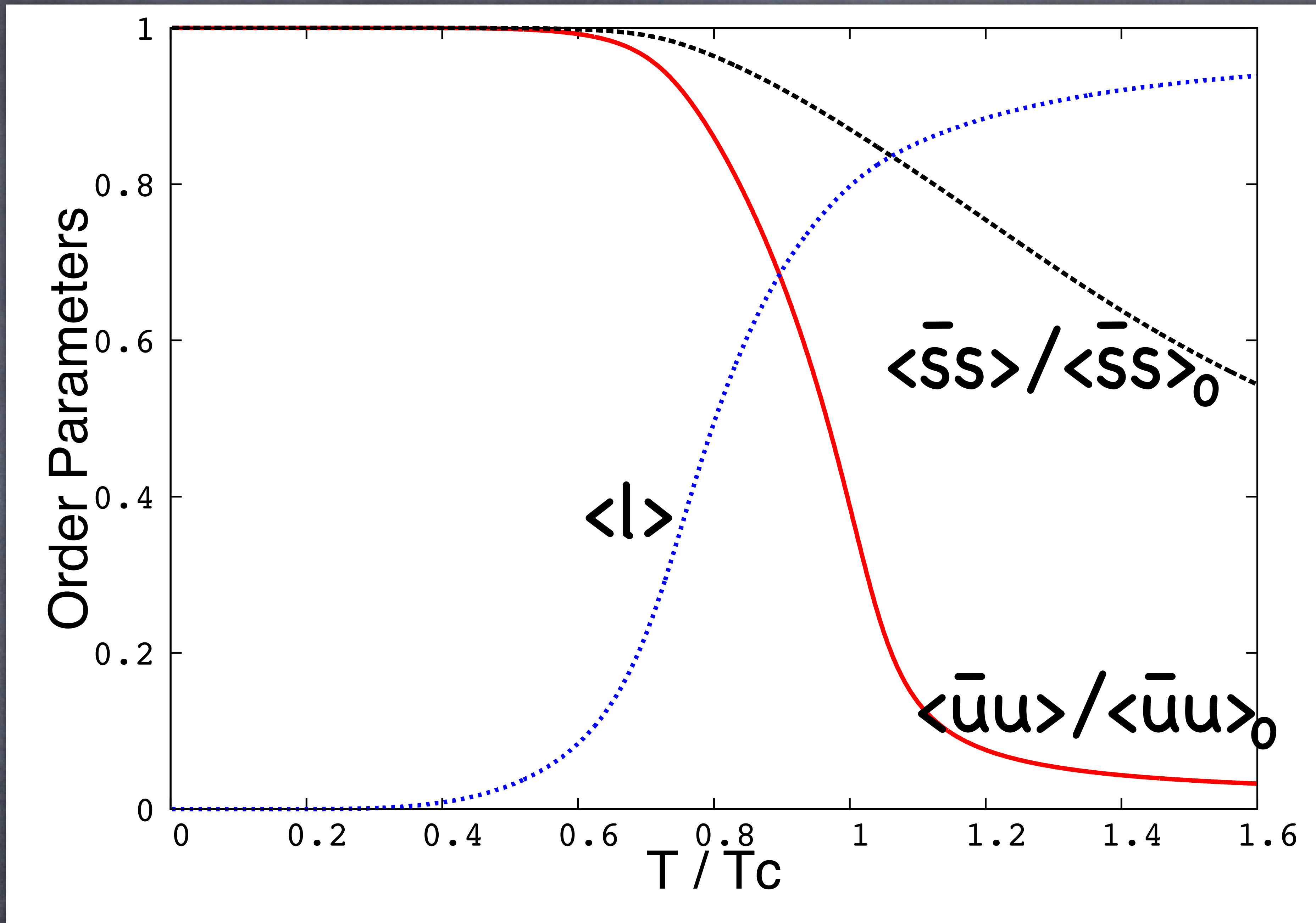
- Constituent quark masses are determined by solving **gap equations** :

$$\begin{cases} M_u = m_u - 4G\langle \bar{u}u \rangle + 2K\langle \bar{d}d \rangle \langle \bar{s}s \rangle \\ M_s = m_s - 4G\langle \bar{s}s \rangle + 2K\langle \bar{u}u \rangle \langle \bar{d}d \rangle \end{cases}$$



- Chiral condensates : $\langle \bar{u}u \rangle (= \langle \bar{d}d \rangle), \langle \bar{s}s \rangle$

Order Parameters



<l> : Expectation value of Polyakov loop

Tc : pseudo critical temperature

Tc $\sim$ 220MeV

Pressure

- mean field

$$p_{MF}(T) = \sum_f p^0_{M_f} + 4N_c \sum_f \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_f} f_{\langle l \rangle}(E_f) - \mathcal{U}(T)$$

- mesonic correlations

$$p_M = -\sum_n \int \frac{d^3q}{(2\pi)^3} \Big\{ 3\ln\mathcal{M}_\pi(\omega_n, q) + 4\ln\mathcal{M}_K(\omega_n, q) + \ln\mathcal{M}_\eta(\omega_n, q) + \ln\mathcal{M}_{\eta'}(\omega_n, q)$$
$$+\ln\mathcal{M}_\sigma(\omega_n, q) + 4\ln\mathcal{M}_\kappa(\omega_n, q) + 3\ln\mathcal{M}_{a_0}(\omega_n, q) + \ln\mathcal{M}_{f_0}(\omega_n, q) \Big\}$$

$$\mathcal{M}_\alpha(\omega_n, q) = \frac{1}{2G'_\alpha} - \Pi_\alpha(\omega_n, q)$$

$\Pi(\omega_n, q)$ =

Pressure

- mean field

$$p_{MF}(T) = \sum_f p^0_{M_f} + 4N_c \sum_f \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_f} f_{\langle l \rangle}(E_f) - \mathcal{U}(T)$$

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Pseudo scalar

$$\mathcal{M}_\alpha(\omega_n,q) = \frac{1}{2G'_\alpha} - \Pi_\alpha(\omega_n,q)$$

$\Pi(\omega_n,q) =$

Pressure

- mean field

$$p_{MF}(T) = \sum_f p^0_{M_f} + 4N_c \sum_f \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_f} f_{\langle l \rangle}(E_f) - \mathcal{U}(T)$$

- mesonic correlations

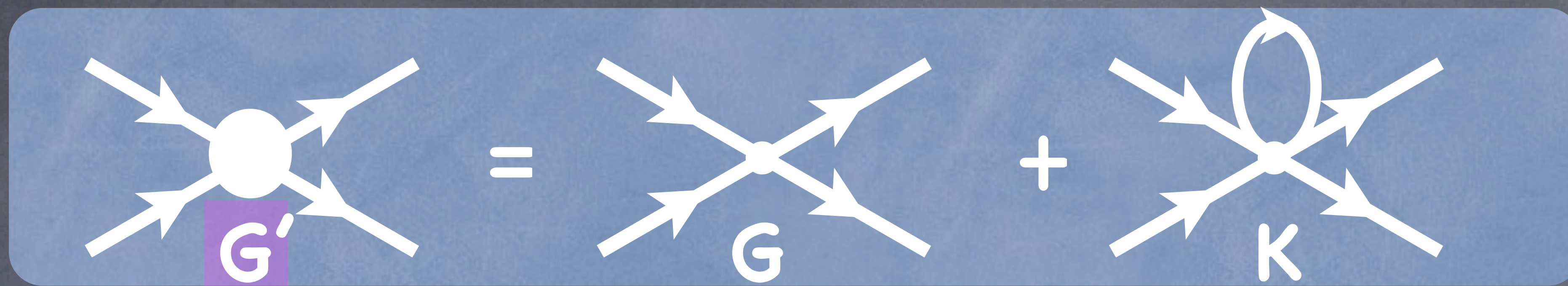
Scalar

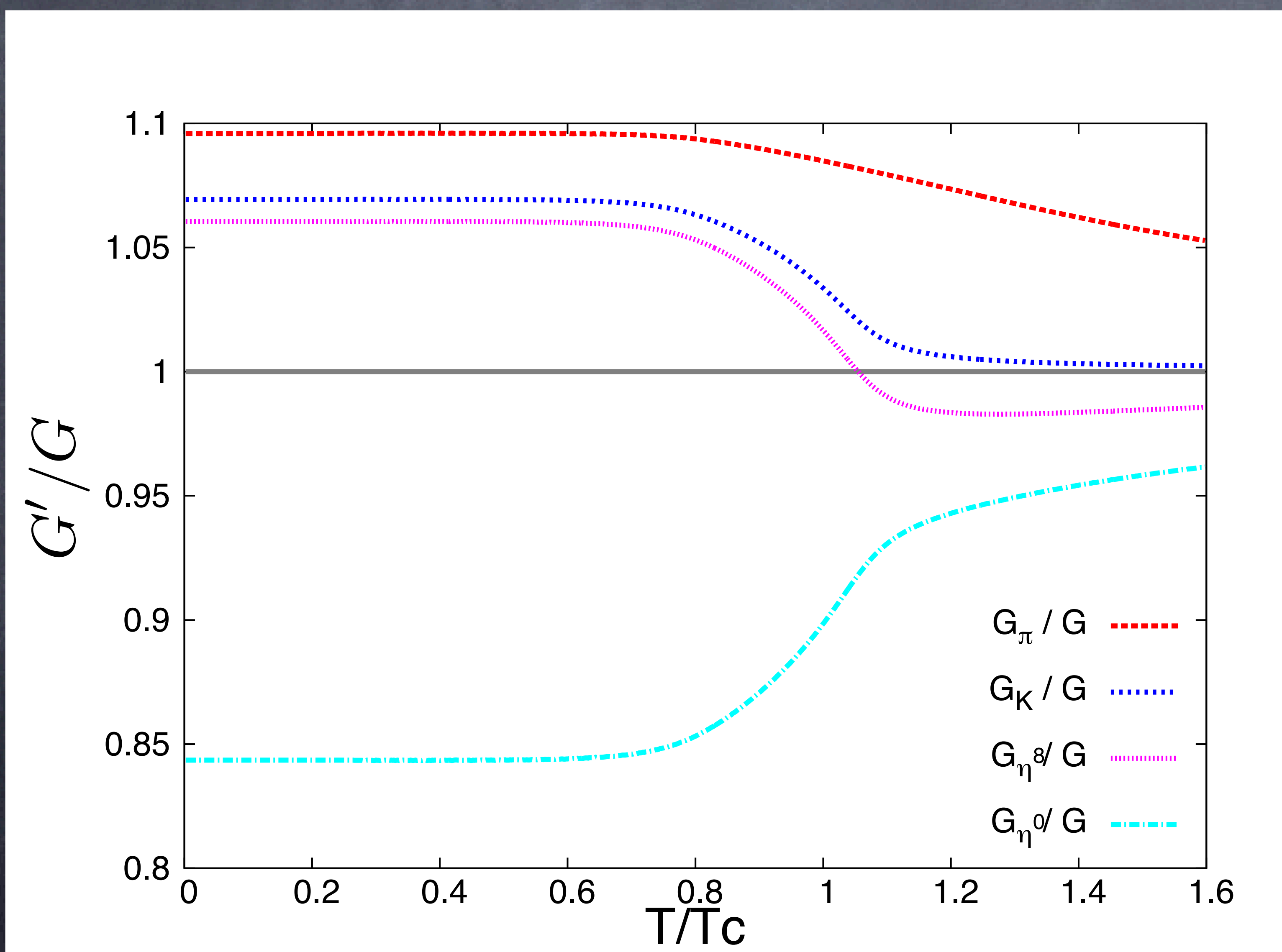
$$p_M = -\sum_n \int \frac{d^3q}{(2\pi)^3} \Big\{ 3\ln\mathcal{M}_\pi(\omega_n, q) + 4\ln\mathcal{M}_K(\omega_n, q) + \ln\mathcal{M}_\eta(\omega_n, q) + \ln\mathcal{M}_{\eta'}(\omega_n, q)$$
$$+ \ln\mathcal{M}_\sigma(\omega_n, q) + 4\ln\mathcal{M}_\kappa(\omega_n, q) + 3\ln\mathcal{M}_{a_0}(\omega_n, q) + \ln\mathcal{M}_{f_0}(\omega_n, q) \Big\}$$

$$\mathcal{M}_\alpha(\omega_n, q) = \frac{1}{2G'_\alpha} - \Pi_\alpha(\omega_n, q)$$

$\Pi(\omega_n, q) =$

Effective coupling G'





$$G'_\pi = G - \frac{K}{2}\langle\bar{s}s\rangle$$

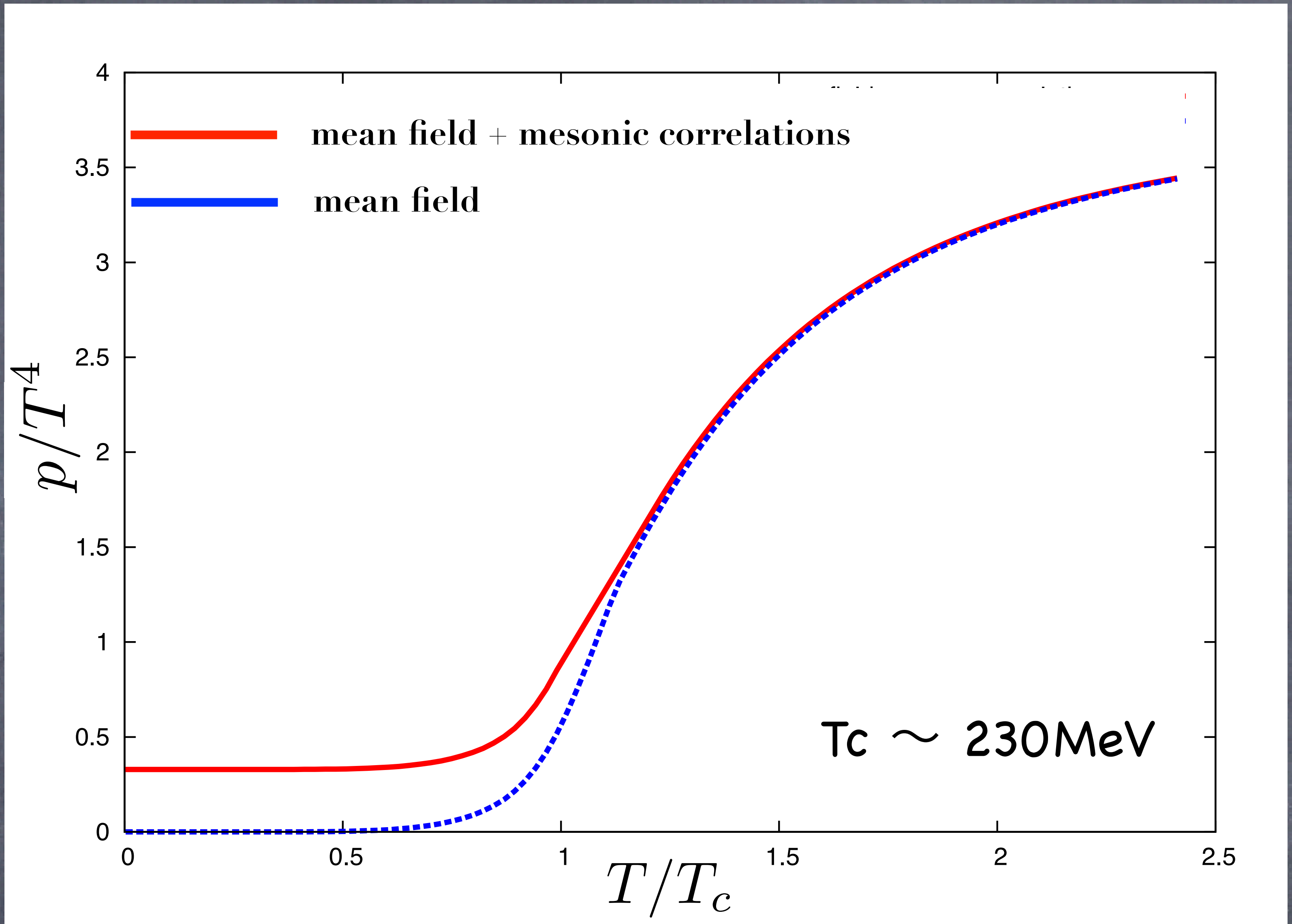
$$G'_K = G - \frac{K}{2}\langle\bar{u}u\rangle$$

$$G'_{\eta^8} = G + \frac{K}{6}\Big(\langle\bar{s}s\rangle - 4\langle\bar{u}u\rangle\Big)$$

$$G'_{\eta^0} = G + \frac{K}{3}\Big(\langle\bar{s}s\rangle + 2\langle\bar{u}u\rangle\Big)$$

Numerical Results

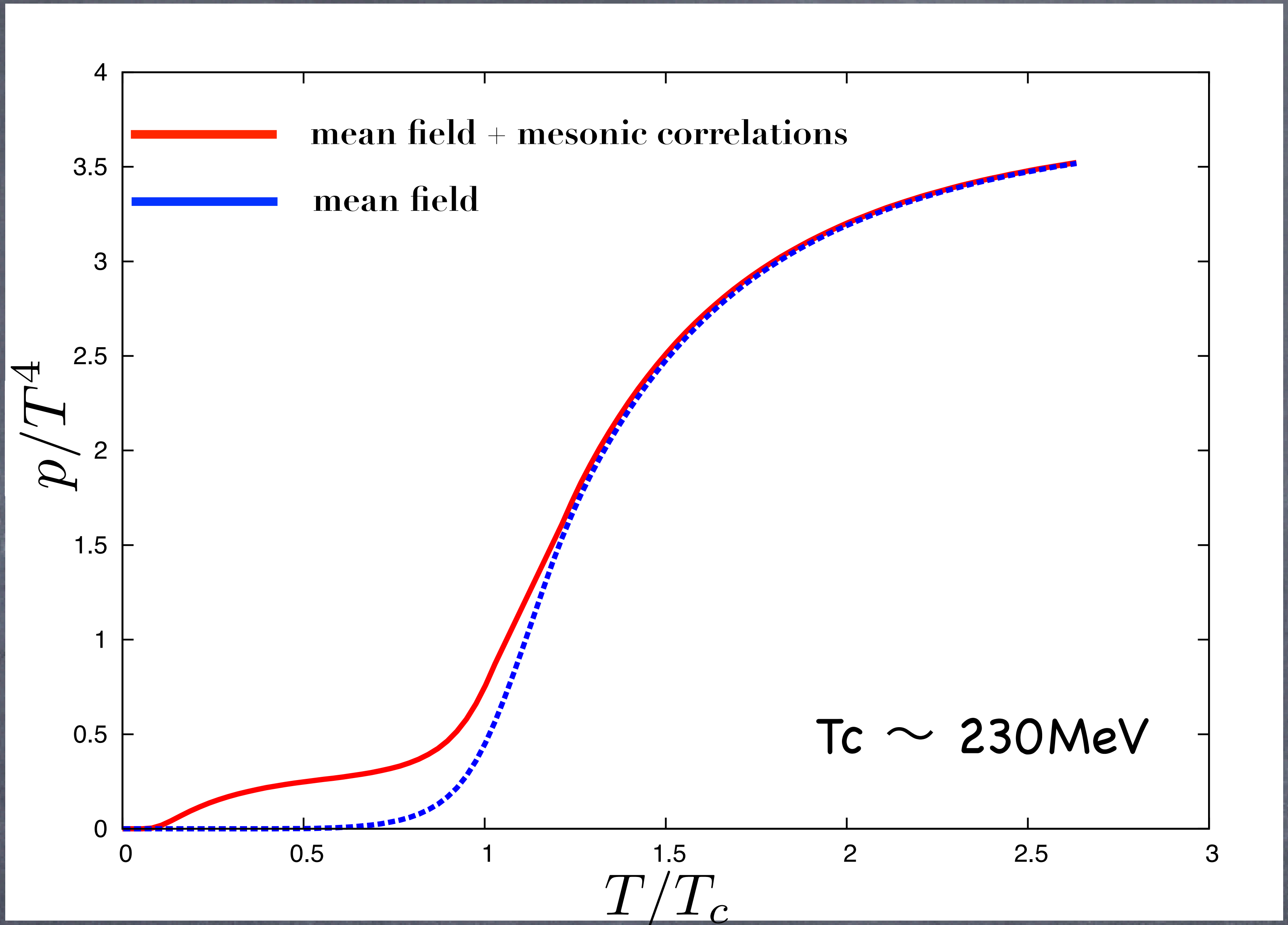
Pressure in the chiral limit (2-flavor)



- At low T, pressure from quark excitation is suppressed by Polyakov loop.

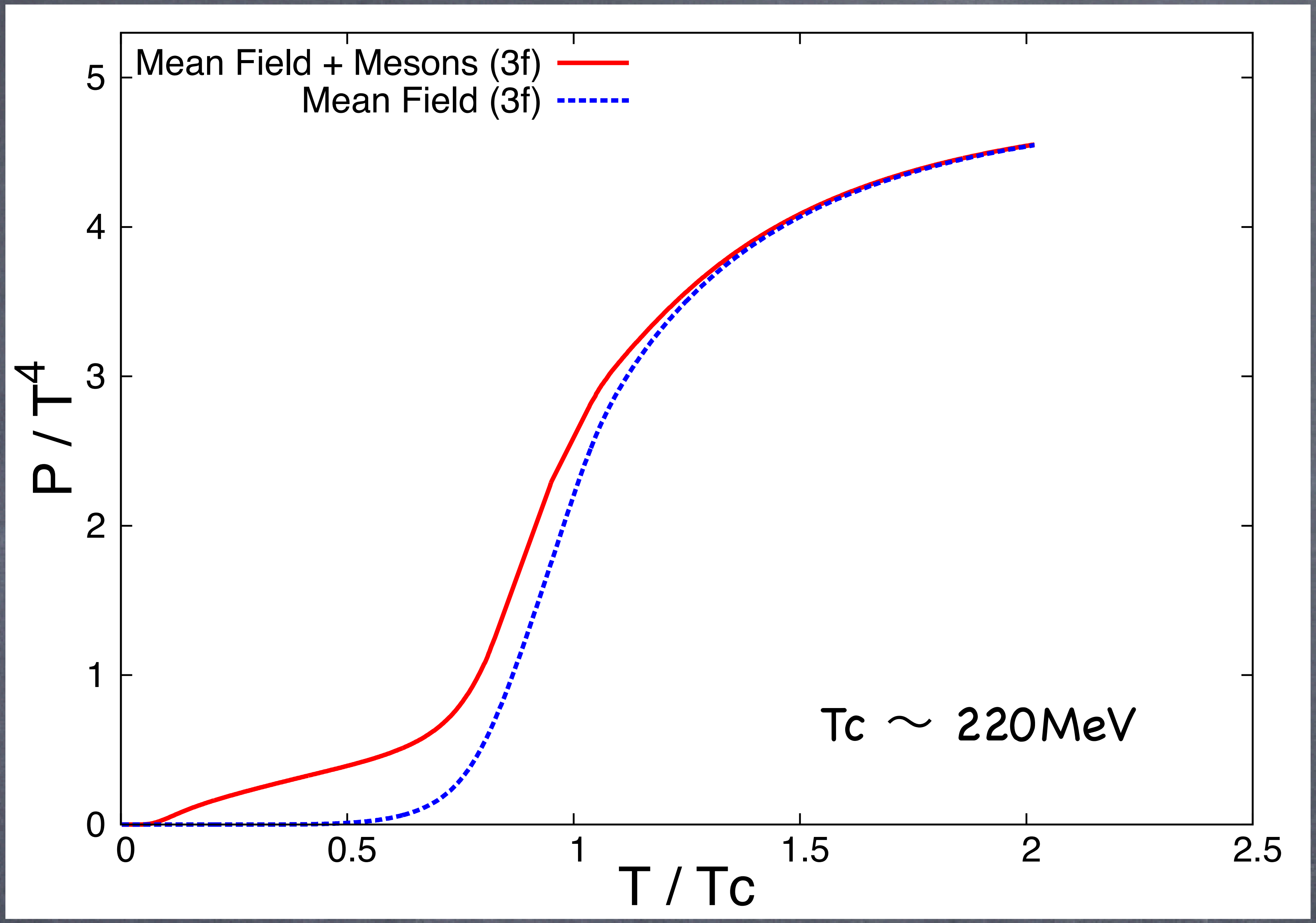
- At high T, massless quark excitations dominate the pressure.

Pressure with finite bare quark mass (2-f)



- At **low T**, pressure from quark excitation is suppressed by **Polyakov loop**.
- At **high T, massless quark** excitations dominate pressure.

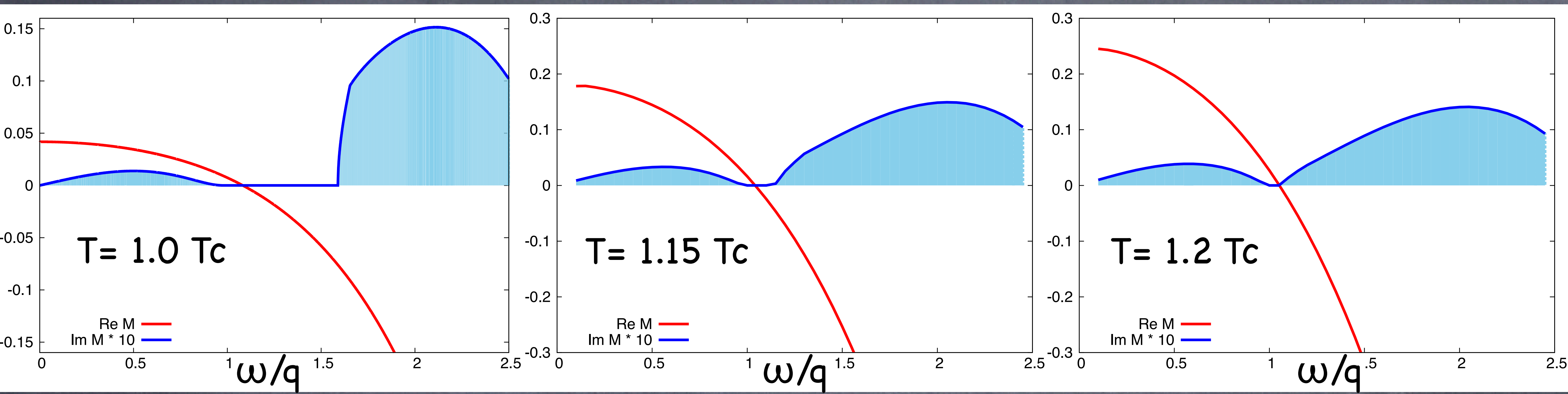
Pressure (3-flavor)



- π, K and σ are taken into this calculation.
- At low T, pion and kaon dominate pressure.
- At high T, pressure approaches to quark mean field pressure.

Collective modes

Pion



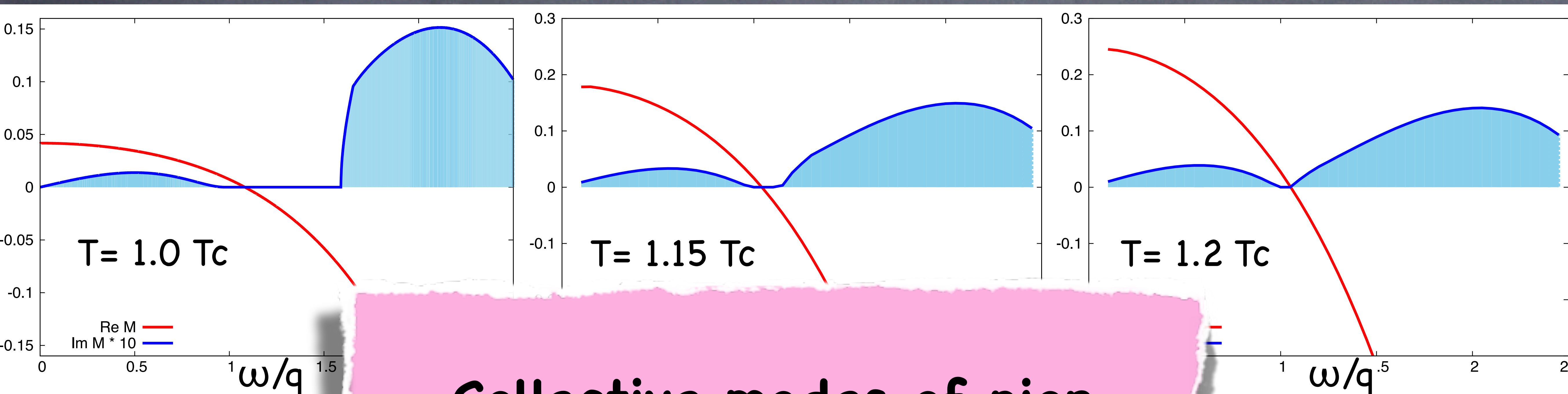
Real part of M

Imaginary part of M

$$\mathcal{M}(\omega, q) = \frac{1}{2G'} - \Pi(\omega, q)$$

Collective modes

Pion



- Collective modes of pion disappear at T=1.2Tc

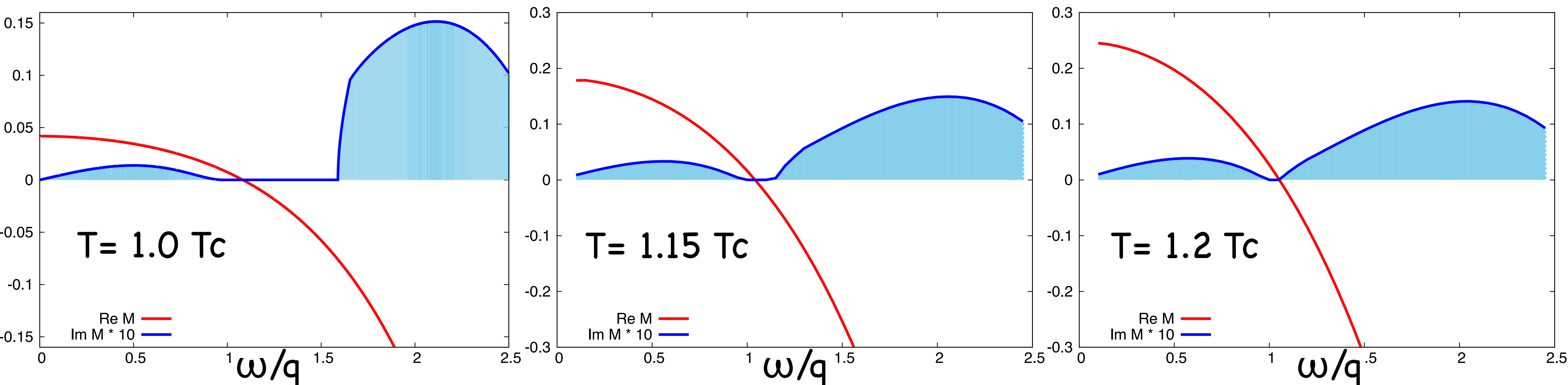
Real

Imag

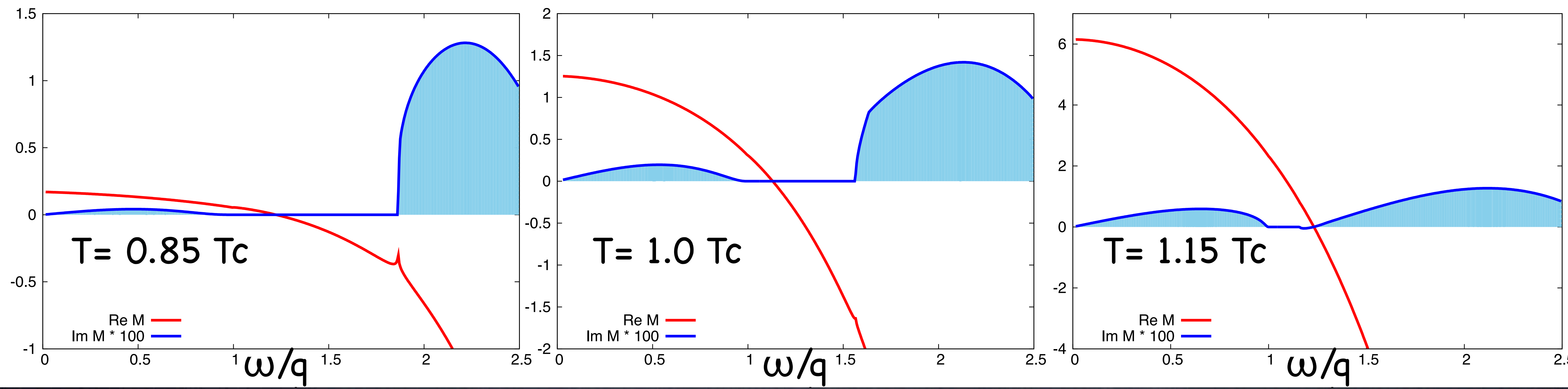
$\Pi(\omega, q)$

Collective modes

Pion

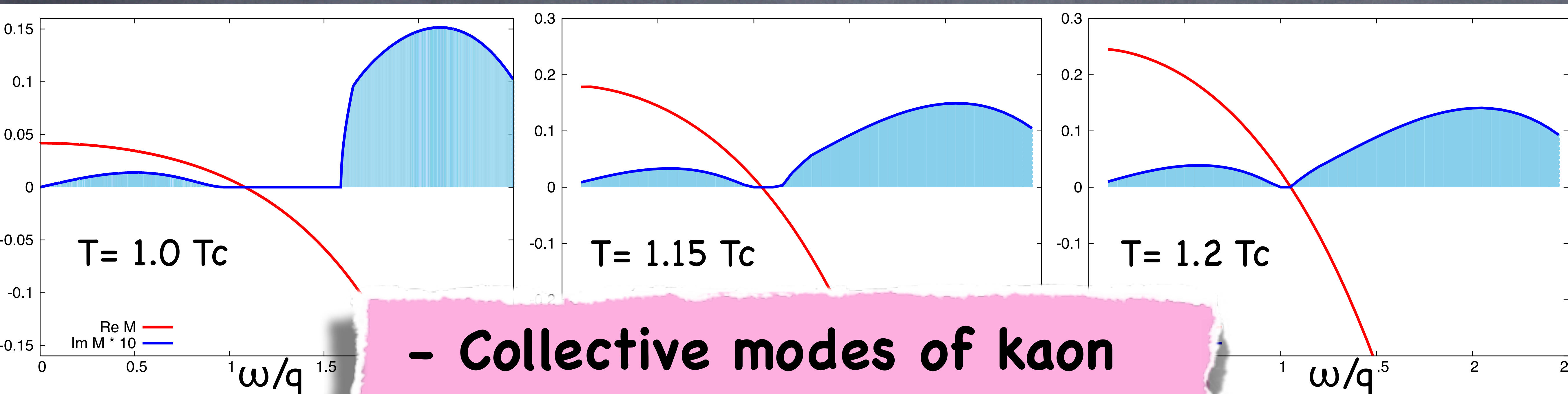


Kaon



Collective modes

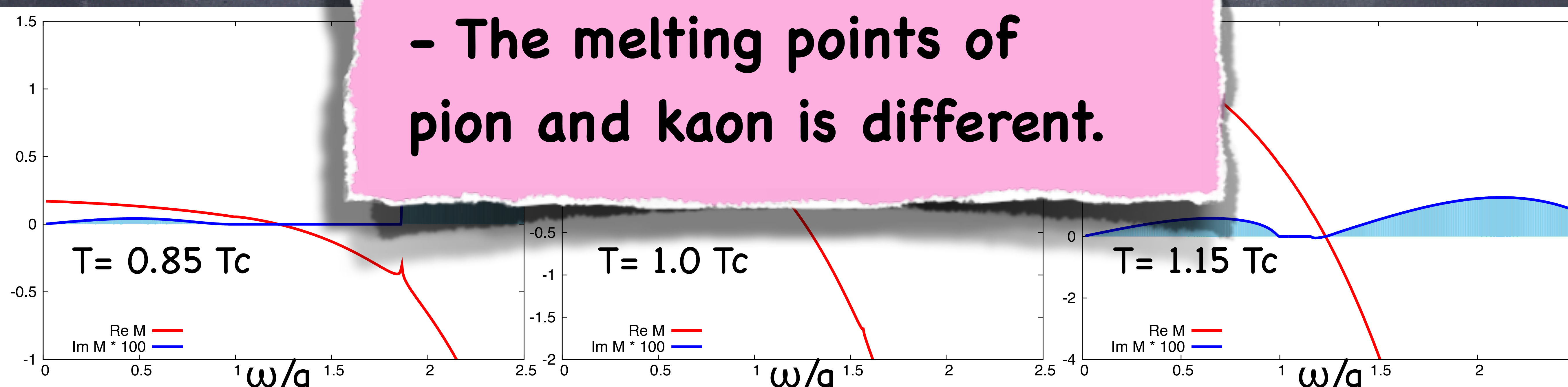
Pion



- Collective modes of kaon disappear at T=1.15 Tc

- The melting points of pion and kaon is different.

Kaon



Summary

Summary and Outlook

- We have described a quark-hadron phase transition from interacting quarks.

- Collective modes of mesons at low T melt with increasing temperature, and resolve to quarks.

- Melting temperature of pion is higher than that of kaon.

- How to put baryonic correlation at finite chemical potential.