Anomalous hydrodynamic simulation and charge-dependent elliptic flow

[arXiv: 1309.2823]

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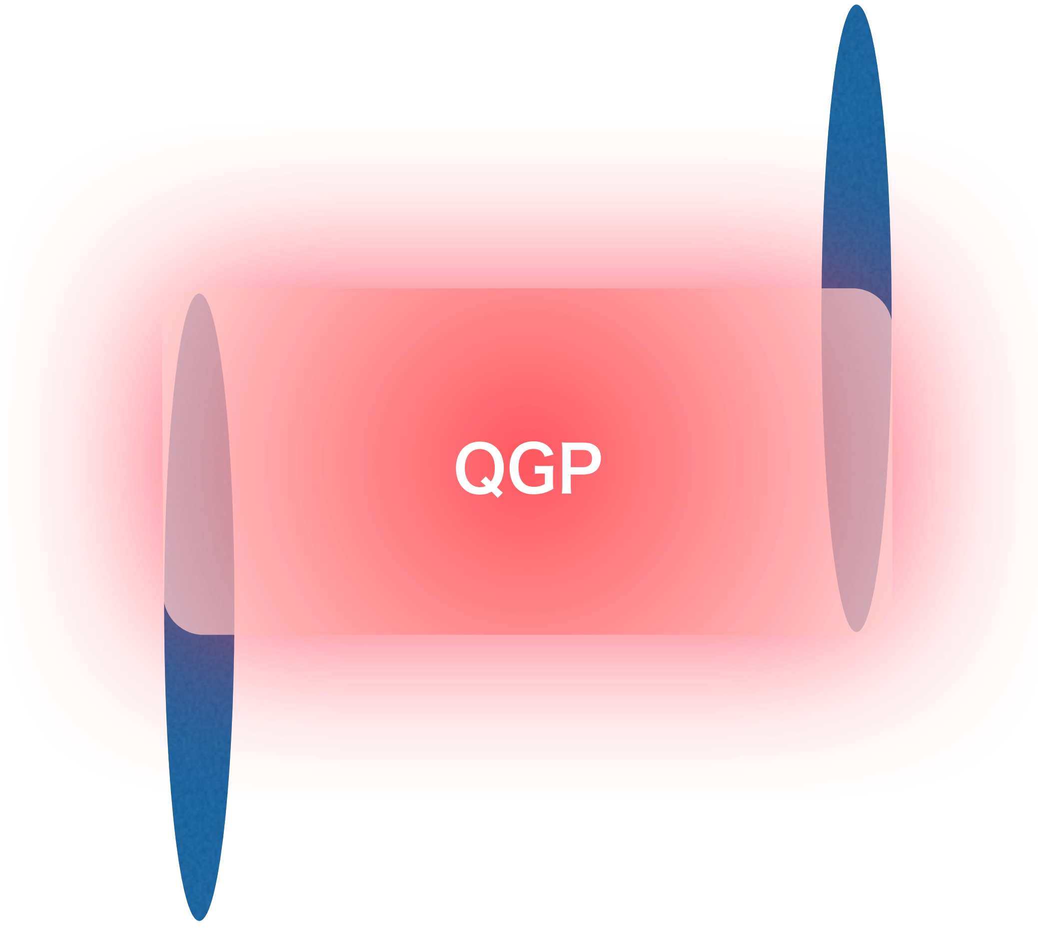
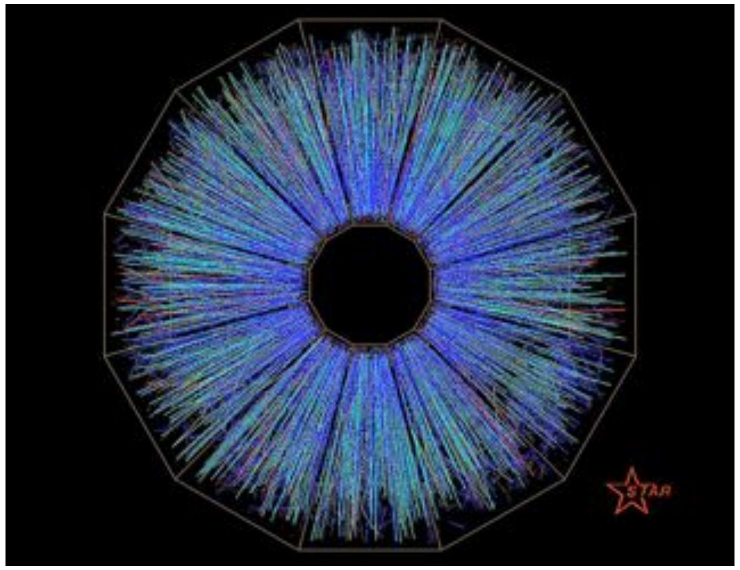
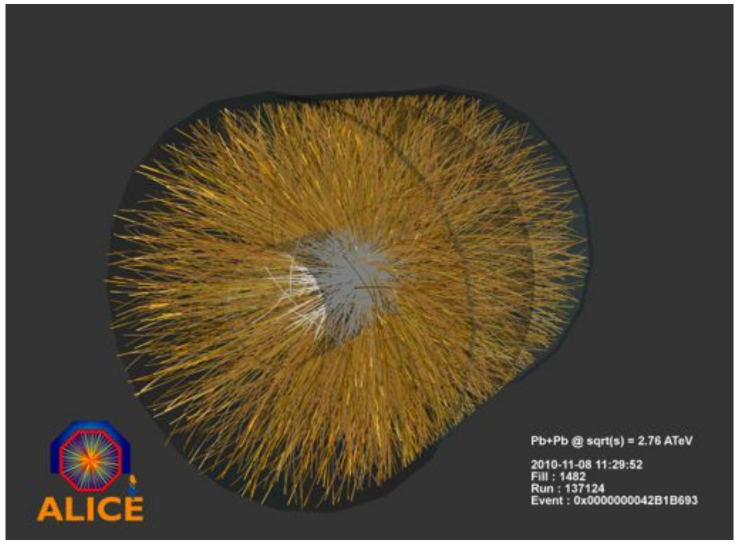
Quark Gluon Plasma and Heavy Ion Collision





Au+Au $\sqrt{s_{NN}} = 200$ GeV @BNL

[http://www.flickr.com/photos/brookhavenlab/3112770151/in/set-72157613690851651/]



Pb+Pb $\sqrt{s_{NN}} = 2.76$ TeV @LHC

[http://aliceinfo.cern.ch/Public/en/Chapter1/fstablebeams.html]

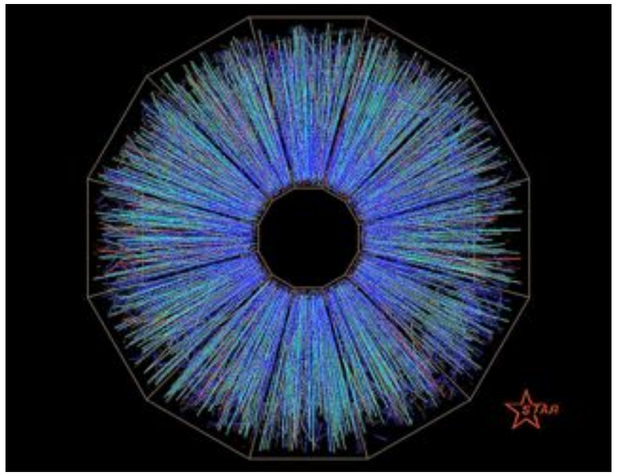
Properties of QGP

Temperature?

hadron spectra, thermal photon···

Equation of state?

collective flow



Viscosity?

elliptic flow, triangular flow···

Phase transition?

fluctuation of charge

Diffusion constant?

nuclear modification factor
for heavy quark

Stopping power?

nuclear modification factor for jet

Electric conductivity?

dilepton spectra, charge difference in v_1

Anomaly induced transport?
(Chiral Magnetic Effect)

Possible signal?

Outline

I. Introduction
- ◇ Electromagnetic Field in Heavy Ion Collisions
- ◇ Anomaly induced transport (CME/CSE, CMW)

II. Anomalous Hydrodynamics
- ◇ Setup for Anomalous Hydrodynamic Simulation

III. Results of Numerical simulation
- ◇ Chiral Magnetic Wave in uniform/expanding plasma
- ◇ Charge dependent elliptic flow

IV. Summary

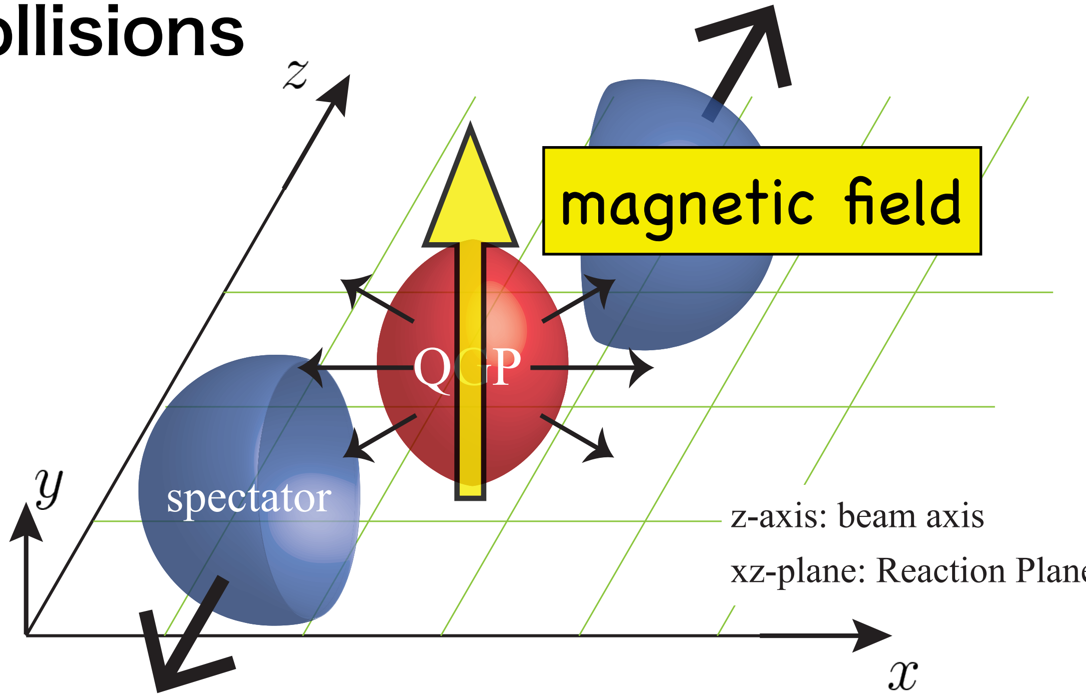
Introduction

Heavy Ion Collisions

■ Quark Gluon Plasma in Heavy Ion Collisions

LHC(CERN) : Pb+Pb $\sqrt{s_{NN}} = 2.76$ TeV

RHIC(BNL) : Au+Au $\sqrt{s_{NN}} = 200$ GeV



◇ Observation of Elliptic flow v_2

➡ QGP is like a perfect fluid, $\frac{\eta}{s} \sim \frac{1}{4\pi}$

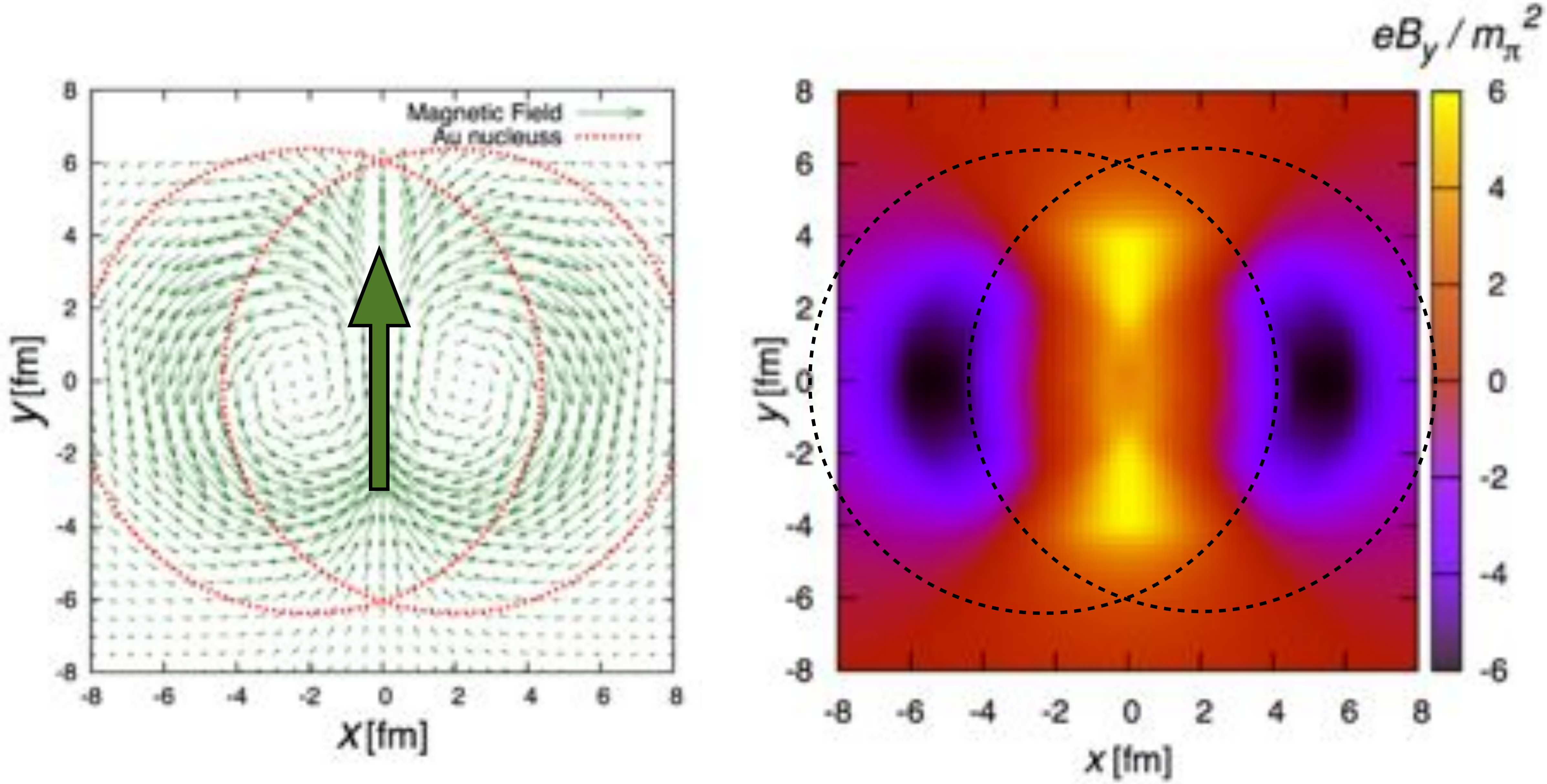
■ Intense magnetic field in peripheral collisions

$$\boxed{e|\vec{B}| \sim m_\pi^2 \sim 10^{14}\ \mathrm{T}}$$

cf.) Compact Star : $10^8 - 10^{11}$ T

Intense Magnetic field in HIC

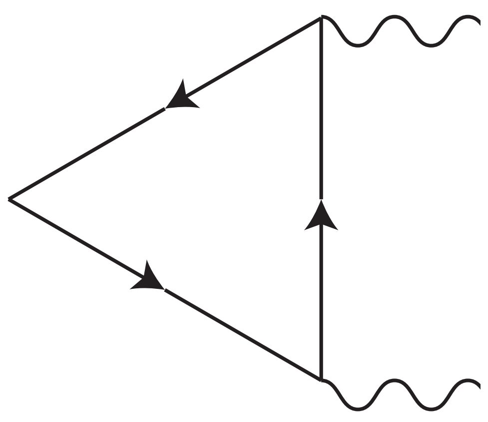
■ **Event-averaged magnetic field with impact parameter** $b = 4 \text{ fm}$



QED Chiral Anomaly

■**Axial current non-conservation** (Adler, Bell, Jackiw (1969))

$$\partial_\mu j_5^\mu = \boxed{-\frac{N_c e^2}{16\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}}$$



Chiral Magnetic/Separation Effect

■ Chiral Magnetic Effect(CME)

Fukushima et al. (2008), Vilenkin(1980)

Magnetic field+chirality imbalance

$$\vec{j} = \frac{N_c e}{2\pi^2}\mu_5 \vec{B}$$

chiral chemical potential:

$$\mu_5 \equiv \mu_R - \mu_L$$

$$n_5 \equiv n_R - n_L \neq 0$$

vector current || magnetic field

■ Chiral Separation Effect(CSE)

Metlitski et al. (2005)

Magnetic field+charge density

$$\vec{j}_5 = \frac{N_c e}{2\pi^2}\mu \vec{B}$$

charge chemical potential:

$$\mu \equiv \mu_R + \mu_L$$

$$n \equiv n_R + n_L \neq 0$$

axial current || magnetic field

Chiral Magnetic Wave

Chiral Separation Effect

$$\vec{j}_5 = \frac{N_c e}{2\pi^2}\underline{\mu}\vec{B}$$

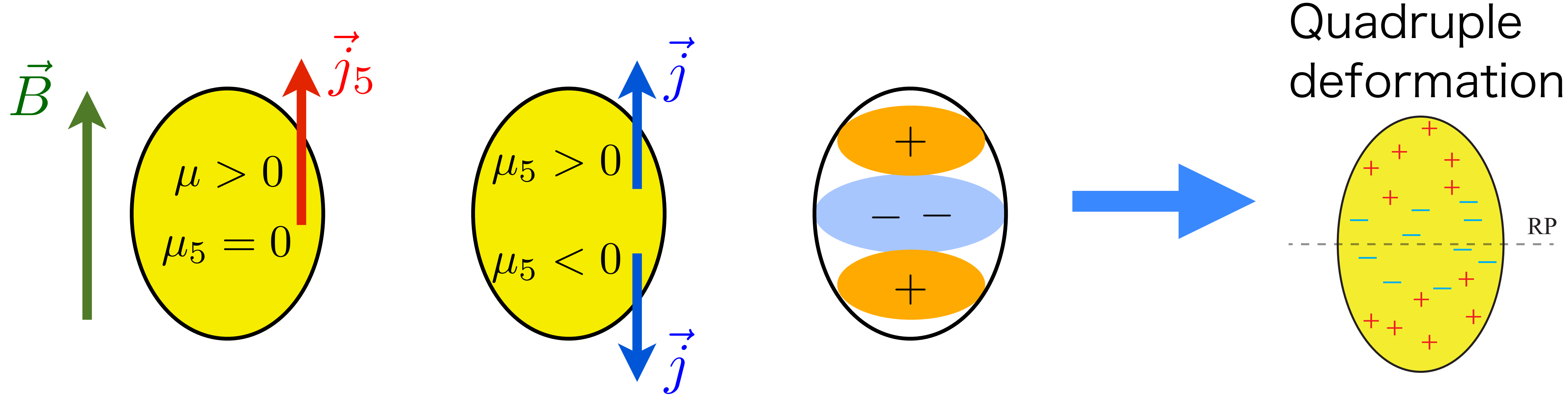
Chiral Magnetic Effect

$$\vec{j} = \frac{N_c e}{2\pi^2}\underline{\mu_5}\vec{B}$$

Chiral Magnetic Wave

Kharzeev et al, (2011)

◇ In the case of finite chemical potential (low energy collisions)



(Burnier et al, (2012))

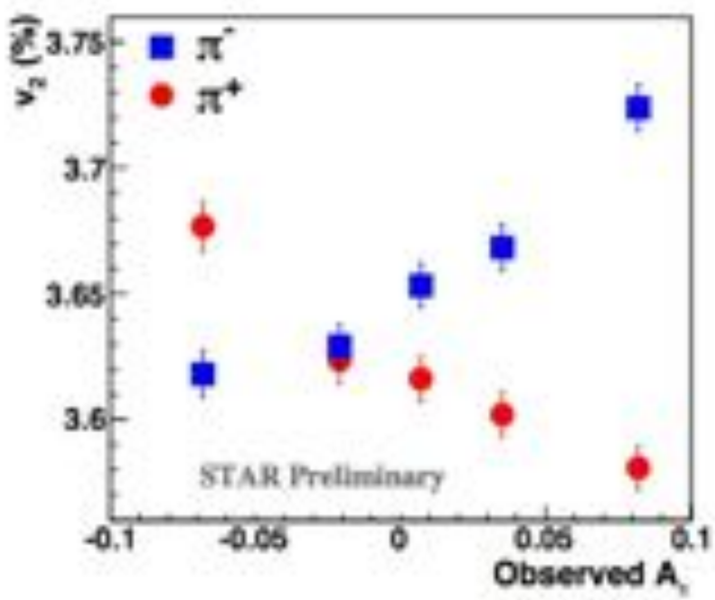
Charge Dependent Elliptic Flow

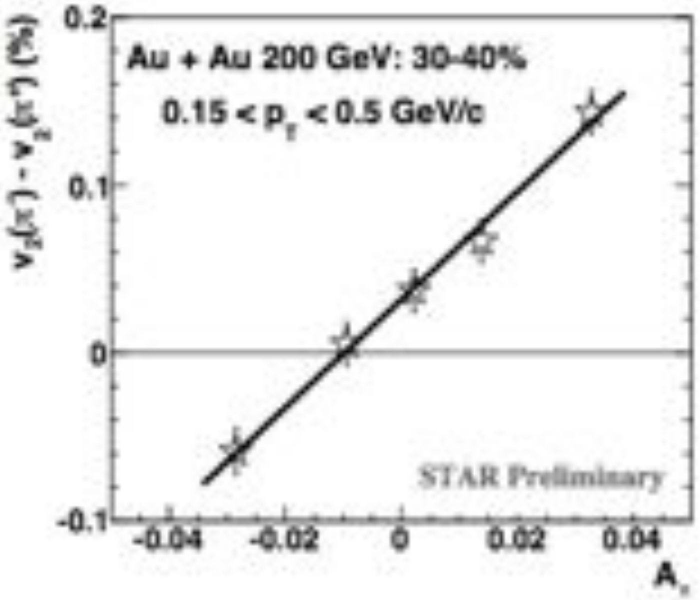
■**Observable :**

$$\frac{d\,(N_+ + N_-)}{d\phi} = (\overline{N}_+ + \overline{N}_-)\,[1 + 2v_2\cos(2\phi)]\,,$$

$$\frac{d\,(N_+ - N_-)}{d\phi} = (\overline{N}_+ - \overline{N}_-)\,[1 + 2r_e\cos(2\phi)]$$

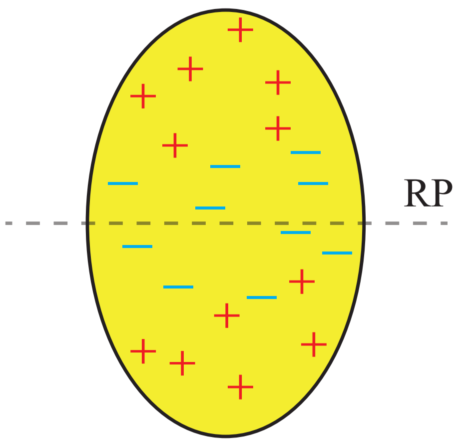
Charge dependent v_2 **:** $v_2^{\pm} = v_2 \mp r_e A$ $\quad A \equiv \frac{\overline{N}_+ - \overline{N}_-}{\overline{N}_+ + \overline{N}_-}$





(arXiv:1210.5498)

Quadrupole deformation



$$\boxed{r_e \sim 10^{-2} > 0}$$

Effect of the CME/CSE？

(Burnier et al, (2012))

Origins of $\Delta v_2^{\pm}$

charge dependent v_2

- Chiral Magnetic Wave
- Initial chemical potential
- Conducting current
- Local Charge Conservation

full-3D anomalous hydrodynamic simulation

Anomalous Hydrodynamics

Hydrodynamics in External Fields

■ **Relativistic hydrodynamic equation**

– Energy-momentum conservation of the fluid

$$\partial_\mu T^{\mu\nu} = 0$$

$T^{\mu\nu}$: energy-momentum tensor of the QGP fluid

■ **Relativistic hydrodynamic equation under external fields**

$$\boxed{\partial_\mu T^{\mu\nu} = \underline{F^{\nu\lambda} j_\lambda}\,, \qquad \partial_\mu j^\mu = 0}$$

$F^{\mu\nu}$: field strength tensor

j^μ : $U(1)_V$ current

Energy-momentum source due to **Lorentz force**

Anomaly and Entropy Production

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda \qquad T^{\mu\nu} = (e+p)u^\mu u^\nu - p g^{\mu\nu}$$

$$\partial_\mu j^\mu = 0 \qquad j^\mu = n u^\mu + \nu^\mu$$

$$\partial_\mu j_5^\mu = -C E^\mu B_\mu \qquad j_5^\mu = n_5 u^\mu + \nu_5^\mu$$

1st law of thermodynamics

$$e + p = Ts + \mu n + \mu_5 n_5$$

$$de = T ds + \mu dn + \mu_5 dn_5$$

$$\partial_\mu \left(s u^\mu - \frac{\mu}{T}\nu^\mu - \frac{\mu_5}{T}\nu_5^\mu \right) = -\nu^\mu \left(\partial_\mu \frac{\mu}{T} + \frac{E_\mu}{T} \right) - \nu_5^\mu \partial_\mu \frac{\mu_5}{T} + C\frac{\mu_5}{T} E^\mu B_\mu$$

can make quadratic form and positive

can't make quadratic form and can be negative

Correction induced by Anomaly

◇ Entropy principle

(Son, Surowka(2009))

$$\partial_\mu s^\mu \geq 0$$

should be realized at all times

◇ Correction of currents

$$\boxed{\begin{aligned} s^\mu &\to s^\mu + D_B B^\mu + D_\omega \omega^\mu \\ \nu^\mu &\to \nu^\mu + \kappa_B B^\mu + \kappa_\omega \omega^\mu \\ \nu_5^\mu &\to \nu_5^\mu + \xi_B B^\mu + \xi_\omega \omega^\mu \end{aligned}}$$

4-magnetic field/vorticity

$$B^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\sigma\rho} u_\nu F_{\sigma\rho}$$

$$\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\sigma\rho} u_\nu \partial_\sigma u_\rho$$

$$\partial_\mu s^\mu = \ldots = (\ldots)E^\mu B_\mu + (\ldots)\omega^\mu E_\mu + (\ldots)B^\mu + (\ldots)\omega^\mu$$

(. . .)should be zero! → Determine coefficients

Anomalous Hydrodynamics

(Son, Surowka(2009))

◇ **Conservation law :**

$$\partial_\mu T^{\mu\nu} = eF^{\nu\lambda} j_\lambda, \quad \partial_\mu j^\mu = 0\,, \quad \partial_\mu j_5^\mu = -CE^\mu B_\mu$$

◇ **Constitutive equation :**

$$j^\mu = nu^\mu + \underbrace{\kappa_B B^\mu}_{\text{CME}} + \underbrace{\kappa_\omega \omega^\mu}_{\text{CVE}}$$

Chiral Magnetic/Vortical Effect(CME/CVE)

$$j_5^\mu = n_5 u^\mu + \underbrace{\xi_B B^\mu}_{\text{CSE}} + \underbrace{\xi_\omega \omega^\mu}_{\text{CVE}}$$

Chiral Separation Effect ... (CSE)

◇ **Coefficients :**

$$e\kappa_B = C\mu_5 \left(1 - \frac{\mu n}{\varepsilon + p} \right)$$

$$e^2\kappa_\omega = 2C\mu\mu_5 \left(1 - \frac{\mu n}{\varepsilon + p} \right)$$

$$e\xi_B = C\mu \left(1 - \frac{\mu_5 n_5}{\varepsilon + p} \right)$$

$$e^2\xi_\omega = C\mu^2 \left(1 - \frac{2\mu_5 n_5}{\varepsilon + p} \right)$$

Simulation Setup (1)

◇ **Conservation law**

$$\partial_\mu T^{\mu\nu} = eF^{\nu\lambda} j_\lambda, \quad \partial_\mu j^\mu = 0, \quad \partial_\mu j_5^\mu = -CE^\mu B_\mu \qquad \left(C = \frac{N_c e^2}{2\pi^2} \right)$$

◇ **Constitutive equation (CME and CSE)**

$$j^\mu = nu^\mu + \kappa_B B^\mu, \qquad j_5^\mu = n_5 u^\mu + \xi_B B^\mu$$

◇ **EoS: ideal gas (Gluons +1-component Fermion)**

$$p = \frac{1}{3} e = \frac{g_{\mathrm{qgp}} \pi^2}{90} T^4 + \frac{N_c}{6} \left(\mu^2 + \mu_5^2 \right) T^2 + \frac{N_c}{12\pi^2} \left(\mu^4 + 6\mu^2 \mu_5^2 + \mu_5^4 \right)$$

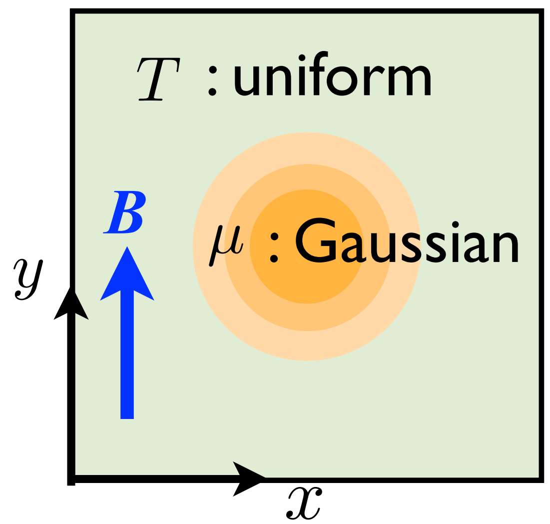
$$n = \frac{N_c}{3\pi^2} \mu^3 + \frac{N_c}{3} \mu \left(T^2 + \frac{3}{\pi^2} \mu_5^2 \right), \quad n_5 = \frac{N_c}{3\pi^2} \mu_5^3 + \frac{N_c}{3} \mu_5 \left(T^2 + \frac{3}{\pi^2} \mu^2 \right)$$

→ **Solve these equations numerically without linearization**

Simulation Setup (2)

1. Test case : Charge in uniform temperature

Initial Condition :

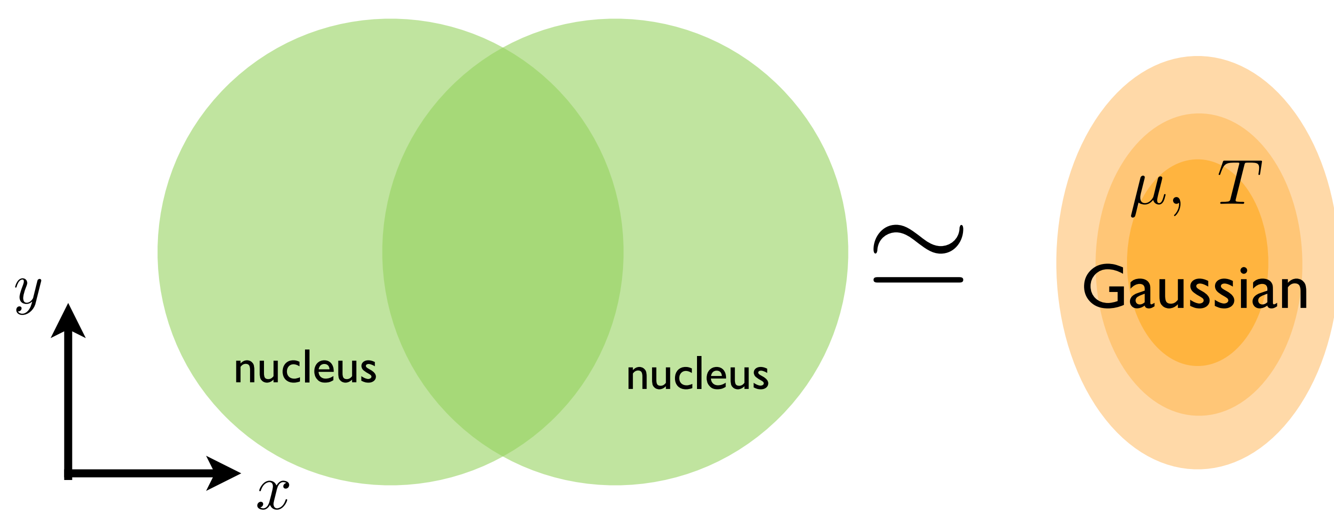


anomalous

hydro simulation

- time evolution of charge?
- CMW in uniform plasma?

2. More realistic case : Expanding geometry



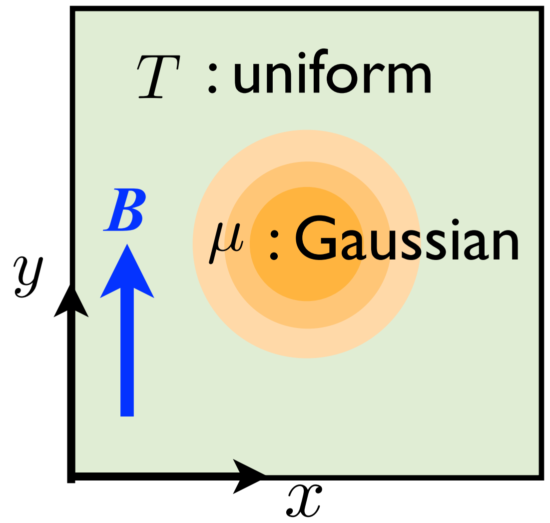
- CMW in expanding plasma
- Source term by anomaly
- Lorentz force

Simulation (1)
Uniform Plasma

Case-1 Uniform Plasma

1. Test case : Charge in uniform temperature

Initial Condition :



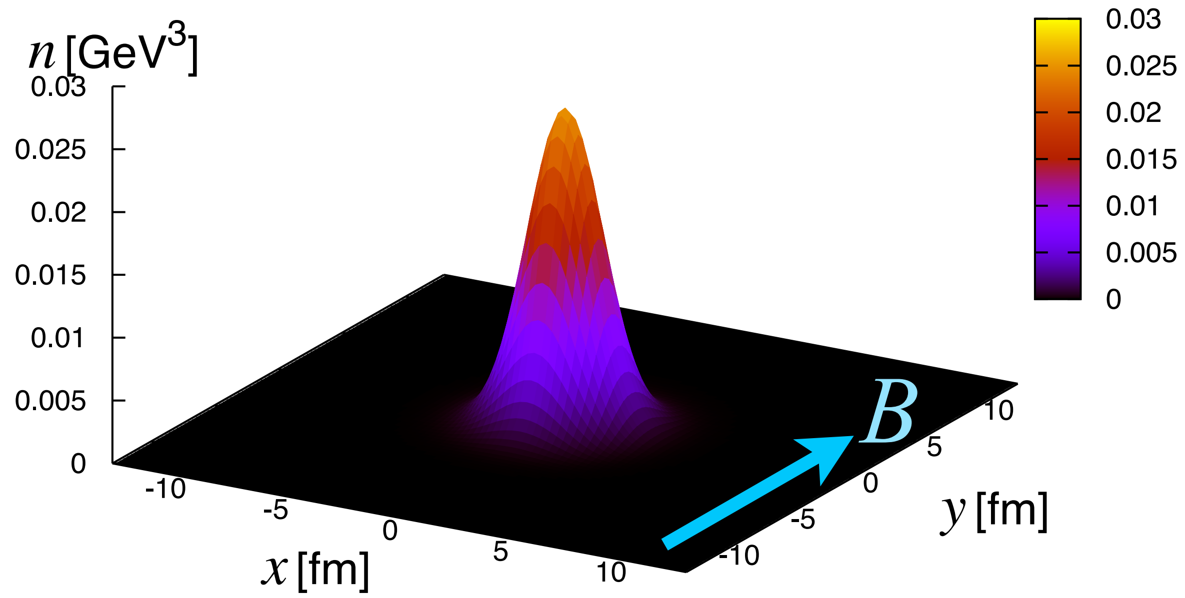
anomalous

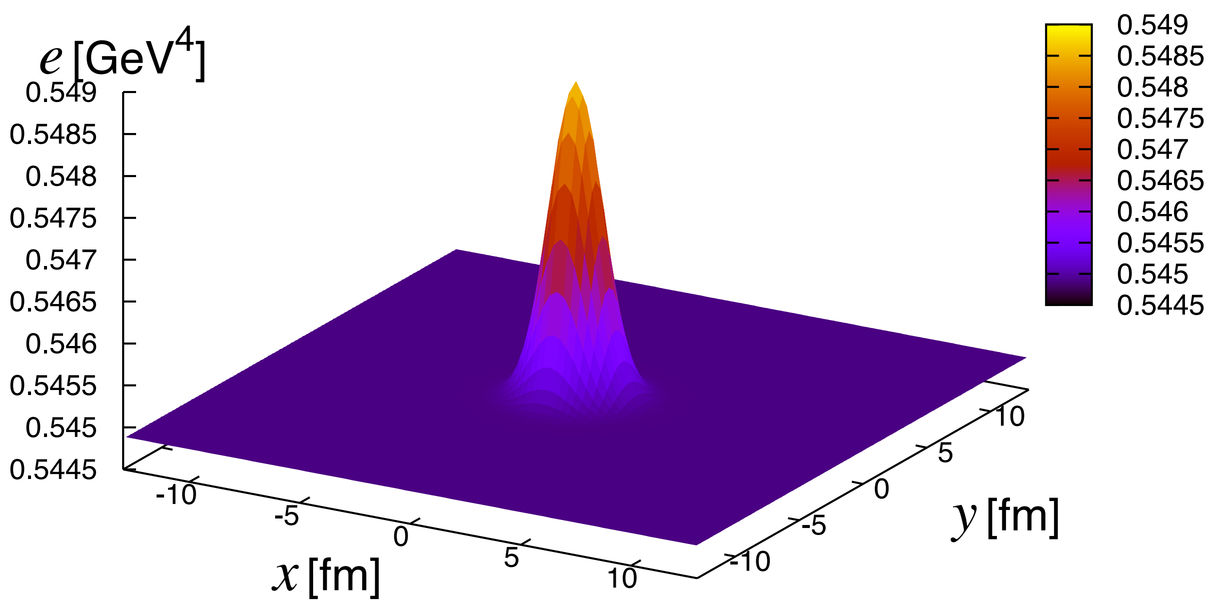
hydro simulation

- time evolution of charge?
- CMW in uniform plasma?

Charge in uniform plasma

■ Initial Condition





Parameters (IC & EM field)

- Temperature $T_0 = 0.5\ \mathrm{GeV}$
- Chemical potential

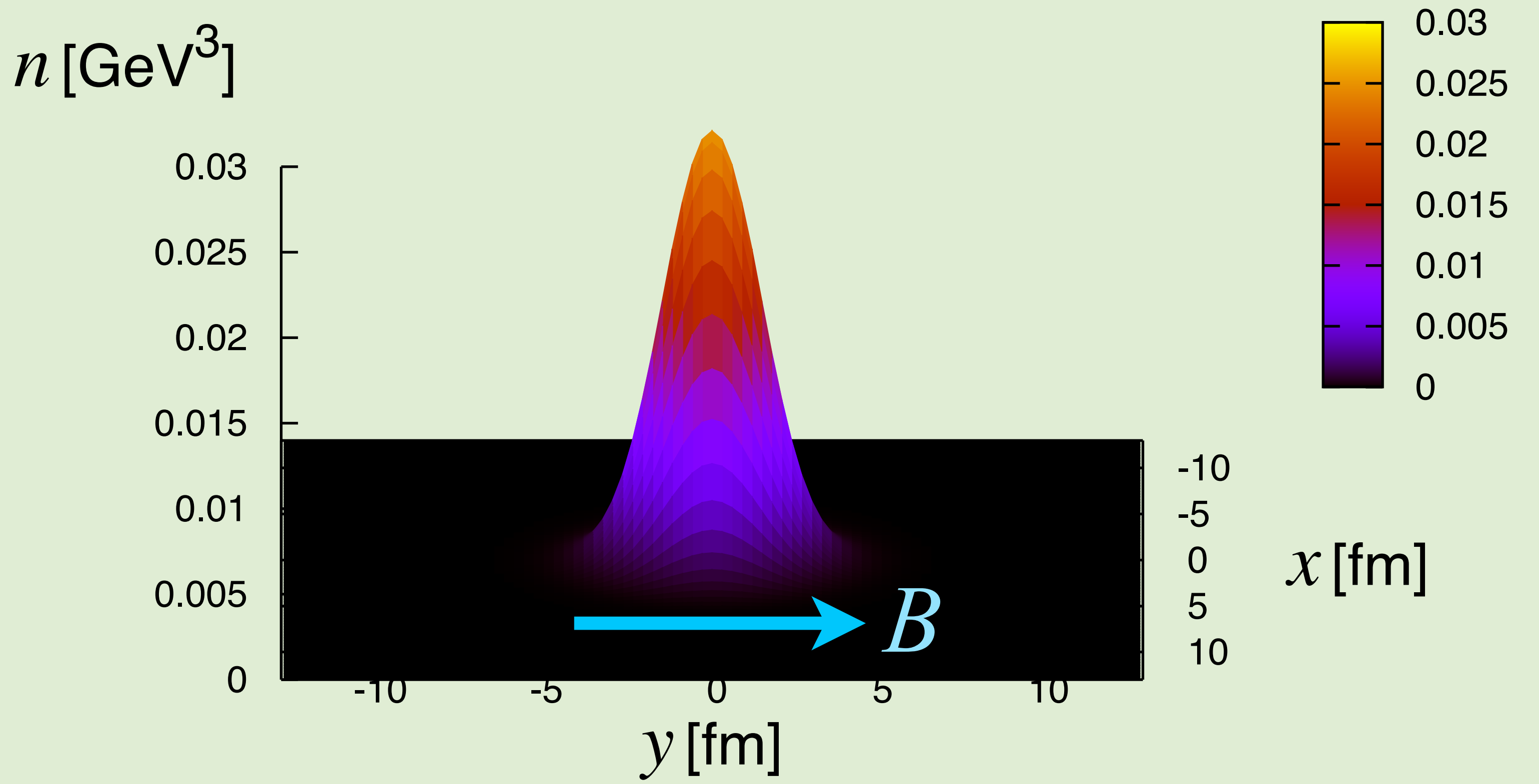
$$\mu(\vec{r}) = \mu_0 \exp\left(-\frac{\vec{r}^{\,2}}{2\sigma^2}\right)$$

$$\mu_0 = 0.1\ \mathrm{GeV} \qquad \sigma = 1.5\ \mathrm{fm}$$

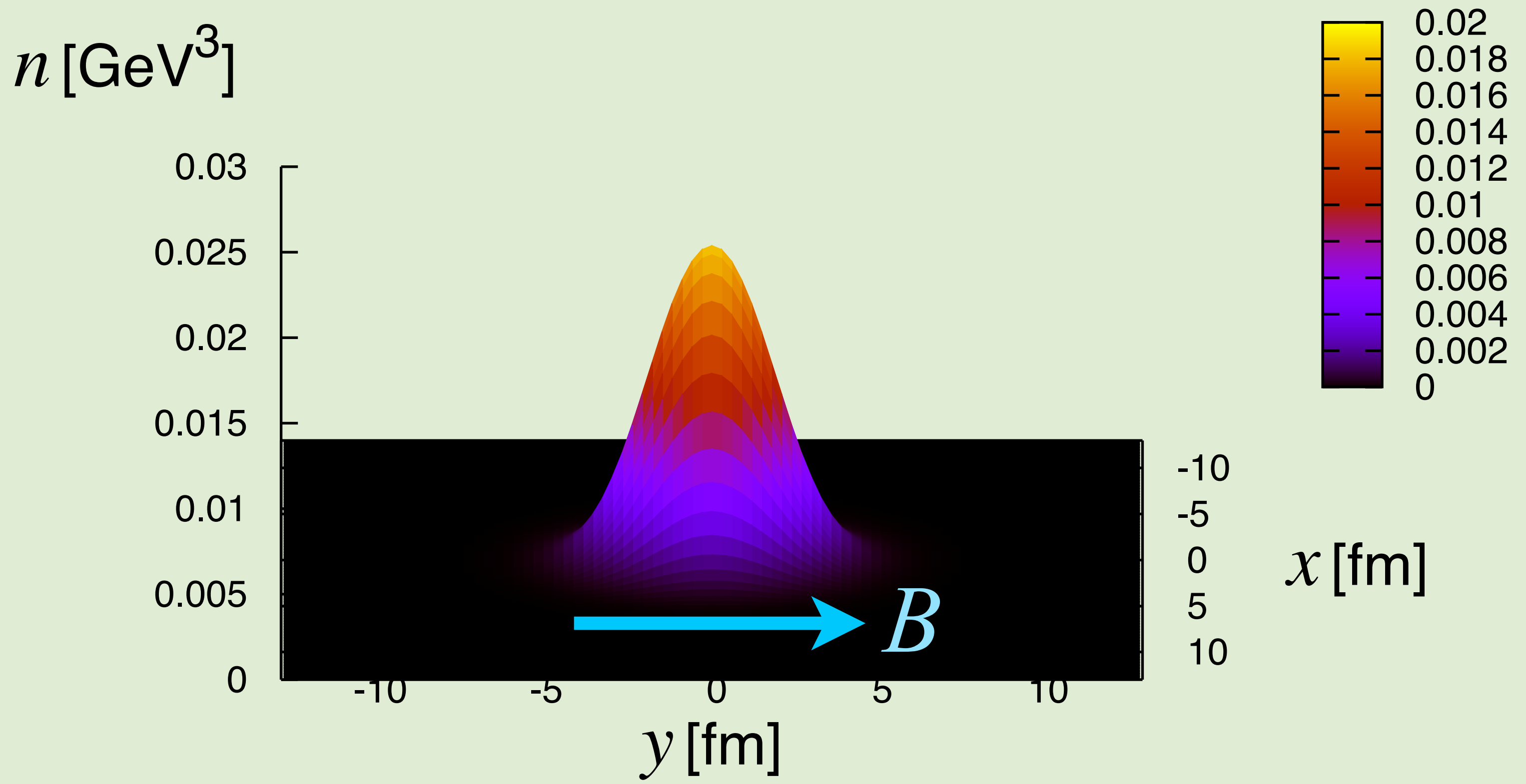
- EM field

$$eB_y = 1.0\ \mathrm{GeV}^2$$

■ Time evolution of the charge distribution ($t = 0 - 9.0$ fm)



■ Time evolution of the charge distribution ($t = 0 - 9.0$ fm)



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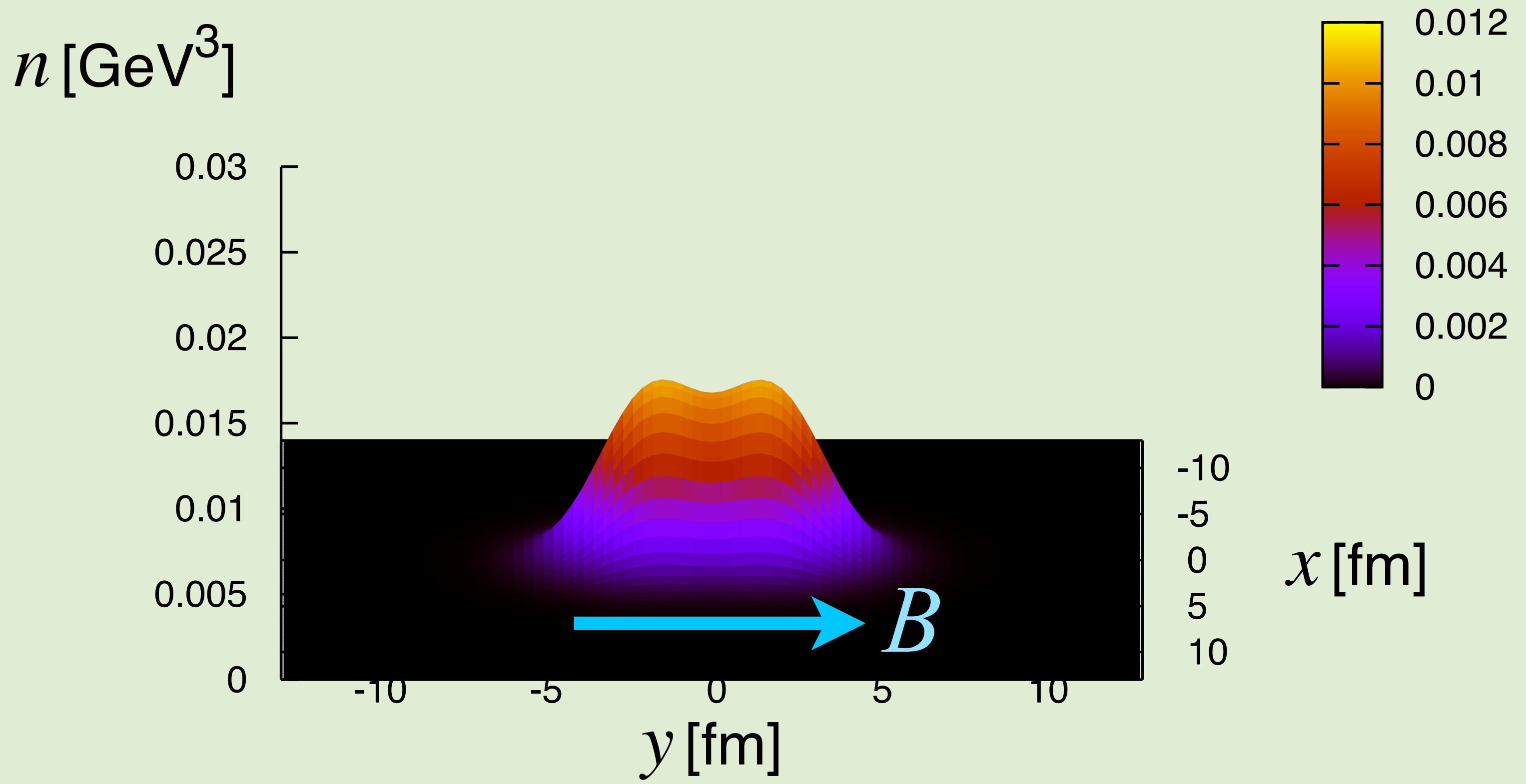

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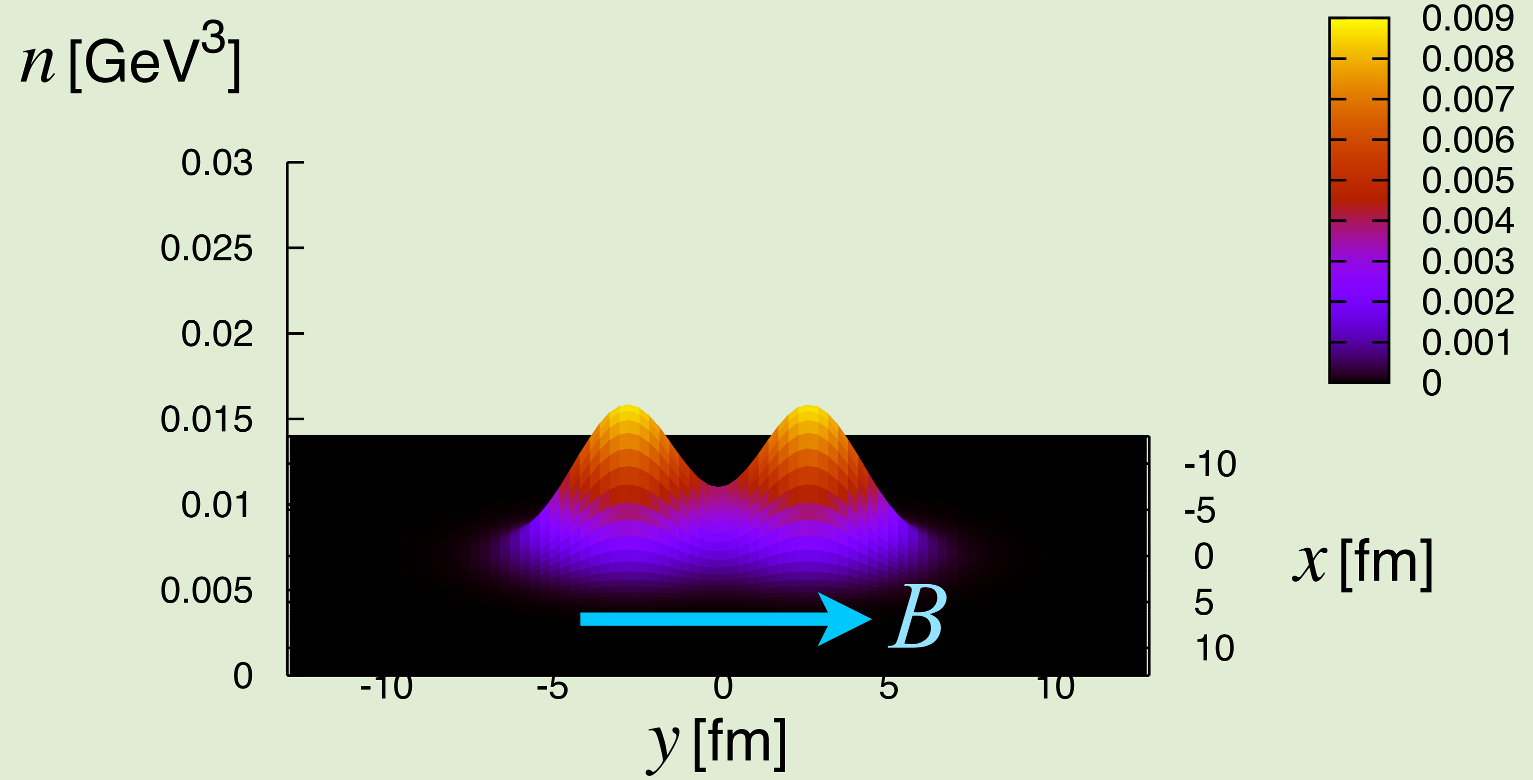
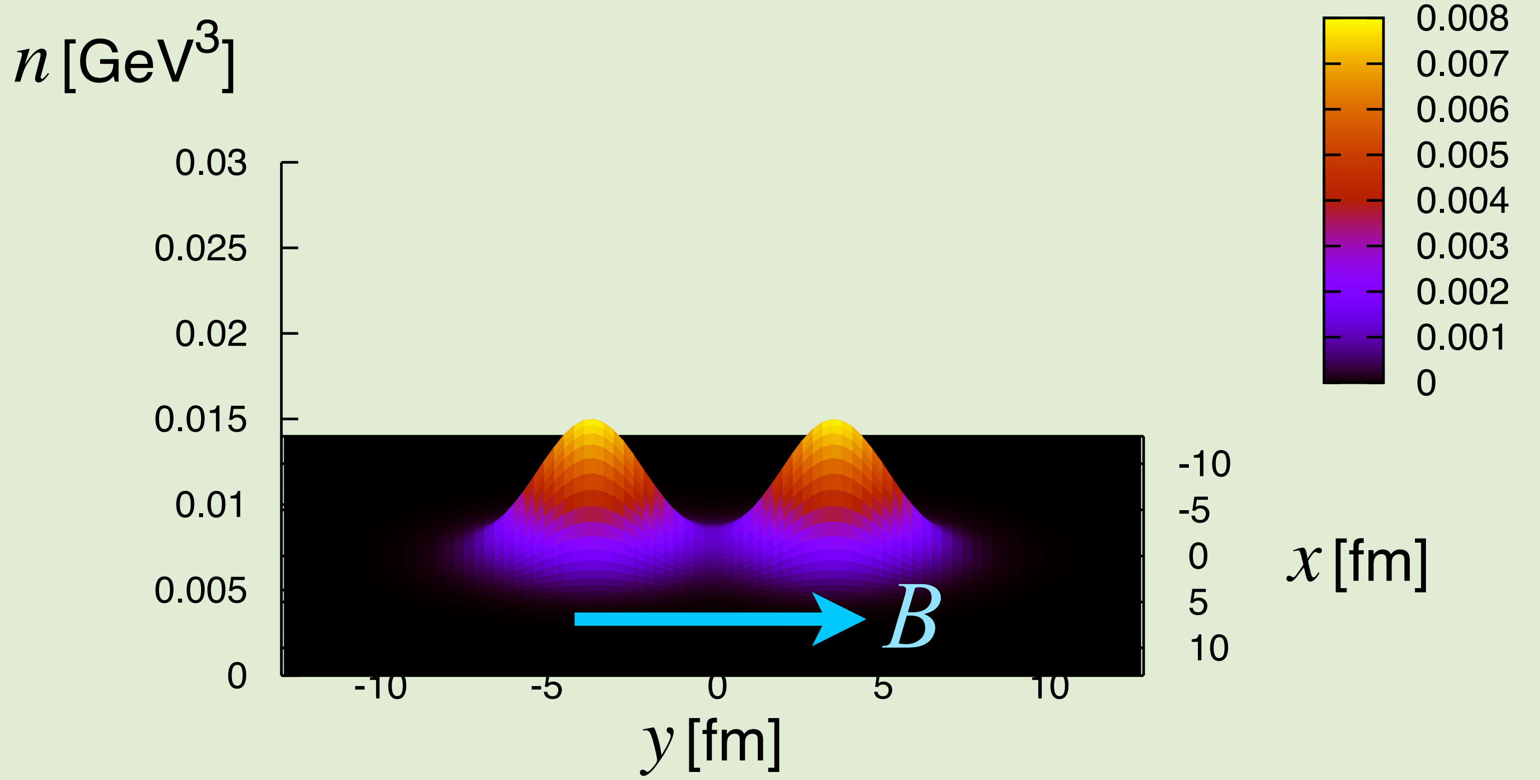
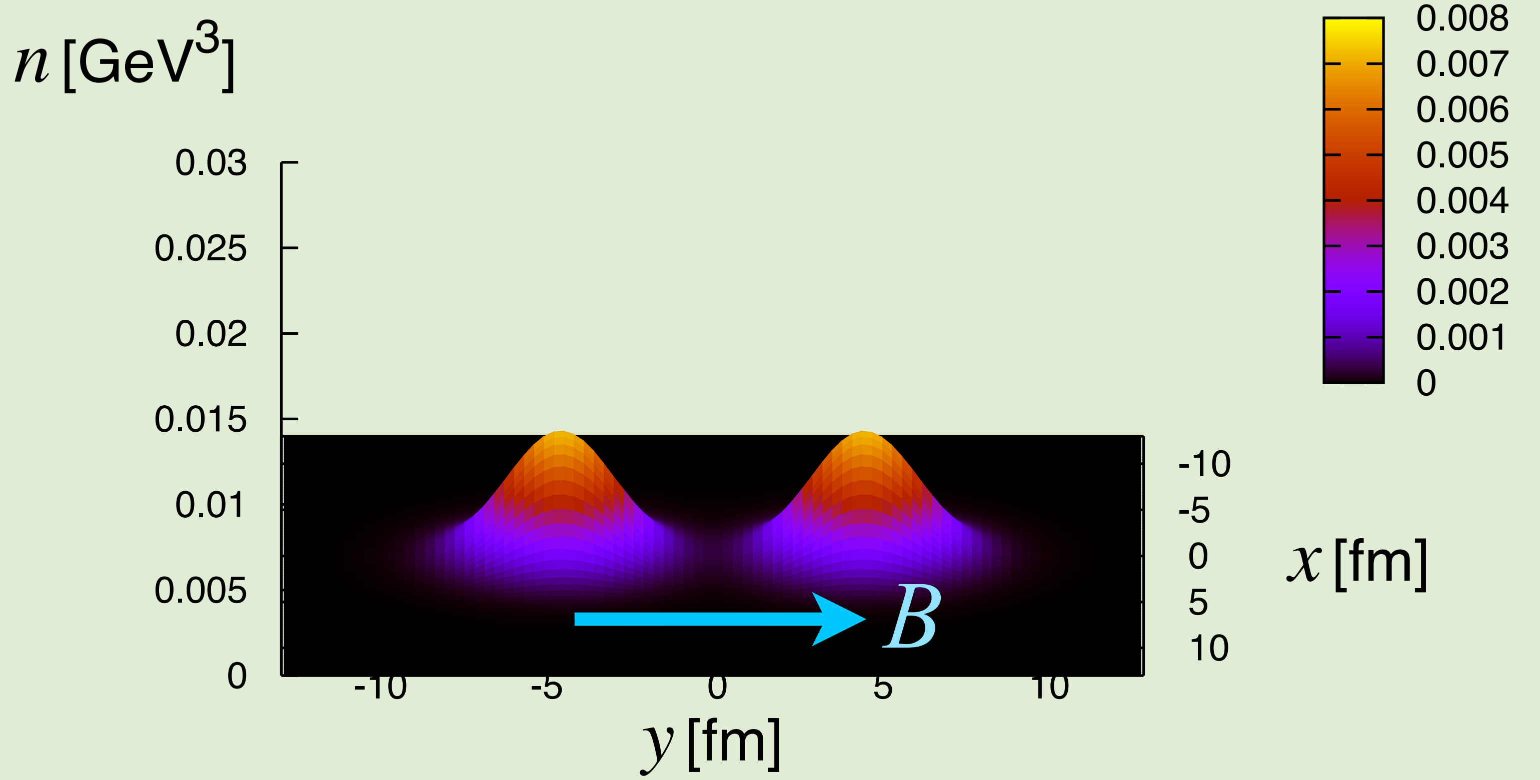


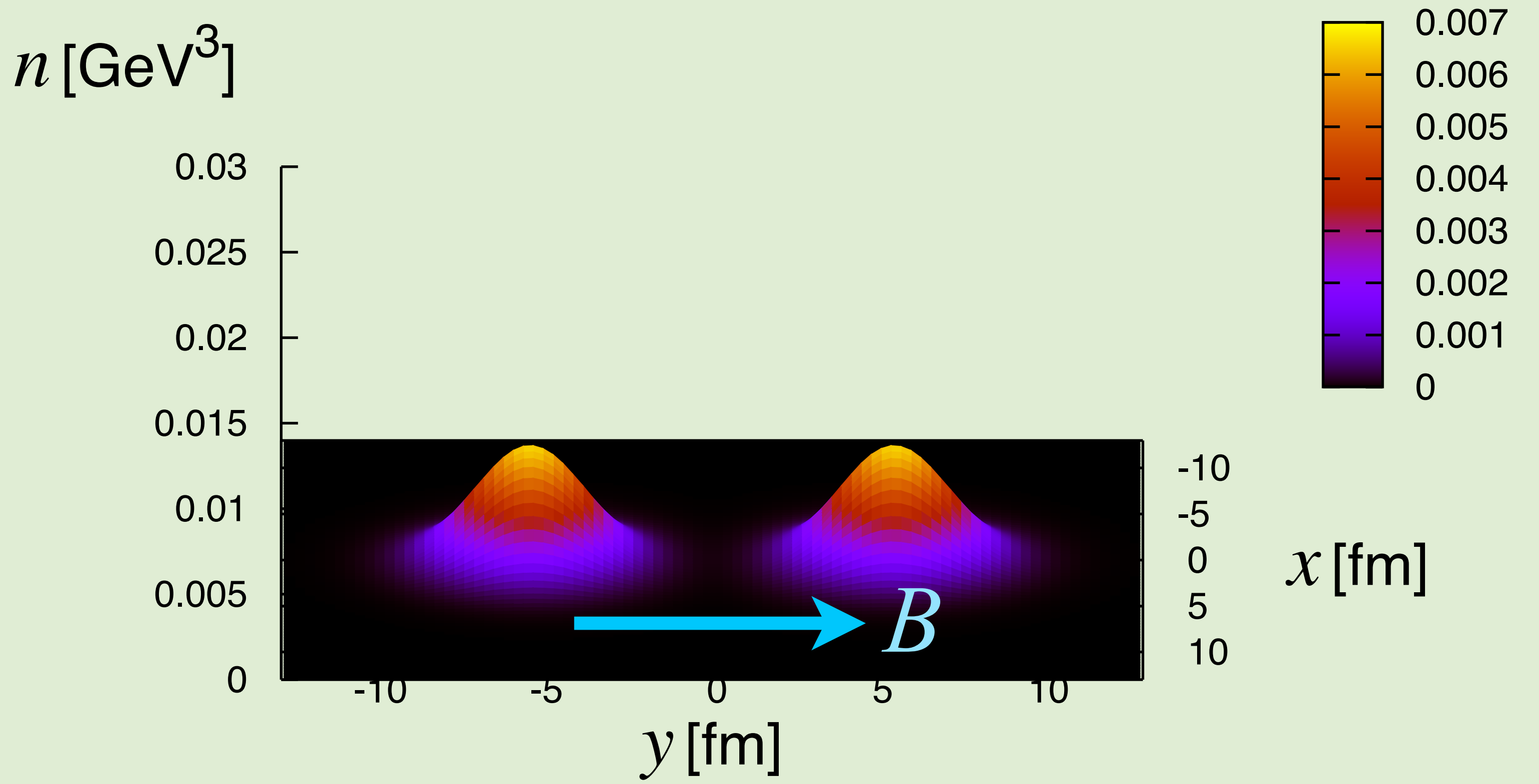
■ Time evolution of the charge distribution ($t = 0 - 9.0\ \mathrm{fm}$)



■ Time evolution of the charge distribution ($t = 0 - 9.0$ fm)



■ Time evolution of the charge distribution ($t = 0 - 9.0\ \mathrm{fm}$)



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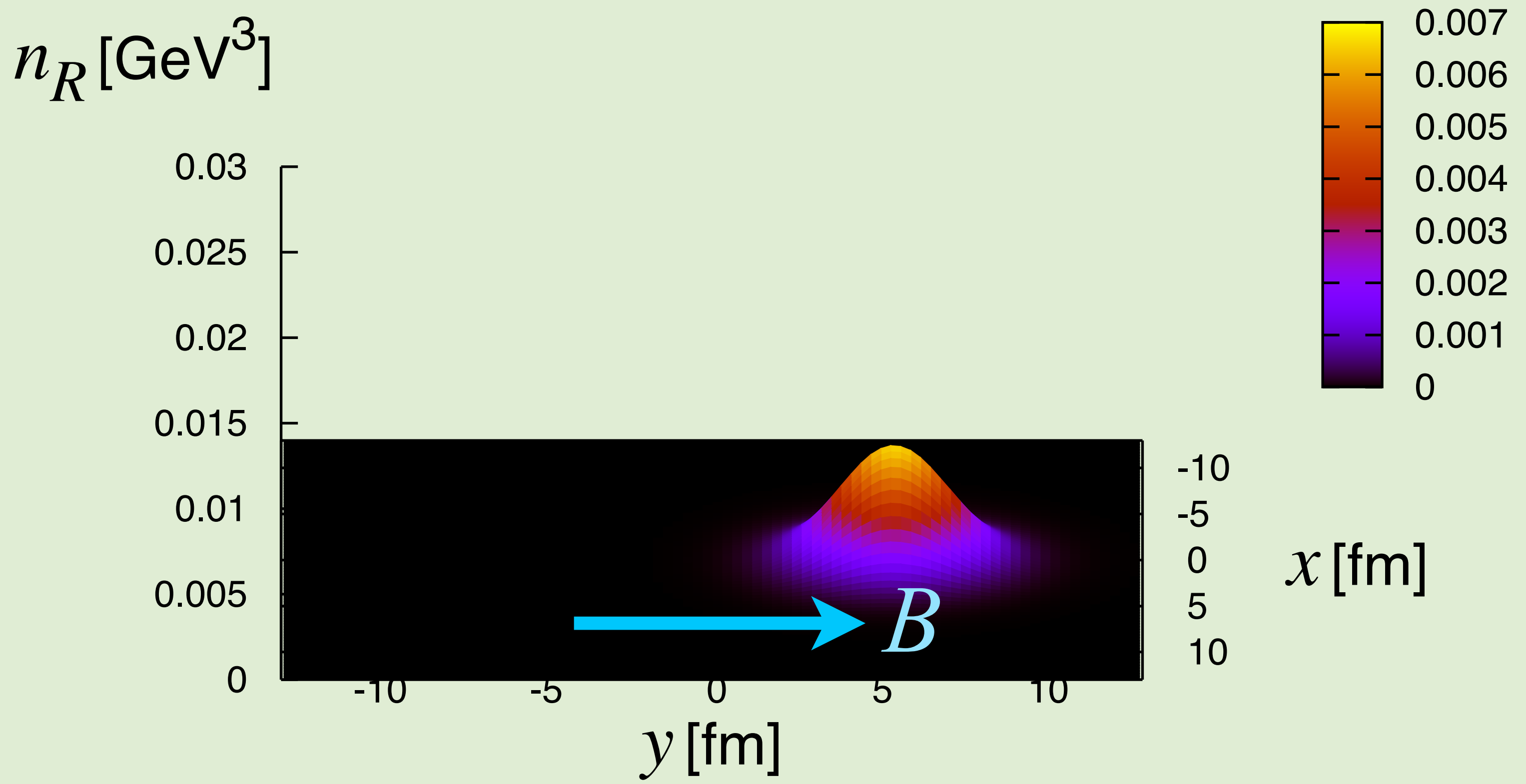

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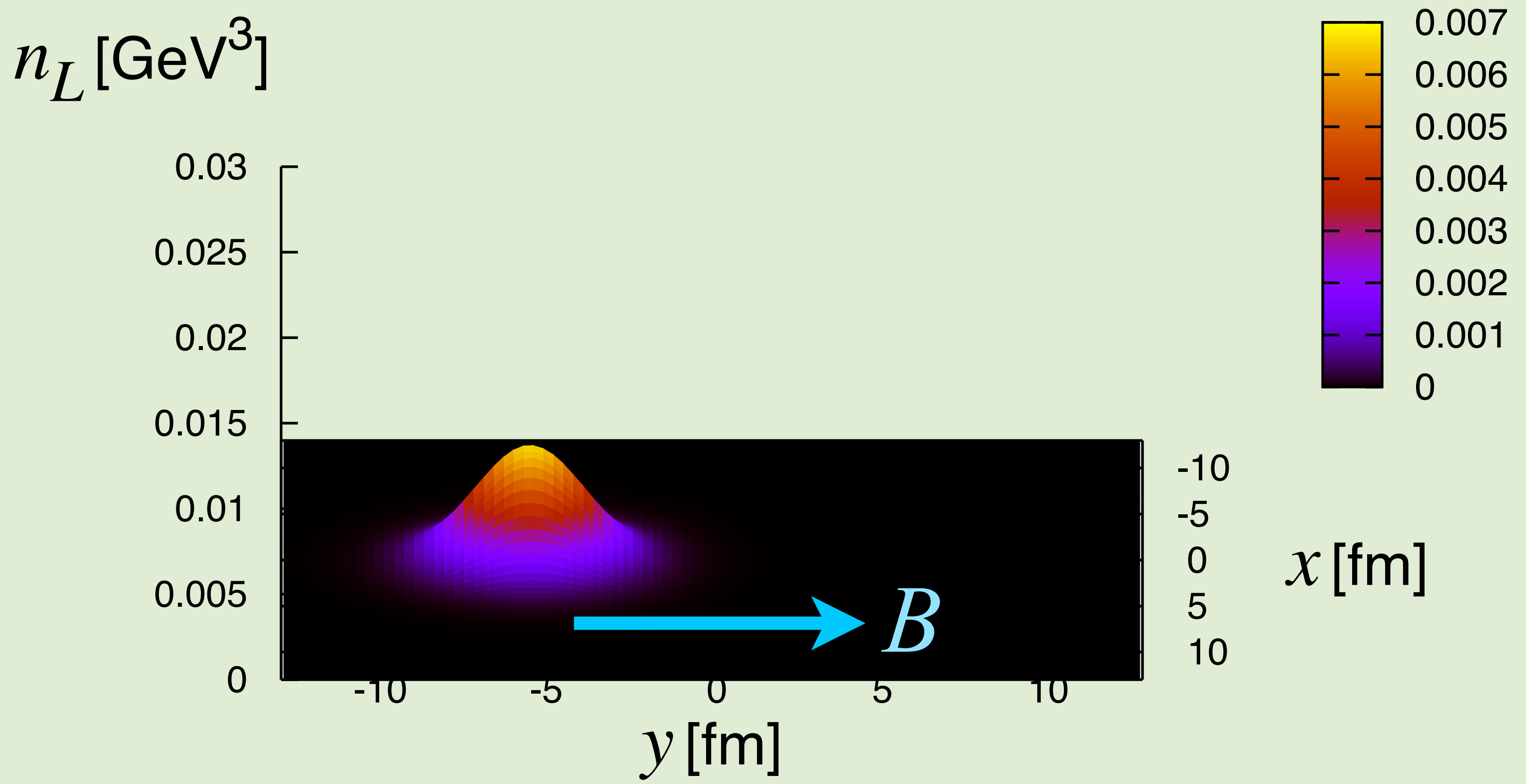
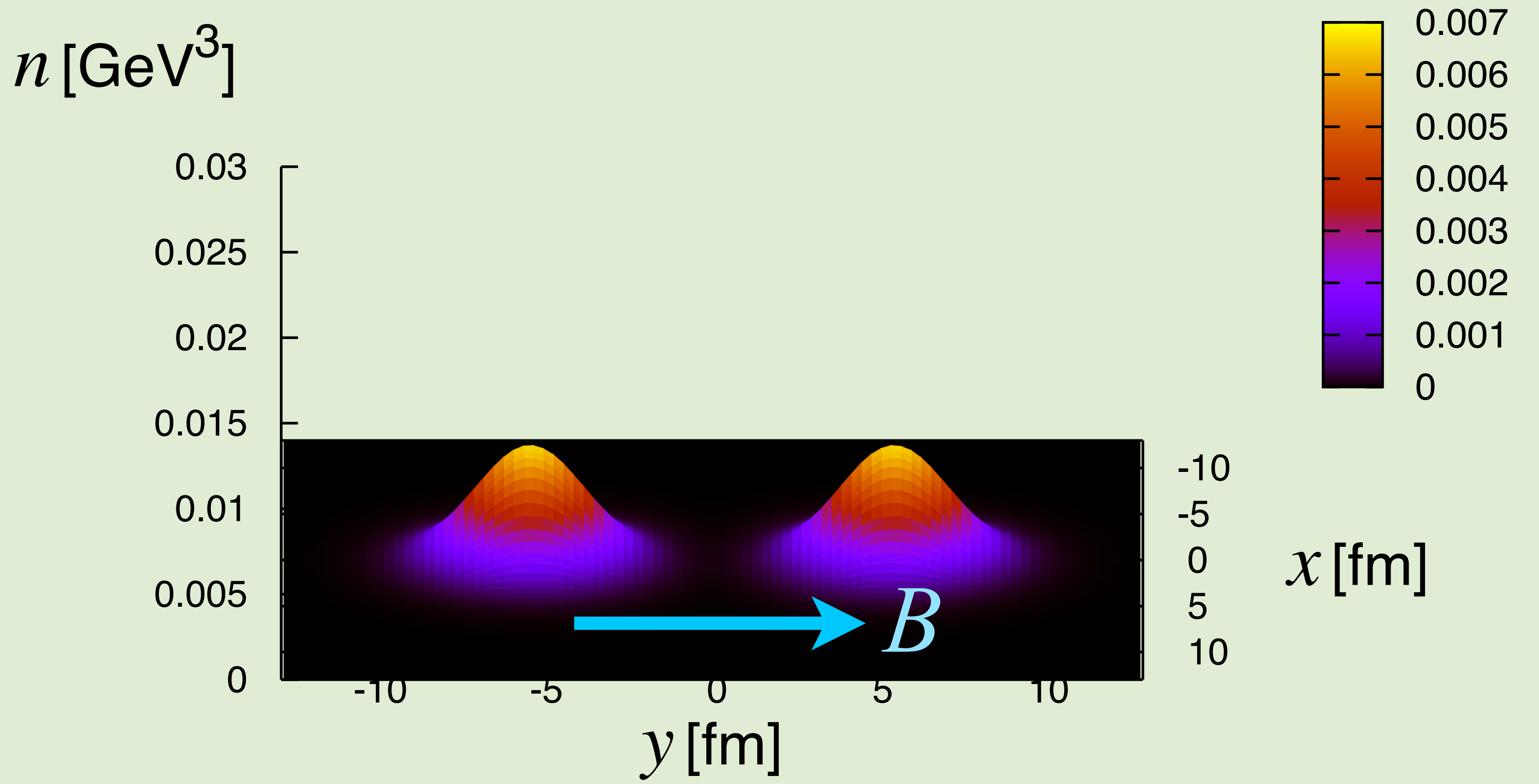


■ Time evolution of the charge distribution ($t = 0 - 9.0\ \mathrm{fm}$)



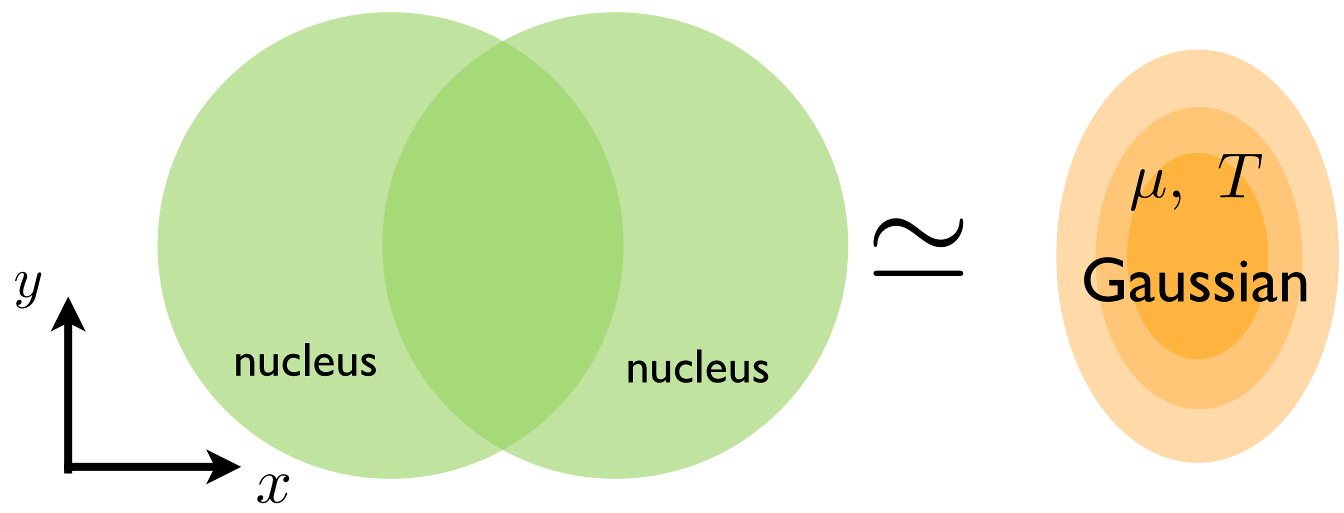
– Charge propagation along magnetic field = Chiral Magnetic Wave

– Estimated propagation speed is consistent with a linearized calculation

Simulation (2)
Expanding Plasma

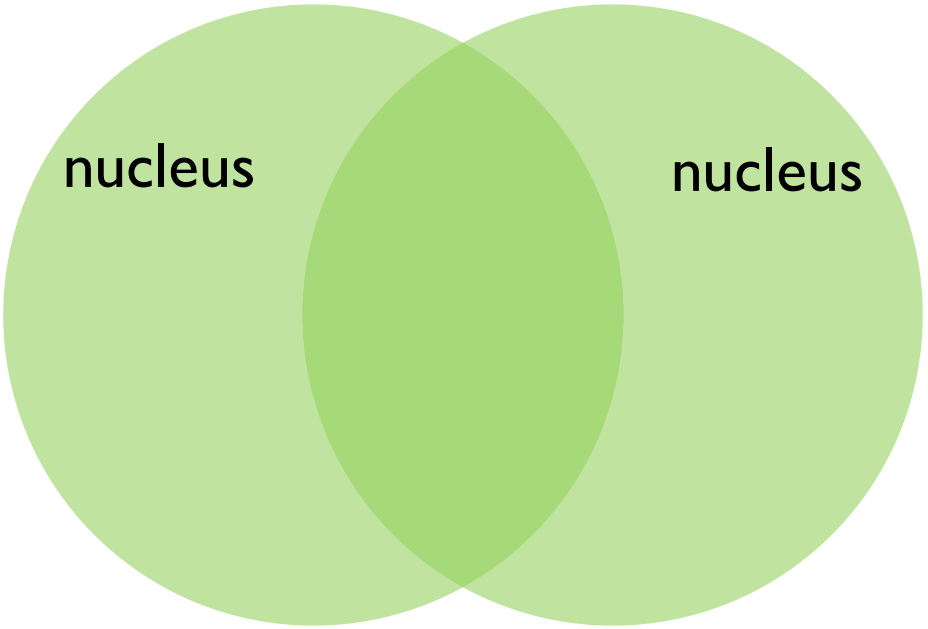
Case-2 Expanding Plasma

2. More realistic case : Expanding geometry



- CMW in expanding plasma
- Source term by anomaly
- Lorentz force

Initial Condition

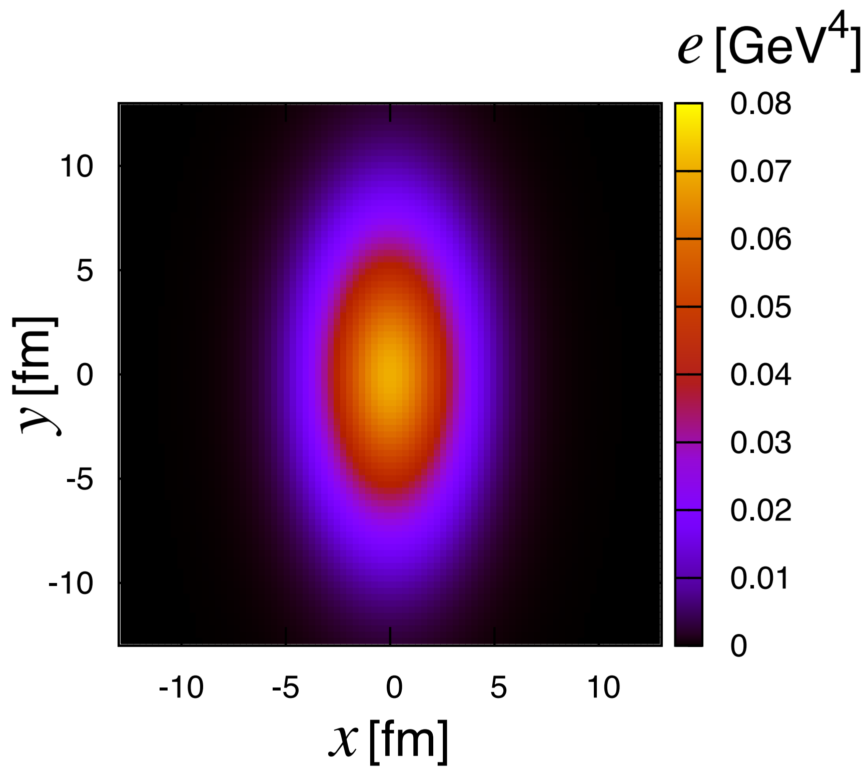


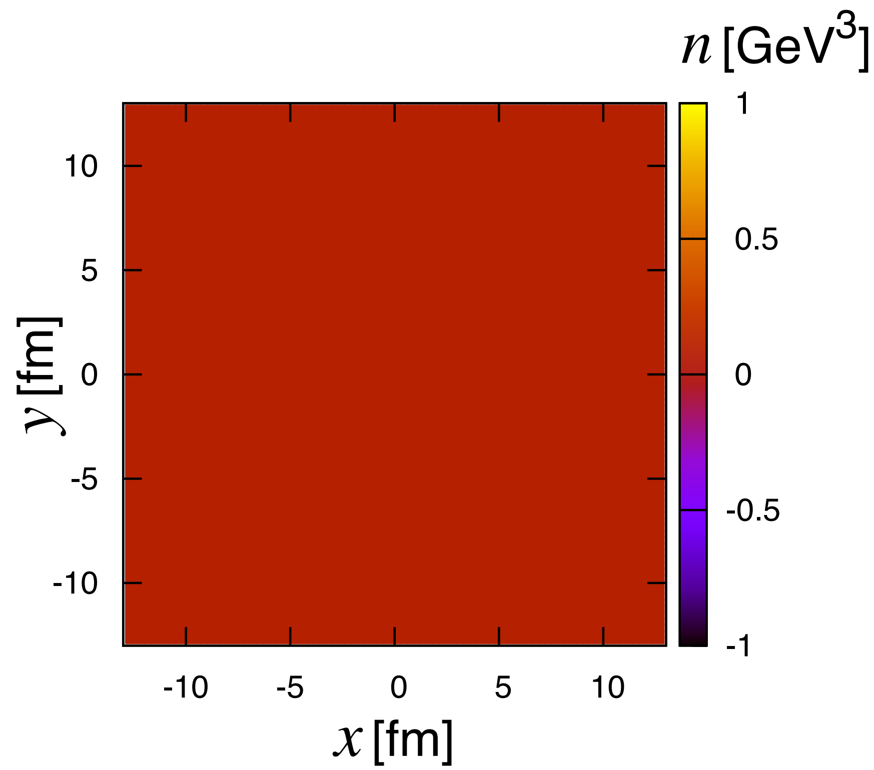
Parameters

$$T(\vec{r}) = T_0 \exp\left(-\frac{r_y^2}{2\sigma_T^2}\right) \qquad T_0 = 0.3\ \text{GeV},\quad \sigma_T = 5\ \text{fm}$$

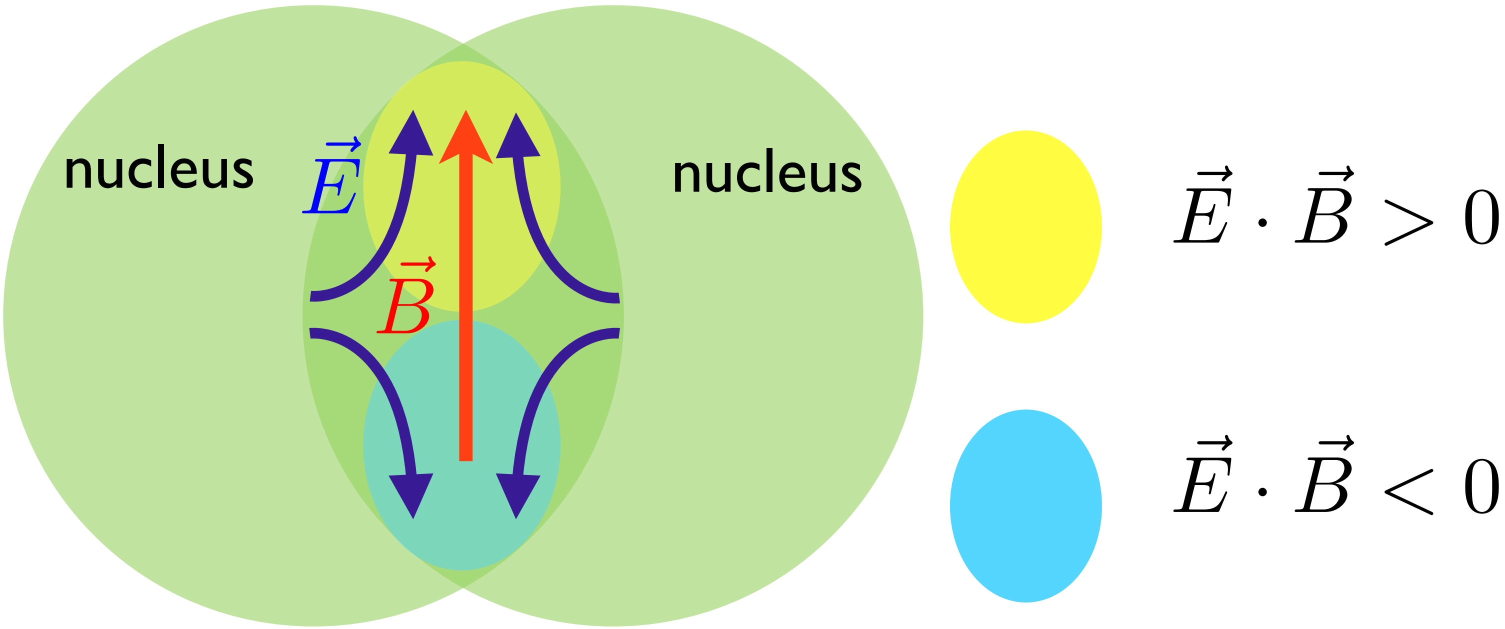
$$\mu(\vec{r}) = \mu_0 \exp\left(-\frac{r_y^2}{2\sigma_\mu^2}\right) \qquad \mu_0 = 0\ \text{GeV},\quad \sigma_\mu = 5\ \text{fm}$$

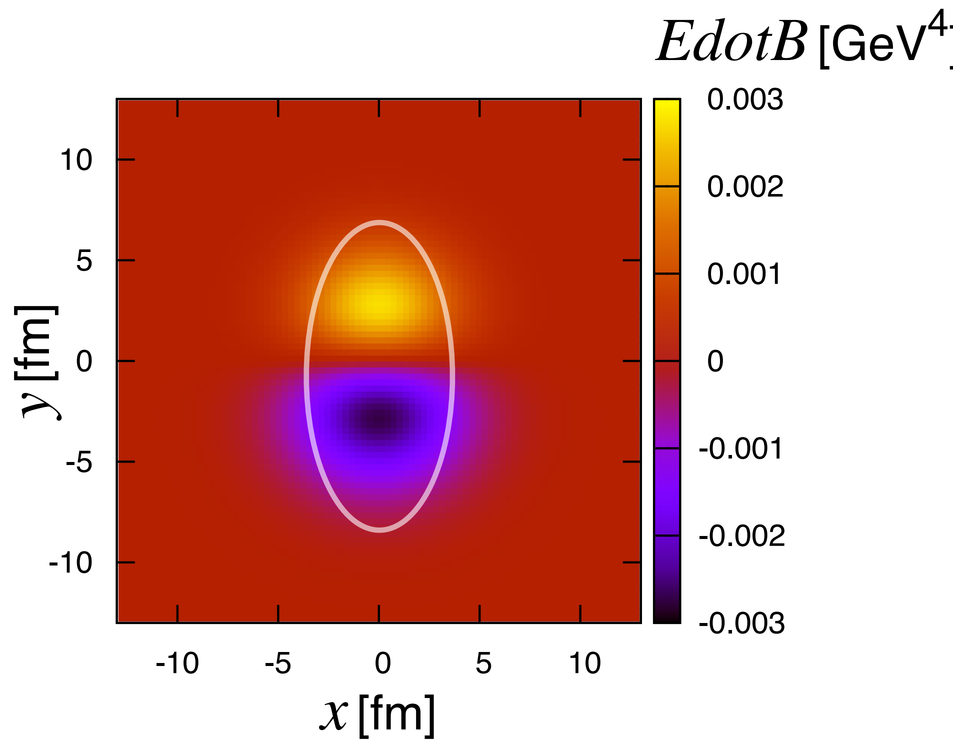
$$r_y = \sqrt{x^2 + (y/2)^2 + z^2}$$





External EM Fields





Parameters

$$eB_y(\vec{r}) = eB_0 \exp\left(-\frac{r_z^2}{2\sigma_{EB}^2}\right) \times \exp\left(-\frac{t}{\tau_B}\right)$$

$$eB_0 = 0.08 \text{ GeV}^2$$

$$\sigma_{EB} = 4 \text{ fm} \quad \tau_B = 3 \text{ fm}$$

$$eE_y(\vec{r}) = y \times eE_0 \exp\left(-\frac{r_z^2}{2\sigma_{EB}^2}\right) \times \exp\left(-\frac{t}{\tau_E}\right)$$

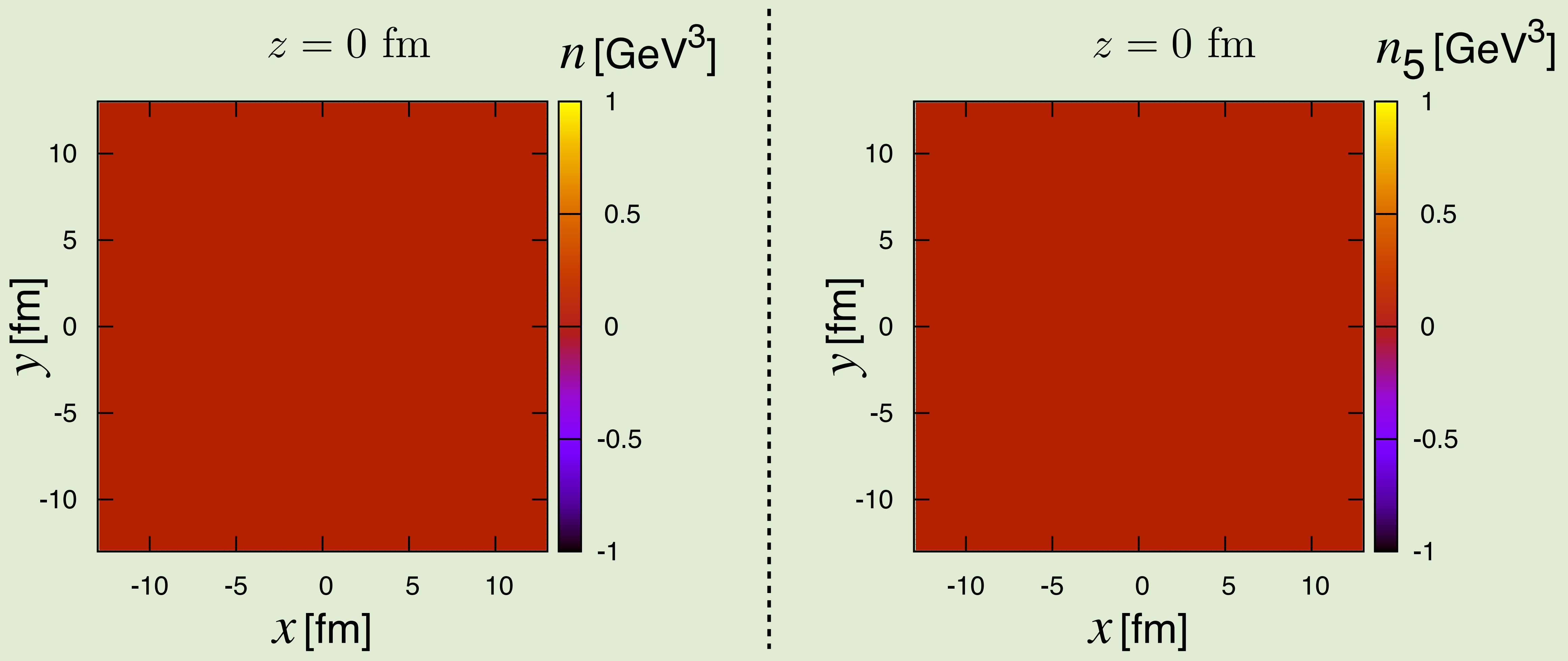
$$eE_0 = 0.02 \text{ GeV}^2$$

$$\sigma_{EB} = 4 \text{ fm} \quad \tau_E = 1 \text{ fm}$$

$$r_z = \sqrt{x^2 + y^2 + (2z)^2}$$

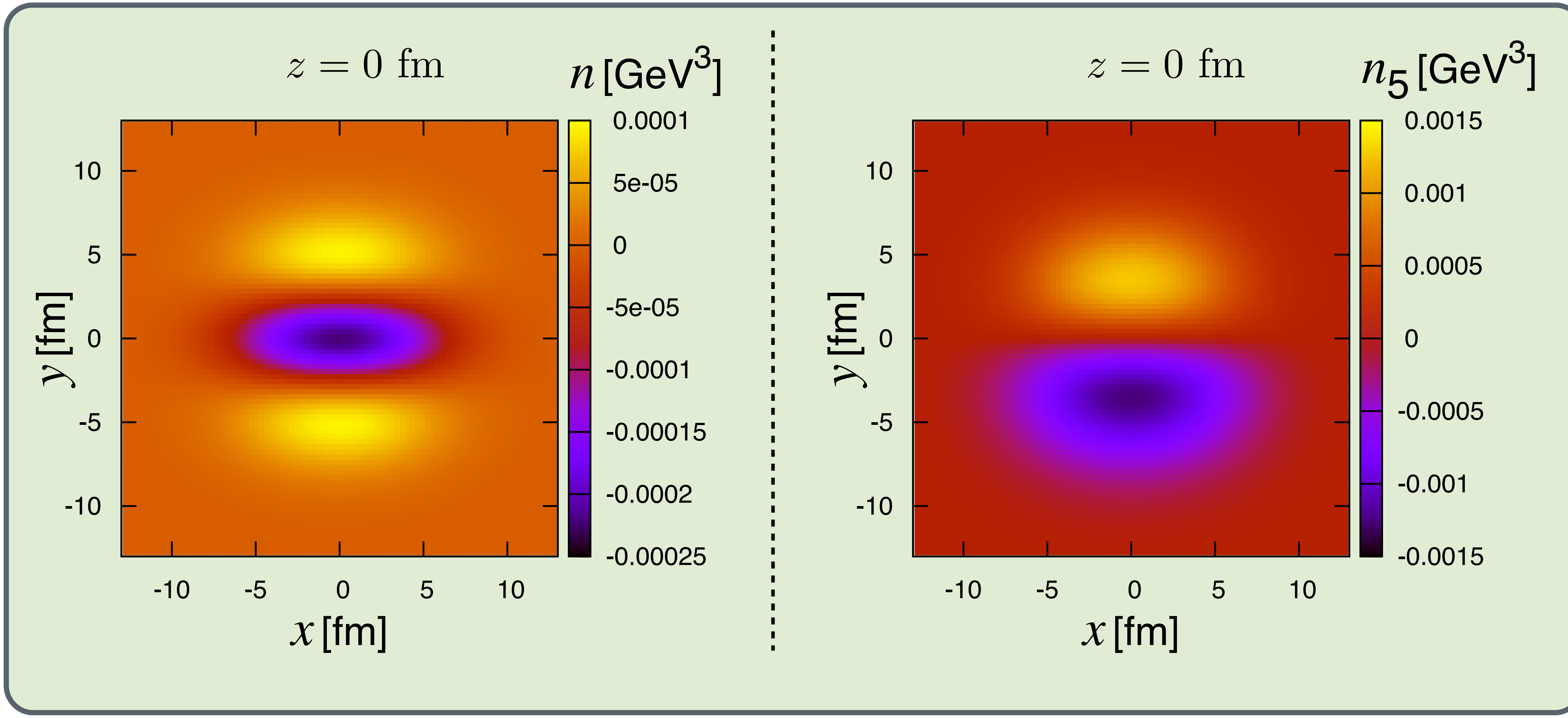
Charge distributions at initial time

◇ Vector/Axial charge distribution($t = 0.0$ fm)

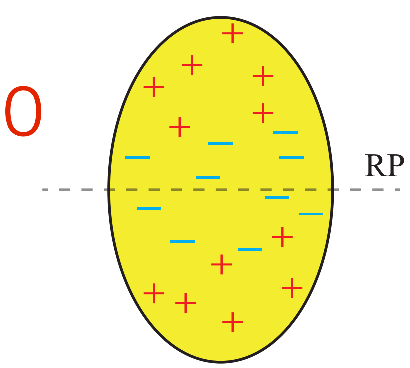


■ Deformation of the charge distribution

◇ Vector/Axial charge distribution($\boxed{t = 6.0\ \mathrm{fm}}$)



– Quadrupole deformation of charge distribution even if $A_{\pm}$=0



Origin of $v_2(\pi^-) > v_2(\pi^+)$ in high energy collision?

Charge dependent elliptic flow $\Delta v_2^{\pm}$

Numerical Simlation for v_2

Initial condition: $T(\vec{r}),\ \mu(\vec{r}),\ \mu_5(\vec{r}),$ ... **at t = 0 fm**

Anomalous hydro simulation

Freeze out: $T(\vec{r}),\ \mu(\vec{r}),\ \mu_5(\vec{r}),$ ... **at t = 6 fm**

Cooper-Frye formula

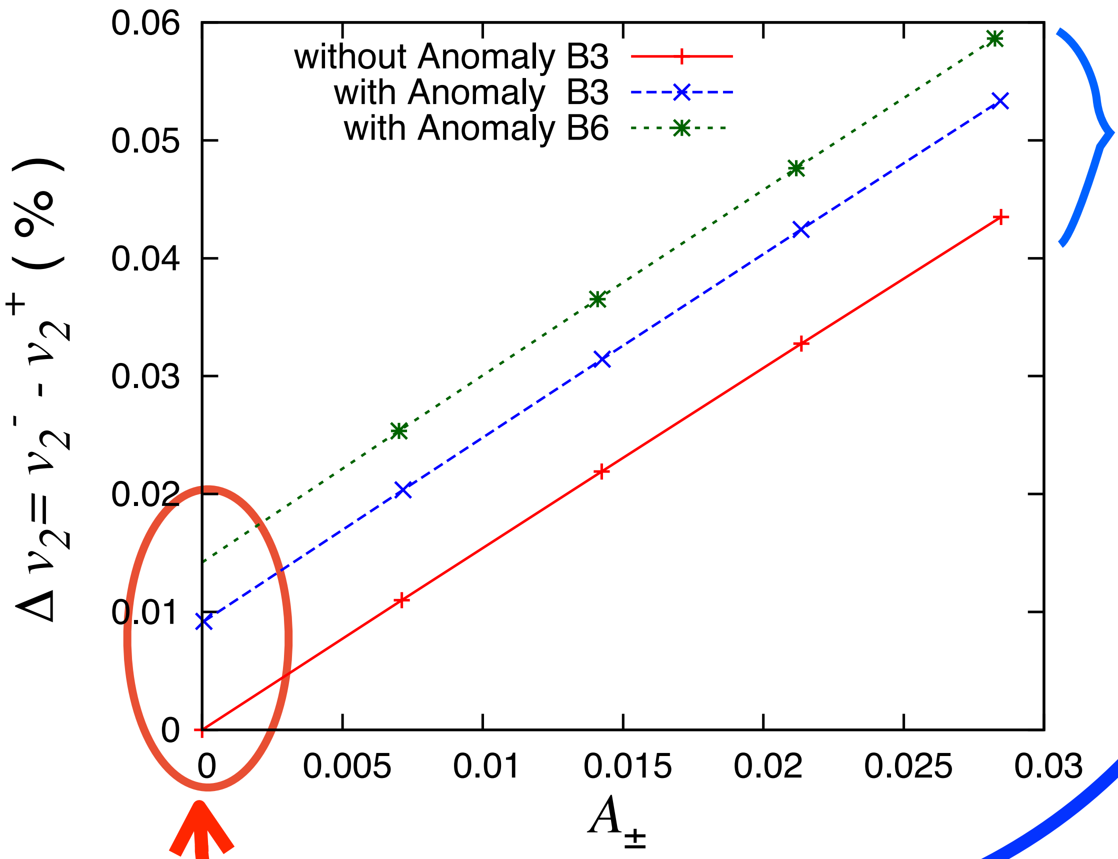
Azimuthal particle distribution: $\frac{dN_\pm}{d\phi},\ A_\pm \equiv \frac{\overline{N}_+ - \overline{N}_-}{\overline{N}_+ + \overline{N}_-}$

Charge dependent v_2

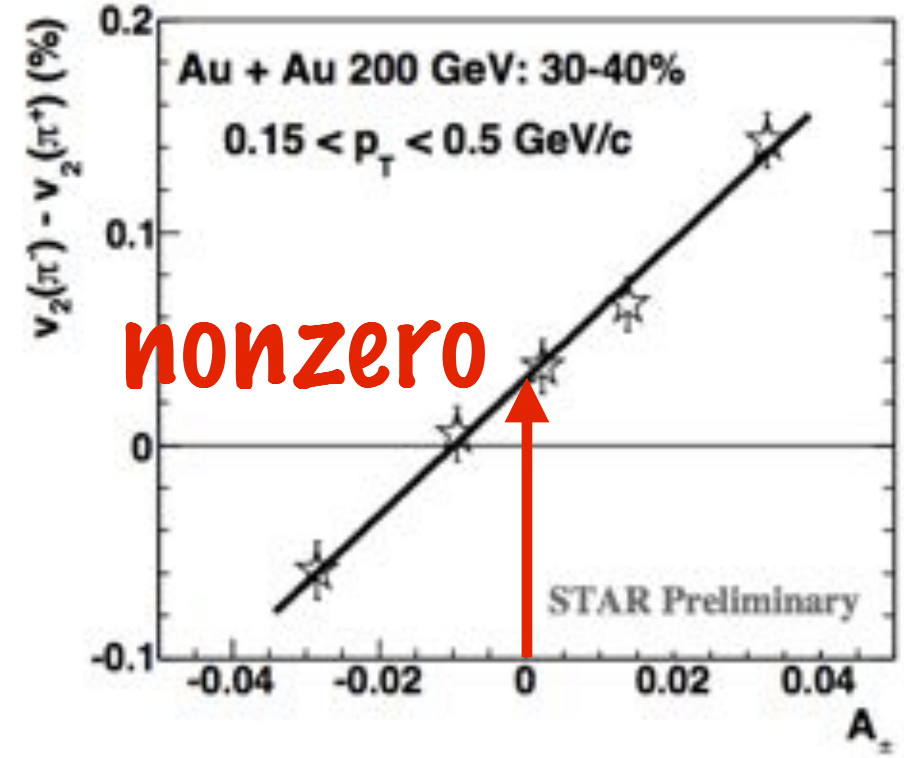
$\mu_0 = 0 - 10\ \mathrm{MeV}$

$A_\pm(\mu_0) = 0, \ldots$

Results of $\Delta v_2^{\pm}$



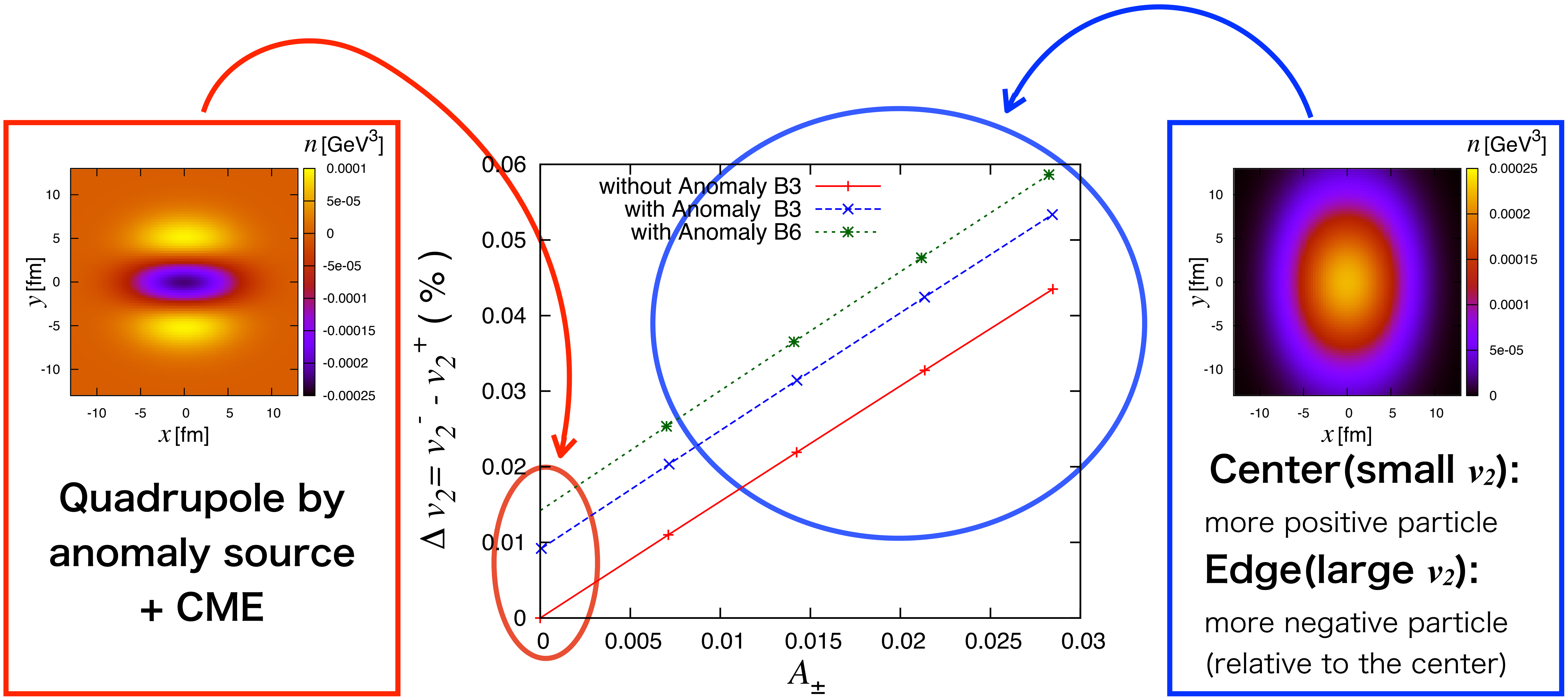
cf. Experimental result



– slope r_e **is not sensitive** to the existence of anomaly

– intercept $\Delta v_2^{\pm}(A_\pm = 0)$ **is sensitive** to the anomaly

Physical Interpretation for $\Delta v_2^{\pm}$



Quadrupole by anomaly source + CME

Center(small v_2):

more positive particle

Edge(large v_2):

more negative particle (relative to the center)

Need more realistic study by anomalous hydro

Summary

Summary

■ Anomalous hydrodynamic simulation

– Chiral Magnetic Wave in an expanding plasmas

■ Proposal in observables

– Charge dependent elliptic flow : $\Delta v_2^{\pm}(A_{\pm}) = r_e A_{\pm} + \Delta v_2^{\pm}(A_{\pm} = 0)$

slope parameter r_e

→ **not** sensitive to the existence of the anomaly

intercept $\Delta v_2^{\pm}(A_{\pm} = 0)$

→ **sensitive** to the existence of the anomaly

Outlook

- Realistic setup for heavy ion collisions
- Dissipative effects (conductivity, diffusion)
- Chiral vortical effect
- Dynamical electromagnetic field