Dynamical freeze-out in event-by-event hydrodynamics

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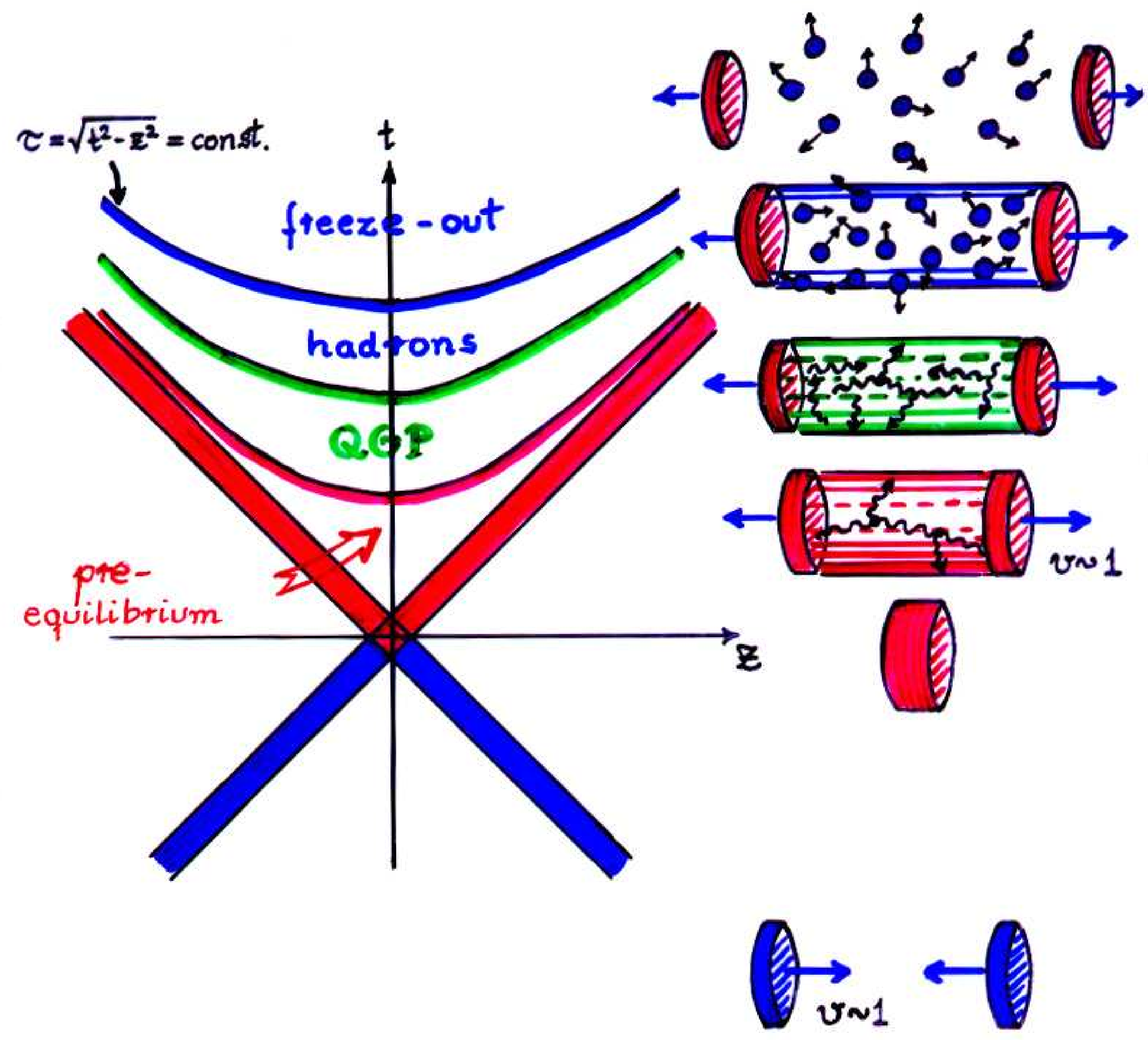
The space-time picture:



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Hydrodynamics

local conservation of energy, momentum and baryon number:

$$\partial_\mu T^{\mu\nu}(x) = 0$$
$$\partial_\mu N^{\mu}(x) = 0$$

partial differential equations
$\Longrightarrow$ **what are the boundary conditions?**

– ***"Hydro doesn't know where to start nor where to end"***

Unknowns: initial state, final state

Freeze-out

- **Kinetic equilibrium requires scattering rate $\gg$ expansion rate**
- **this not valid $\rightarrow$ system behaves as free streaming particles**
- **momentum distributions cease to evolve $\rightarrow$ they "freeze-out"**

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$$\boxed{\frac{1}{K_n} = \frac{\tau_{\mathrm{scat}}^{-1}}{\partial_\mu u^\mu} \approx 1}$$

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- $\tau_{\mathrm{scat}}^{-1} \propto T^4 \rightarrow$ rapid transition to free streaming
- **Approximation:** decoupling takes place on constant temperature hypersurface $T = T_{\mathrm{fo}}$

Dynamical criterion

- **need to evaluate**

$$\frac{1}{K_n} = \frac{\tau_{\rm scat}^{-1}}{\partial_\mu u^\mu}$$

- $\partial_\mu u^\mu$ **known from hydro**
- $\tau_{\rm scat}^{-1}$**?**

Dynamical criterion

- **need to evaluate**

$$\frac{1}{K_n} = \frac{\tau_{\text{scat}}^{-1}}{\partial_\mu u^\mu}$$

- $\partial_\mu u^\mu$ **known from hydro**
- τ_{scat}^{-1}**?**
  - **Prakash *et al.*, Phys. Rept. 227, 321 (1993):**

    Parametrization: Daghigh & Kapusta, Phys. Rev. D 65, 064028 (2002)

$$\tau_{\pi\pi}^{-1}(T) \approx 16 \left(\frac{T}{100\ \text{MeV}}\right)^4 \text{MeV}$$

- **pions only, chemical equilibrium**

Scattering rates

- **evaluate scattering rate of pions in thermal hadron gas**
  - **number of scatterings:** $N = F_1 N_2 \sigma_{12} = n_1 |\vec{v}_{12}| N_2 \sigma_{12}$
  - $|\vec{v}_{12}| = \sqrt{(s - s_a)(s - s_b)}/(2E_a E_b)$
    where $s_a = (m_1 + m_2)^2$ **and** $s_b = (m_1 - m_2)^2$
  - **fold over thermal distributions**
  - **sum over all scattering partners**
  - **scatterings per pion** $\rightarrow$ **divide by pion density**

$$\tau_{\rm scat}^{-1} = \frac{1}{n_\pi(T,\mu_\pi)} \sum_i \int {\rm d}^3 p_\pi\, {\rm d}^3 p_i\, f_\pi(T,\mu_\pi) f_i(T,\mu_\pi) \frac{\sqrt{(s-s_a)(s-s_b)}}{2E_\pi E_i} \sigma_{\pi i}(s)$$

- **what is $\sigma_{\pi i}$?**

Cross sections

- **as in UrQMD:**
  - $\sigma_{\pi i}(s)$ **for resonance formation using Breit-Wigner**

$$\sigma_{\pi i}(s) = \sum_R \sigma_{\pi i \to R}(s)$$

  - **estimate** $\sigma_{\pi m}(s)$ **for elastic** π**-meson scattering**

$\Rightarrow$ **check that the result fits the cross section data**

Cross sections

$$\sigma_{\pi i\to R}(s) = \frac{2S_R+1}{(2S_\pi+1)(2S_i+1)}\,\frac{\pi}{{p_{\rm CMS}}^2}\,\frac{\Gamma_{R\to\pi i}(\sqrt{s})\,\Gamma_{tot}(\sqrt{s})}{(m_R-\sqrt{s})^2+{\Gamma_{tot}}^2(\sqrt{s})/4}$$

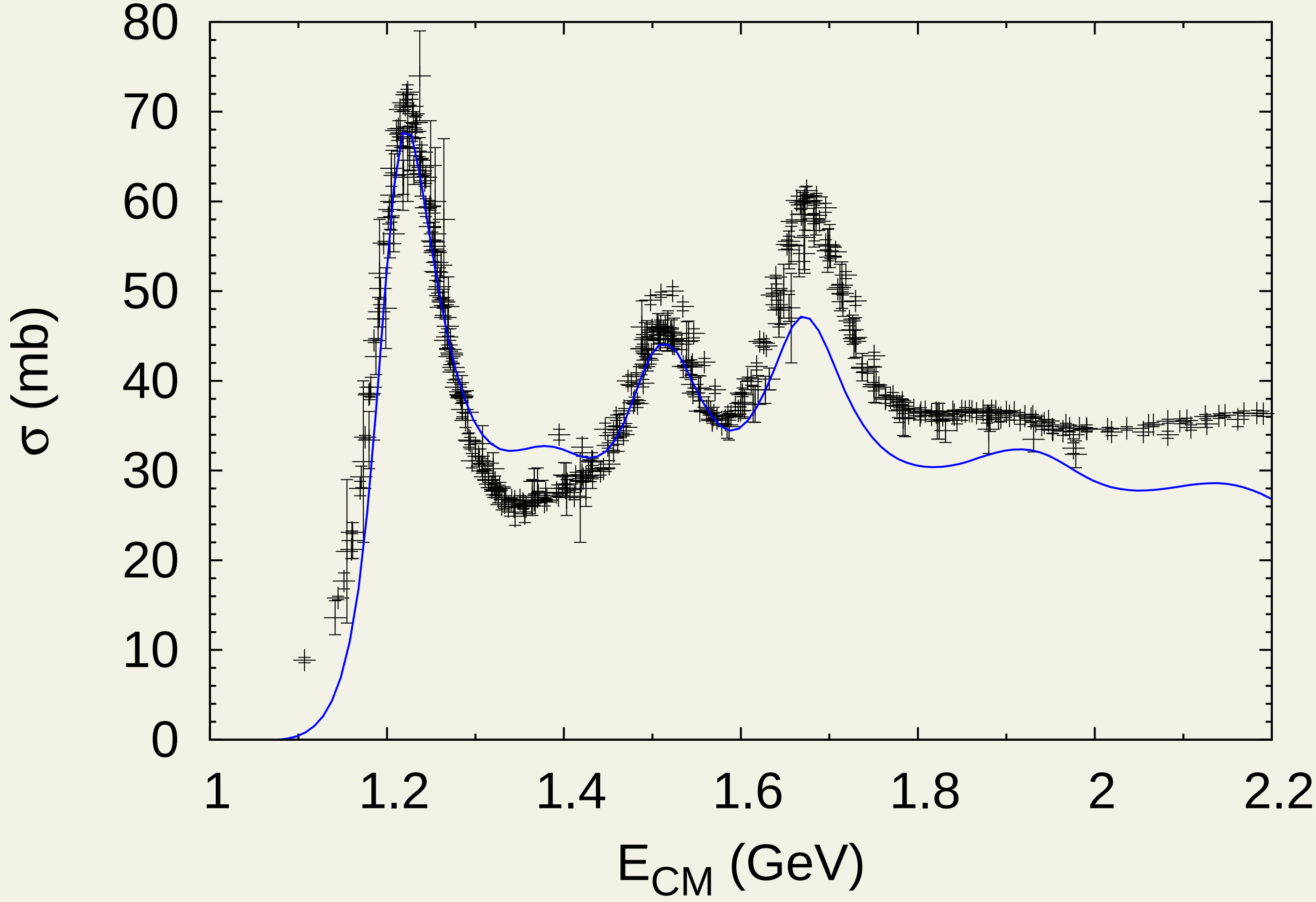
where

- S_j **is spin**
- $p_{\rm CMS}$ **is particle momenta in CMS**
- Γ_{tot} **and** $\Gamma_{R\to\pi i}$ **total and partial decay widths:**

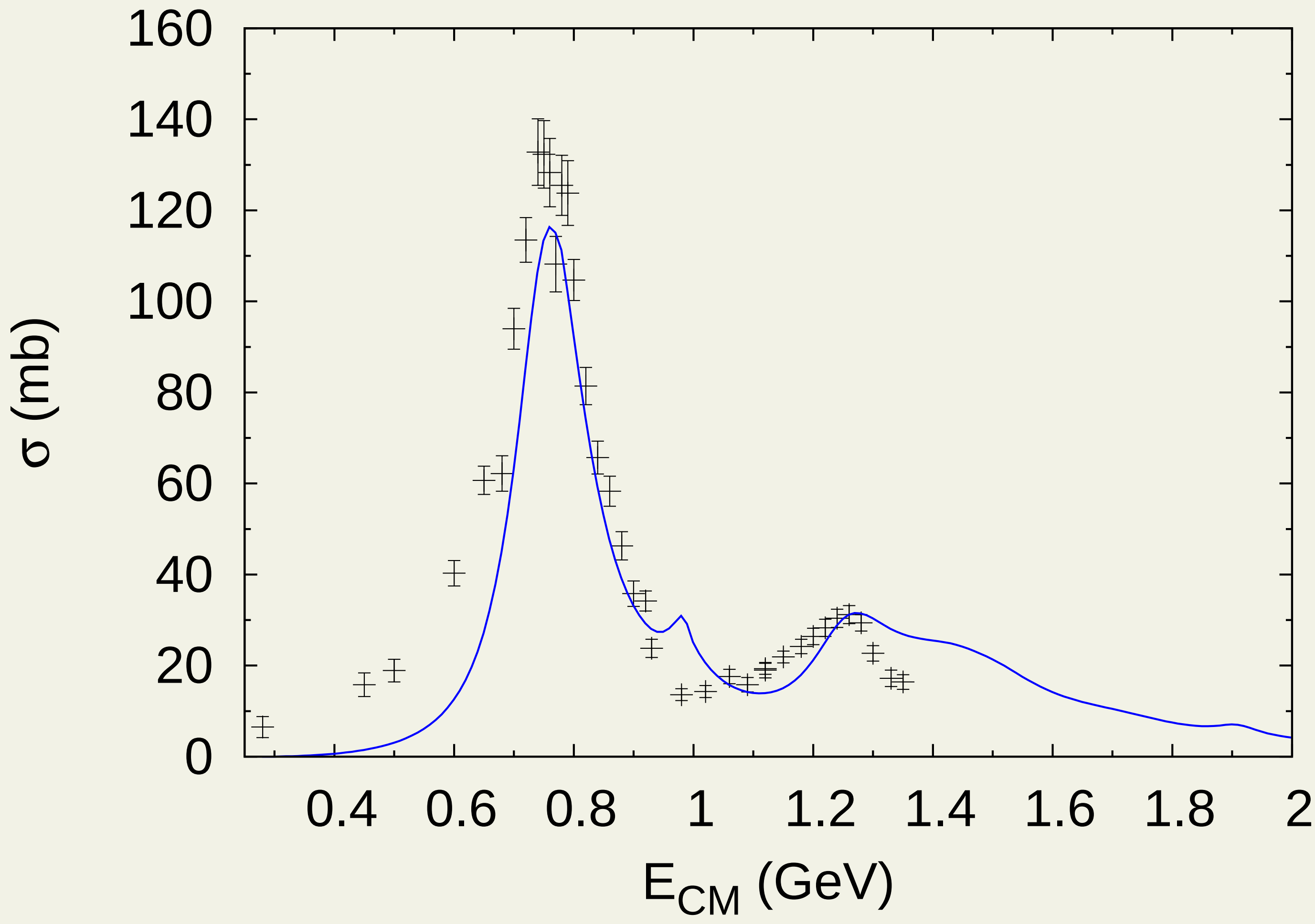
$$\Gamma_{R\to\pi i}(M) = \Gamma_R^{\pi i}\,\frac{m_R}{M}\left(\frac{p_{\rm CMS}(M)}{p_{\rm CMS}(m_R)}\right)^{2l+1}\frac{1.2}{1+0.2\left(\frac{p_{\rm CMS}(M)}{p_{\rm CMS}(m_R)}\right)^{2l}}$$

- **Note: scattering partner** i **can be a resonance!**

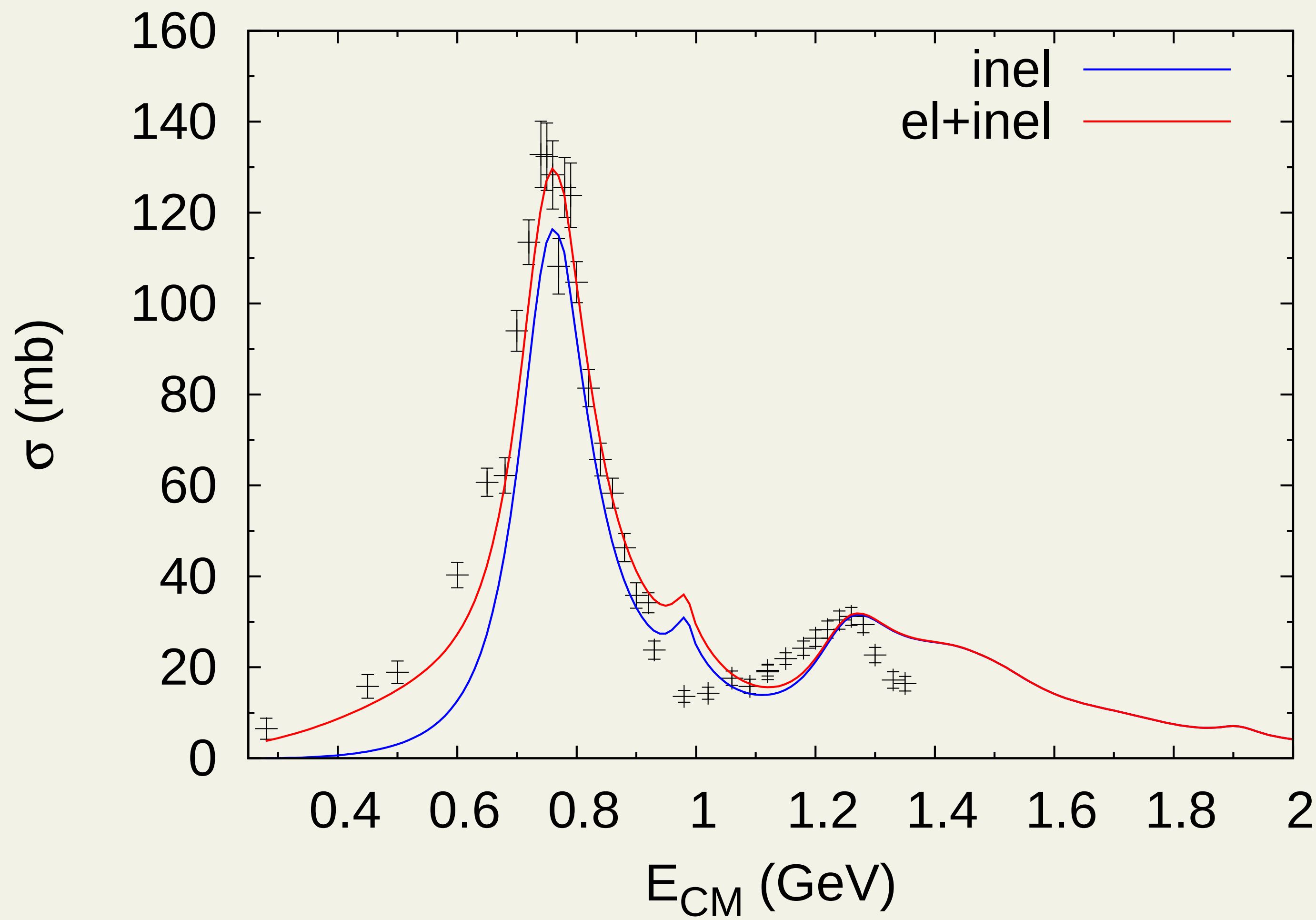
$\sigma_{\pi^- p}$



$\sigma_{\pi^+\pi^-}$



$\sigma_{\pi^+\pi^-}$

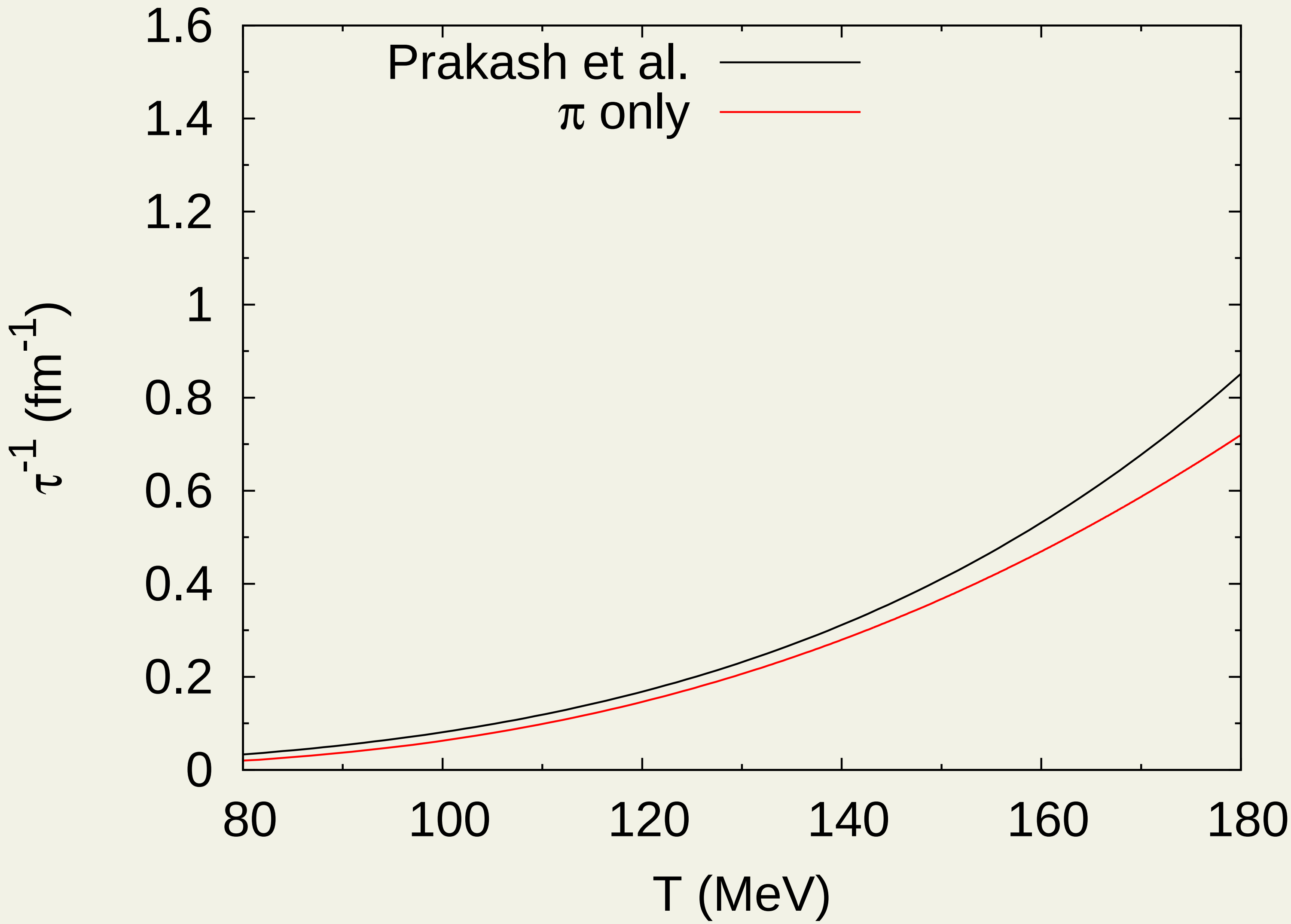


– **elastic meson-meson scattering** $\sigma_{mm} = 5$ **mb**

– **elastic $\pi\pi$ scattering** $\sigma_{\pi\pi} = \sigma_0 e^{-(\sqrt{s}-m_0)^2/w}$

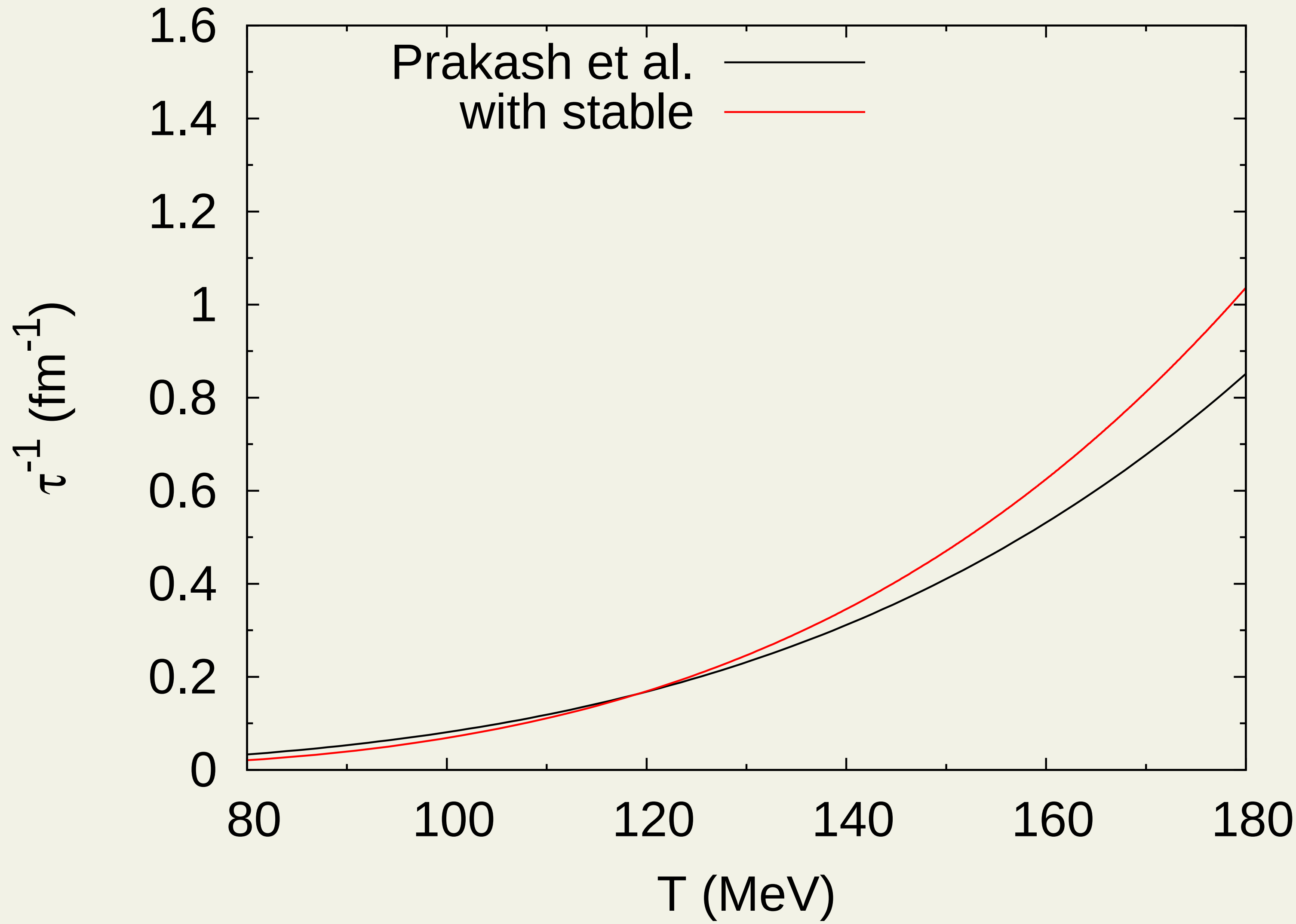
$\sigma_0 = 15$ **mb,** $m_0 = 0.65$ **GeV,** $w = 0.1$ **GeV**

Pions only

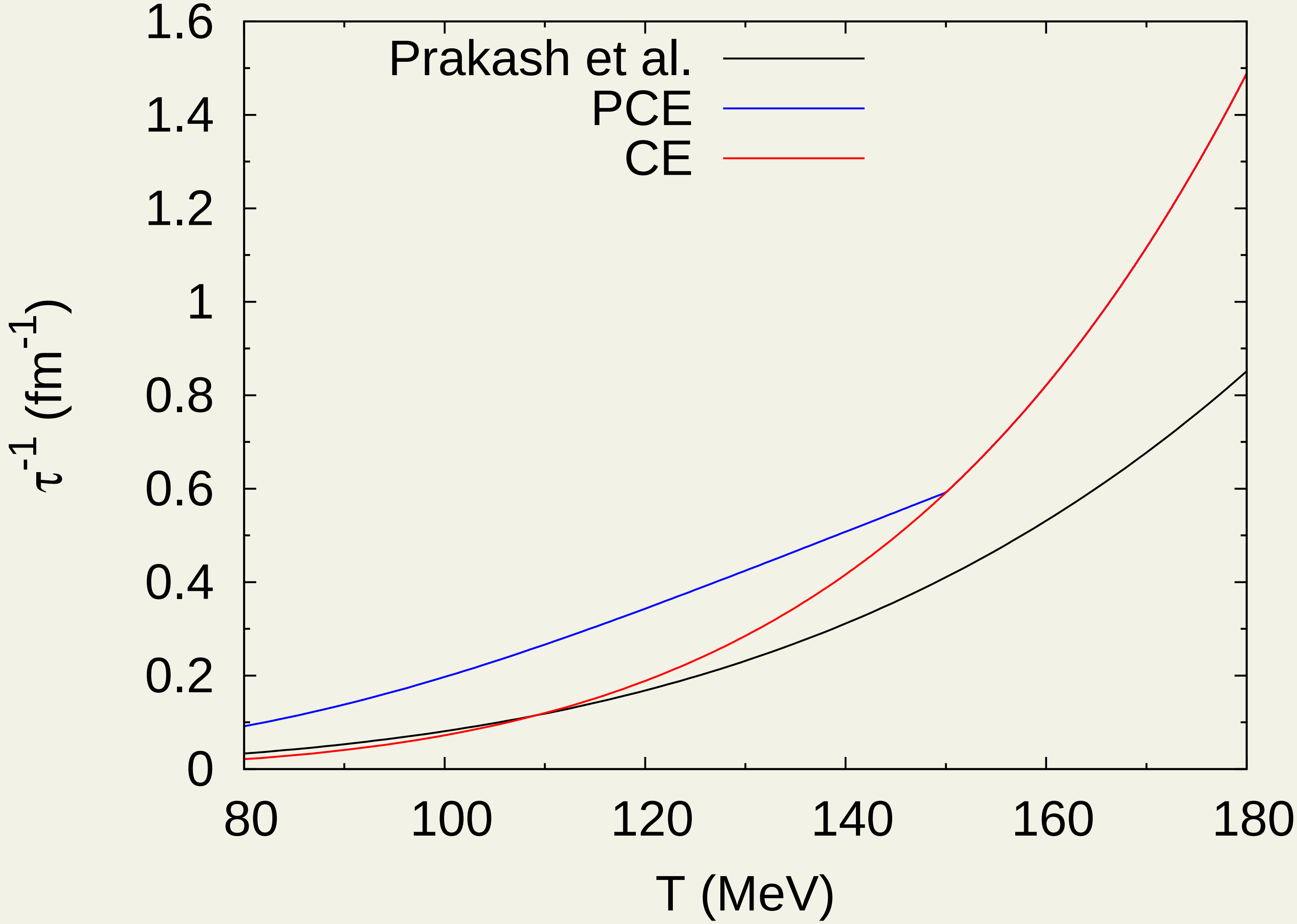


Prakash et al.
π only
τ^{-1} (fm^{-1})
T (MeV)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
80
100
120
140
160
180

Scattering with stable particles

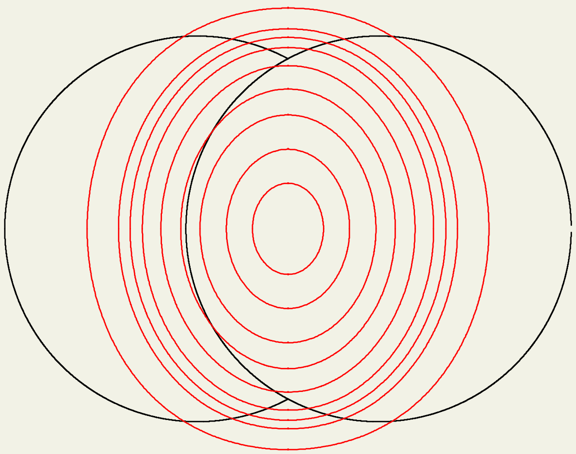


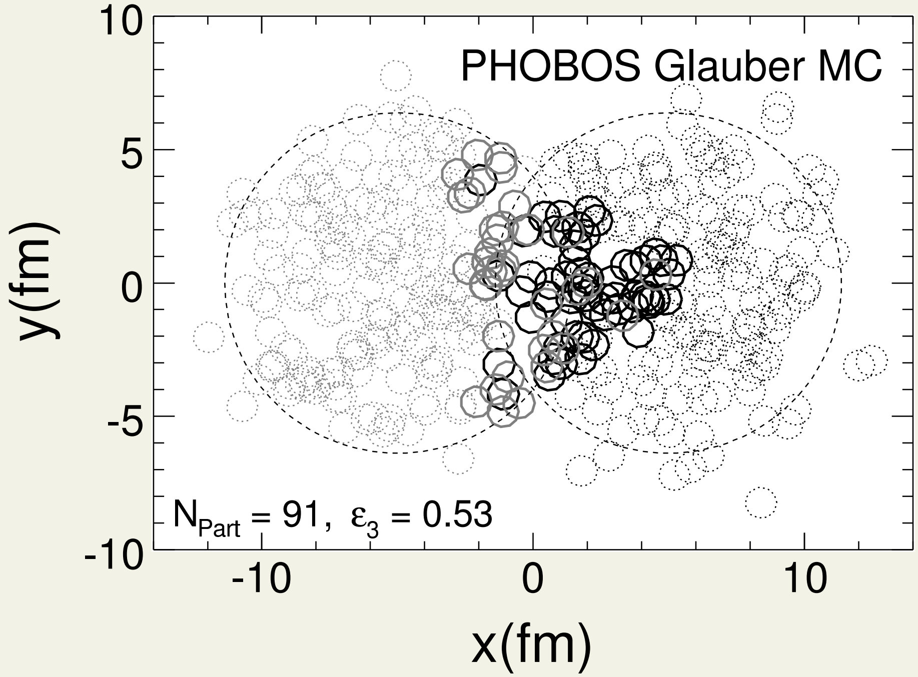
Total rate



Prakash et al.
PCE
CE
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
80
100
120
140
160
180
T (MeV)
τ^{-1} (fm^{-1})

event-by-event



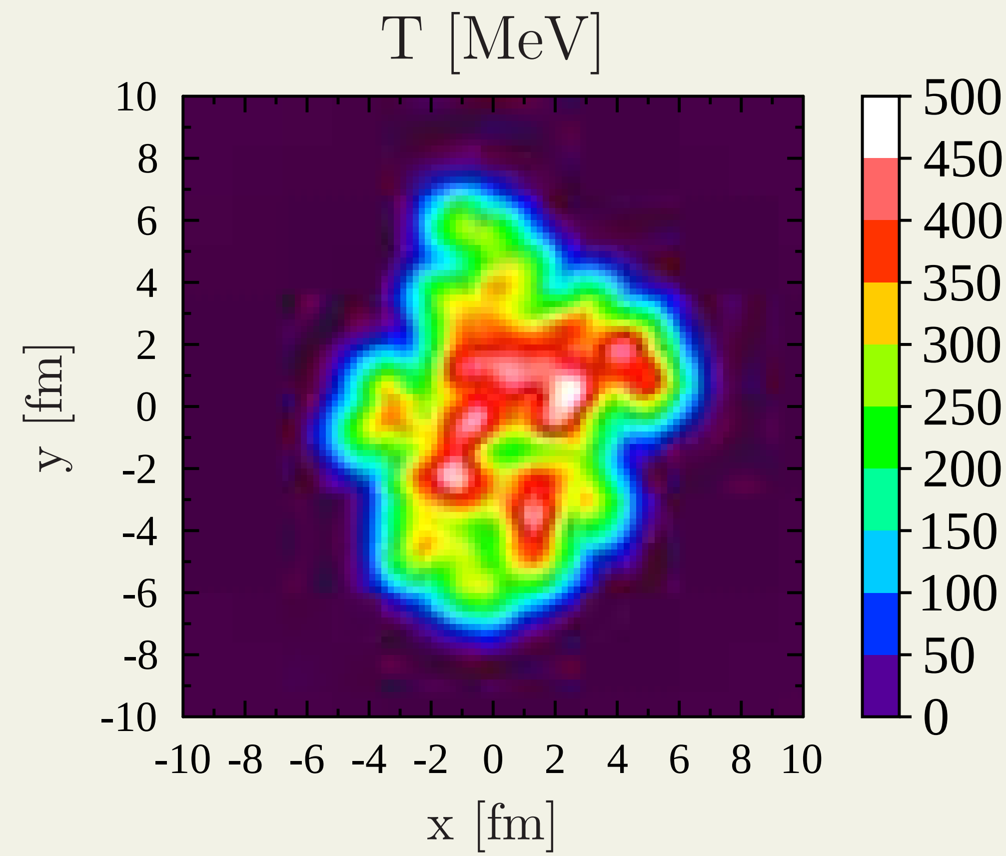


Alver and Roland, Phys.Rev.C81:054905,2010

- **shape fluctuates event-by-event**

Fluctuating initial states

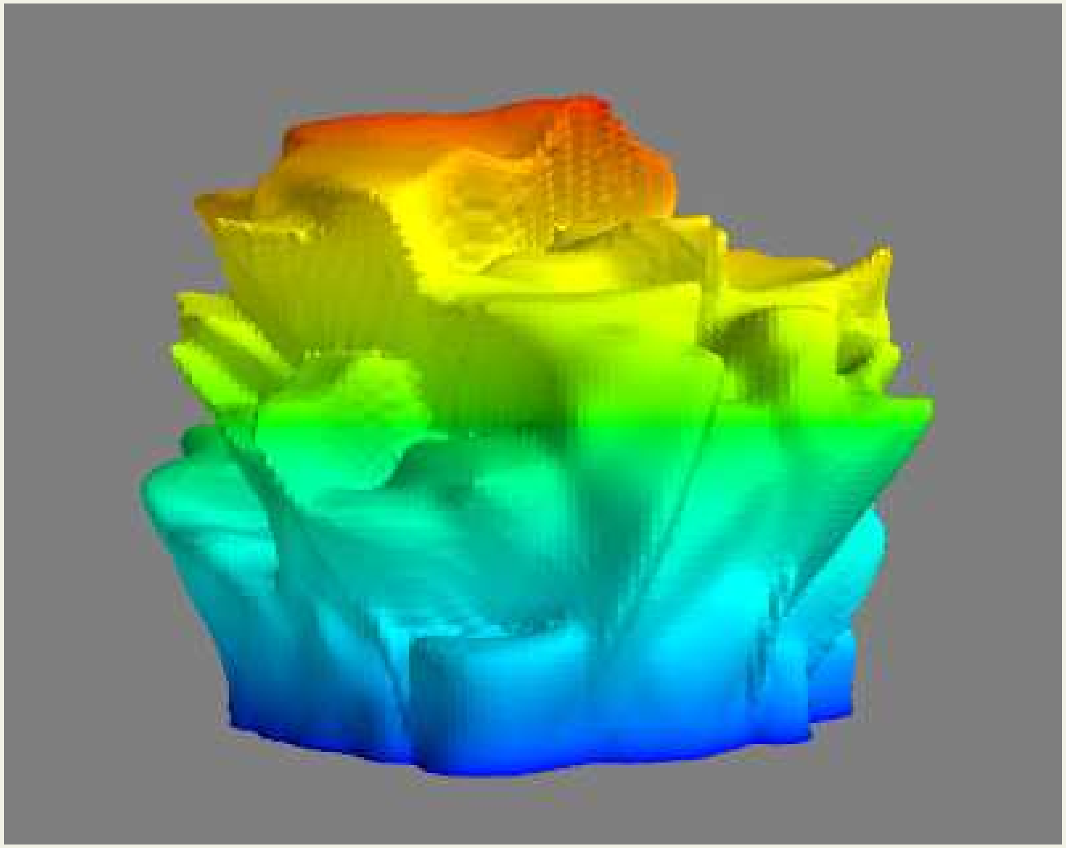
- **Monte Carlo Glauber**
- **Entropy density is distributed around the positions of WN and BC**
- **When distributing entropy we use Gaussian smearing:**



$$s(x,y) = \text{const.} \sum_{\text{wn,bc}} \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma^2}\right]$$

Constant temperature surface

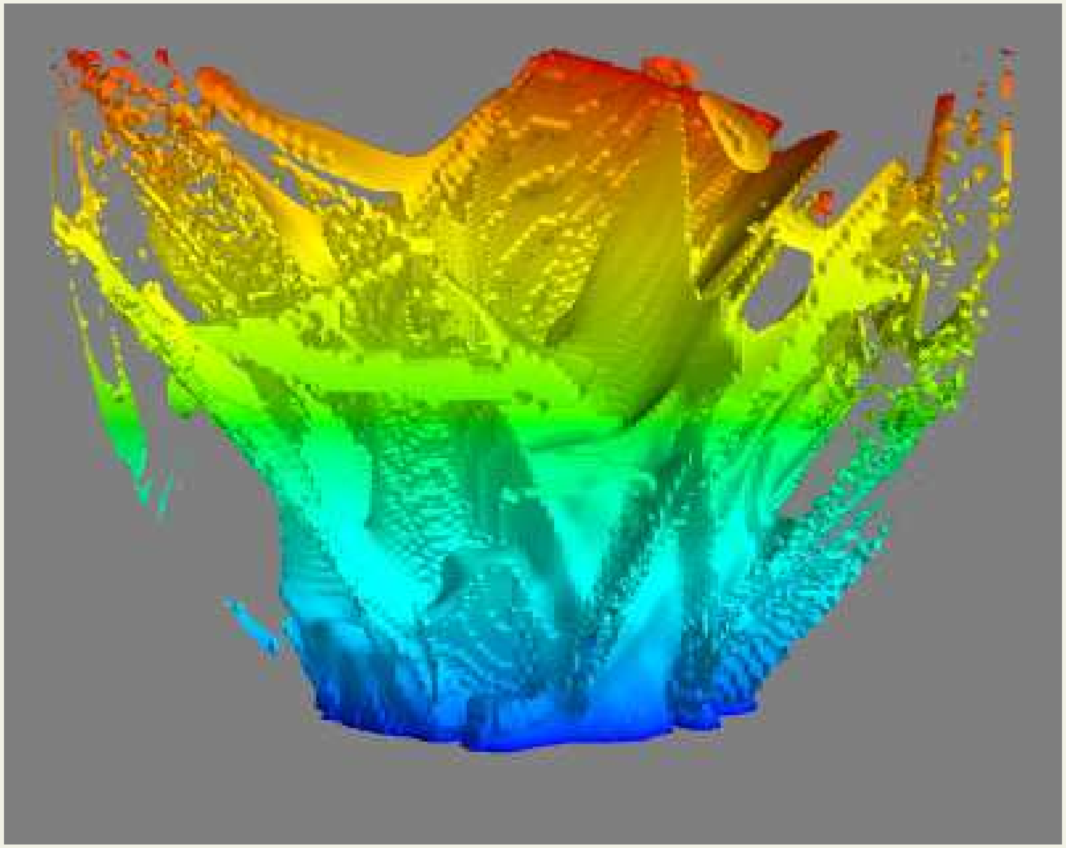
more structure than in the averaged case



Constant K_n surface

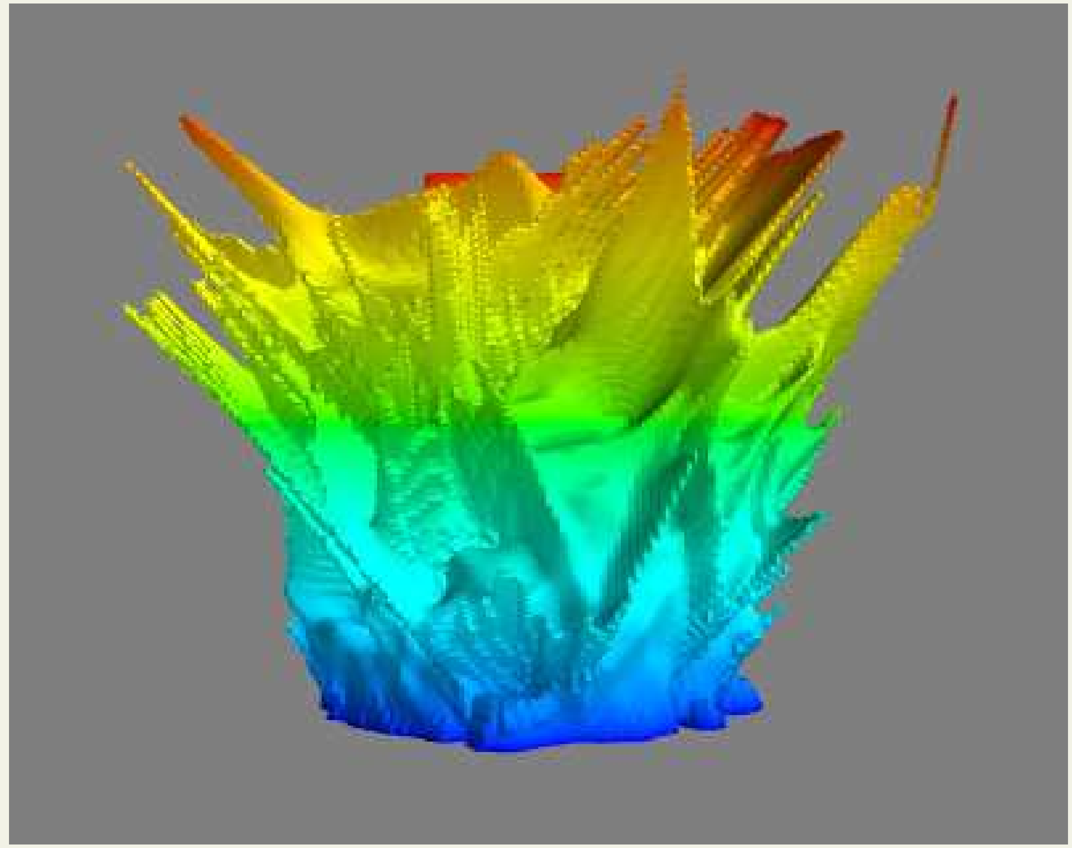
- many separate surfaces
- temperatures ~ 50 MeV
- fluid dynamics outside the surface

$\Longrightarrow$ compression, $K_n < 1$



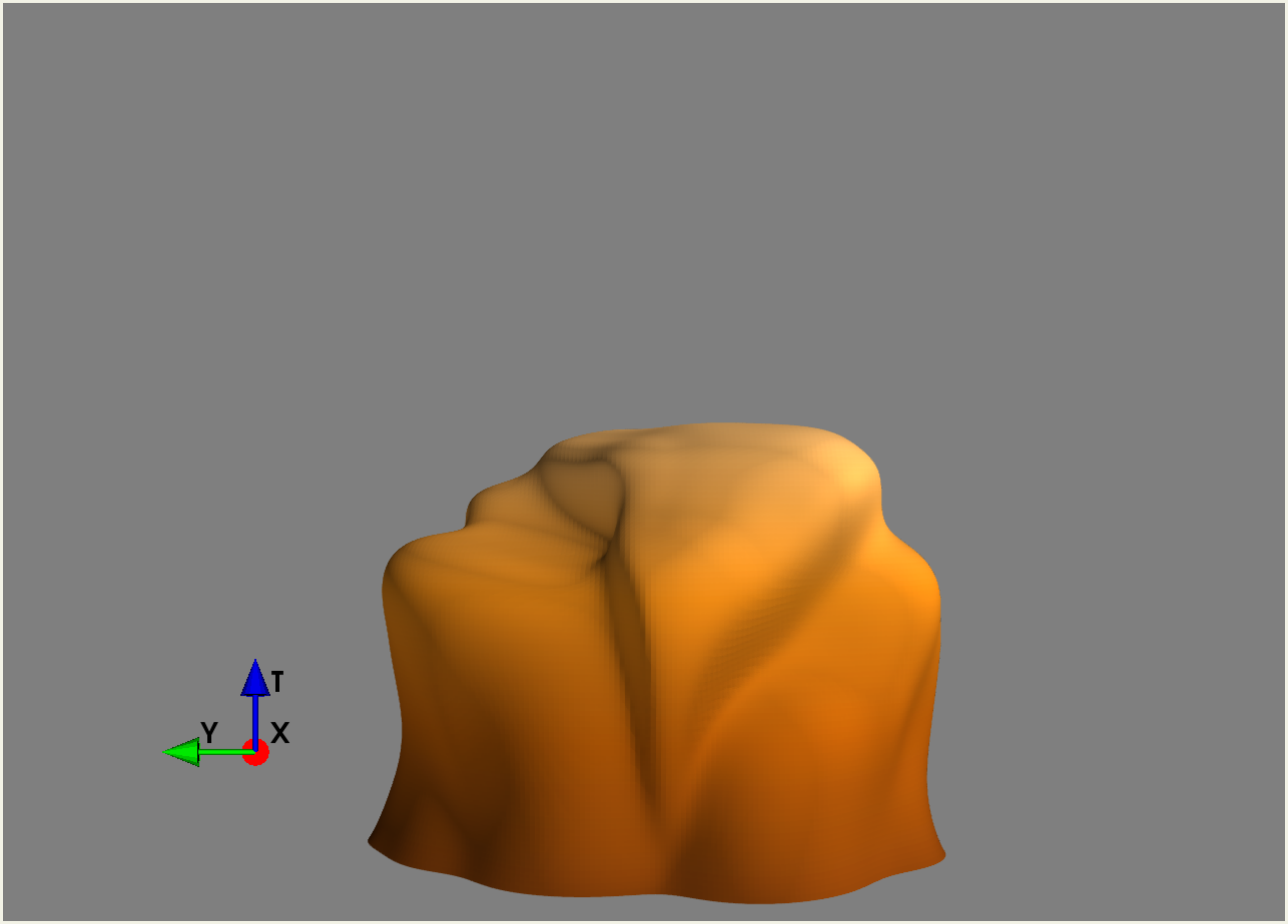
Constant K_n + no "backflow"

- once frozen out may not re-enter
- no droplets
- still fins and horns

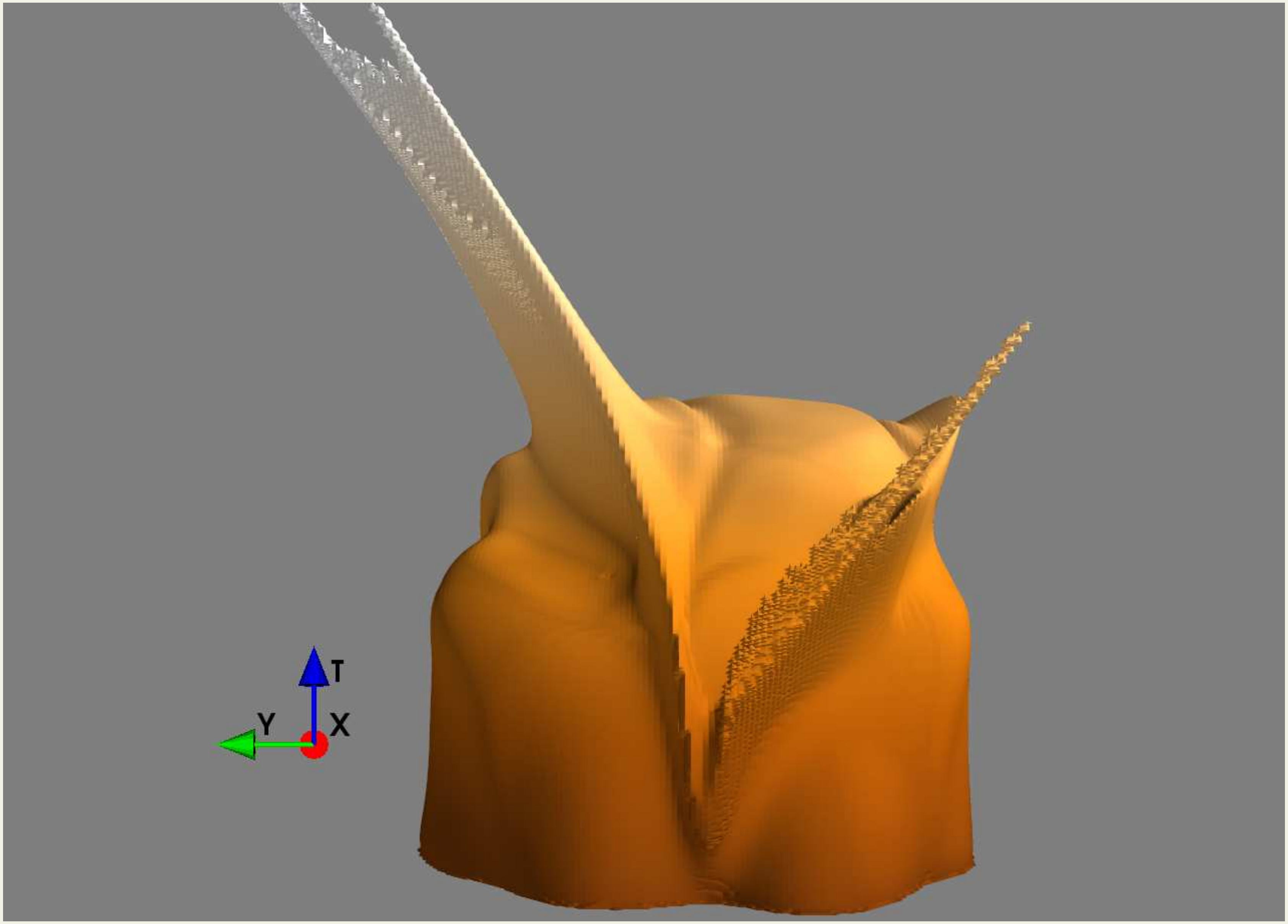


$\Longrightarrow$ **wider gaussian needed!** $\sigma = 0.8$ **fm instead of 0.4 fm**

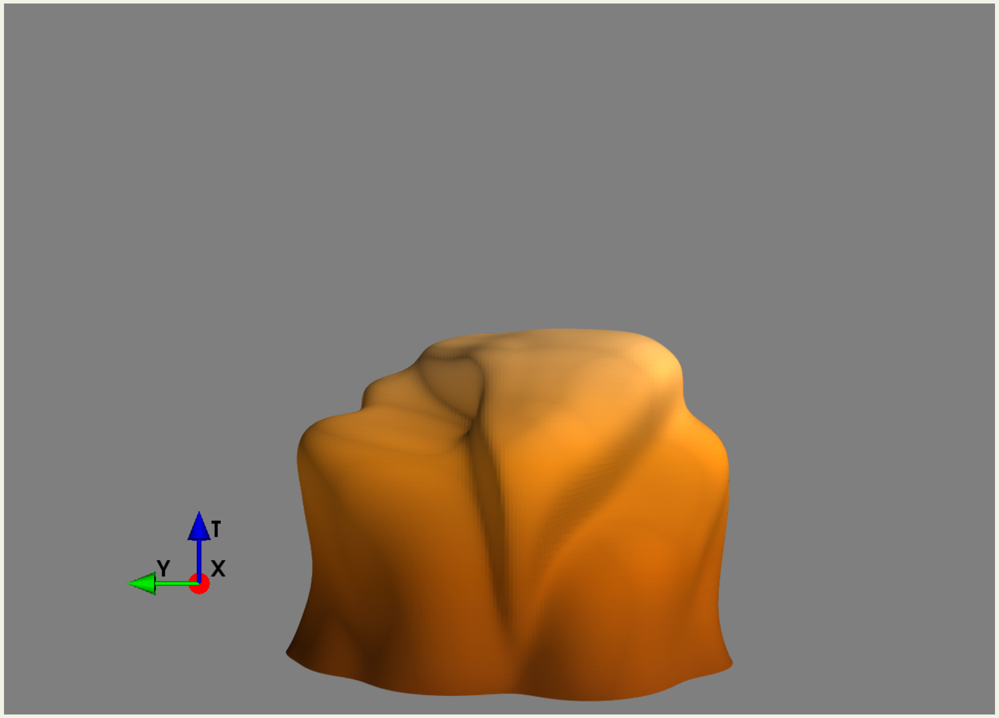
Constant temperature surface

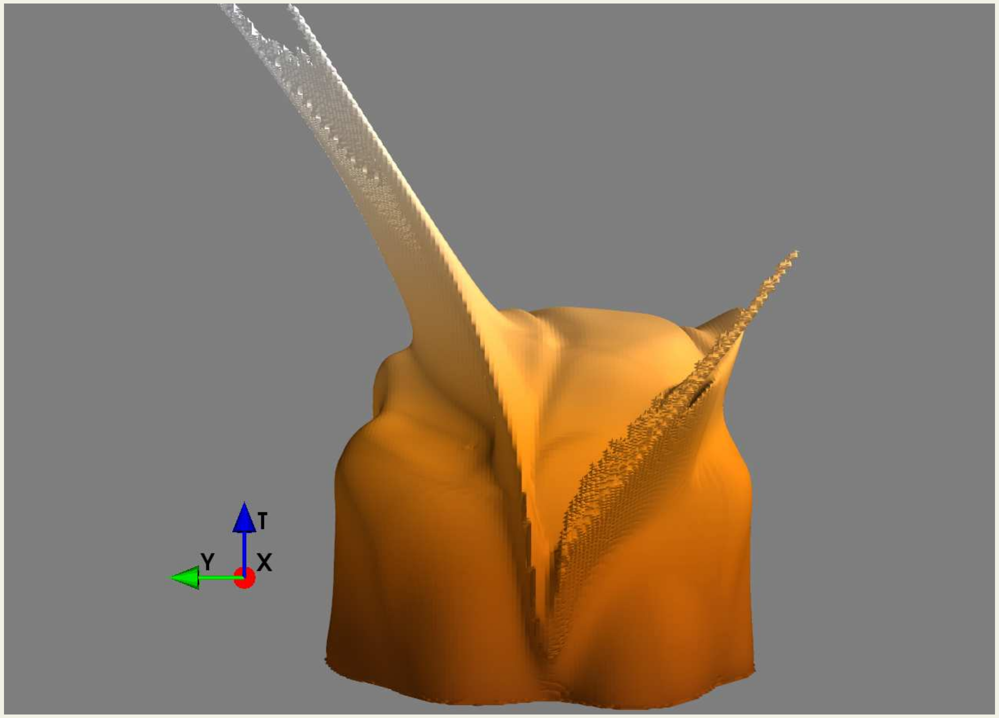


Constant K_n surface

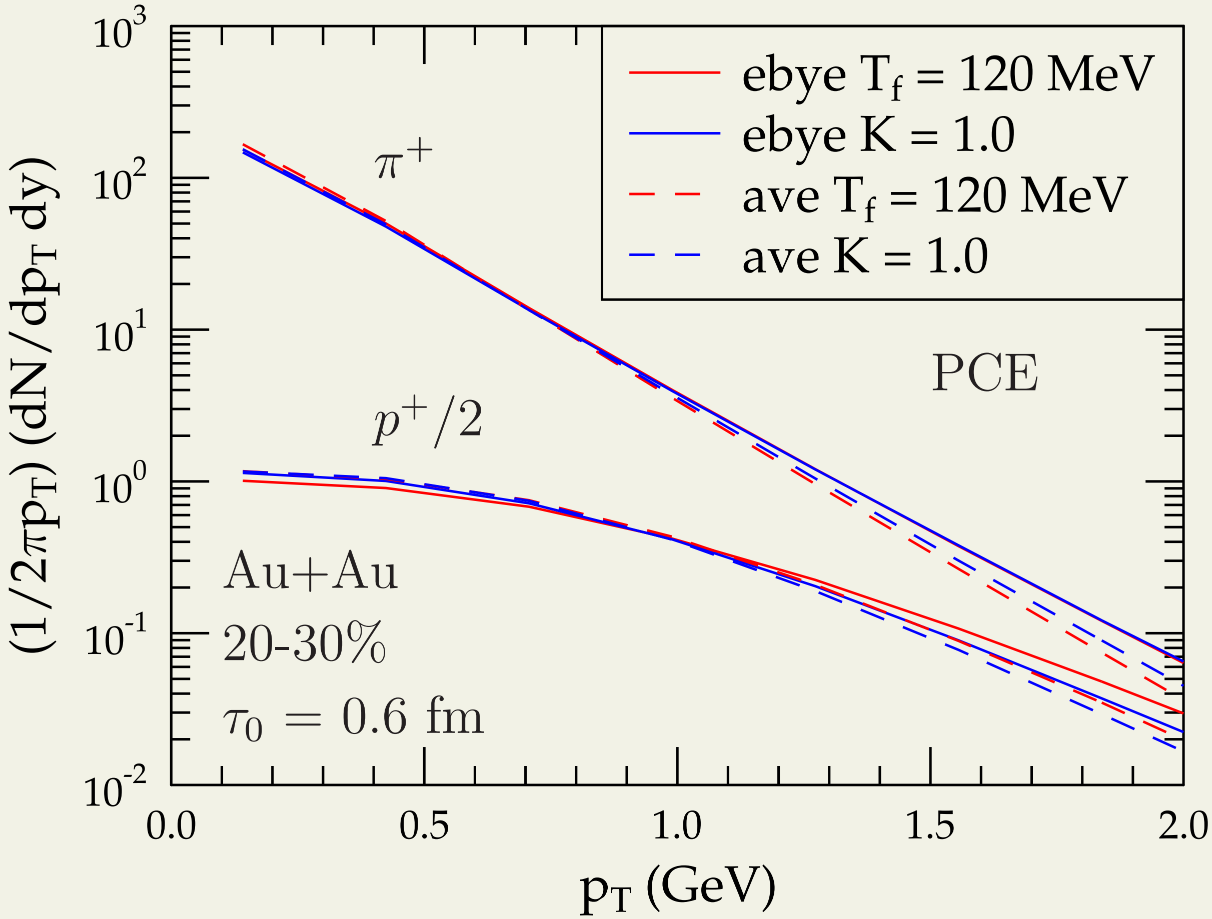


Constant T vs. constant K_n





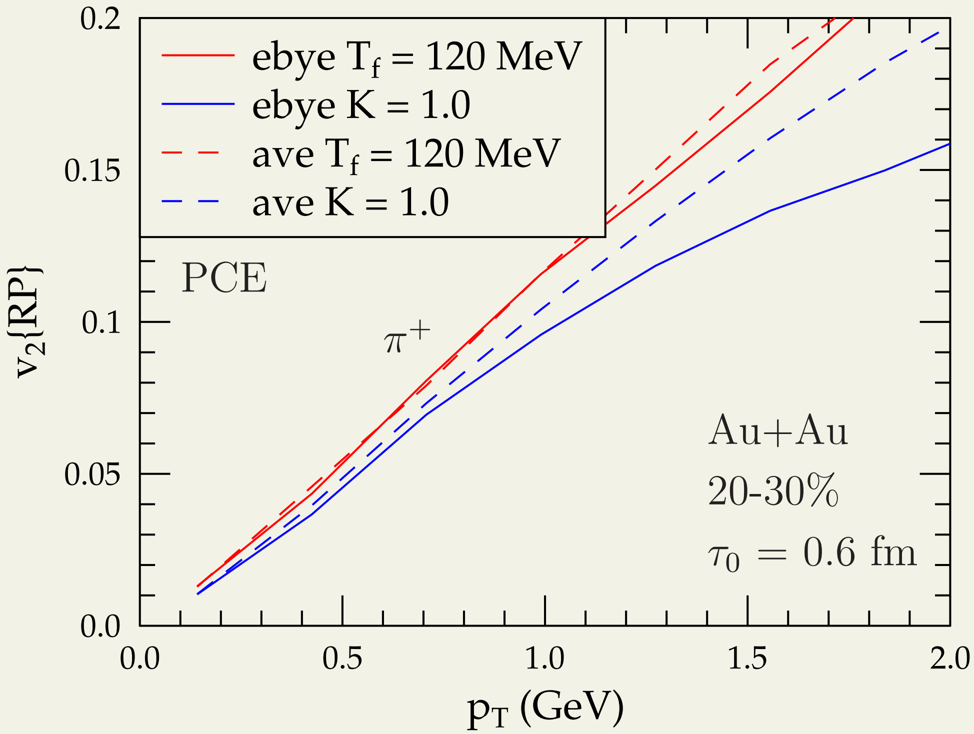
p_T spectra

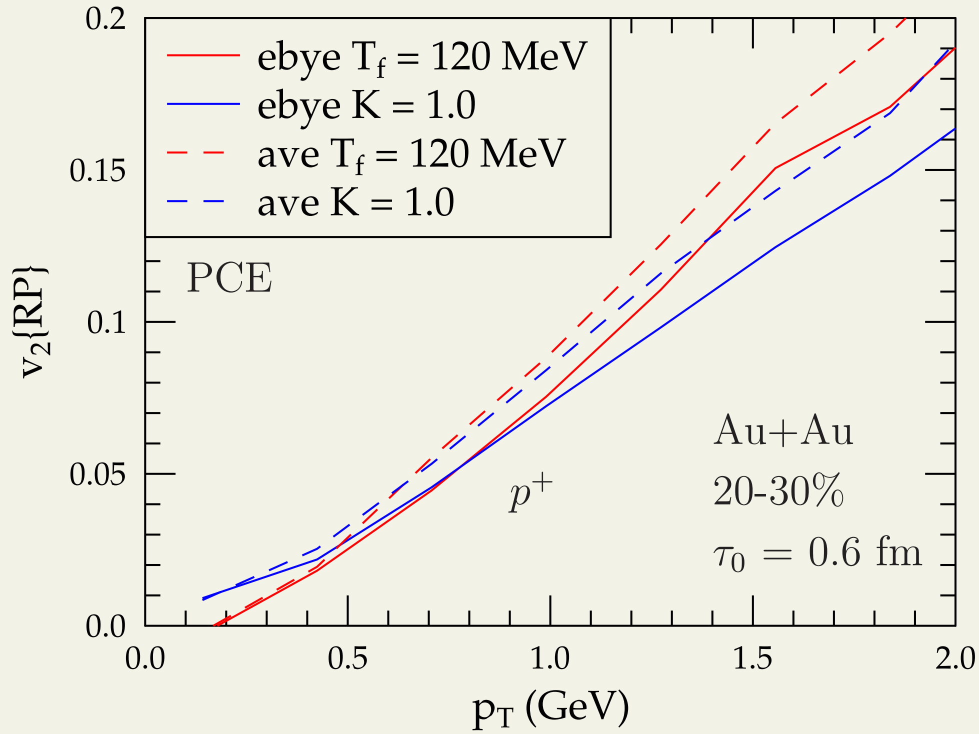


Main effect:

- **pions: fluctuations**
- **protons: both**

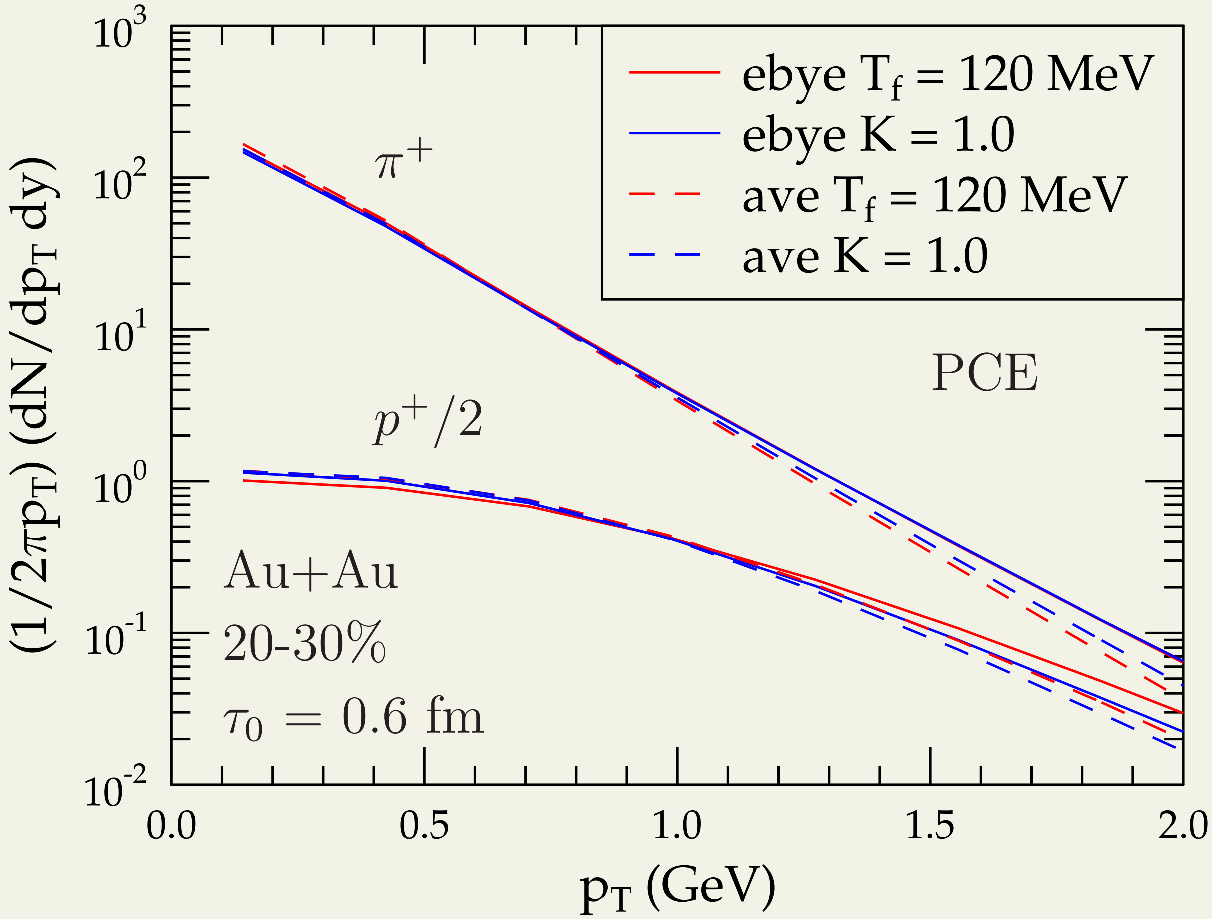
$v_2(p_T)$





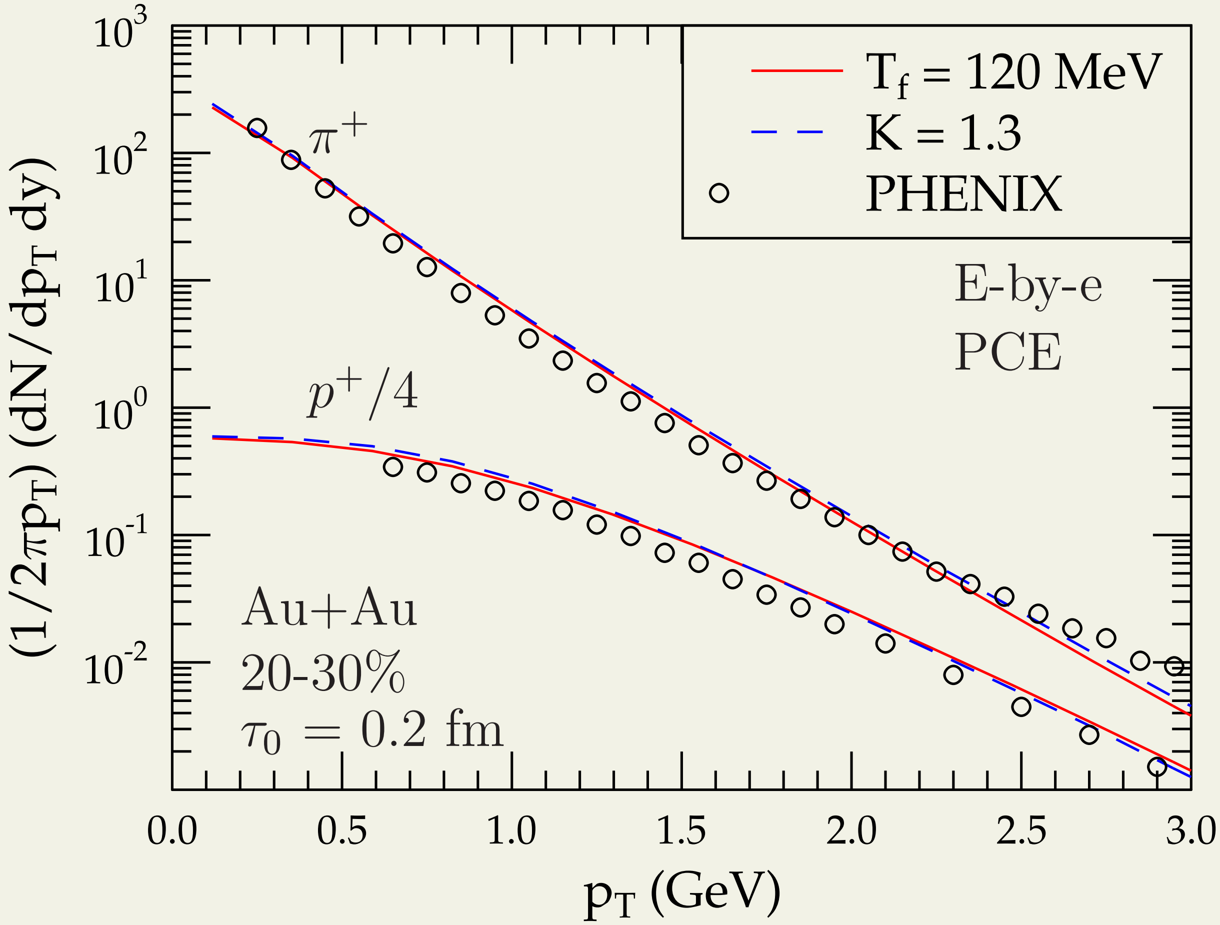
- **pion $v_2(p_T)$ reduced by $\sim$10%!**

p_T spectra



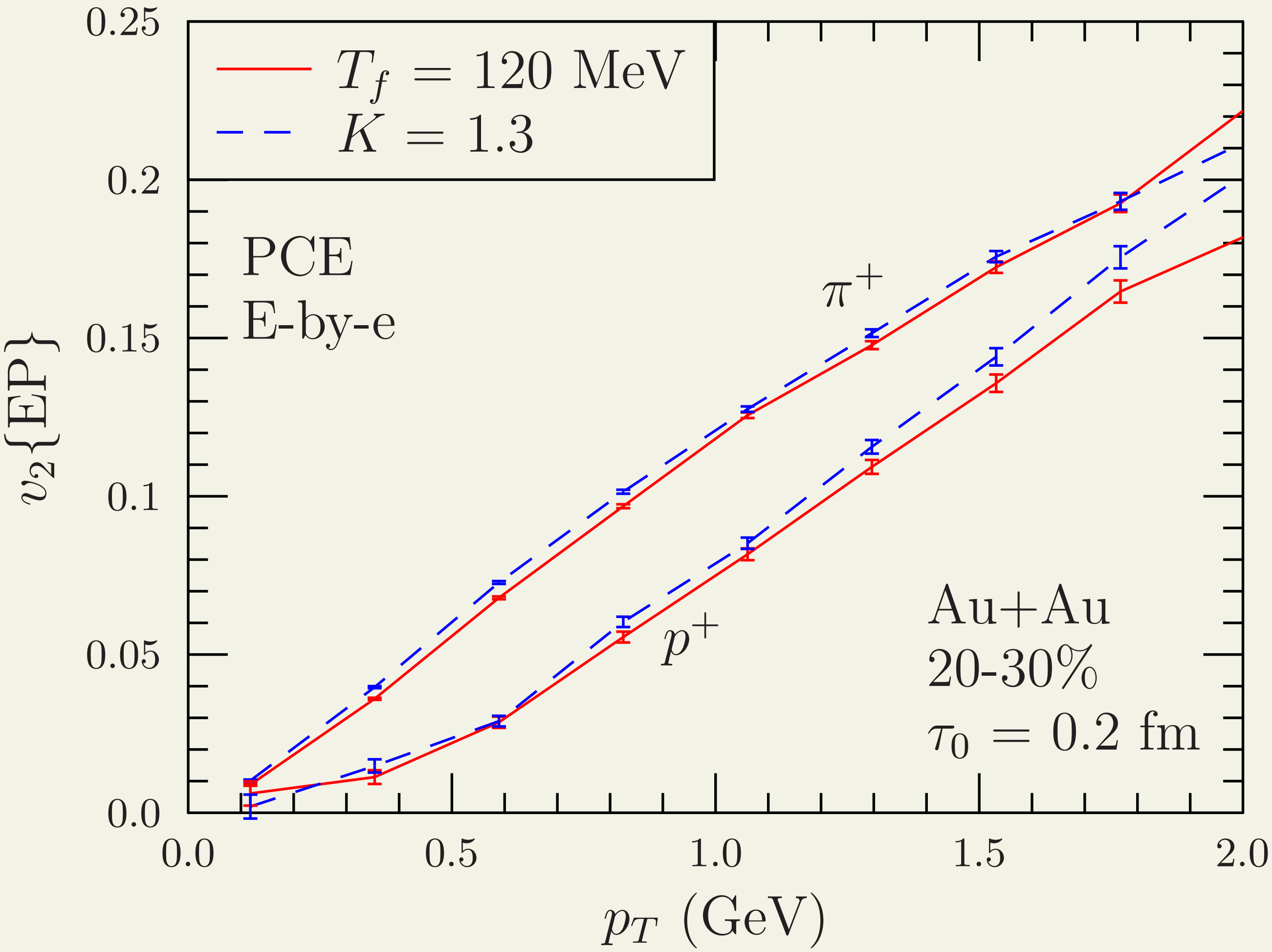
What if spectra are fixed to be same?

adjust parameters



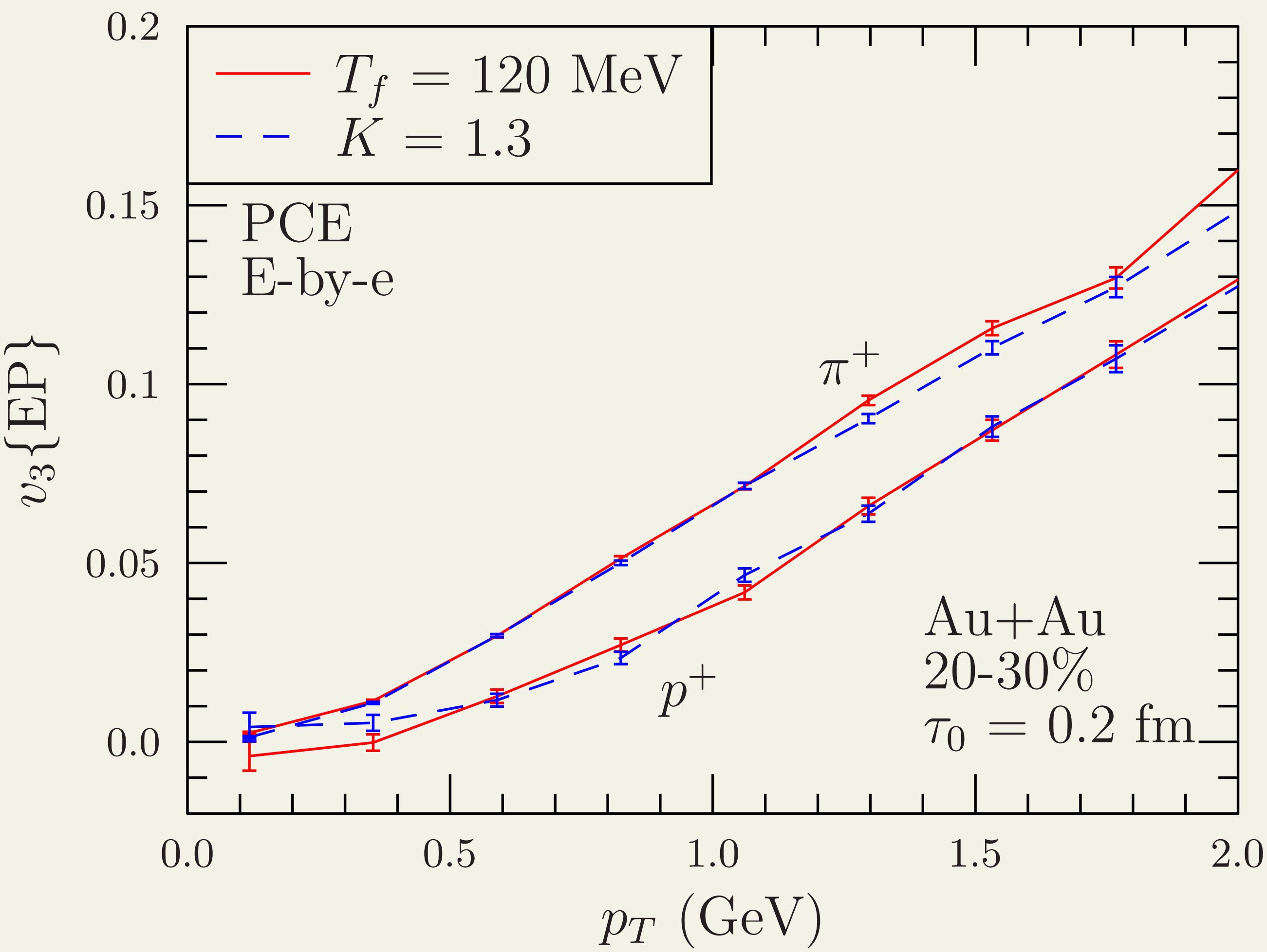
- $\tau_0 = 0.6 \rightarrow 0.2\,\mathrm{fm}$
- $K_n = 1 \rightarrow 1.3$

$v_2(p_T)$



- **difference disappeared!**

$v_3(p_T)$

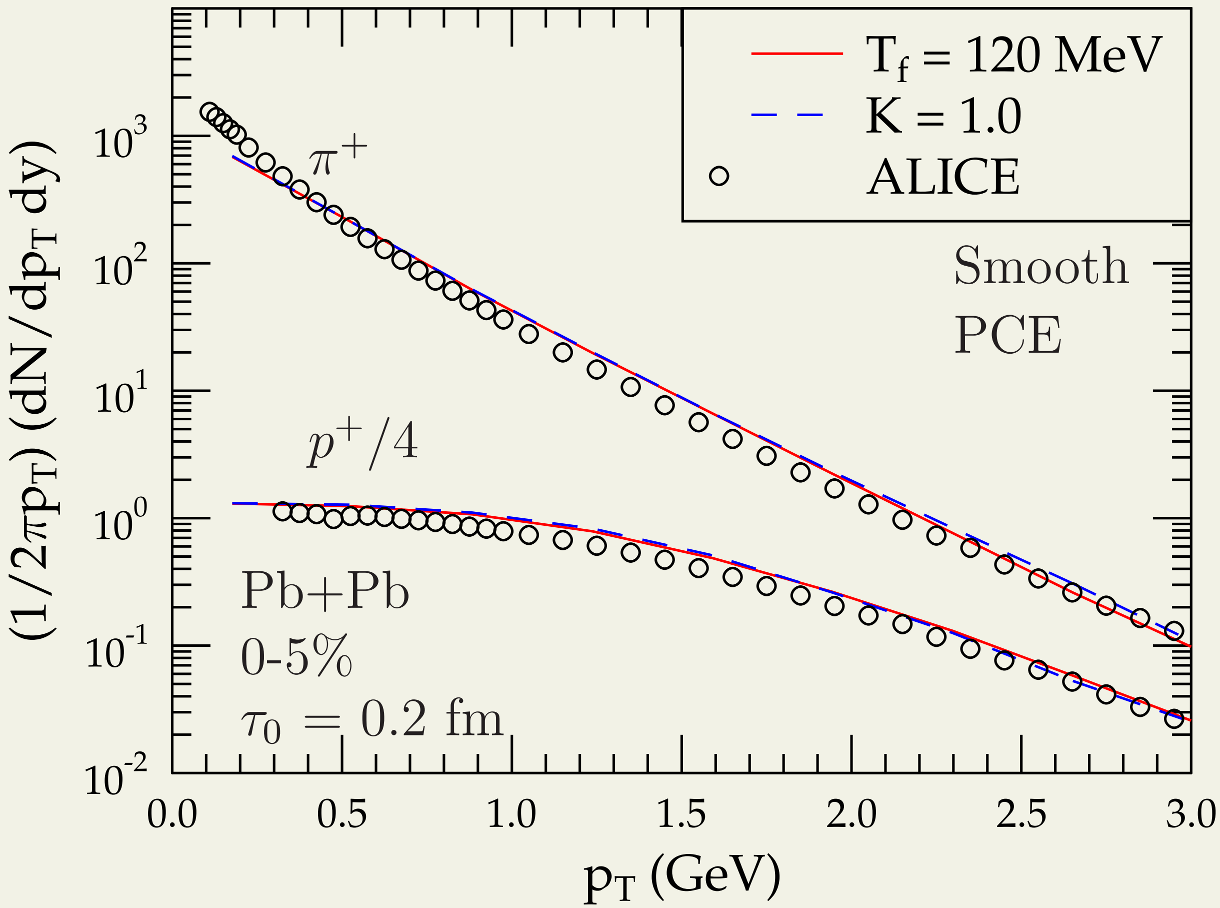


- **similar too. . .**

and LHC?

averaged initial state only

p_T spectra

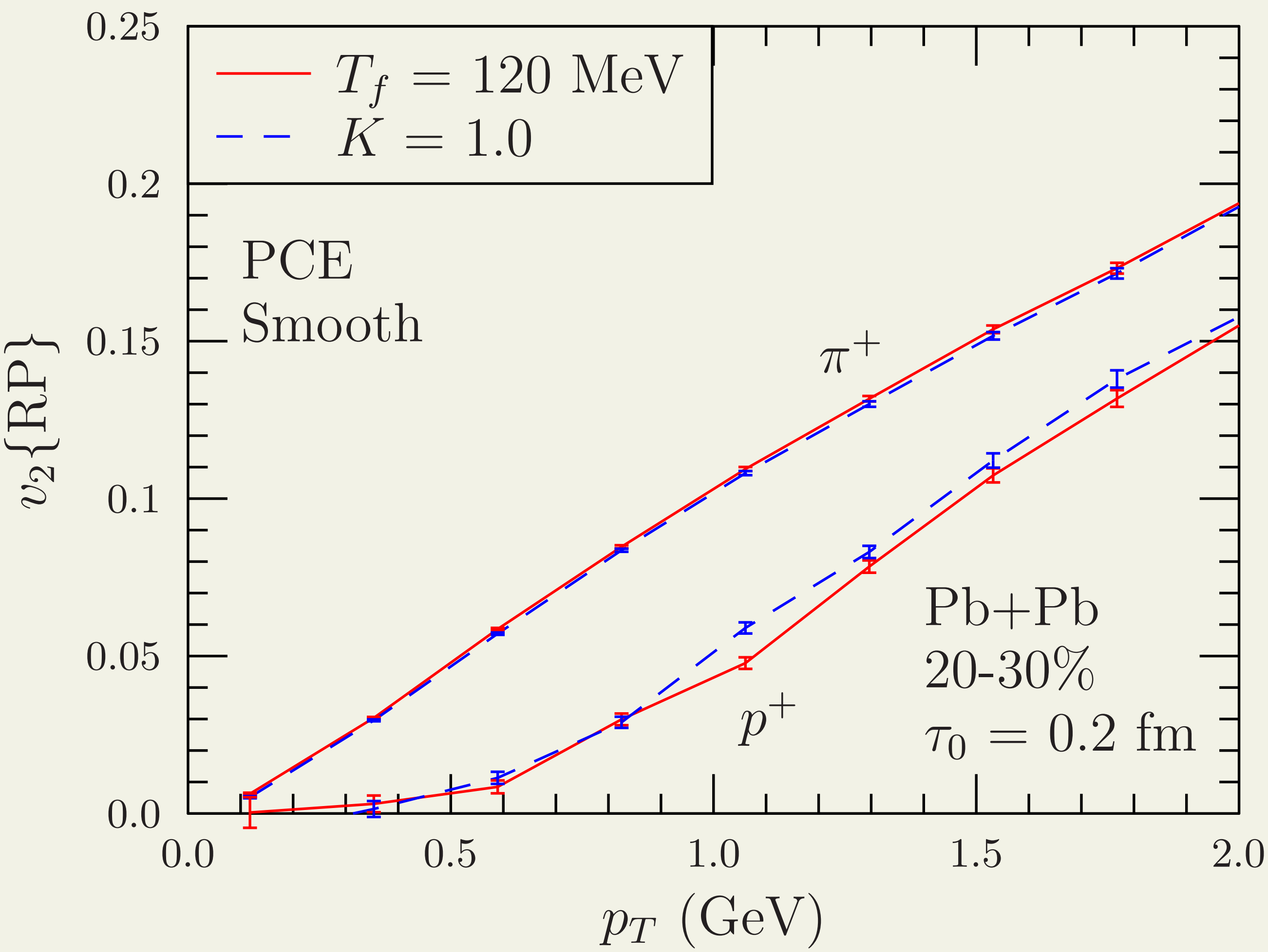


Why $T_f = 120\,\mathrm{MeV}$ **at RHIC and LHC**

but

$K = 1.3$ **vs.** 1**?**

$v_2(p_T)$ LHC



- **similar too. . .**

Conclusions

- constant T freeze-out is an oversimplification
  - but it does not matter!
  - if p_T-spectra are the same, $v_n(p_T)$ are the same!
  - is this universal??
- effect on femtoscopy or δf ?