Resummed QCD thermodynamics at finite temperature and chemical potential

Michael Strickland

Primary References and Collaborators

1309.3968: N. Haque, J.O. Andersen, M.G. Mustafa, MS, and N. Su (3-loop HTLpt)

1307.8098: S. Mogliacci, J.O. Andersen, N. Su, MS, and A. Vuorinen (4-loop resummed DR)

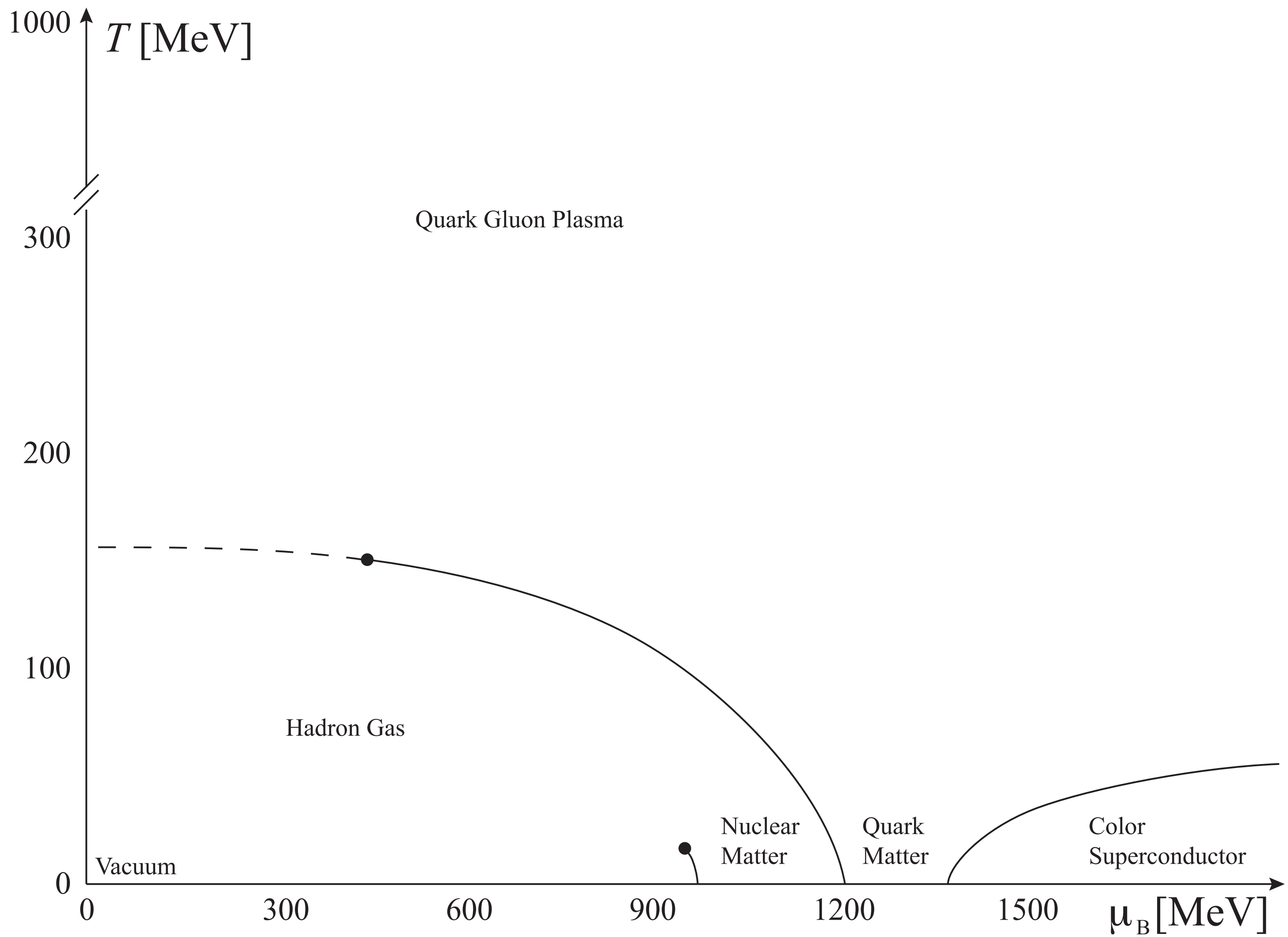
New Frontiers in QCD 2013

--- *Insight into QCD matter from heavy-ion collisions* ---

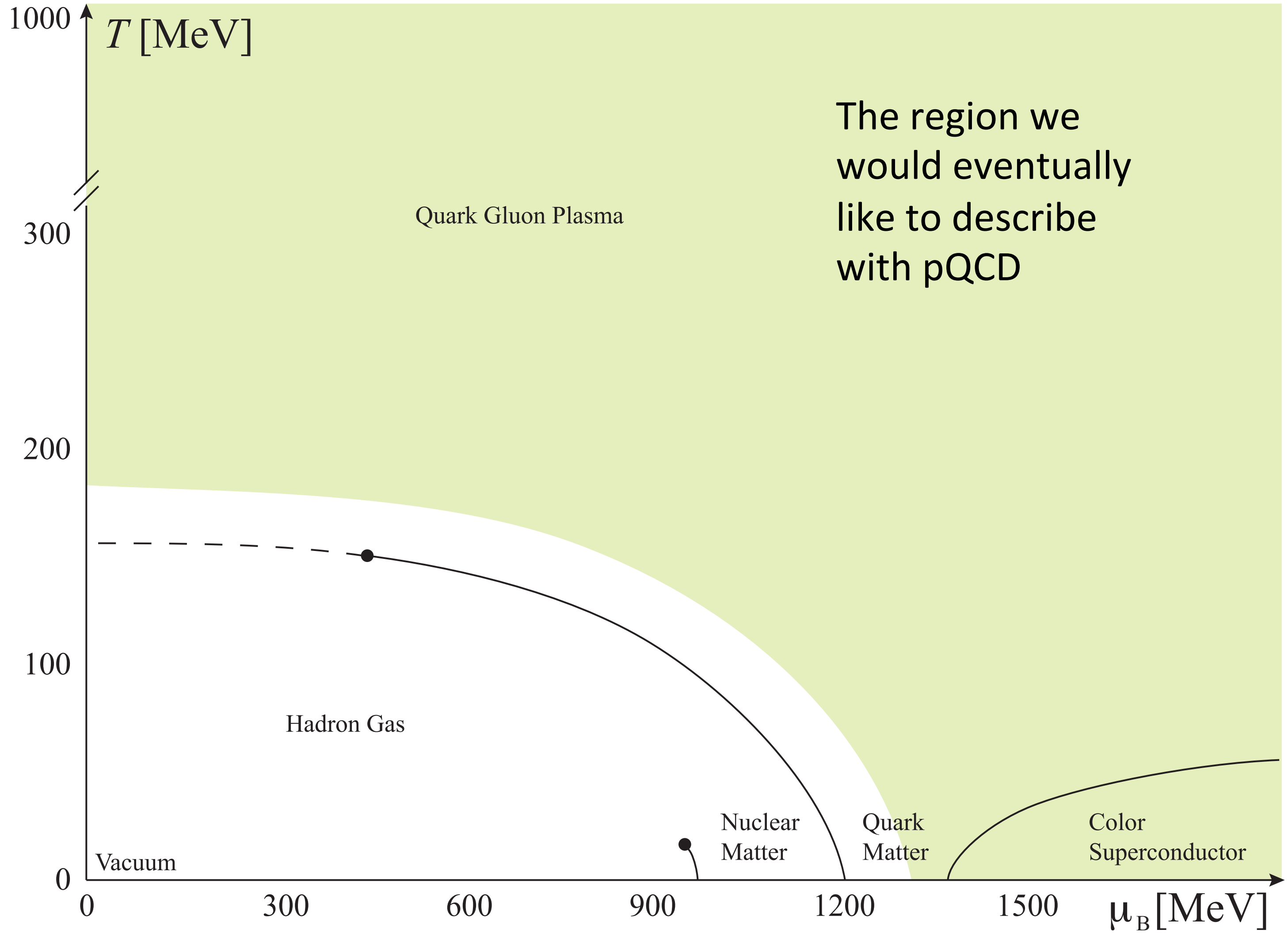
Outline

- Motivation
- Hard-thermal-loop perturbation theory (HTLpt)
- Comparison of 3-loop HTLpt results for pressure and susceptibilities with lattice data
- Resummed dimensional reduction (DR)
- Comparison of 4-loop resummed DR results for susceptibilities with lattice data and 3-loop HTLpt
- Conclusions and outlook

Obligatory QCD Phase Diagram

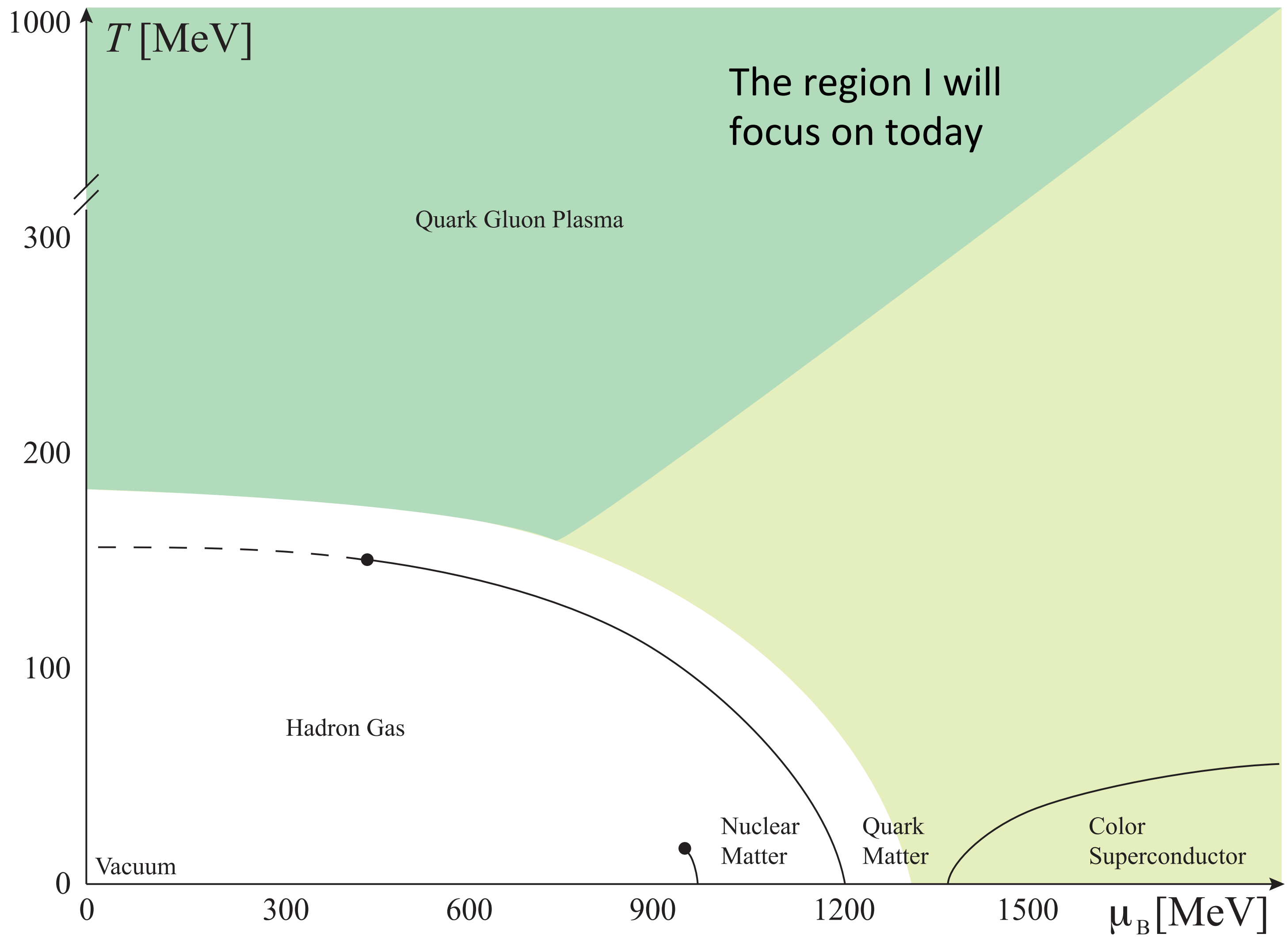


Obligatory QCD Phase Diagram



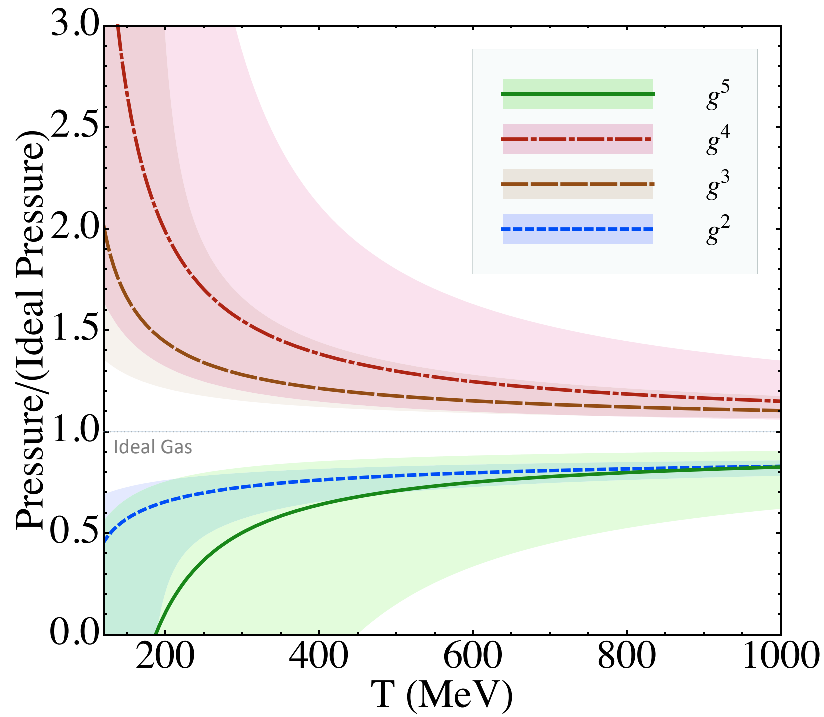
The region we would eventually like to describe with pQCD

Obligatory QCD Phase Diagram



Naïve pQCD thermodynamics

- QCD free energy known up to three loops (g^5) since 1994
  (Arnold, Zhai, and Khastening 94)
- Series in g (not g^2) due to plasma screening effects: Debye mass $m_D \sim gT$
- Very poorly convergent: need temperatures on the order of $T \sim 10^5$ GeV for convergence
- Similar problem in QED and scalar theories → <u>the problem is not specific to QCD</u>
- Our goal: Find a more convergent gauge-invariant scheme for T > $2T_c$
- Resulting framework should also be able to describe dynamical properties of the QGP



Simple Case – Anharmonic Oscillator

- Consider quantum mechanics in an anharmonic potential

$$V(x) = \frac{1}{2}\omega^2 x^2 + \frac{g}{4}x^4 \qquad (\omega^2, g > 0)$$

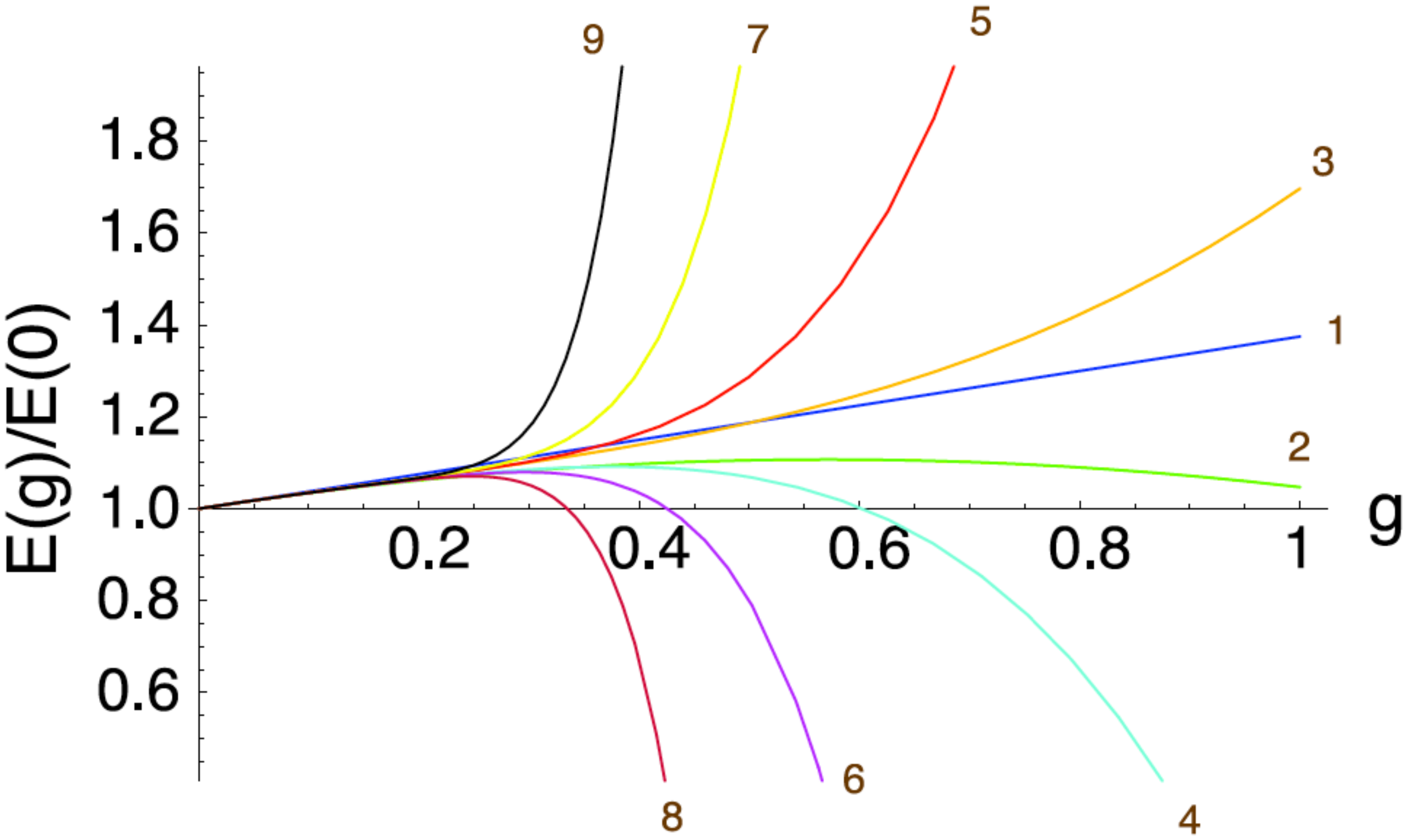
- Weak-coupling expansion of the ground state energy is known up to all orders (Bender and Wu 69/73)

$$E(g) = \omega \sum_{n=0}^{\infty} c_n^{\rm BW} \left(\frac{g}{4\omega^3}\right)^n , \quad c_n^{\rm BW} = \left\{\frac{1}{2}, \frac{3}{4}, -\frac{21}{8}, \frac{333}{16}, -\frac{30885}{16}, \ldots\right\}$$

$$\lim_{n\to\infty} c_n^{\rm BW} = (-1)^{n+1}\sqrt{\frac{6}{\pi^3}}\, 3^n (n - \tfrac{1}{2})!$$

- Factorial growth → expansion is an asymptotic series with zero radius of convergence!

Asymptotic Series



Variational Perturbation Theory

- Split the harmonic term into two pieces and treat the second as part of the interaction (Janke and Kleinert 95)

$$\omega^2 \to \Omega^2 + \left(\omega^2 - \Omega^2\right) \quad \Longrightarrow \quad E_N(g,r) = \Omega \sum_{n=0}^{N} c_n(r) \left(\frac{g}{4\Omega^3}\right)^n$$

$$r \equiv \tfrac{2}{g}\left(\omega^2 - \Omega^2\right)$$

- The coefficients c_n can be computed analytically in this case

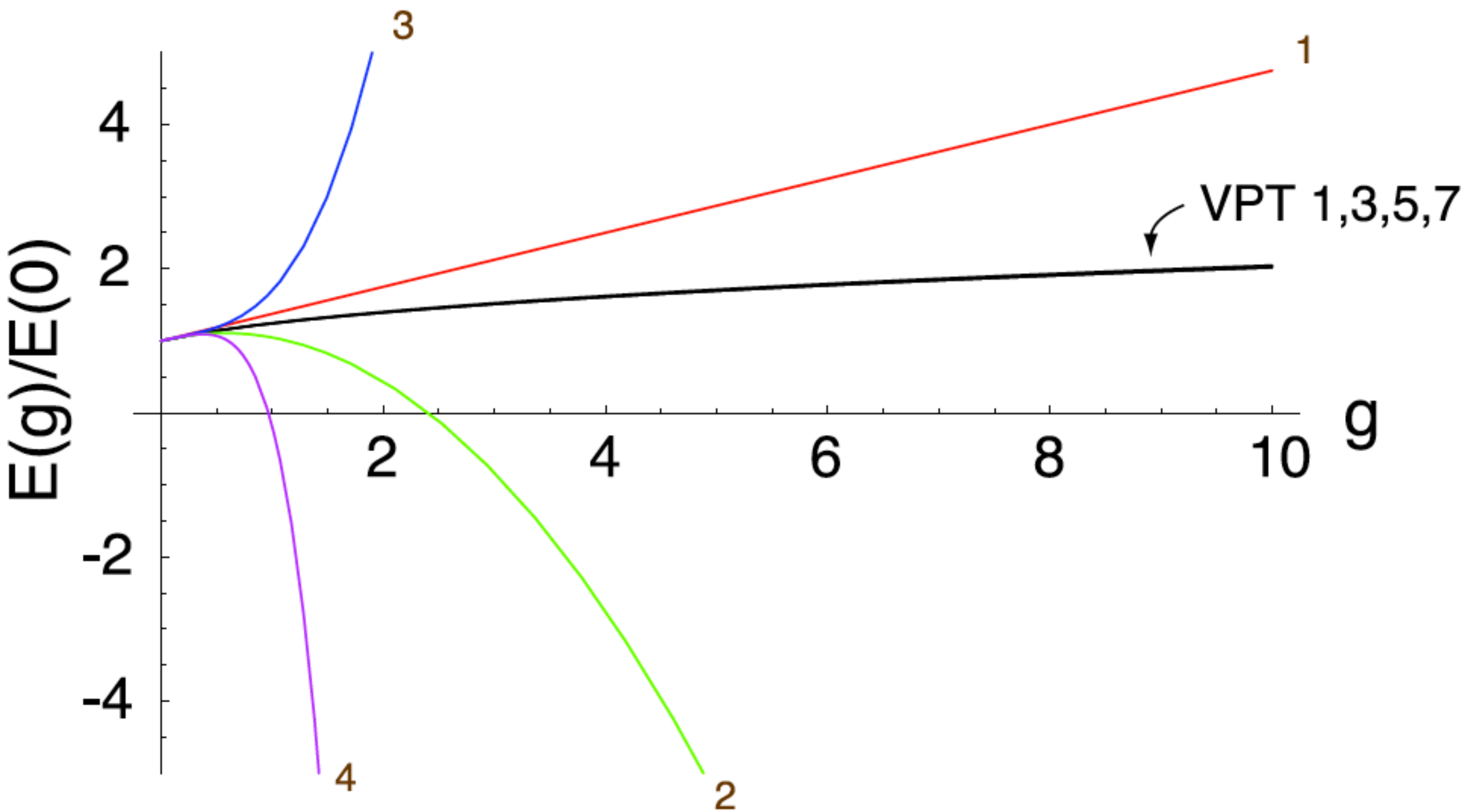
$$c_n(r) = \sum_{j=0}^{n} c_j^{\mathrm{BW}} \begin{pmatrix} (1-3j)/2 \\ n-j \end{pmatrix} (2r\Omega)^{n-j}$$

- Fix Ω by imposing variational condition that ground state energy is minimized

$$\left.\frac{\partial E_N}{\partial \Omega}\right|_{\Omega=\Omega_N} = 0$$

→ "Gap equation"

Variational Perturbation Theory



Hard Thermal Loop Perturbation Theory (HTLpt)

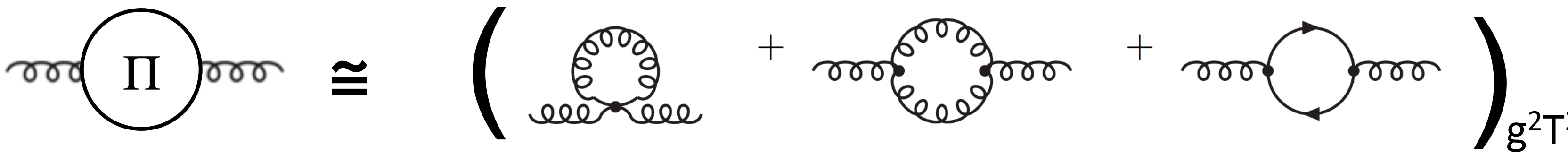
High temperature scale separation

At very high temperature there is a hierarchy of length scales in the QGP:

- Hard Scale, $\lambda \sim 1/T$
  - $n_b(E)\, g^2(T) \sim g^2(T)$
  - Wavelength of thermal fluctuations
  - Inverse mass of non-static field modes ($p_0 \neq 0$)
  - Purely perturbative contribution to QCD thermodynamics (g^{2n})
- Electric Scale, $\lambda \sim 1/gT$
  - $n_b(E)\, g^2(T) \sim g(T)$
  - Screening scale for static chromoelectric fluctuations
  - Inverse Debye mass of the A_0
  - Resummation of an infinite subset of diagrams necessary
  - Odd powers of g and logarithms (e.g. g^3, $g^4 \log g$, etc)
- Magnetic Scale, $\lambda \sim 1/g^2T$
  - $n_b(E)\, g^2(T) \sim g^0(T)$
  - Screening scale for static chromomagnetic fluctuations
  - Inverse "magnetic mass"
  - Generates non-perturbative contribution to pressure starting at 4-loop order ("Linde Problem")

Hard Thermal Loops

In a high temperature system we must resum a certain class of diagrams which have hard internal (loop) momentum $p_{\text{hard}} \sim T$ and soft external momentum $p_{\text{soft}} \sim gT$



$$\Pi_T(\omega,p) = \frac{m_D^2}{2}\frac{\omega^2}{p^2}\left[1 + \frac{p^2-\omega^2}{2\omega p}\log\frac{\omega+p}{\omega-p}\right]$$

$$\Pi_L(\omega,p) = m_D^2\left[1 - \frac{\omega}{2p}\log\frac{\omega+p}{\omega-p}\right]$$

$$\lim_{\omega\to 0}\Pi_T(\omega,p) = 0$$

$$\lim_{\omega\to 0}\Pi_L(\omega,p) = m_D^2$$

At finite temperature there are transverse and longitudinal gluons

$$\Delta_T(p) = \frac{1}{p^2 - \Pi_T(p)}$$

$$\Delta_L(p) = \frac{1}{\mathbf{p}^2 + \Pi_L(p)}$$

Gluons acquire a temperature dependent mass which is proportional to the temperature. At LO one has

$$m_D^2 = \frac{1}{3}\left(N_c + \frac{1}{2}N_f\right)g^2T^2$$

HTLpt Action

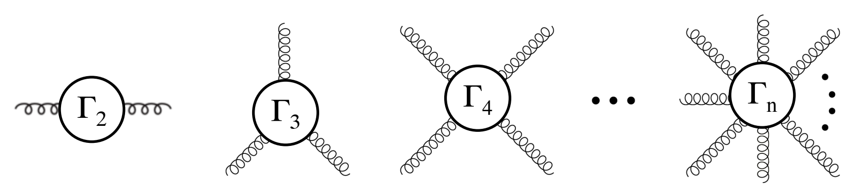
- Can express an infinite number of HTL-dressed n-point functions concisely in terms of an HTL effective action, $\mathcal{L}_{\text{HTL}}$
- Expanding $\mathcal{L}_{\text{HTL}}$ to quadratic order in A gives dressed propagator (2-point function)
- Expanding to cubic order in A gives the dressed gluon three-vertex
- Expanding to quartic order in A gives dressed gluon four-vertex
- And so on . . . contains an infinite number of higher order vertices which all exactly satisfy the appropriate Slavnov-Taylor identities

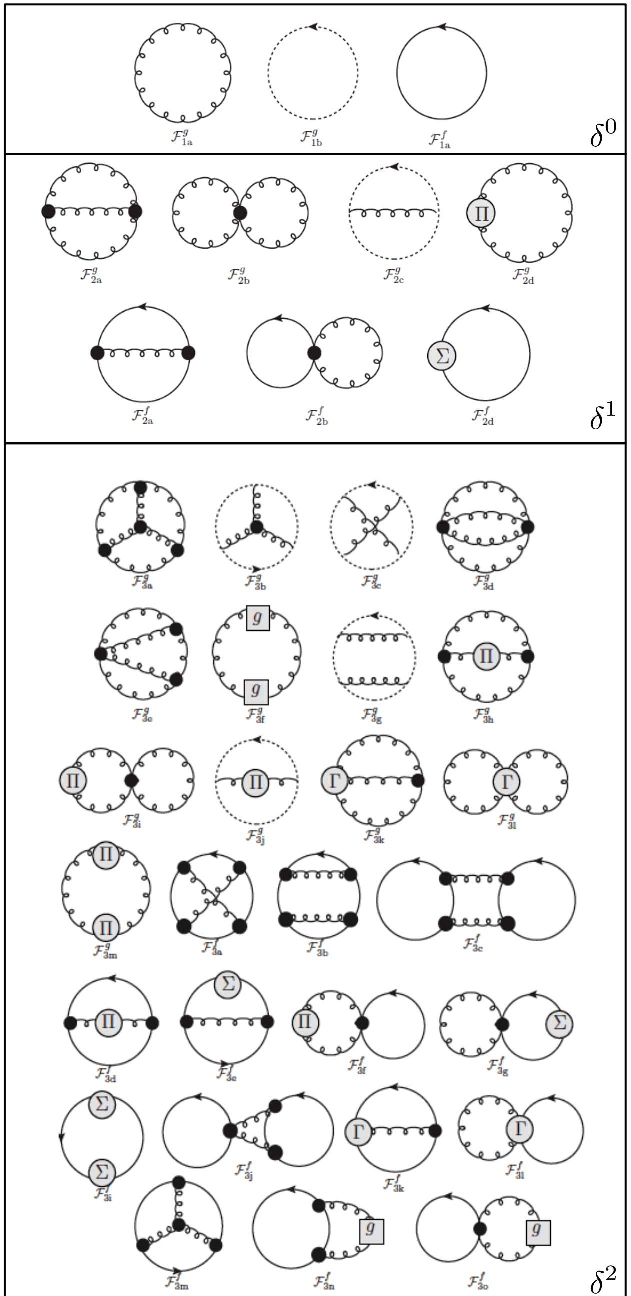
$$\mathcal{L} = (\mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{HTL}})\Big|_{g_s \to \sqrt{\delta} g_s} + \Delta\mathcal{L}_{\text{HTL}}$$

[Andersen, Braaten, and MS, 99 → 02]

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\text{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] + i\bar{\psi}\gamma^\mu D_\mu \psi + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \Delta\mathcal{L}_{\text{QCD}}$$

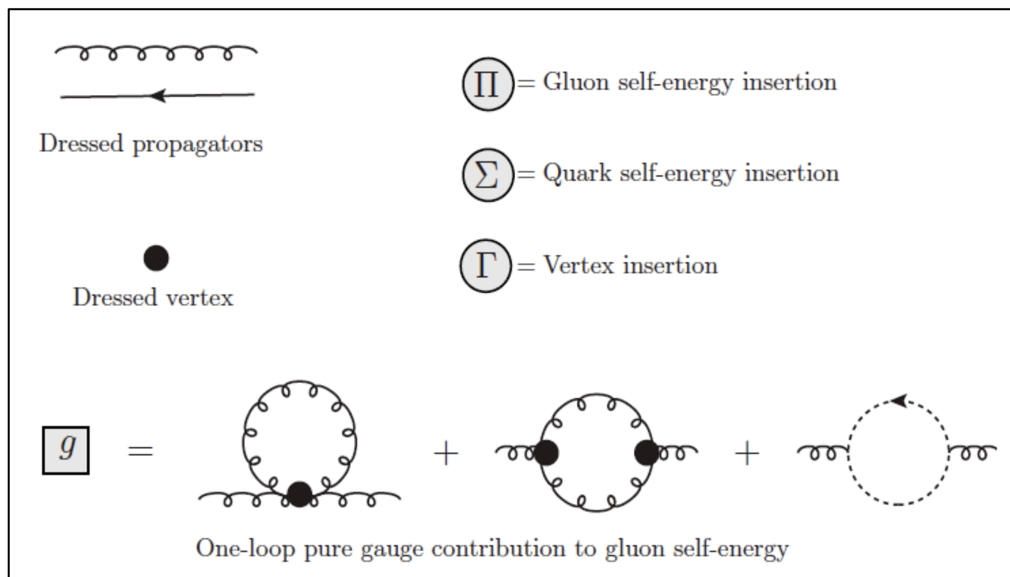
$$\mathcal{L}_{\text{HTL}} = -\frac{1}{2}(1-\delta)m_D^2 \,\text{Tr}\left(G_{\mu\alpha}\left\langle \frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y G^\mu{}_\beta\right) + (1-\delta)\, i m_q^2 \bar{\psi}\gamma^\mu \left\langle \frac{y_\mu}{y\cdot D}\right\rangle_y \psi \,,$$



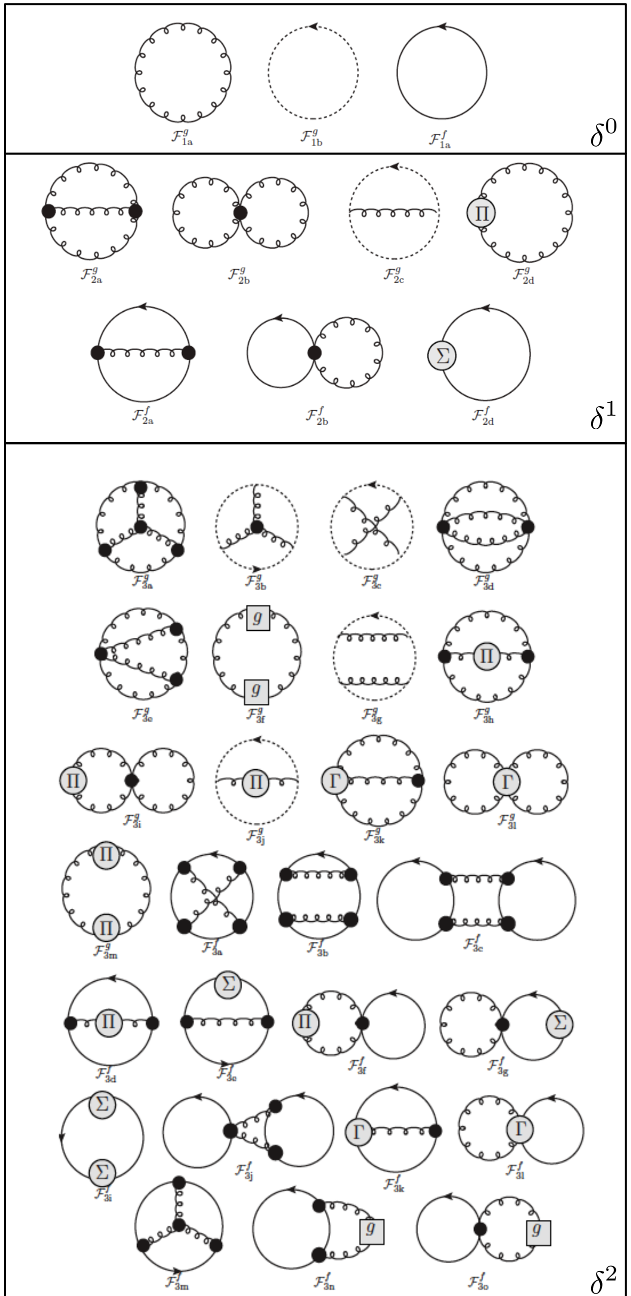


3-loop Calculation

- Now “simply” compute all contributions up to three loops (49 diagrams) using HTL-dressed propagators and vertices.
- Finite T and zero quark chemical potential 3-loop result: Andersen, Leganger, Su, and MS 2011



Finite μ and T Result



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$$
\begin{aligned}
\frac{\Omega_{\rm NNLO}}{\Omega_0} = {} & \frac{7}{4}\frac{d_F}{d_A}\left(1+\frac{120}{7}\hat\mu^2+\frac{240}{7}\hat\mu^4\right)+\frac{s_F\alpha_s}{\pi}\Bigg[-\frac{5}{8}\left(1+12\hat\mu^2\right)\left(5+12\hat\mu^2\right)+\frac{15}{2}\left(1+12\hat\mu^2\right)\hat m_D \\
& +\frac{15}{2}\left(2\ln\frac{\hat\Lambda_q}{2}-1-\aleph(z)\right)\hat m_D^3-90\hat m_q^2\hat m_D\Bigg]+s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\Bigg[\frac{15}{64}\Bigg\{35-32\left(1-12\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}+472\hat\mu^2 \\
& +1328\hat\mu^4+64\Big(-36i\hat\mu\aleph(2,z)+6(1+8\hat\mu^2)\aleph(1,z)+3i\hat\mu(1+4\hat\mu^2)\aleph(0,z)\Big)\Bigg\}-\frac{45}{2}\hat m_D\left(1+12\hat\mu^2\right)\Bigg] \\
& +\left(\frac{s_F\alpha_s}{\pi}\right)^2\Bigg[\frac{5}{4\hat m_D}\left(1+12\hat\mu^2\right)^2+30\left(1+12\hat\mu^2\right)\frac{\hat m_q^2}{\hat m_D}+\frac{25}{12}\Bigg\{\left(1+\frac{72}{5}\hat\mu^2+\frac{144}{5}\hat\mu^4\right)\ln\frac{\hat\Lambda_q}{2} \\
& +\frac{1}{20}\left(1+168\hat\mu^2+2064\hat\mu^4\right)+\frac{3}{5}\left(1+12\hat\mu^2\right)^2\gamma_E-\frac{8}{5}(1+12\hat\mu^2)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{34}{25}\frac{\zeta'(-3)}{\zeta(-3)} \\
& -\frac{72}{5}\Big[8\aleph(3,z)+3\aleph(3,2z)-12\hat\mu^2\aleph(1,2z)+12i\hat\mu\left(\aleph(2,z)+\aleph(2,2z)\right)-i\hat\mu(1+12\hat\mu^2)\,\aleph(0,z) \\
& -2(1+8\hat\mu^2)\aleph(1,z)\Big]\Bigg\}-\frac{15}{2}\left(1+12\hat\mu^2\right)\left(2\ln\frac{\hat\Lambda_q}{2}-1-\aleph(z)\right)\hat m_D\Bigg] \\
& +\left(\frac{c_A\alpha_s}{3\pi}\right)\left(\frac{s_F\alpha_s}{\pi}\right)\Bigg[\frac{15}{2\hat m_D}\left(1+12\hat\mu^2\right)-\frac{235}{16}\Bigg\{\left(1+\frac{792}{47}\hat\mu^2+\frac{1584}{47}\hat\mu^4\right)\ln\frac{\hat\Lambda_q}{2}-\frac{144}{47}\left(1+12\hat\mu^2\right)\ln\hat m_D \\
& +\frac{319}{940}\left(1+\frac{2040}{319}\hat\mu^2+\frac{38640}{319}\hat\mu^4\right)-\frac{24\gamma_E}{47}\left(1+12\hat\mu^2\right)-\frac{44}{47}\left(1+\frac{156}{11}\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{268}{235}\frac{\zeta'(-3)}{\zeta(-3)} \\
& -\frac{72}{47}\Big[4i\hat\mu\aleph(0,z)+\left(5-92\hat\mu^2\right)\aleph(1,z)+144i\hat\mu\aleph(2,z)+52\aleph(3,z)\Big]\Bigg\}+90\frac{\hat m_q^2}{\hat m_D}+\frac{315}{4}\Bigg\{\left(1+\frac{132}{7}\hat\mu^2\right)\ln\frac{\hat\Lambda_q}{2} \\
& +\frac{11}{7}\left(1+12\hat\mu^2\right)\gamma_E+\frac{9}{14}\left(1+\frac{132}{9}\hat\mu^2\right)+\frac{2}{7}\aleph(z)\Bigg\}\hat m_D\Bigg]+\frac{\Omega^{\rm YM}_{\rm NNLO}(\Lambda_g)}{\Omega_0}\,.
\end{aligned}
$$

$$
\begin{aligned}
\hat m_D^2 = \frac{\alpha_s}{3\pi}\Bigg\{ & c_A+\frac{c_A^2\alpha_s}{12\pi}\left(5+22\gamma_E+22\ln\frac{\hat\Lambda_g}{2}\right)+s_F\left(1+12\hat\mu^2\right)+\frac{c_As_F\alpha_s}{12\pi}\Bigg(\left(9+132\hat\mu^2\right)+22\left(1+12\hat\mu^2\right)\gamma_E \\
& +2\left(7+132\hat\mu^2\right)\ln\frac{\hat\Lambda_q}{2}+4\aleph(z)\Bigg)+\frac{s_F^2\alpha_s}{3\pi}\left(1+12\hat\mu^2\right)\left(1-2\ln\frac{\hat\Lambda_q}{2}+\aleph(z)\right)-\frac{3}{2}\frac{s_{2F}\alpha_s}{\pi}\left(1+12\hat\mu^2\right)\Bigg\}
\end{aligned}
$$

Use one-loop running with $\alpha_s(1.5\text{ GeV}) = 0.326$ taken from state-of-the-art lattice measurement (Bazazov et al 12)

Finite μ and T Result

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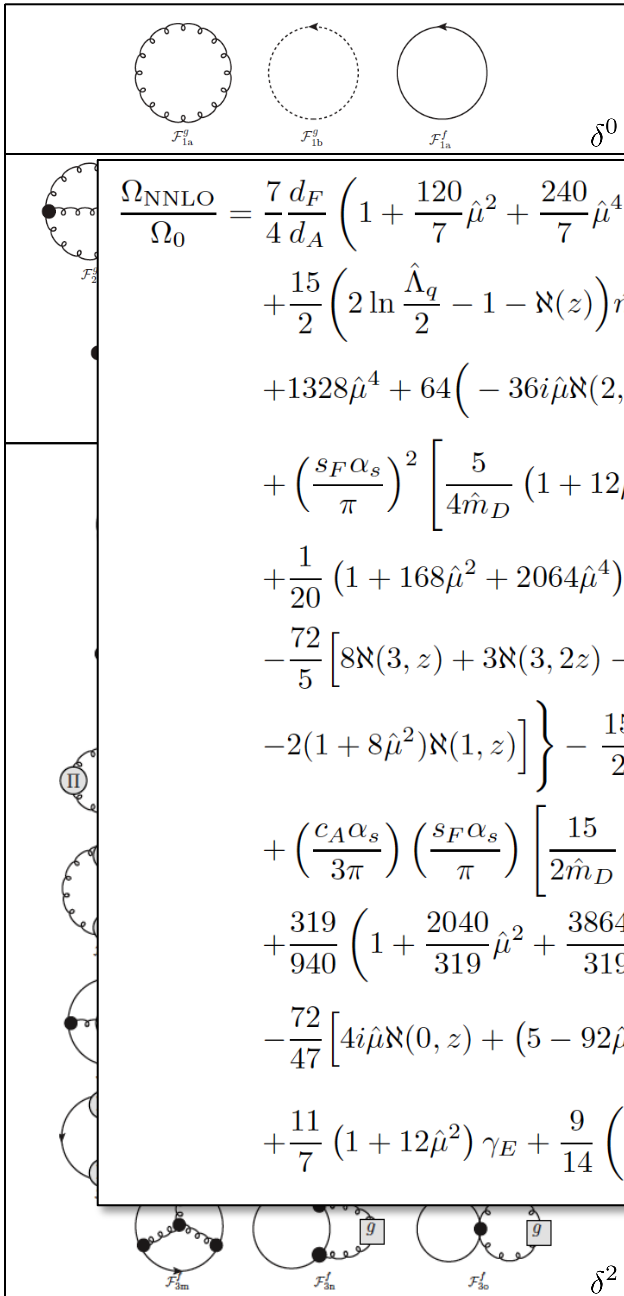

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\end{aligned}
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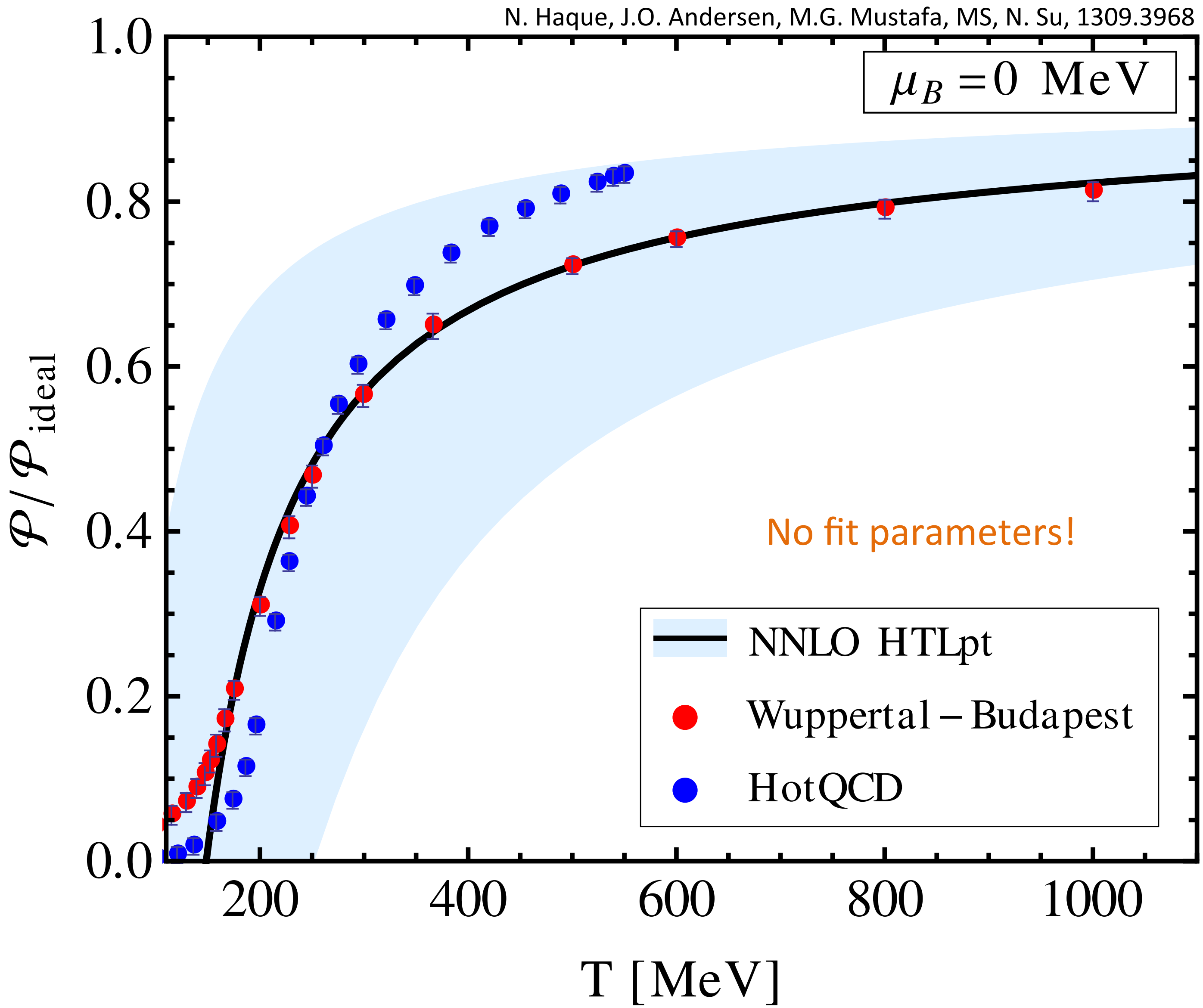
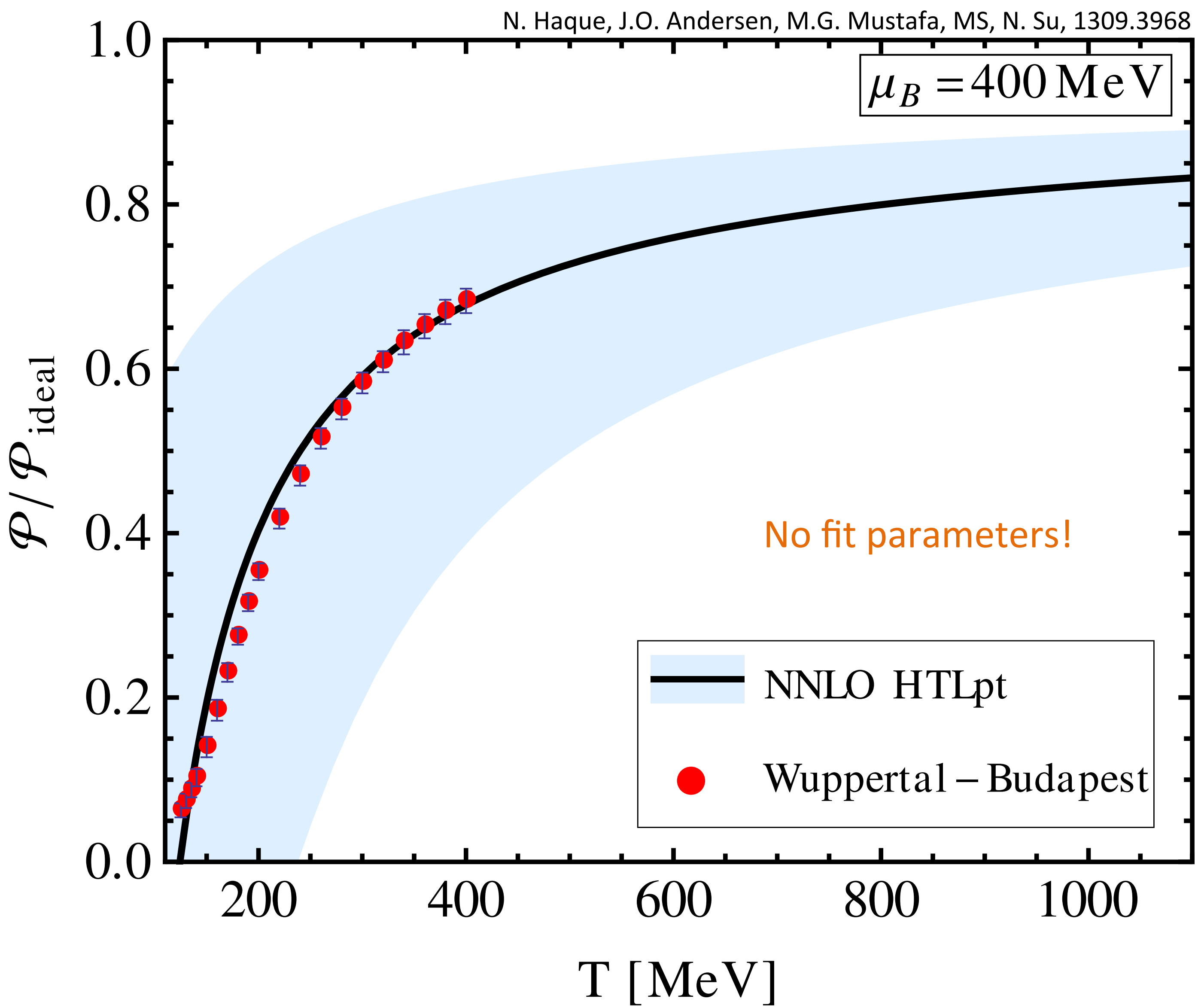
Use one-loop running with $\alpha_s(1.5\ \text{GeV}) = 0.326$ taken from state-of-the-art lattice measurement (Bazazov et al 12)

Pressure vs Temperature – $\mu_B = 0$ MeV



Pressure vs Temperature – μ_B = 400 MeV



$\chi_{2(B,u)}$ vs Temperature

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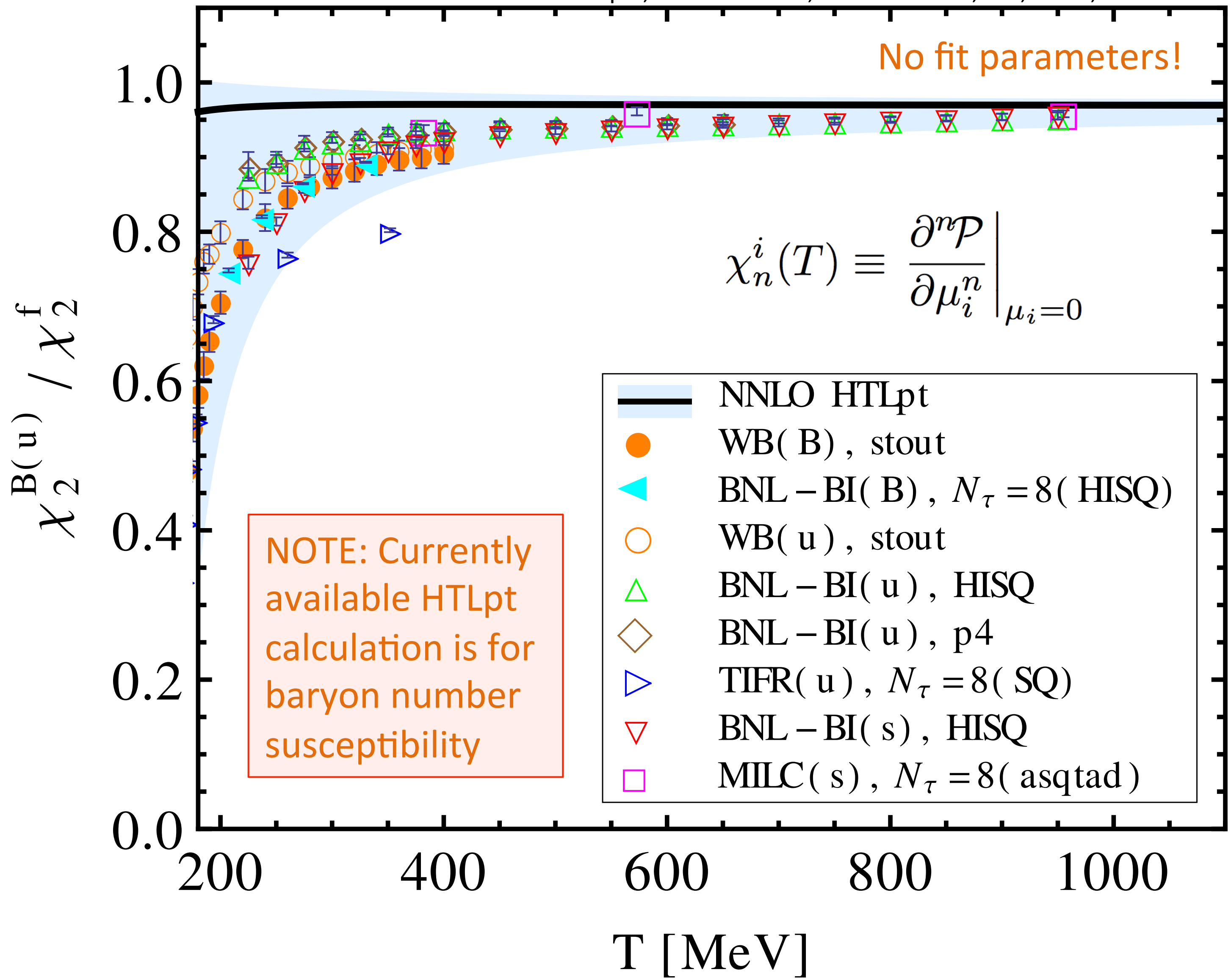
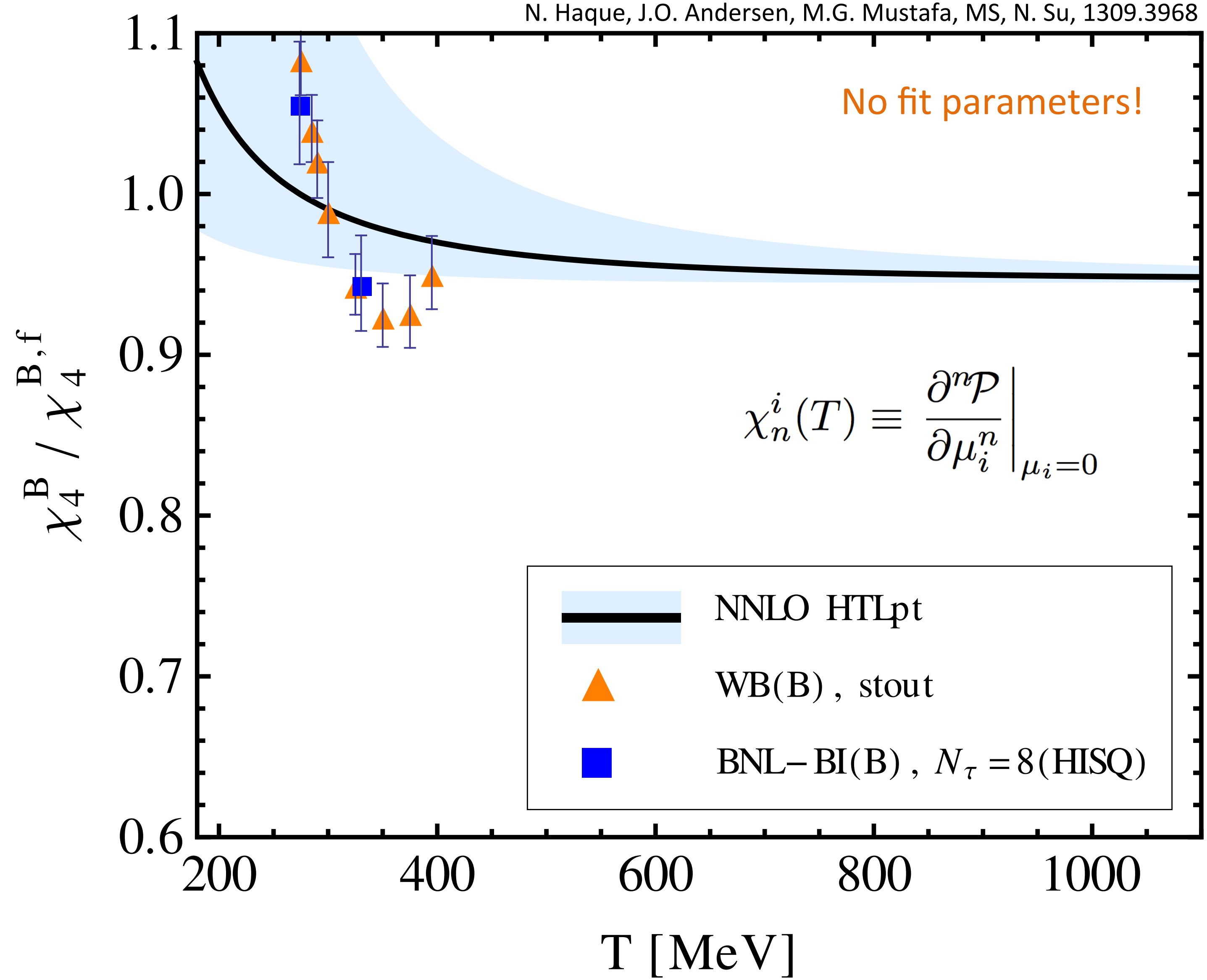


χ_{4B} vs Temperature



HTLpt Conclusions

- 3-loop HTLpt seems to work reasonably well and it is completely analytic and gauge invariant!
- Suggests that existing and future applications of HTLs to real time dynamics in heavy ion collisions may not be a complete waste of time
- Results for single quark susceptibilities are forthcoming
- Finite quark mass corrections needed (so far $m_q=0$)
- We are also working on extending the result to the full μ-T plane (need ring resummation at low T and high μ)
- Need log resummation in HTLpt to further reduce scale variation bands
- Next question: Calculation was difficult, is there another way to obtain perturbatively resummed susceptibilities which we might push to 4-loop order?

Resummed Dimensional Reduction

High temperature scale separation

At very high temperature there is a hierarchy of length scales in the QGP:

- Hard Scale, $\lambda \sim 1/T$
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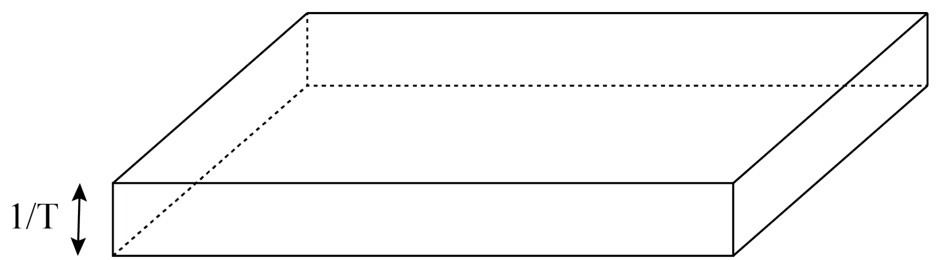
  - Screening scale for static chromomagnetic fluctuations
  - Inverse “magnetic mass”
  - Generates non-perturbative contribution to pressure starting at 4-loop order (“Linde Problem”)

Dimensional Reduction

- Scale hierarchy → Integrate out massive (non-static) modes (Ginsparg 80; Gross, Pisarski, and Yaffe 81; Appelquist and Pisarski, 81)
- Results in effective description for scales $\Delta x \gtrsim 1/gT$



- In the high temperature limit one obtains a 3d effective theory for static electric modes (Braaten and Nieto 1995, Kajantie et al 2003)

$$\mathcal{L}_{\rm EQCD} = \frac{1}{g_E^2}\left(\frac{1}{2}{\rm Tr}[F_{ij}^2] + {\rm Tr}[(D_i A_0)^2] + m_E^2 {\rm Tr}[A_0^2] + \lambda_E {\rm Tr}[A_0^4]\right) + \delta\mathcal{L}_{\rm E}$$

$$g_{\rm E} \equiv \sqrt{T}g, \quad m_{\rm E} \sim gT, \quad \lambda_{\rm E} \sim g^2$$

Resummed EQCD

- Completely non-perturbative magnetic (MQCD) contribution enters at 4-loop order; can be reduced to one number that can be computed using a 3d lattice calculation (still hard!)
- For the results shown herein this number is <u>not needed</u>
- Hard scale contributions are strictly perturbative
- Soft scale contributions involve inverse powers of m_{D}
- If you expand out the soft sector perturbatively, you come back to square one (pQCD result on slide 3 is obtained)
- In order to fully resum soft sector contributions, one should not Taylor expand the Debye mass contributions in g, but instead keep the full g-dependence
- Similar to HTLpt, this results in an expression which contains terms of all orders in the strong coupling constant → Resummed EQCD

Resummed DR Results

- Resummed "3.5" loop DR Pressure @ T≠0 μ=0 (undetermined g^6 coeff. fit to lattice):
  Kajantie, Laine, Rummukainen and Schroder, hep-ph/0211321 (see also hep-ph/0007109)
- Perturbative DR Pressure and Susceptibilities @ T≠0, μ≠0:
  A. Vuorinen, hep-ph/0305183
- **Resummed 4-loop DR Susceptibilities @ T≠0, μ≠0:**
  S. Mogliacci, J.O. Andersen, N. Su, MS, and A. Vuorinen, 1307.8098

$$p_{\rm QCD}(T,\boldsymbol{\mu}) \equiv p_{\rm HARD}(T,\boldsymbol{\mu}) + T\, p_{\rm SOFT}(T,\boldsymbol{\mu})$$

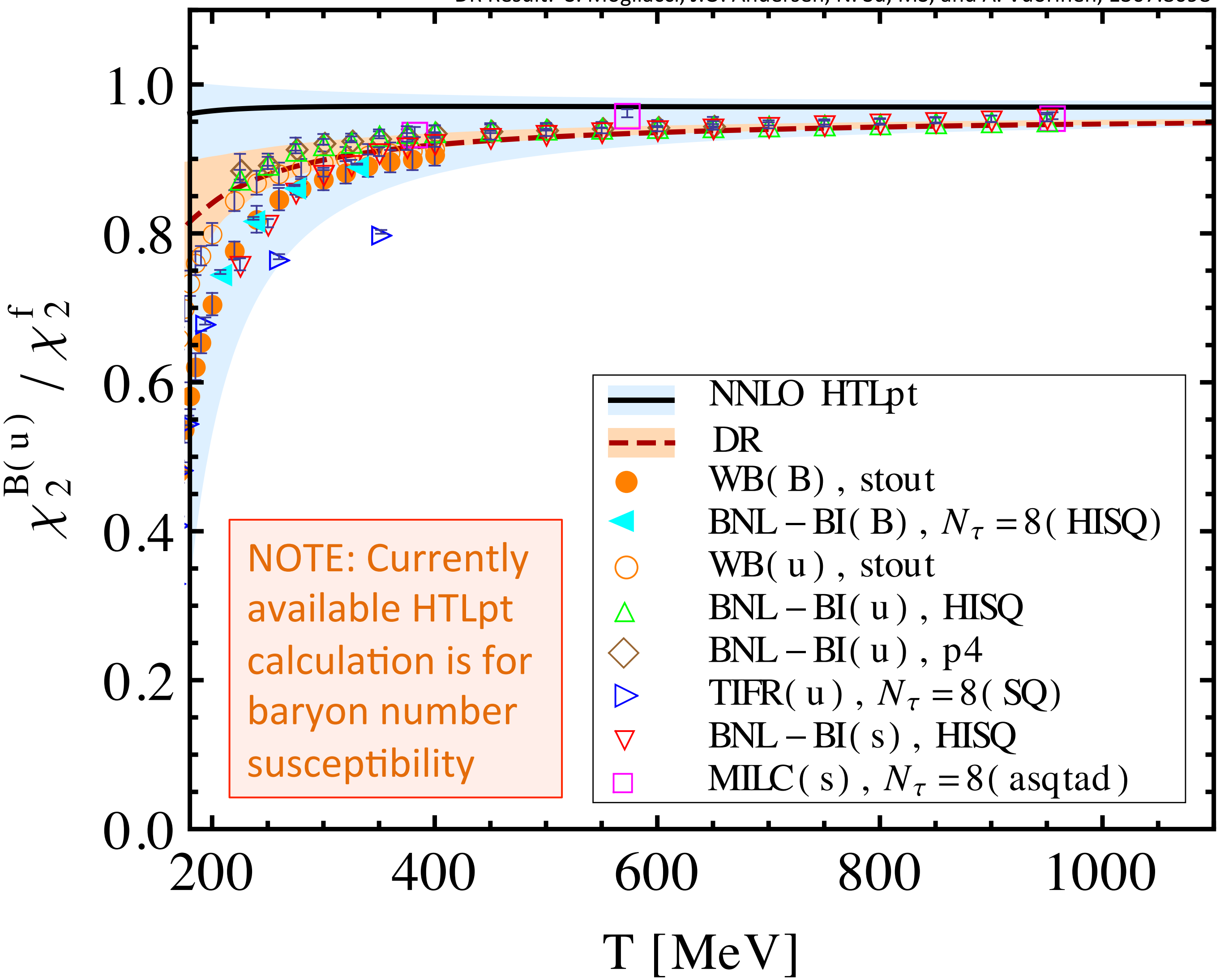
$$\frac{p_{\rm HARD}(T,\boldsymbol{\mu})}{T^4} = \alpha_{\rm E1} + \hat{g}_3^2\,\alpha_{\rm E2} + \frac{\hat{g}_3^4}{(4\pi)^2}\left(\alpha_{\rm E3} - \alpha_{\rm E2}\,\alpha_{\rm E7} - \frac{1}{4}d_{\rm A}C_{\rm A}\,\alpha_{\rm E5}\right) + \frac{\hat{g}_3^6}{(4\pi)^4}\left[d_{\rm A}C_{\rm A}\left(\alpha_{\rm E6} - \alpha_{\rm E4}\,\alpha_{\rm E7}\right) - d_{\rm A}C_{\rm A}^3\left(\frac{43}{3} - \frac{27}{32}\pi^2\right)\right]\log\frac{\bar{\Lambda}}{4\pi T} + \mathcal{O}(g^6)\,, \tag{2.20}$$

$$\frac{p_{\rm SOFT}(T,\boldsymbol{\mu})}{T^3} = \frac{\hat{m}_{\rm E}^3}{12\pi}\,d_{\rm A} - \frac{\hat{g}_3^2\,\hat{m}_{\rm E}^2}{(4\pi)^2}\,d_{\rm A}C_{\rm A}\left(\log\frac{\bar{\Lambda}}{2T\hat{m}_{\rm E}} + \frac{3}{4}\right) - \frac{\hat{g}_3^4\,\hat{m}_{\rm E}}{(4\pi)^3}\,d_{\rm A}C_{\rm A}^2\left(\frac{89}{24} + \frac{\pi^2}{6} - \frac{11}{6}\log 2\right) + \frac{\hat{g}_3^6}{(4\pi)^4}\,d_{\rm A}\left[C_{\rm A}^3\left(\frac{43}{4} - \frac{491}{768}\pi^2\right)\log\frac{\bar{\Lambda}}{2T\hat{m}_{\rm E}} + C_{\rm A}^3\left(\frac{43}{12} - \frac{157}{768}\pi^2\right)\log\frac{\bar{\Lambda}}{2C_{\rm A}T\hat{g}_3^2} - \frac{4}{3}\frac{N_c^2-4}{N_c}\left(\sum_f \hat{\mu}_f\right)^2\log\frac{\bar{\Lambda}}{2T\hat{m}_{\rm E}}\right] + \mathcal{O}(g^6)\,, \tag{2.21}$$

Analytic coefficients etc. are complicated (see the paper above for the gory details)

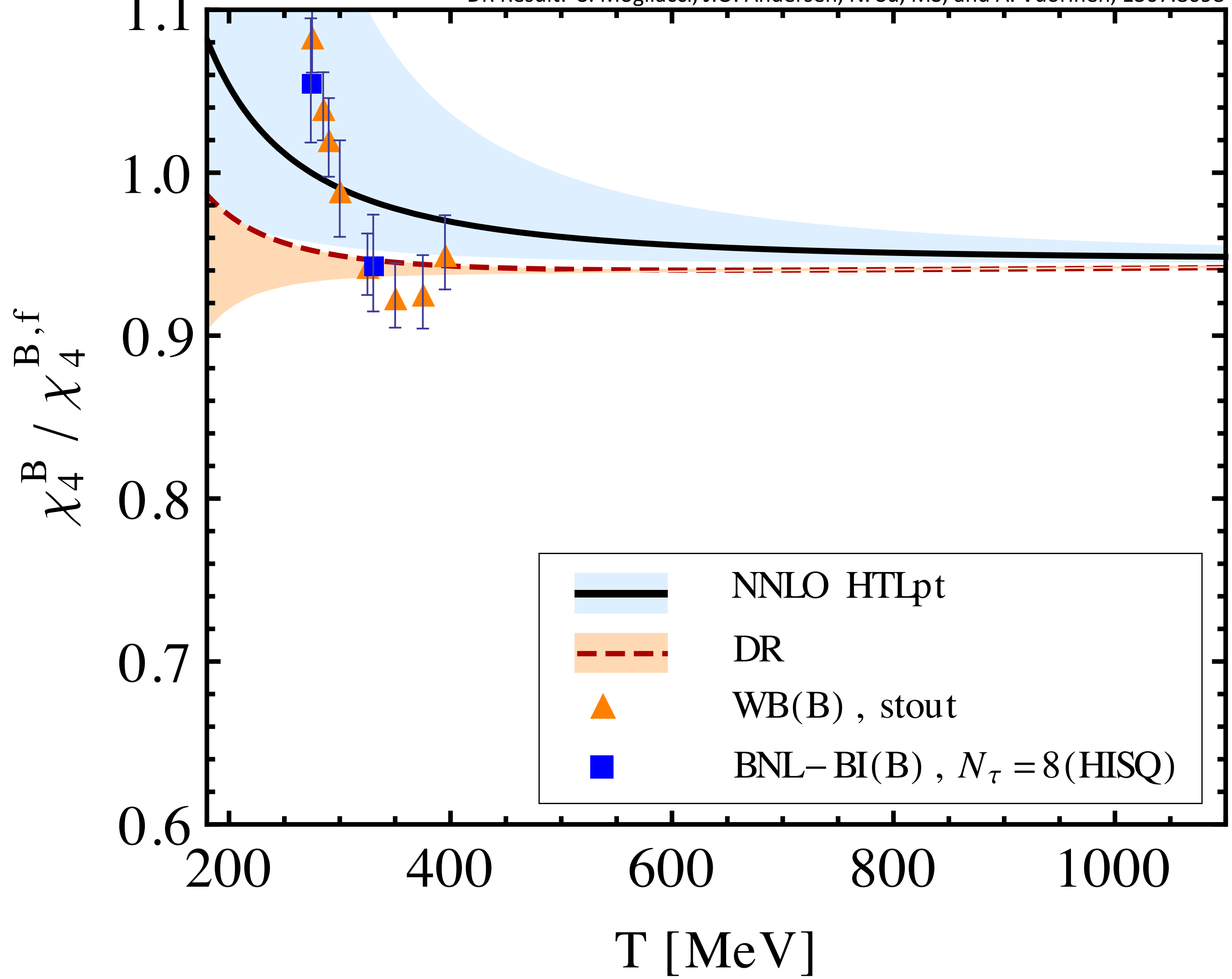
4-loop Resummed DR Result – $\chi_{2(B,u)}$

DR Result: S. Mogliacci, J.O. Andersen, N. Su, MS, and A. Vuorinen, 1307.8098

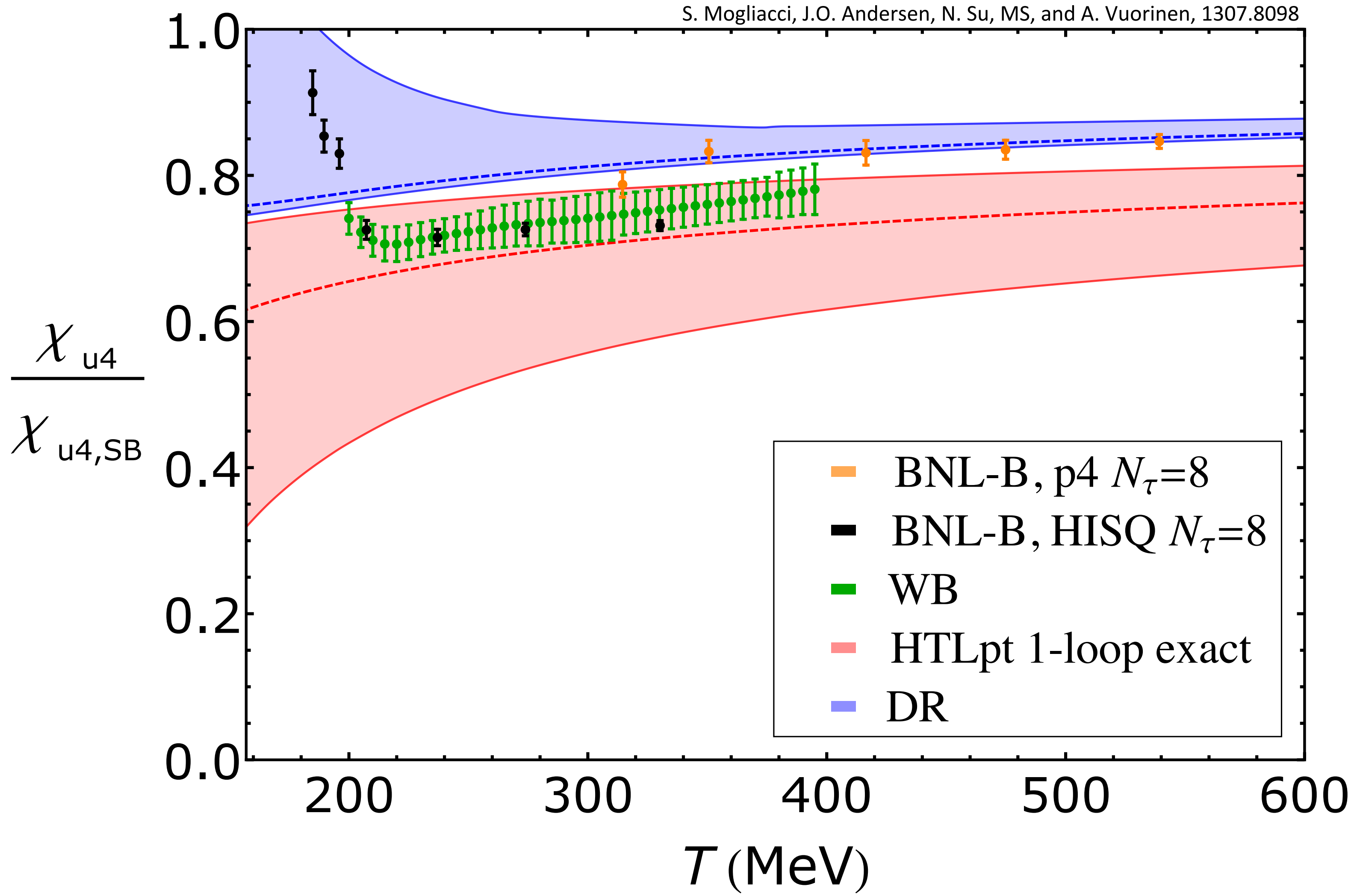


4-loop Resummed DR Result – χ_{4B}

DR Result: S. Mogliacci, J.O. Andersen, N. Su, MS, and A. Vuorinen, 1307.8098



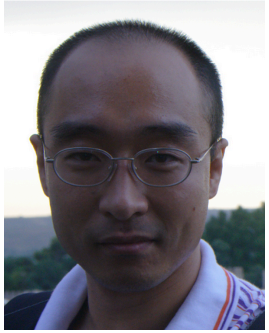
4-loop Resummed DR Result – χ_{4u}



Thanks to my collaborators

Najmul Haque

Sylvain Mogliacci



Nan Su

Jens Andersen

Munshi Mustafa

Aleksi Vuorinen

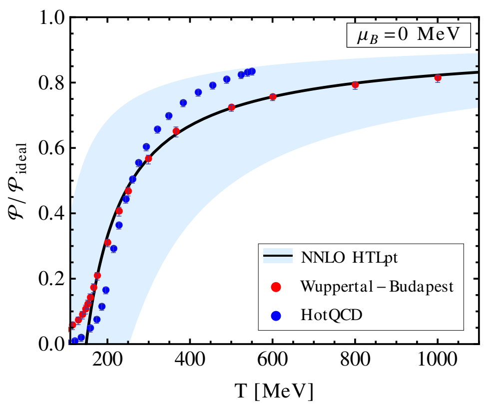
Conclusions and Outlook

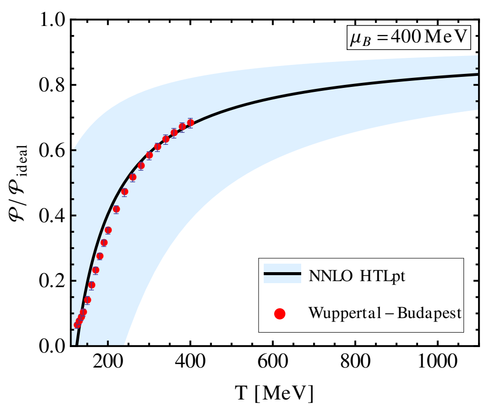
- 3-loop HTLpt and 4-loop resummed DR results for the susceptibilities are in good agreement with one another within theoretical uncertainties
- There seems to be some "advantage" for HTLpt in the fourth-order susceptibility when comparing to the lattice data at low temperatures, but the lattice data error bars are still large
- For the second-order susceptibility the 4-loop resummed DR has the "advantage" since it lies inside the 3-loop HTLpt band and seems to be in very good agreement with available lattice data
- We plan to continue these developments to include large chemical potential and small temperatures; need ring resummation, inclusion of gapped phases, etc.
- The success of HTLpt is encouraging for our field since it is formulated in Minkowski space and is not limited to the calculation of static QGP properties

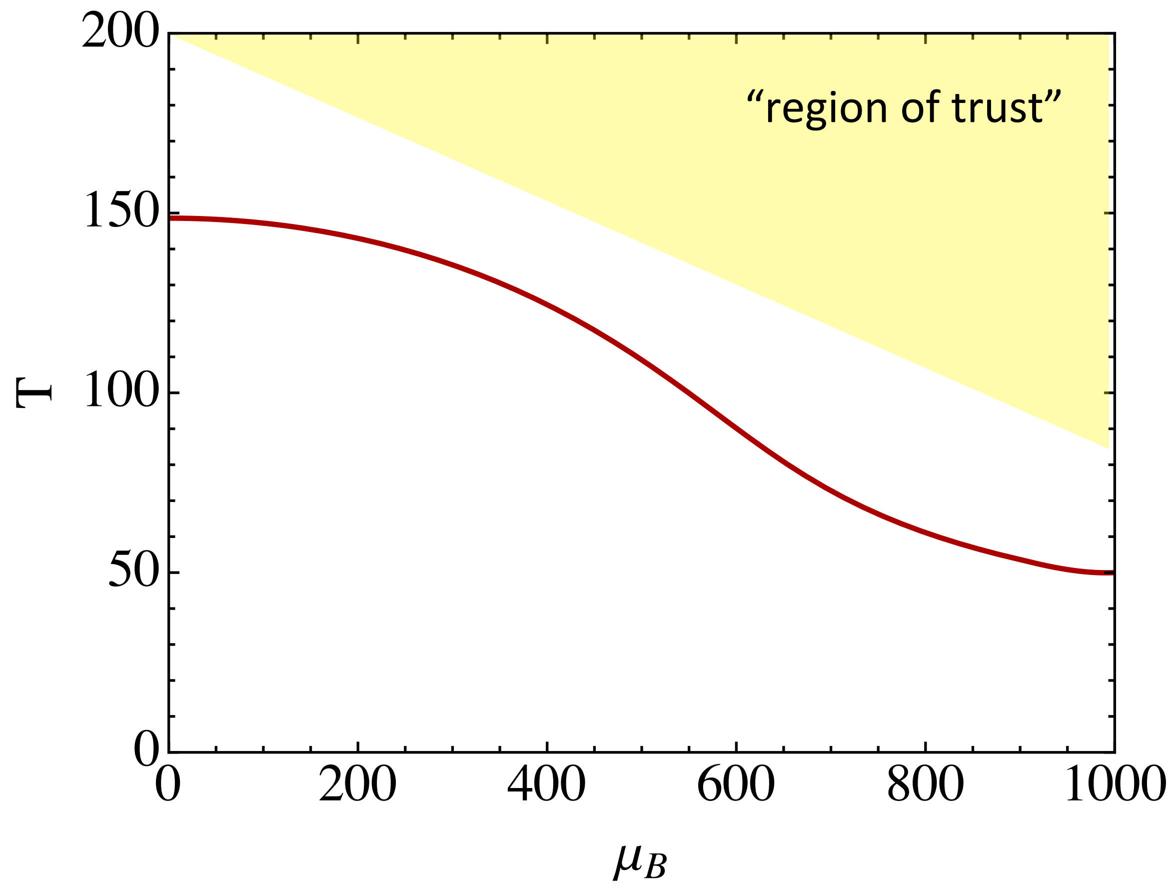
Backup Slides

HTLpt "Phase Transition Line"

Although we probably shouldn't do it since we are extrapolating out of our "region of trust," we can solve for the point where the HTLpt pressure goes to zero in order to extract a kind of phase transition line.

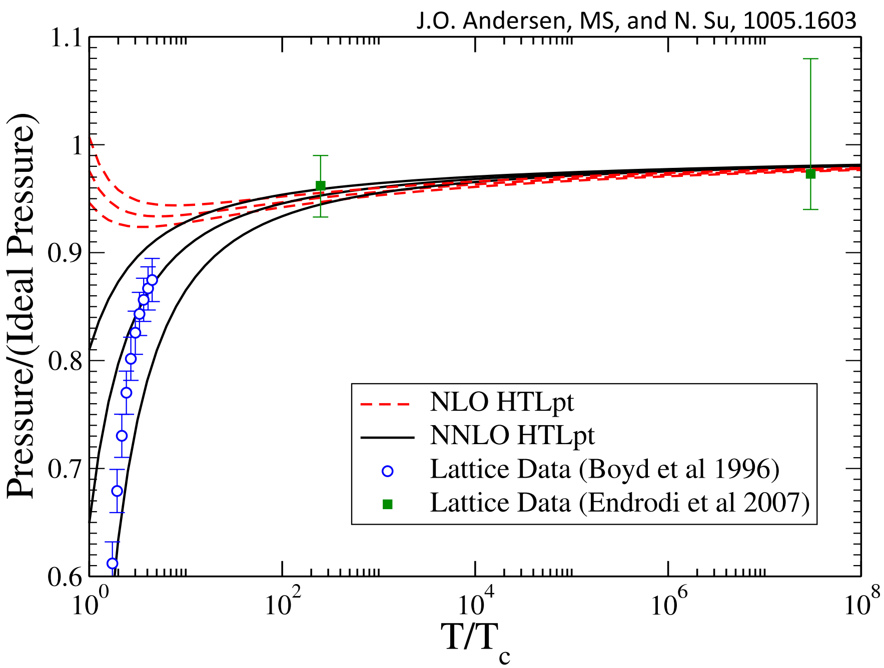


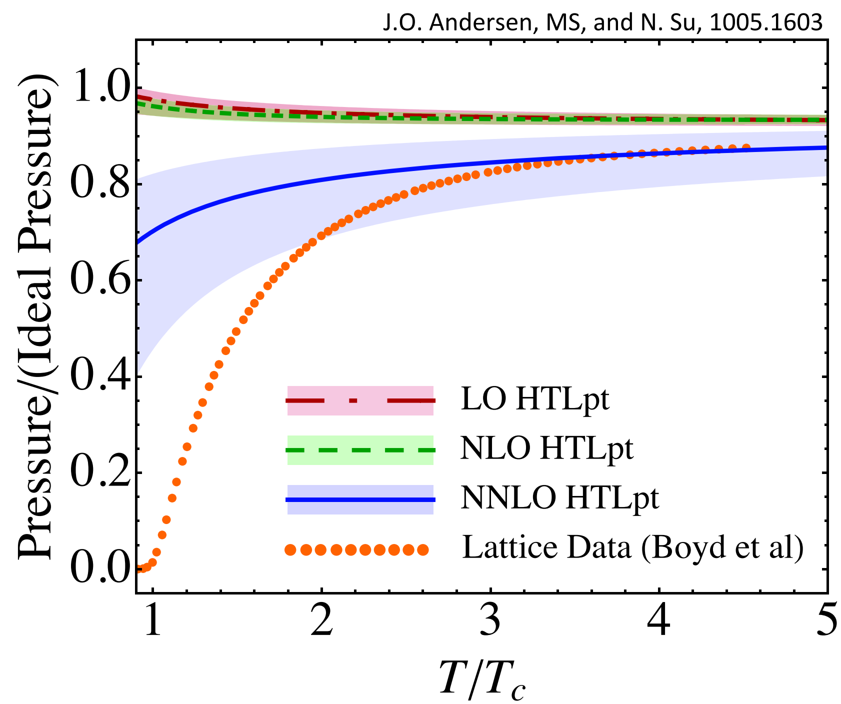




HTLpt Convergence?

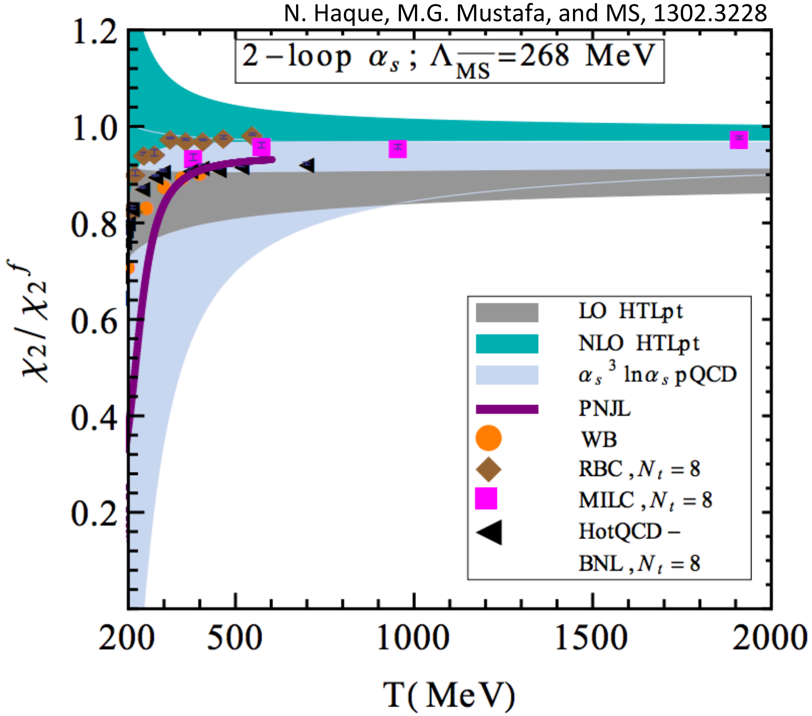
Pure Glue @ zero chemical potential



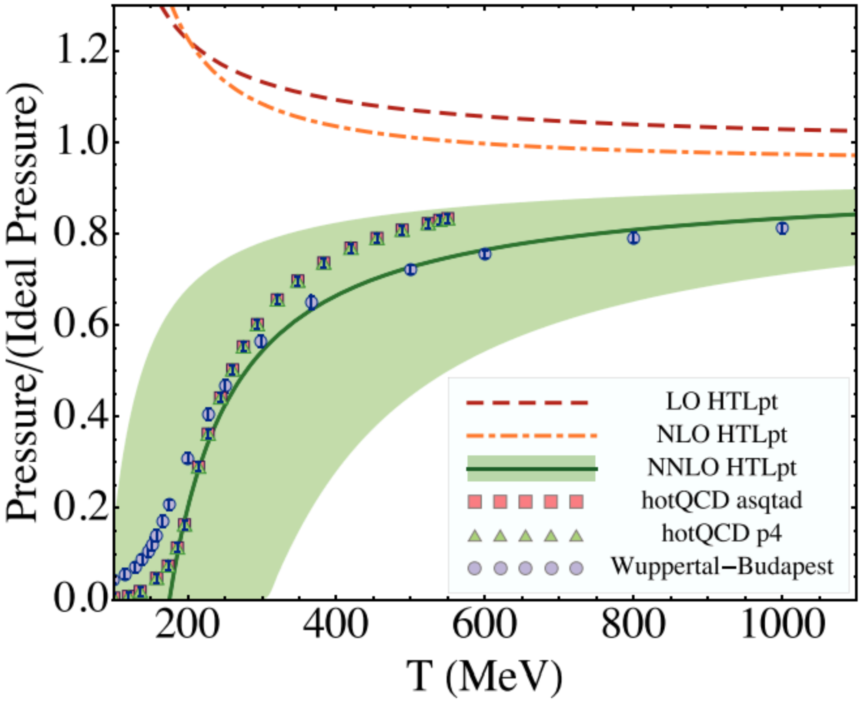


HTLpt Convergence?

N. Haque, M.G. Mustafa, and MS, 1302.3228



J.O Andersen, L.E. Leganger, MS, and N. Su., 1103.2528



Trace Anomaly